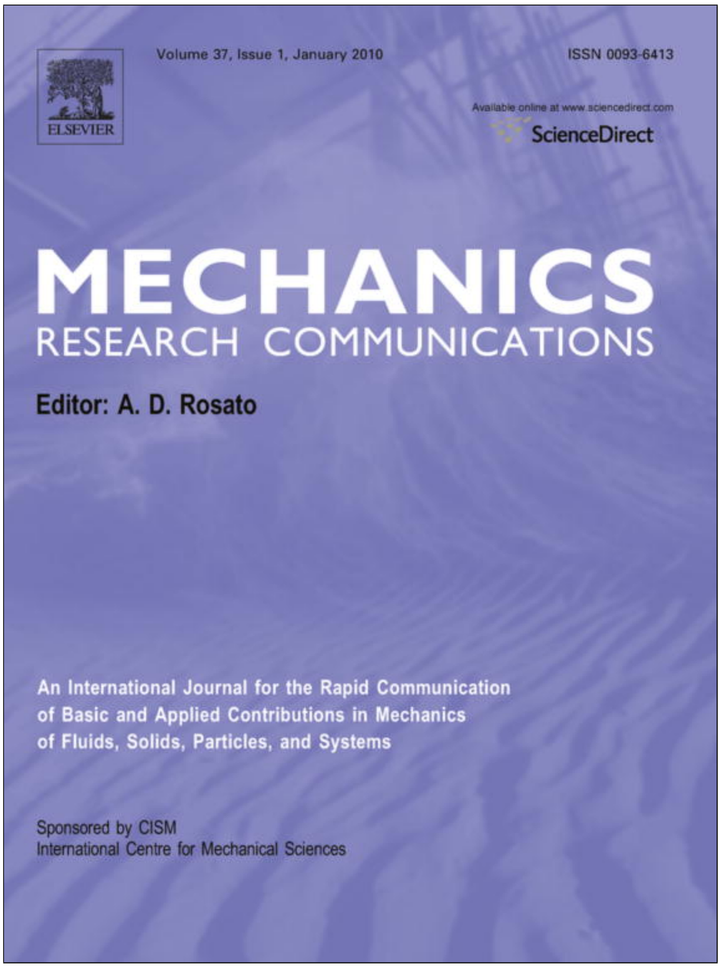
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On a class of micromechanical damage models with initial stresses for geomaterials

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ABSTRACT

In this paper, we extend a class of micromechanical damage models by including initial stresses. The proposed approach is based on the solution of the Eshelby inhomogeneous inclusion problem in the presence of a pre-stress (in the matrix), adapted for elastic voided media. The closed form expression of the corresponding energy potential is used as the basis of various isotropic damage models corresponding to three standard homogenization schemes. These models are illustrated by considering isotropic tensile loadings with different initial stresses. Finally, still in the isotropic context, we provide an interpretation of the macroscopic damage model formulated by Halm and Dragon (1996) by briefly connecting it to the present study.

1. Introduction

The mechanical behavior of engineering materials and in particular geomaterials are significantly affected by the presence of voids or crack-like defects. The modeling of such behavior is classically performed by considering purely macroscopic or micromechanically-based damage models (see for instance Andrieux et al., 1986; Halm and Dragon, 1996; Krajcinowic, 1996, etc.). Recent developments in homogenization of microcracked media provides now physical and mathematical arguments for the description of damage-induced anisotropy, as well as crack closure effects (Pensée et al., 2002; Dormieux et al., 2006). The above models have been applied for geomaterials including concrete or rock-like media. However, except an interesting attempt to incorporate damage-induced residual stresses by Halm and Dragon (1996) in the context of purely macroscopic modeling, most of the damage models proposed in literature are not generally able to account for in situ initial stresses which are crucial in geomechanics (tunneling, compaction of petroleum reservoir, waste storage...). It is convenient to emphasize that pre-stresses in geotechnical problems can also originate from the loading conditions (gravity in most cases), and as such, should be handled at the macroscopic scale. In the present work, no attempt is done to account for these kinds of pre-stresses which are different in nature from those introduced by means of homogenization techniques as components of the material behavior.

