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Volatility Factors of Returns and Risk Analysis Using Quantile Risk Measures in the Gold and Silver Market

Abstract: The aim of the paper is to identify unobservable factors that may significantly determine the level of gold and silver returns and to assess the risk of investment in these metals. To measure risk, the value at risk and other, less popular measures are used: the ES, MS, Rachev ratio and GlueVaR risk measure. Normal and Student's t-distributions are used as theoretical distributions. The results of the study show that we can identify latent factors based on observable variables that have a significant impact on the level of gold and silver returns. In addition, it was observed that the risk measures would vary depending on the period of research. It was shown that the estimates of the risk measures using Student's t-distribution have a lower estimation error than those based on the normal distribution.

Keywords: risk, gold, silver, precious metals, quantile risk measures

JEL: G01, G11, G31

1. Introduction

The contemporary economy is an extremely complicated construct, despite numerous regulations. Its proper recognition allows us to understand the nature of corresponding processes. The possibilities of modelling economic phenomena should be considered in the context of the market using multidimensional approach, as their volatility and nature are usually influenced by more than one factor. Such factors can be used to build an econometric model that aims to synthetically describe the studied phenomena, which in turn leads to a tool for forecasting its values.

The commodity market is an alternative to the classical capital market. Investments in commodities can hedge capital market investments, especially in the times of economic crises. In this paper, we try to reveal some factors determining volatility observed in returns on investment in these metals, and then use these factors to build statistical models describing returns changes. The returns for gold and silver are also analysed in terms of risk of their volatility. Such analysis is particularly important from the investment point of view. Each investor strives to maximise expected profit while minimising risk associated with the considered investment. Of course, this is only one of the possible investment strategies. The main aim of the analysis is to assess the impact of factors on changes in the level of gold and silver returns and to compare investment risk related to these metals. It is assumed that unobservable factors influence volatility of returns at different levels. Similarly, in the case of risk, it is assumed that investments in gold and silver differ due to the level of risk. To evaluate risk of unpredictable and significant changes observed within returns, quantile risk measures have been proposed.

We divided our study into two parts. The first one is related to modelling returns for gold and silver using some unobservable factors, whereas the second one is related to comparative analysis of risk in terms of volatility observed in returns on investment in gold and silver. The choice of these two metals is dictated by their investing nature. Gold is an ore with a history dating back to the ancient times and has always been a symbol of power, wealth, and economic stability. As a metal, gold has a very wide range of practical applications. It is used in industry, electronics, dentistry, medicine, cosmetics, and even in gastronomy. From the investment point of view, gold is a good traded on the commodity market (mainly as a base of futures contracts), it is also a component of different structural products, ETFs, etc. traded on the capital market. In addition, it is widely used in jewellery as a precious metal. The human attitude to gold is also very specific – gold is seen as 'safe haven,' which means that, especially during an economic crisis, the possession of gold is associated with wealth and security (stability). From the analytical point of view, a negative correlation between stock indices, exchange rates and the price of gold is observed. Silver, just like gold, has a wide range of applications. It is used in industry more often than gold. In addition, it is used in jewellery, electronics, photography, and medicine. Silver is a by-product of copper, therefore the chemical properties of these both metals are similar. To a lesser extent, it serves to accumulate wealth, but it does not mean that silver is less popular than gold.

The analysis of gold and silver returns (as well as returns for other metals) is determined by several factors directly or indirectly related to the metals market. One of the most important features distinguishing commodities from other assets (including stocks, bonds, units in investment funds, etc.) is resource limit. The level of production of gold and silver has its upper boundary, and this is an extremely important factor because it determines futures contracts for the physical delivery of these metals. Some key factors determining the level of gold and silver returns (Borowski, 2008; Kasprzak-Czelej, 2016) are as follows:

– exchange rates,
– inflation and interest rates,
– increase of income and the global economic situation,
– financial market disturbances,
– political risk,
– volatility of prices of strategic raw materials (i.e., crude oil),
– speculation,
– supply.

All above-presented factors have a significant impact on the returns on investments in analysed metals. All of these factors are unpredictable, therefore they generate risk associated with volatility observed in returns.

2. Literature review

Nowadays alternative markets are a very attractive area for investing, especially in the times of increasing uncertainty in financial markets. Such type of investing can be defined as non-classical compared to traditional forms observed in the financial market (Dębski, 2006). An alternative is the commodity market and the metal market as its part. In this paper, we try to assess the risk of volatility observed in returns on investments made on the gold and silver market, two most popular precious metals. While analysing this problem, it was observed that scientific publications concerned mostly their application in industry or medicine. There are few studies considering an analysis of precious metals from the investment point of view, so this study fills the gap in this field.

Giot and Laurent (2003) and Krężołek (2015; 2017a) have shown that the time series in the metals market exhibit similar characteristics as the financial time series. The clustering of variance, asymmetry, leptokurtosis, and heavy tails was observed in empirical distributions. The efficiency of gold and silver market was examined by Solt and

Swanson (1981). They found out that there appeared some positive dependencies in the series of gold and silver prices. These results have important implications for the role of these metals as investment assets. Szczygielski, Enslin, and du Toit (2018) investigate whether an investment in gold mining stocks provides gold price-linked safe haven benefits to investors in an emerging economy. The results indicate that there is a strong, yet changing, relationship between gold price, dollar exchange rate and gold mining returns. Khair-Afham, Law Siong-Hook, and Azman-Saini (2017) examined the relationship between investments in gold on the Malaysian market and the level of inflation. They found that during high momentum regimes, gold return was able to hedge against inflation in Malaysia better than during low momentum regimes.

The relationship between prices of gold and silver was examined by Conover et al. (2009) and Draper, Faff, and Hillier (2010). They showed that metal prices were usually negatively correlated with exchange rates and financial market assets. In the Polish market, the issue of investments in metals was researched by Gierałtowska (2012). She pointed out the need to diversify the financial portfolio with the assets from alternative markets. Kasprzak-Czelej (2018) tested the hypothesis that precious metals were an alternative class of assets. This hypothesis was tested for those who invested their funds in PLN, finally, however, it was not confirmed. In the papers of Krężołek (2017a; 2017b) and Krężołek and Trzpiot (2017), an investment risk assessment was carried out in the metals market using quantile risk measures. They observed significant differences in these assessments determined by the class of risk measures. It was also pointed out that it was justified to use the family of heavy-tailed theoretical distributions in estimation of extreme risk.

3. Measures of extreme risk

The variety of economic factors, often unrelated to the investigated area, is a source of uncertainty observed in the market. Uncertainty results directly from the volatility of assets over time and from the lack of knowledge about the set of causative factors for this volatility. Indeed, such a set might be determined, but will not be clearly definable.

Both uncertainty and volatility generate risk which can be defined as the difference between the present and future value of an asset. In addition, this definition shows that the risk (as the quantitative difference) can be expressed numerically. In the literature, we can point out a specific type of risk that should undoubtedly be considered – the extreme risk (Jajuga, 2008). That type of risk (called also a catastrophic risk) is related to an event with low probability of occurrence, but if such an event does take place, then it can produce large losses. Among this kind of events, one can indicate those that, on the one hand, are expected by the market participant (profit) and, on the other hand, are very undesirable (loss).

Risk measurement for extremely rare events is the topic of analysis in the area of theoretical statistics called the Extreme Value Theory (EVT), pioneered by Leonard Trippet. Within ETV, there are basically two approaches. The first one is based on the Fisher-Trippet theorem and relies on modelling the maxima distribution using the Generalised Extreme Value distribution (GEV). Based on this theorem, three probability distributions were considered: by Weibull, Gumbel and Fréchet. The other approach is based on the assessment of the distribution of a random variable under the condition that its value exceeds a certain threshold. This method is based on the Pickands-Balkem-de Hanna theorem and defines the Peaks Over Threshold (POT) model using the family of Generalised Pareto Distributions (Generalised Pareto Distributions, GPD) (Fałdziński, 2014).

Both scientists and practitioners involved in risk measurement use many different measures of risk. One of the most popular measures is Value-at-Risk defined as the α – quantile of the distribution of return which indicates the level of potential loss that can result from the difference between the present and future value of the asset in a given time horizon and with a fixed tolerance level 1 – α. For the random variable X, we can define VaR as:

$$VaR_{\alpha}(X) = \inf\{x | F_X(x) \geq \alpha\} = F_X^{-1}(\alpha). \quad (1)$$

Value-at-Risk, as a measure, is very intuitive in interpretation, therefore it is so popular. Nevertheless, it has a certain disadvantage, namely it does not satisfy the axiom of sub-additivity of the coherent risk measure (Artzner et al., 1999), which means that it should not be used in risk assessment if the portfolio of assets is taken into account. An alternative measure for VaR is the expected value beyond VaR in terms of mean (called Expected Shortfall, ES or Conditional VaR, CVaR) or in terms of median (Median Shortfall, MS). Mathematical formulas for these risk measures are as follows:

$$ES_{\alpha}(X) = CVaR_{\alpha}(X) = E\left[X | X > VaR_{\alpha}(X)\right]. \quad (2)$$

$$MS_{\alpha}(X) = Median\left[X | X > VaR_{\alpha}(X)\right]. \quad (3)$$

Both Expected Shortfall and Median Shortfall satisfy all axioms of coherent risk measure and have an intuitive interpretation as well. Among other measures used to assess extreme risk, we can mention the Rachev ratio and the GlueVaR risk measure. The Rachev ratio is the ratio of expected profit to expected loss at a given VaR level, simultaneously in the area of profits (the right tail of the distribution) and losses (the left tail of the distribution) (Rachev et al., 2004). It is expressed by the following formula:

$$R_{\alpha_1,\alpha_2}(X)=\frac{E\left[|X| \mid X\geq -VaR_{\alpha_1}(-X)\right]}{E\left[|X| \mid X\leq VaR_{\alpha_2}(X)\right]}, \tag{4}$$

where α_1 and α_2 stand for significance levels for profits and losses, respectively. An interesting property of Rachev ratio is the possibility of assessing the tails' distribution asymmetry (assuming a symmetrical level of significance, for example, the left tail at the level 0.05 and the right tail at the level 0.95 or the left tail at the level 0.01 and the right tail at the level 0.99). On the other hand, the GlueVaR risk measure, as a coherent measure of risk, was proposed by Belles-Sampera, Guillén, and Santolino (2014) as a measure that considers the attitude toward a hypothetical risky event. GlueVaR is a linear combination of VaR and CVaR and is expressed by the following formula:

$$GlueVaR_{\beta,\alpha}^{\omega_1,\omega_2}(X)=\omega_1 CVaR_\beta+\omega_2 CVaR_\alpha+(1-\omega_1-\omega_2)VaR_\alpha, \tag{5}$$

where α, β stand for confidence levels satisfying the condition $(0<\alpha\leq\beta<1)$, and ω_1, ω_2 reflect weights given by an investor to hypothetical extremely rare events. In this paper, we decided to modify the original formula for GlueVaR (5) deleting the part related to VaR. The final formula is as follows:

$$mGlueVaR_{\beta,\alpha}^{\omega_1,\omega_2}(X)=\omega_1 CVaR_\beta(X)+\omega_2 CVaR_\alpha(X). \tag{6}$$

The mGlueVaR risk measure informs about the expected level of loss for two extremely risky events with an additional assumption of the importance of a given event for a market participant. This importance is considered in terms of significance given to the consequences of a risky event.

The conditional VaR(ES) corresponds to the average risk level beyond the value of VaR. It is possible to assess the volatility in the area exceeding VaR using conditional variance (CTV) and conditional standard deviation (CTD) computed for the data in the tail of the distribution. Mathematical expressions are as follows:

$$CTV_\alpha(X)=E\left[\left(X-ES_\alpha(X)\right)^2 \middle| X>VaR_\alpha(X)\right]. \tag{7}$$

$$CTD_\alpha(X)=\sqrt{E\left[\left(X-ES_\alpha(X)\right)^2 \middle| X>VaR_\alpha(X)\right]}. \tag{8}$$

The above-presented formulas allow us to assess the average level of volatility in the tail of the distribution.