In the above-mentioned applications, initial stresses, which appear as in situ stresses and exist before any underground excavation, can have a magnitude of several MPa. Mainly from the perspective of concerned applications in geomechanics,[1] it is desirable to formulate a micromechanical model and determine how initial stresses affect the response of material sustaining damage by voids growth. Before presenting the developments in the present study, it is convenient to note that although the use of the concept of pre-stress in the context of mechanical damage modeling with pre-stress is at several aspects original, various micromechanics-based works dealing with poroelastic damage, strength and/or poroplasticity already exist in literature (see for instance among others (Dormieux et al., 2001; Barthélémy and Dormieux, 2004; Dormieux et al., 2006; Maghous et al., 2009, and references cited herein).

The main purpose of the present study is to derive from homogenization techniques a new class of micro–macro damage models which incorporates initial stresses and couples them to an evolving damage. Simple examples highlight the role of the homogenization scheme in this coupling. Finally, on the basis of the present study, an interpretation of the macroscopic damage model formulated by Halm and Dragon (1996) will be provided.

[1] Many other domains of applications include damage of quasi brittle materials such as ceramics or brittle matrix composites in which initial stresses can be induced by the formation process. Damage in porous bone is also concerned (Lennon and Prendergast, 2002).

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2. Principle of the modeling including initial stresses

Consider a representative elementary volume (*rev*, Ω) made up of a solid matrix *s* (occupying a domain Ω^s) and a family of inhomogeneous inclusions denoted *I* and occupying a domain Ω^I. The matrix and the inclusions are considered to behave elastically. Moreover, an initial uniform stress field $\boldsymbol{\sigma}_0$ is assumed in Ω^s. The quantity $\underline{z}$ denotes the vector position, $\underline{\xi}$ the displacement vector, and $\mathbf{E}$ the macroscopic strain tensor. The *rev* is subjected, as classically, to uniform strain boundary conditions:

$$\partial\Omega : \underline{\xi} = \mathbf{E} \cdot \underline{z} \tag{1}$$

A convenient way to formulate the problem of homogenization with initial stresses in a unified way is to consider the stress tensor field $\boldsymbol{\sigma}(\underline{z})$, everywhere in the *rev*, in an affine form:

$$(\forall \underline{z} \in \Omega) \quad \boldsymbol{\sigma}(\underline{z}) = \mathbb{C}(\underline{z}) : \boldsymbol{\varepsilon}(\underline{z}) + \boldsymbol{\sigma}^p(\underline{z}) \tag{2}$$

where $\mathbb{C}(\underline{z})$ is a heterogeneous stiffness tensor, and $\boldsymbol{\sigma}^p(\underline{z})$ a prestress tensor such as:

$$\mathbb{C}(\underline{z}) = \begin{cases} \mathbb{C}^I & \text{in } (\Omega^I) \\ \mathbb{C}^s & \text{in } (\Omega^s) \end{cases} \quad \boldsymbol{\sigma}^p(\underline{z}) = \begin{cases} \boldsymbol{\sigma}_0 & \text{in } (\Omega^s) \\ 0 & \text{in } (\Omega^I) \end{cases} \tag{3}$$

In this form, the problem can be solved by using the classical Levin's theorem (Levin, 1967) (see also Laws, 1973). This yields the following constitutive equation (see Dormieux et al., 2006 in a general context of poroelasticity):

$$\boldsymbol{\Sigma} = \mathbb{C}^{hom} : \mathbf{E} + \overline{\boldsymbol{\sigma}^p : \mathbb{A}} \tag{4}$$

in which the overbar represents the average of any considered quantity over the *rev*. The fourth order tensor $\mathbb{A}$ is the so-called heterogeneous strain localization tensor which relates the microscopic strain tensor and the macroscopic strain tensor $\mathbf{E}$ in absence of initial stress: $\boldsymbol{\varepsilon}(\underline{z}) = \mathbb{A}(\underline{z}) : \mathbf{E}$. Tensor $\mathbb{C}^{hom}$ is the macroscopic stiffness tensor which can be obtained from any homogenization scheme of the standard linear elasticity (e.g. without prestress), and $\boldsymbol{\Sigma}$ is the stress averaged over the *rev*, i.e. $\boldsymbol{\Sigma} = \overline{\boldsymbol{\sigma}(\underline{z})}$