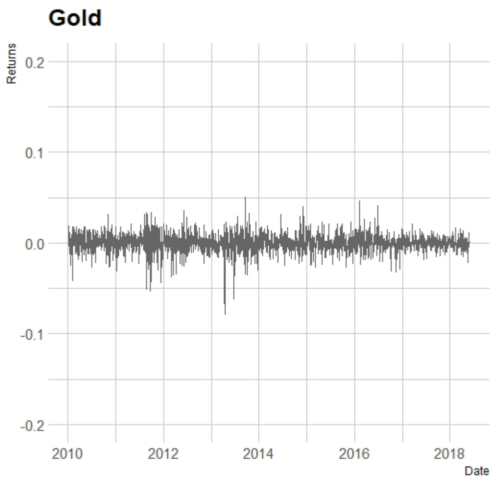
However, the results should be interpreted carefully due to the assumption of symmetry for variance as a dispersion measure.

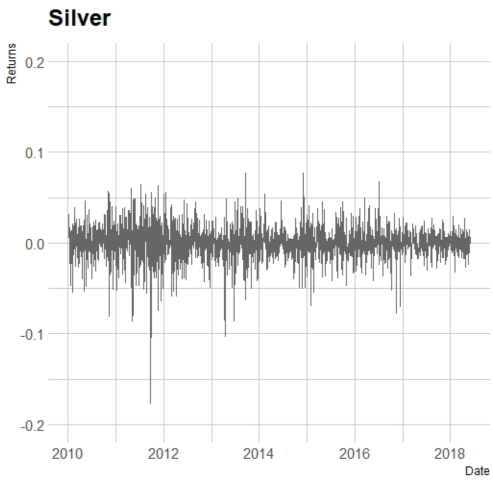
4. Empirical study of gold and silver markets

The entire analysis is conducted for the data from the London Metal Exchange. Based on gold and silver spot prices, the daily log-returns have been calculated. The period of analysis is from January 2010 to May 2018. In the first part of analysis, the relationship between the volatility of gold and silver returns is examined with the volatility of selected market indices. The following indices have been used:

- DXY – Dollar Index Spot,
- LMEX – LMEX Metals Index,
- MSCIWMMI – MSCI World Metals & Mining Index,
- DJUSTMBMI – Dow Jones US Total Market Basic Materials Index,
- MSCIWII – MSCI World Industrials Index,
- MSCIWREI – MSCI World Real Estate Index.

It has been hypothesised that these market indices exhibit correlation with returns on investment in gold and silver, and therefore unobservable factors that affect the returns on investment in analysed metals exist. The figures below present time series of returns for gold and silver as well as industrial indices within the entire period of analysis.





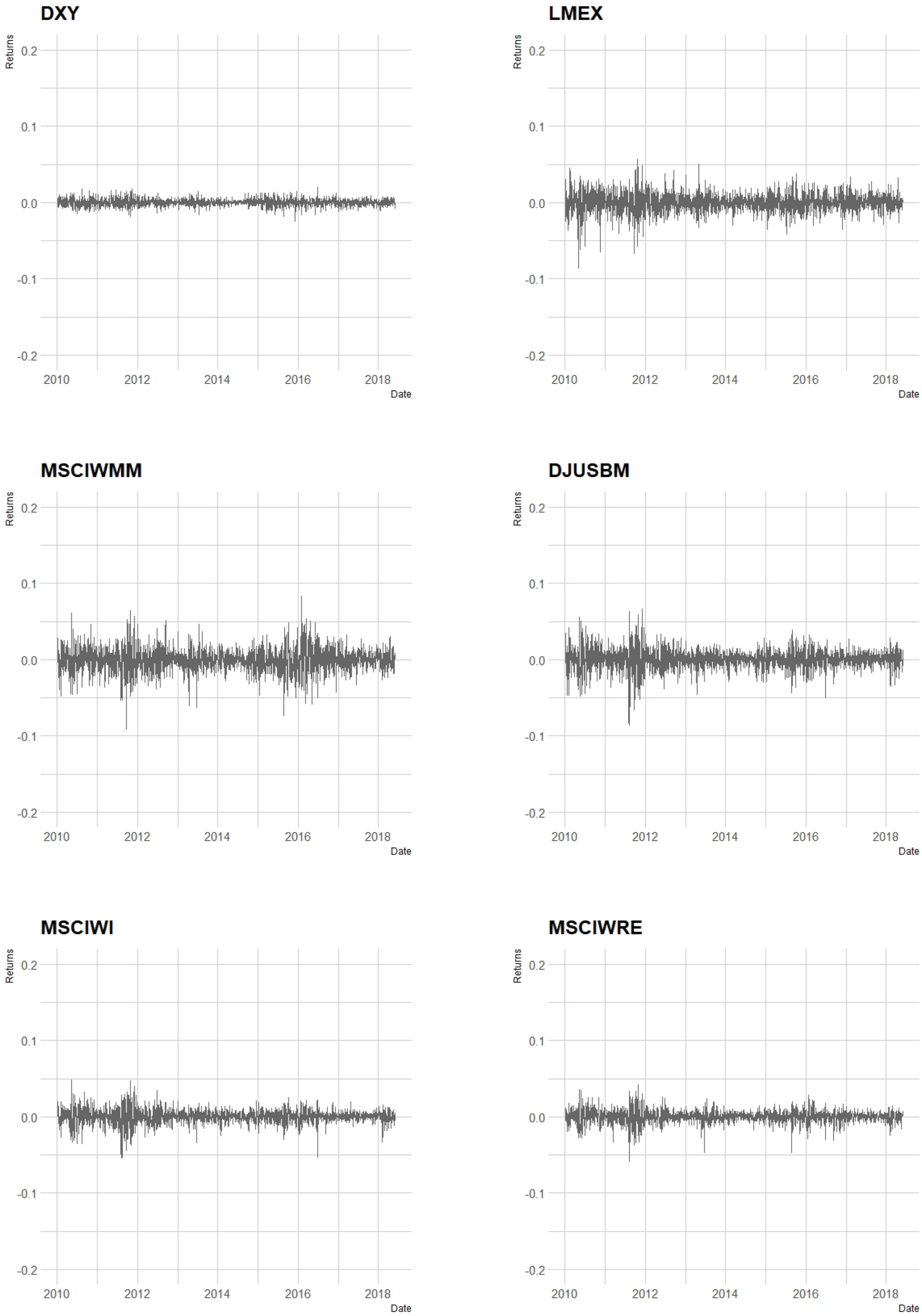


Figure 1. Time series of returns for gold and silver as well as industrial indices – the entire period

Source: own calculations

As can be observed, time series are characterised by different levels of volatility. It is easy to identify clusters of variability which may suggest the existence of long memory effect in the series of returns.

At the beginning, the stationarity of time series has been verified. We use two tests: the unit roots test of Dickey-Fuller (ADF) and the stationarity test of Kwiatkowski-Philips-Schmidt-Shin (KPSS). Both tests indicate that analysed time series are stationary (Table 1).

Table 1. Stationarity tests

Asset	ADF		KPSS	
	Test statistics	p-value	Test statistics	p-value
GOLD	–47.609	0.0001	0.181	> 0.10
SILVER	–46.398	0.0001	0.188	> 0.10
DXY	–46.626	0.0001	0.074	> 0.10
LMEX	–10.702	0.0000	0.116	> 0.10
MSCIWMM	–17.481	0.0000	0.133	> 0.10
DJUSBM	–23.017	0.0000	0.031	> 0.10
MSCIWI	–22.668	0.0000	0.029	> 0.10
MSCIWRE	–26.492	0.0000	0.043	> 0.10

Source: own calculations

To test the hypothesis of existence of unobservable factors which can determine the level of gold and silver returns, we use a multivariate statistical method called factor analysis. Factor analysis (FA) is a causal modelling technique that attempts to explain correlations among a set of observed (manifest) variables through the linear combination of a few unknown number of latent (unobserved) random factors.

Following Rencher (2020), we assume a random sample $\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_\mathbf{n}$ from a homogeneous population with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. The factor analysis model defines each variable as a linear combination of underlying common factors $F_1, F_2, \ldots, F_m$, with an accompanying error term to account for that part of the variable that is unique (not in common with the other variables). For $y_1, y_2, \ldots, y_p$, in any observation vector $\mathbf{y}$, the model is as follows:

$$
\begin{aligned}
y_1 - \mu_1 &= \lambda_{11}F_1 + \lambda_{12}F_2 + \ldots + \lambda_{1m}F_m + \varepsilon_1 \\
y_2 - \mu_2 &= \lambda_{21}F_1 + \lambda_{22}F_2 + \ldots + \lambda_{2m}F_m + \varepsilon_2 \\
&\vdots \\
y_p - \mu_p &= \lambda_{p1}F_1 + \lambda_{p2}F_2 + \ldots + \lambda_{pm}F_m + \varepsilon_p
\end{aligned}
\tag{9}
$$

The coefficients λ_{ij} are called loadings and serve as weights, showing how each y_i individually depends on the *F*'s. With appropriate assumptions, λ_{ij} indicates the importance of the *j*-th factor F_j to the *i*-th variable y_i and can be used in the interpretation of F_j. We can easily rewrite the equations given by (9) in the matrix notation and the final factor model is of the following form:

$$\mathbf{y} - \mu = \Lambda \mathbf{F} + \varepsilon, \tag{10}$$

where $\mathbf{y} = \left[y_1, y_2, \ldots, y_p \right]^{\mathrm{T}}$, $\mu = \left[\mu_1, \mu_2, \ldots, \mu_p \right]^{\mathrm{T}}$, $\mathbf{F} = \left[F_1, F_2, \ldots, F_p \right]^{\mathrm{T}}$, $\varepsilon = \left[\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_p \right]^{\mathrm{T}}$, and Λ is defined as:

$$\Lambda = \begin{pmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1m} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_{p1} & \lambda_{p2} & \cdots & \lambda_{pm} \end{pmatrix}.$$

We use the factor analysis model to identify the hidden structure within the variability of returns of selected indices. At the beginning, we examine whether there are reasons to use factor analysis. We applied two tests: the Kaiser-Meyer-Olkin measure of sampling adequacy (KMO) and Bartlett's test of sphericity. The measure of adequacy of sample selection (0.837) and Bartlett's test (statistics χ^2 = 7550, p-value = 0.000) indicate the existence of a latent factor structure within analysed variables. Two extracted factors explain about 75.42% of the total variance of the input dataset (factor 1 –54.53%, factor 2 –20.89%). The matrix of final factor loadings after Varimax rotation[1] is shown in Table 2.

Table 2. Factor loadings[2], Varimax rotation

Index	Factors	
	F1	**F2**
MSCIWI	0.915	
MSCIWMM	0.866	
DJUSBM	0.836	
MSCIWRE	0.785	
LMEX	0.589	
DXY		.0.950

Source: own calculations

1 Varimax rotation is used to ensure orthogonality of factors.
2 Extraction method: Principal Component Analysis.

The results show that there exist two unobservable factors. The first factor (F1) is represented by all analysed industrial indices, while the second factor (F2) is represented mostly by the dollar index. Factor loadings represent the correlation between observed variables and latent factors. The analysed returns have a very strong impact on creating these two factors. The USD index (DXY) shows a strong negative correlation with factor F2 (–0.950), whereas correlations between industrial indices and F1 are strongly positive. The perception map presented in Figure 2 graphically shows the location of analysed variables in a factorial system.