Recalling that, the prestress is null in Ω^I and is equal to $\boldsymbol{\sigma}_0$ in Ω^s, it is readily seen that:

$$\boldsymbol{\Sigma} = \mathbb{C}^{hom} : \mathbf{E} + (1-\varphi)\boldsymbol{\sigma}_0 : \mathbb{A}^s = \mathbb{C}^{hom} : \mathbf{E} + \boldsymbol{\sigma}_0 : (\mathbb{I} - \varphi\mathbb{A}^I) \tag{5}$$

$\mathbb{A}^s$ and $\mathbb{A}^I$ are the averages of concentration tensor over the matrix and the inclusion phase, respectively. φ denotes the volume fraction of the considered inclusions, i.e. the porosity when dealing with voided materials (as it will be the case in Section 3). Since $\mathbb{C}^{hom} = \mathbb{C}^s : (\mathbb{I} - \varphi\mathbb{A}^I)$, (5) can be also put in the form:

$$\boldsymbol{\Sigma} = (\mathbb{C}^s : \mathbf{E} + \boldsymbol{\sigma}_0) : (\mathbb{I} - \varphi\mathbb{A}^I) \tag{6}$$

Eq. (6) shows how the initial stress simply combines with $\mathbb{C}^s : \mathbf{E}$ in the expression of the macroscopic stress of the heterogeneous material.

3. A basic isotropic damage model accounting for initial stress

We consider now a *rev* constituted of an elastic matrix and voids; the matrix is still submitted to the uniform initial stress $\boldsymbol{\sigma}_0$. The main purpose of this section is to micromechanically derive a simple elastic damage model due to void growth. To this end, the localization tensor $\mathbb{A}^p$ corresponding to the pores is required. Obviously, the expression of $\mathbb{A}^p$ depends on the considered homogenization scheme: for the matrix/inclusion morphology studied here, an Hashin–Shtrikhman upper bound is appropriate, while the dilute scheme is restricted to very low porosities $\varphi = \varphi^p$. For simplicity, spherical voids will be considered in the following and the porosity will then play the role of scalar damage variable for the isotropic medium. For convenience, this scalar damage variable will be denoted *d*, as in standard literature. For completeness, the formulation of the elastic isotropic damage model with initial stresses, will be done also using the first order bound of Voigt; this corresponds to an extension of the standard Lemaître-Chaboche model (Lemaitre and Chaboche, 1990) (see also Marigo, 1985).

3.1. An energy approach of the isotropic damage in presence of initial stress

We start from the definition of the potential energy of the solid phase with prestress, $\Psi(\mathbf{E}, d) = \frac{1}{2|\Omega|}\int_{\Omega^s} \boldsymbol{\varepsilon} : \mathbb{C}^s : \boldsymbol{\varepsilon} dV_z + \boldsymbol{\sigma}_0 \frac{1}{|\Omega|}\int_{\Omega^s} \boldsymbol{\varepsilon} dV_z$, which reads:

$$\Psi(\mathbf{E}, d) = \frac{1}{2}\mathbf{E} : \mathbb{C}^{hom}(d) : \mathbf{E} + \boldsymbol{\sigma}_0 : (\mathbb{I} - d\mathbb{A}^p) : \mathbf{E} \tag{7}$$

and corresponds to the first state law (5), rewritten here in the form:

$$\boldsymbol{\Sigma} - \boldsymbol{\sigma}_0 = \mathbb{C}^{hom} : \mathbf{E} - d\boldsymbol{\sigma}_0 : \mathbb{A}^p \tag{8}$$

$\mathbb{A}^I$ being now denoted $\mathbb{A}^p$.