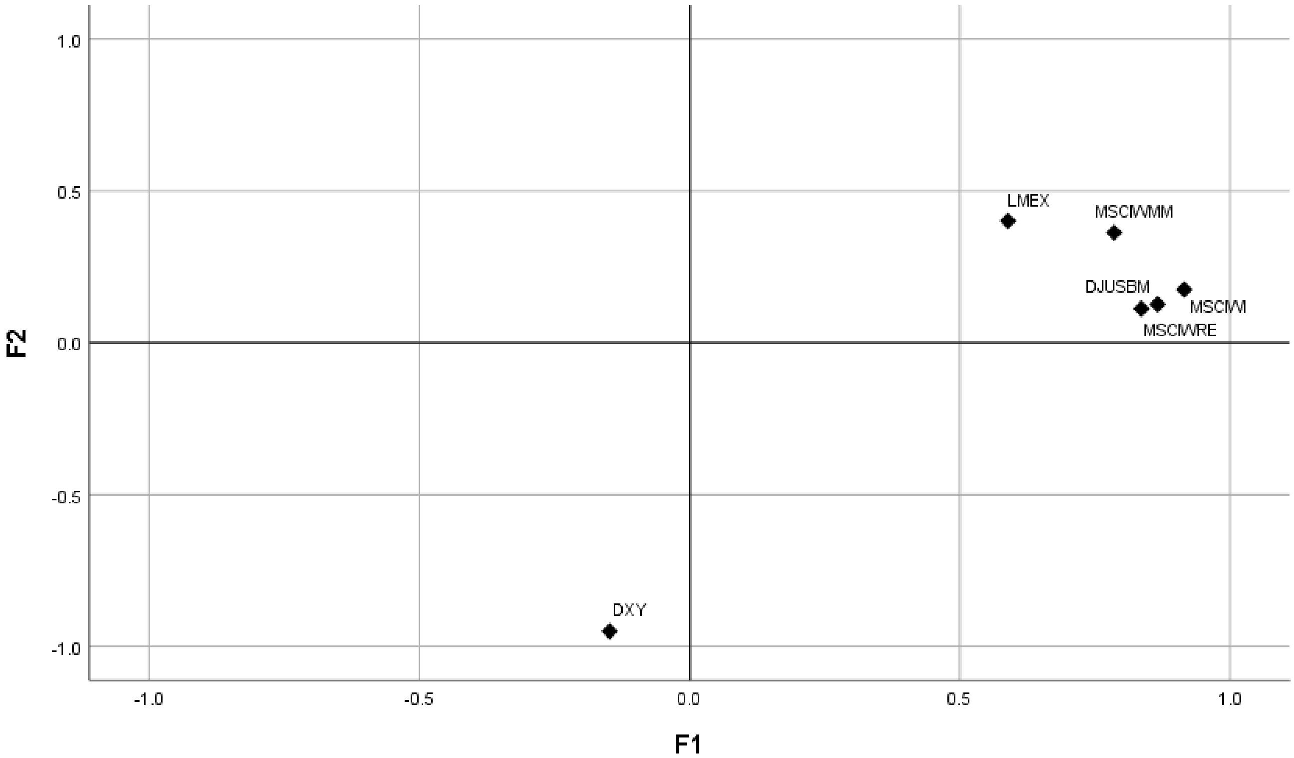


Figure 2. Factors plot in rotated space
Source: own calculations

In the next step of analysis, a factor regression model for returns was estimated using two revealed factors. Mathematically, the factor regression model can be expressed in the following form (Sharpe, 1964):

$$r_i = \beta_0 + \beta_1 F_{i1} + \beta_2 F_{i2} + \ldots + \beta_k F_{ik} + \varepsilon_i , \tag{11}$$

where r_i represents return from the series of i observations ($i = 1,2,\ldots,n$), F_k is the k-th factor determining values of dependent variable r_i, and ε_i is an error term. Unknown parameters β_k's are estimated using the OLS method.

Before the regression model is applied, some basic characteristics of returns must be commented on. In Table 3, descriptive statistics of gold and silver returns are presented.

Table 3. Descriptive statistics of gold and silver returns

Statistics	GOLD	SILVER
Mean	0.00008	–0.00001
Median	0.00022	0.00028
Standard deviation	0.00992	0.01816
Coefficient of variation	12738.72%	–168533.35%
Kurtosis	4.98153	7.00467
Skewness	–0.55990	–0.83812
Min	–0.07800	–0.17700
Max	0.05066	0.07677

Source: own calculations

The average return for both gold and silver returns is around zero, with very high values of coefficient of variation (especially for silver). Empirical distributions (Figure 3) are leptokurtic and skewed to the left, which may suggest the existence of outliers.

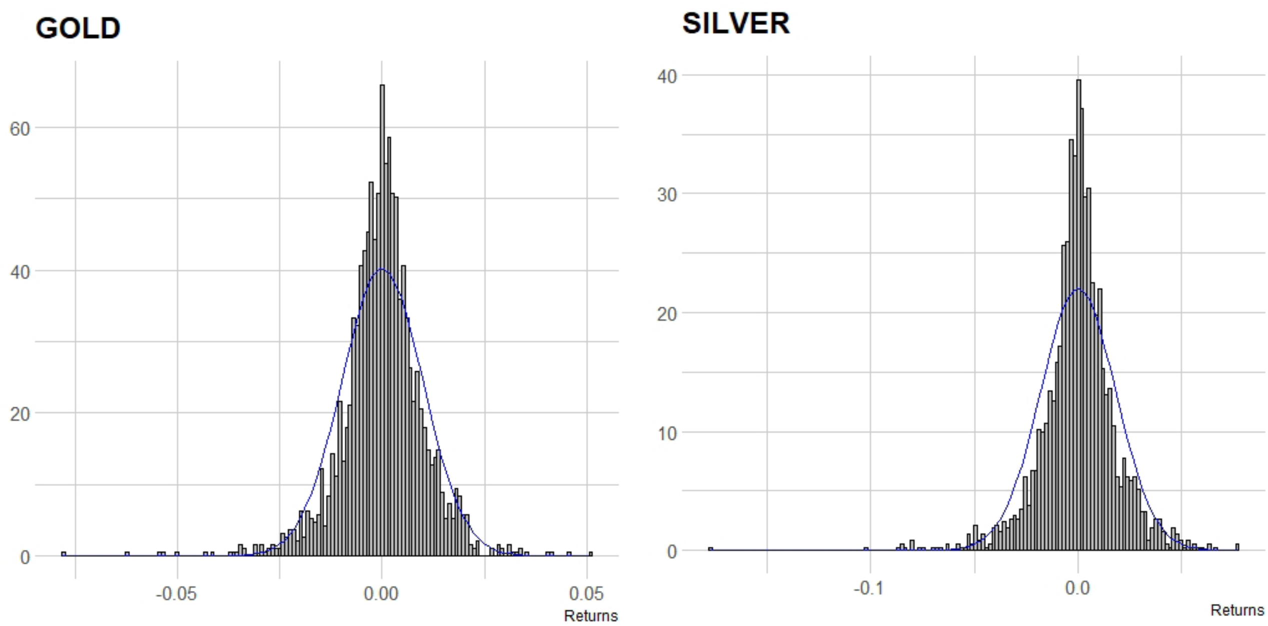


Figure 3. Empirical distributions of gold and silver returns with normal densities
Source: own calculations

In the case of gold returns, the coefficient of determination is at the level of about 16.2% and indicates the percentage of variability of gold returns explained by the estimated model (statistics $F = 212.165$, p-value = 0.000). Estimators of model parameters for gold are presented in Table 4.