The second state law gives the damage energy release rate $\mathcal{F}^d$ (obtained as the negative of the derivative of Ψ with respect to *d*):

$$\mathcal{F}^d = -\frac{\partial\Psi}{\partial d} = -\frac{1}{2}\mathbf{E} : \frac{\partial\mathbb{C}^{hom}}{\partial d} : \mathbf{E} + \boldsymbol{\sigma}_0 : \left(\mathbb{A}^p + d\frac{\partial\mathbb{A}^p}{\partial d}\right) : \mathbf{E} \tag{9}$$

This clearly shows that $\mathcal{F}^d$ is a priori affected by the initial stress through combination with the damage variable. Moreover, it strongly depends on $\mathbb{A}^p$ and then on the chosen homogenization scheme. The next step for the derivation of the damage model is the consideration of a damage yield function. Following Marigo (1985), the yield function is taken in the form:

$$f = \mathcal{F}^d - \mathcal{R}^d(d) = 0 \tag{10}$$

Which reads:

$$f = -\frac{1}{2}\mathbf{E} : \frac{\partial\mathbb{C}^{hom}}{\partial d} : \mathbf{E} + \boldsymbol{\sigma}_0 : \left(\mathbb{A}^p + d\frac{\partial\mathbb{A}^p}{\partial d}\right) : \mathbf{E} - \mathcal{R}^d(d) = 0 \tag{11}$$

Assuming the normality rule, $\dot{d} = \dot{\Lambda}\frac{\partial f}{\partial\mathcal{F}^d} = \dot{\Lambda}$ yields:

$$\dot{d} = \frac{(\mathbb{C}'^{hom} : \mathbf{E} - X) : \dot{\mathbf{E}}}{\frac{1}{2}\mathbf{E} : \mathbb{C}''^{hom} : \mathbf{E} + \mathcal{R}'^d(d) - Y} \tag{12}$$

in which

$$X = \boldsymbol{\sigma}_0 : (\mathbb{A}^p + d\mathbb{A}'^p) \quad \text{and } Y = \boldsymbol{\sigma}_0 : (2\mathbb{A}'^p + d\mathbb{A}''^p) : \mathbf{E} \tag{13}$$

This equation explicitly shows that, in addition to modifying the damage yield function, $\boldsymbol{\sigma}_0$ affects also the rate of damage.

The rate form of the damage law is:

$$\dot{\boldsymbol{\Sigma}} = \mathbb{C}_t^{hom} : \dot{\mathbf{E}} \tag{14}$$

with

$$\mathbb{C}_t^{hom} = \begin{cases} \mathbb{C}^{hom} & (\text{if } f < 0 \text{ or if } f = 0 \text{ and } \dot{f} < 0) \\ \mathbb{C}^{hom} - \frac{(\mathbb{C}'^{hom}:\mathbf{E}-X)\otimes(\mathbb{C}'^{hom}:\mathbf{E}-X)}{\frac{1}{2}\mathbf{E}:\mathbb{C}''^{hom}:\mathbf{E}+\mathcal{R}'^d(d)-Y} & (\text{if } f = 0 \text{ and } \dot{f} = 0) \end{cases} \tag{15}$$

In summary, note that that the initial stress $\boldsymbol{\sigma}_0$ affects not only the state laws of the damaged material, but also the elasticity domain of the model (see the damage yield function), as well as the rate of damage and the tangent operator $\mathbb{C}_t^{hom}$ of the extended constitutive law.

3.2. Illustrations of the isotropic damage model based on an Hashin–Shtrikman upper bound

For purpose of illustration, we consider for the elastic damaged medium the well known Hashin–Shtrikman upper bound which appears to be the most adapted homogenization model for the studied *rev*. The corresponding localization tensor of the spherical voids phase reads:

$$\mathbb{A}^p = \mathbb{A}^p_{HS} = (\mathbb{I} - \mathbb{S})^{-1} : \left((1-d)\mathbb{I} + d(\mathbb{I} - \mathbb{S})^{-1}\right)^{-1} \tag{16}$$

where $\mathbb{S}$ is the well known Eshelby tensor associated to spherical inclusions in an isotropic elastic solid matrix ($\mathbb{C}^s = 3k^s\mathbb{J} + 2\mu^s\mathbb{K}$, k^s and μ^s being the bulk modulus and the shear modulus, respectively):

$$\mathbb{S} = S_A\mathbb{J} + S_B\mathbb{K} \quad \text{with} \quad S_A = \frac{3k^s}{3k^s + 4\mu^s} \quad \text{and} \quad S_B = \frac{6(k^s + 2\mu^s)}{3k^s + 4\mu^s} \tag{17}$$