Table 4. Estimated coefficients for the factor regression model – gold

Model	Unstandardised coefficients		Standardised coefficients	t Statistics	p-value
	B	Standard error	Beta		
(Intercept)	0.000	0.000	.	0.402	0.688
Factor 1	0.001	0.000	0.147	7.518	0.000
Factor 2	0.004	0.000	0.375	19.178	0.000

Source: own calculations

The results indicate that both factors stimulate volatility of gold returns. The impact of both factors is statistically significant but stronger for factor 2. The Ljung-Box test for autocorrelation of first order is 2.212 (p-value = 0.137), which means that the effect of autocorrelation for gold does not exist. We can say that both industrial indices and USD index influence the volatility of gold returns.

The estimated factor model for silver exhibits a better fit to empirical returns (the coefficient of determination at the level of about 23.3%). The model turned out to be statistically significant (statistics F = 332.146, p-value = 0.000). The Ljung-Box test for autocorrelation of first order is 2.588 (p-value = 0.108), which means that the effect of autocorrelation for silver does not exist. Estimators of model parameters for silver are presented in Table 5.

Table 5. Estimated coefficients for the factor regression model – silver

Model	Unstandardised coefficients		Standardised coefficients	t statistics	p-value
	B	Standard error	Beta		
(Intercept)	–0.000	0.000	–	–0.032	0.975
Factor 1	0.006	0.000	0.313	16.720	0.000
Factor 2	0.007	0.000	0.367	19.615	0.000

Source: own calculations

The results obtained for the factorial regression analysis for silver returns are similar to those obtained for gold. Similarly, both factors stimulate volatility of returns for silver. Comparing regression results between the two models, the impact of USD index (F2) measured by standardised beta coefficient is similar regardless of the metal, whereas the impact of industrial factor (F1) is stronger for silver. Empirical and predicted returns for gold and silver are presented in Figure 4.

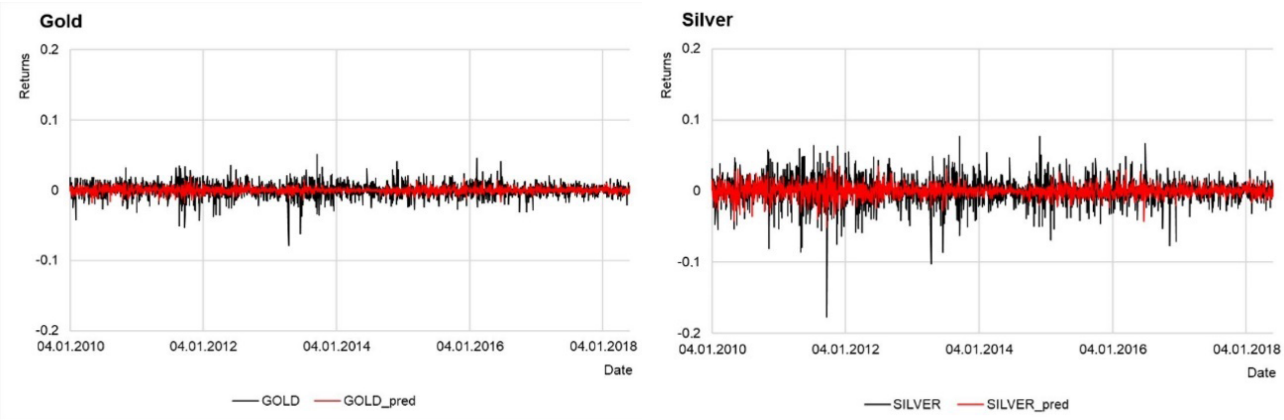


Figure 4. Empirical and predicted returns for gold and silver
Source: own calculations

In the second part of the research, a quantitative assessment of risk of volatility in gold and silver returns was made using the measures proposed in the first part of this paper. The analysed period of January 2010 – May 2018 has been divided into three subperiods, based on the volatility of gold prices (Figure 5):

– Subperiod 1: 2010, January – 2011, August – the upward trend period,
– Subperiod 2: 2011, September – 2015, December – the downward trend period,
– Subperiod 3: 2016, January – 2018, May – the upward trend period after the crisis.

Figure 5. Three sub-periods in gold price changes
Source: own calculations

As we can see, subperiod 1 is characterised by a rather intense upward trend. The average increase in the price of gold in this period was at the level of 0.11% per day, while in silver prices at 0.20% per day. The price of gold at the end of this subperiod was about 64% higher than at the beginning, while in the case of silver there was an increase of nearly 140%. Subperiod 2 shows a moderate long-term downward trend with an average decline in gold prices of about 0.05% and in silver prices of about 0.10%. The price of gold at the end of the second subperiod was over 41% lower than at the beginning, while for silver there was a decrease of over 66%. The last subperiod is characterised by stabilisation and a slight upward trend. The prices of gold and silver changed on average by 0.03%. Gold price at the end of the third subperiod was about 23% higher than at the beginning, while for silver it was around 19% higher than at the beginning. Table 6 presents descriptive statistics for gold and silver returns within each of the examined subperiods.

Table 6. Descriptive statistics of gold and silver prices and returns – three subperiods

Subperiod 1	Prices		Returns	
	GOLD	SILVER	GOLD	SILVER
Mean	1338.88	26.67	0.00119	0.00209
Median	1343.70	24.75	0.00166	0.00348
Standard deviation	175.81	8.99	0.00997	0.02184
Coefficient of variation	13.13%	33.70%	839.35%	1042.73%
Kurtosis	0.03	–1.18	2.61580	2.10703
Skewness	0.62	0.44	–0.63169	–0.75469
Min	1064.09	15.17	–0.05040	–0.08532
Max	1887.43	48.54	0.03407	0.06377
Subperiod 2	**Prices**		**Returns**	
	GOLD	**SILVER**	**GOLD**	**SILVER**
Mean	1401.91	23.29	–0.00048	–0.00098
Median	1316.26	20.98	–0.00032	–0.00057
Standard deviation	221.81	6.85	0.01088	0.01899
Coefficient of variation	15.82%	29.42%	2251.04%	1945.93%
Kurtosis	–1.35	–0.99	5.12806	8.87439
Skewness	0.34	0.47	–0.65714	–0.94466
Min	1052.94	13.74	–0.07800	.0.17700
Max	1900.31	43.25	0.05066	0.07677
Subperiod 3	**Prices**		**Returns**	
	GOLD	**SILVER**	**GOLD**	**SILVER**
Mean	1265.81	17.02	0.00032	0.00027
Median	1271.24	16.93	0.00020	0.00044
Standard deviation	59.58	1.27	0.00781	0.01304
Coefficient of variation	4.71%	7.44%	2418.37%	4744.76%
Kurtosis	0.77	0.84	3.85792	4.86213
Skewness	–0.87	0.37	0.26250	–0.39077
Min	1061.10	13.79	–0.03143	–0.07695
Max	1366.33	20.60	0.04583	0.06715

Source: own calculations

A higher level of volatility for price and return of silver is observed, regardless of the subperiod of analysis. Gold price, despite the downward trend in the second subperiod, reached an average level exceeding its value from the first subperiod by nearly USD 60/ozt. This is mainly due to the high level of gold prices in long-term within this subperiod. On the other hand, the average price of silver exhibits a downward trend from period to period.

The analysis of distribution of returns throughout the entire period as well as in all subperiods shows that, under the Kolmogorov-Smirnov, Cramer-von Misses and Anderson-Darling tests, the hypothesis about the normality of empirical distributions has to be rejected. As the additional theoretical distribution, Student's t-distribution is proposed. The empirical distribution for gold returns over the entire period with theoretical densities (normal and t-Student) is presented in Figure 6.

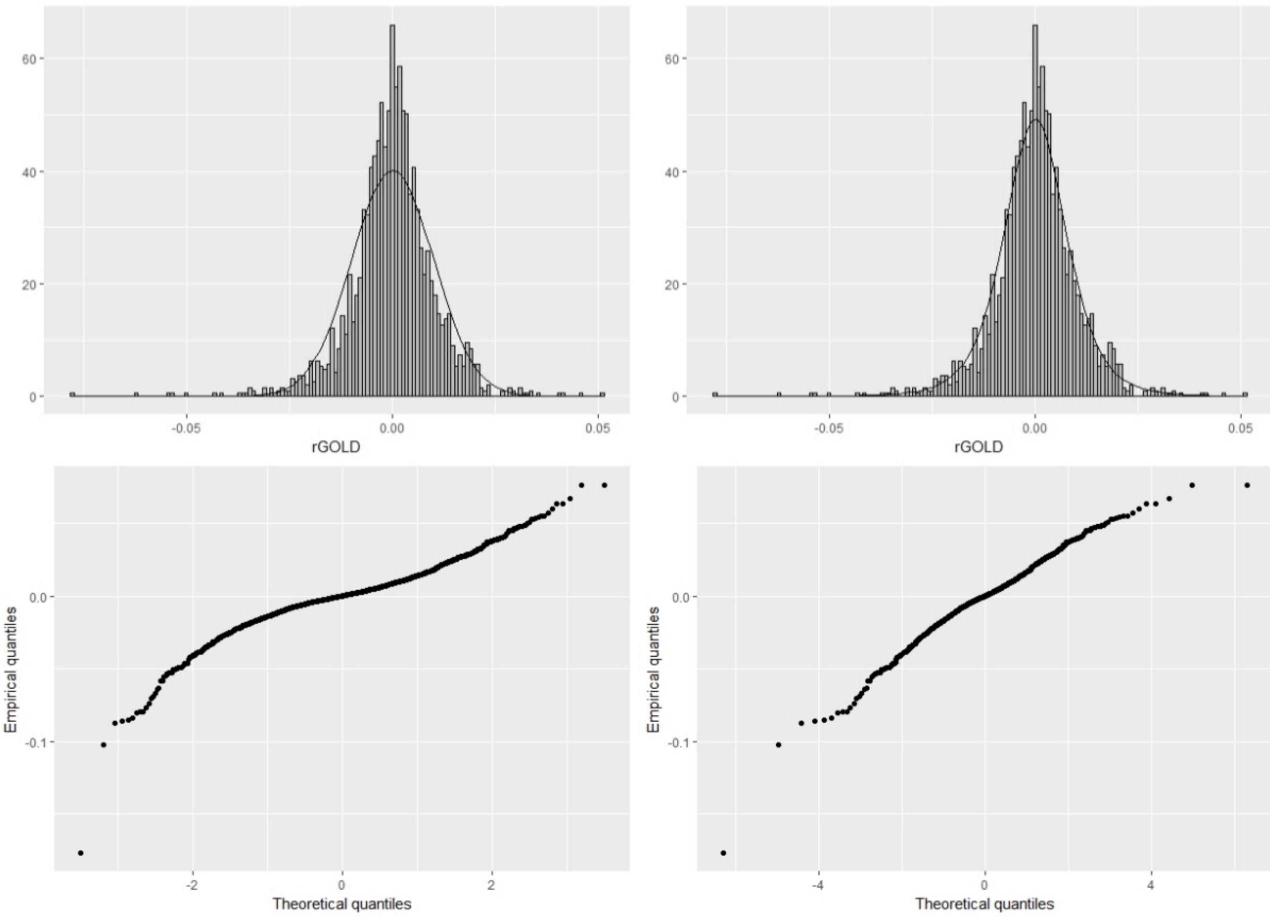


Figure 7. Empirical and theoretical distributions and QQ-plots for gold (normal – left, t-Student – right)
Source: own calculations

The selection of an appropriate probability distribution for the empirical returns allows us to use theoretical density functions to determine values for risk measures proposed by formulas (1–4) and (6). The level of potential profits/losses was compared in accordance with the adopted measures for the entire period and subperiods. The results are presented in Tables 7–10.