It follows that the energy potential, given by (7), can be put in the form:

$$\Psi = \Psi^{HS} = \frac{1}{2}\mathbf{E} : \mathbb{C}^{hom}(d) : \mathbf{E} + \chi tr(\boldsymbol{\sigma}_0)tr(\mathbf{E}) + \kappa\mathbf{E}^d : \boldsymbol{\sigma}_0^d \tag{18}$$

where $\chi = 1 - \frac{d}{1-(1-d)S_A}$ and $\kappa = 1 - \frac{d}{1-(1-d)S_B}$. $\mathbf{E}^d$ and $\boldsymbol{\sigma}_0^d$ represent the deviatoric parts of $\mathbf{E}$ and $\boldsymbol{\sigma}_0^d$, respectively.

It is recalled that $\mathbb{C}^{hom} = \mathbb{C}^s : (\mathbb{I} - d\mathbb{A}^p_{HS})$.

For simplicity, following Marigo (1985), an affine function $\mathcal{R}^d(d) = h_0(1 + \eta d)$ is adopted in what follows. This choice, together with the expression of the potential (18), put in the methodology described in subsection 3, allows to build the complete constitutive damage law with initial stress and based on Hashin–Shtrikman upper bound.

As mentioned before, for completness and for purposes of comparison, we consider two other homogenization schemes: the Voigt model (which coincides with a standard damage model, namely the one of Lemaitre–Chaboche) and the dilute scheme. The corresponding localization tensors are, respectively:

- For the Voigt assumption, $\mathbb{A}^p = \mathbb{I}$; this corresponds to the well known Lemaitre-Chaboche damage type model (Lemaitre and Chaboche, 1990) for which $\mathbb{C}^{hom} = (1-d)\mathbb{C}^s$. Just for convenience, note that the energy potential associated with this model is obtained by puting $S_A = 0$ and $S_B = 0$ in (18).
- For the dilute scheme, $\mathbb{A}^p_{dil} = (\mathbb{I} - \mathbb{S})^{-1}$. This leads to:

$$\Psi = \Psi^{dil} = \frac{1}{2}\mathbf{E} : \mathbb{C}^{hom}(d) : \mathbf{E} + \left(1 - \frac{d}{1-S_A}\right)tr(\boldsymbol{\sigma}_0)tr(\mathbf{E}) + \left(1 - \frac{d}{1-S_B}\right)\mathbf{E}^d : \boldsymbol{\sigma}_0^d \tag{19}$$

in which the homogenized stiffness tensor reads $\mathbb{C}^{hom} = \mathbb{C}^s : (\mathbb{I} - d\mathbb{A}^p_{dil})$.

Note that for these last two models, $\mathbb{A}'^p = 0$ and $\mathbb{A}''^p = 0$ which makes them relatively simple compared with the Hashin–Shtrikman bound-based damage model.

As simple illustrations, an isotropic macroscopic tensile loading path ($\boldsymbol{\Sigma} = \Sigma\mathbf{1}$) is considered as well as an isotropic fixed initial stress field ($\boldsymbol{\sigma}_0 = \sigma_0\mathbf{1}$). Simulations are performed with the following data: matrix Young modulus $E^s = \frac{100}{3}$ GPa, Poisson ratio of the matrix $\nu^s = 0.23$, $h_0 = 10^4 J/m^2$ and $\eta = 32$. For the isotropic tensile loading path, the macroscopic response ($\mathbf{E} = E\mathbf{1}$, the quantity E being a scalar) predicted by the model based on the Hashin–Shtrikman bound is shown on Fig. 1. The considered values of σ_0 are indicated on this figure. It is observed that the magnitude of the initial stress σ_0 clearly affects the overall response of the material undergoing damage: yield stress, peak stress, stress–strain curve. A residual strain (corresponding to $\Sigma - \sigma_0 = 0$), related to the amount of the damage, is observed. In a dual manner, one can observe a residual stress at zero strain. σ_0 also influences the softening regime: the material seems to be more softened when the

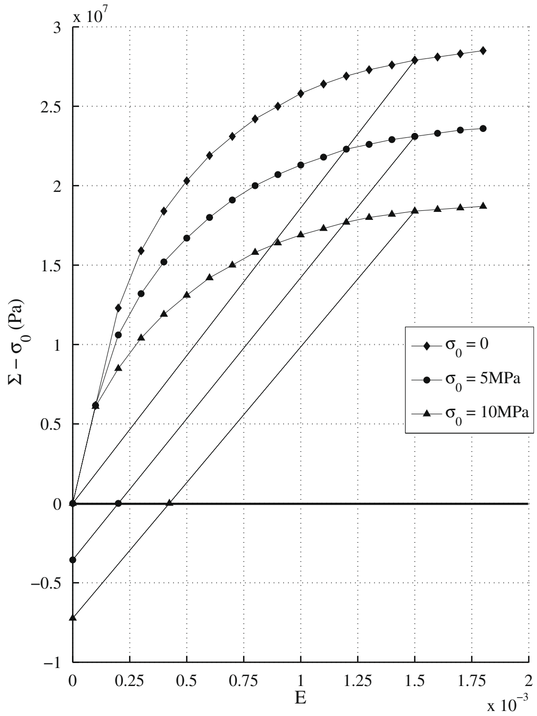

Fig. 1. Hydrostatic curve response predicted by Hashin–Shtrikman upper bound-based model.

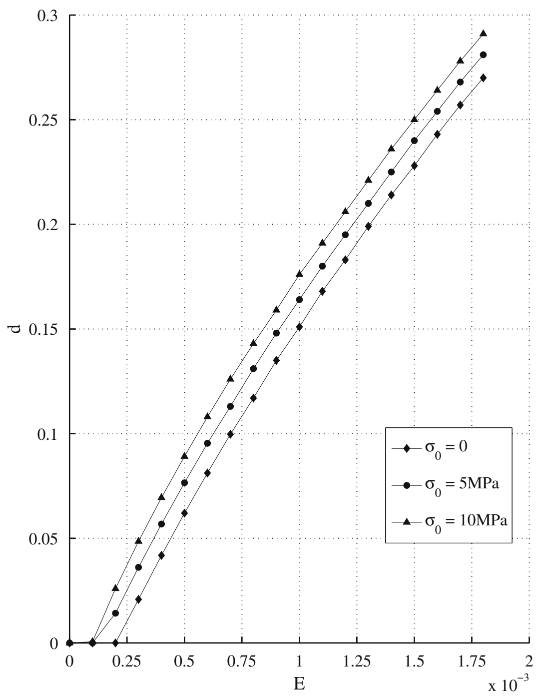

Fig. 2. Damage evolution under hydrostatic loading predicted by Hashin–Shtrikman upper bound-based model.

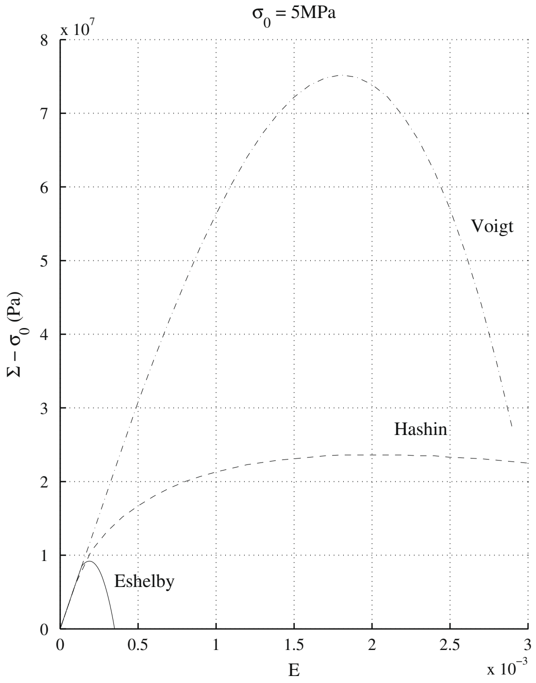

Fig. 3. Comparison of the predicted response of the three micromechanics-based models for an hydrostatic loading.

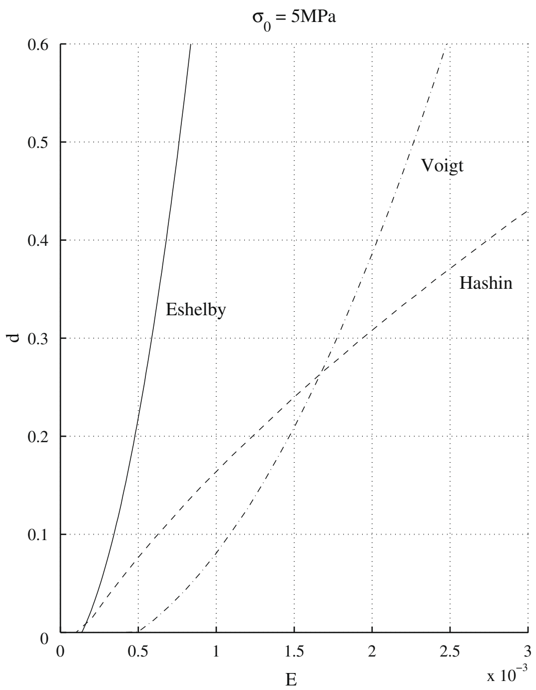

Fig. 4. Comparison of the predicted damage of the three micromechanics-based models for an hydrostatic loading.

tensile residual stress increases. The damage evolution associated with the obtained response is shown on Fig. 2. It appears that the damage evolution is slightly affected by σ_0.

Fig. 3 shows a comparison between the three micromechanics-based damage models for $\sigma_0 = 5$ MPa. The corresponding damage evolution is given on Fig. 4. It is readily seen that the homogenization scheme plays an important role in the prediction of the damage model. In particular, the Lemaitre–Chaboche damage model predicts a significant softening regime due to important damage growth, while the dilute scheme predicts a very brittle macroscopic response. For the Hashin–Shtrikman (HS) upper bound, a less softened response is noted. These observations are confirmed by the comparison of the damage growth predicted by the three models. It must be emphasized that, in contrast to the Lemaitre–Chaboche type model, the results predicted by the dilute scheme-based model as well as the one by HS bound crucially depends on the value of the Poisson ratio ν^s of the matrix.

3.3. Connection with an existing model

It is interesting to connect the proposed approach to existing macroscopic damage models dealing with initial stresses. In particular, the anisotropic model proposed by Halm and Dragon (1996) is considered in the following. The restriction of this model to the case of an isotropic damage leads to the following energy potential:

$$\Psi = \Psi^{HD} = \frac{1}{2}\mathbf{E} : \mathbb{C}^{hom}(d) : \mathbf{E} + g\ d\mathrm{tr}\mathbf{E} \qquad (20)$$

where by adopting the notations introduced by Halm and Dragon (1996) (α and β being two model parameters), one has:

$$\mathbb{C}^{hom} = (3k^s + (6\alpha + 4\beta)d)\mathbb{J} + 2(\mu^s + 2\beta d)\mathbb{K} \qquad (21)$$

As this model predicts a linear dependence of the macroscopic elastic coefficients (bulk and shear moduli), it is comparable to the one associated with the dilute scheme. However, the contribution described by the mean of the term $g\ d\mathrm{tr}\mathbf{E}$ is generally smaller than the one obtained in the present study for the dilute scheme-based constitutive damage law. In particular, it can account only for the effect of isotropic initial stresses.

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