Table 7. Risk measures – entire period

Risk measure	Empirical		Normal		t-Student	
VaR	**GOLD**	**SILVER**	**GOLD**	**SILVER**	**GOLD**	**SILVER**
0.01	−0.0287	−0.0534	−0.0230	−0.0423	−0.0251	−0.0502
0.05	−0.0158	−0.0284	−0.0162	−0.0299	−0.0161	−0.0287
0.95	0.0158	0.0281	0.0149	0.0313	0.0154	0.0299
0.99	0.0229	0.0468	0.0215	0.0524	0.0224	0.0481
ES	**GOLD**	**SILVER**	**GOLD**	**SILVER**	**GOLD**	**SILVER**
0.01	−0.0397	−0.0762	−0.0264	−0.0484	−0.0311	−0.0622
0.05	−0.0242	−0.0459	−0.0204	−0.0375	−0.0227	−0.0403
0.95	0.0213	0.0392	0.0247	0.0427	0.0231	0.0411
0.99	0.0318	0.0555	0.0376	0.0602	0.0326	0.0574
MS	**GOLD**	**SILVER**	**GOLD**	**SILVER**	**GOLD**	**SILVER**
0.01	−0.0346	−0.0723	−0.0227	−0.0447	−0.0318	−0.0612
0.05	−0.0213	−0.0394	−0.0192	−0.0338	−0.0207	−0.0361
0.95	0.0190	0.0375	0.0202	0.0389	0.0199	0.0381
0.99	0.0311	0.0532	0.0341	0.0551	0.0322	0.0539

Source: own calculations

Table 8. Risk measures – subperiod 1

Risk measure	Empirical		Normal		t-Student	
VaR	**GOLD**	**SILVER**	**GOLD**	**SILVER**	**GOLD**	**SILVER**
0.01	−0.0267	−0.0797	−0.0220	−0.0487	−0.0249	−0.0611
0.05	−0.0159	−0.0365	−0.0152	.0.0338	−0.0155	−0.0361
0.95	0.0161	0.0341	0.0163	0.0322	0.0163	0.0338
0.99	0.0199	0.0532	0.0217	0.0513	0.0203	0.0527
ES	**GOLD**	**SILVER**	**GOLD**	**SILVER**	**GOLD**	**SILVER**
0.01	−0.0374	−0.0822	−0.0253	−0.0560	−0.0311	−0.0719
0.05	−0.0236	−0.0549	−0.0193	−0.0429	−0.0224	−0.0501
0.95	0.0203	0.0442	0.0241	0.0507	0.0219	0.0492
0.99	0.0277	0.0577	0.0292	0.0591	0.0280	0.0581
MS	**GOLD**	**SILVER**	**GOLD**	**SILVER**	**GOLD**	**SILVER**
0.01	−0.0363	−0.0820	−0.0247	−0.0517	−0.0302	−0.0674
0.05	−0.0208	−0.0498	−0.0188	−0.0411	−0.0199	−0.0439
0.95	0.0184	0.0403	0.0201	0.0421	0.0197	0.0415
0.99	0.0315	0.0573	0.0243	0.0562	0.0272	0.0569

Source: own calculations

Table 9. Risk measures – subperiod 2

Risk measure	Empirical		Normal		t-Student	
VaR	**GOLD**	**SILVER**	**GOLD**	**SILVER**	**GOLD**	**SILVER**
0.01	−0.0345	−0.0581	−0.0258	−0.0451	−0.0327	−0.0526
0.05	−0.0179	−0.0313	−0.0184	−0.0322	−0.0181	−0.0319
0.95	0.0177	0.0288	0.0192	0.0301	0.0185	0.0292
0.99	0.0281	0.0475	0.0271	0.0422	0.0314	0.0461
ES	**GOLD**	**SILVER**	**GOLD**	**SILVER**	**GOLD**	**SILVER**
0.01	−0.0458	−0.0822	−0.0295	−0.0516	−0.0391	−0.0629
0.05	−0.0274	−0.0482	−0.0229	−0.0401	−0.0311	−0.0393
0.95	0.0226	0.0405	0.0296	0.0427	0.0241	0.0412
0.99	0.0333	0.0567	0.0317	0.0604	0.0328	0.0591
MS	**GOLD**	**SILVER**	**GOLD**	**SILVER**	**GOLD**	**SILVER**
0.01	−0.0370	−0.0686	−0.0271	−0.0476	−0.0353	−0.0612
0.05	−0.0232	−0.0408	−0.0211	−0.0372	−0.0247	−0.0371
0.95	0.0202	0.0383	0.0238	0.0411	0.0223	0.0381
0.99	0.0311	0.0534	0.0299	0.0529	0.0319	0.0531

Source: own calculations

Table 10. Risk measures – subperiod 3

Risk measure	Empirical		Normal		t-Student	
VaR	**GOLD**	**SILVER**	**GOLD**	**SILVER**	**GOLD**	**SILVER**
0.01	−0.0199	−0.0361	−0.0178	−0.0300	−0.0189	−0.0327
0.05	−0.0119	−0.0199	−0.0125	−0.0212	−0.0122	−0.0203
0.95	0.0127	0.0212	0.0131	0.0227	0.0130	0.0209
0.99	0.0190	0.0325	0.0194	0.0361	0.0199	0.0341
ES	**GOLD**	**SILVER**	**GOLD**	**SILVER**	**GOLD**	**SILVER**
0.01	−0.0260	−0.0522	−0.0205	−0.0345	−0.0251	−0.0416
0.05	−0.0173	−0.0306	−0.0158	−0.0266	−0.0163	−0.0301
0.95	0.0177	0.0294	0.0172	0.0311	0.0171	0.0286
0.99	0.0290	0.0433	0.0217	0.0402	0.0302	0.0412
MS	**GOLD**	**SILVER**	**GOLD**	**SILVER**	**GOLD**	**SILVER**
0.01	−0.0261	−0.0467	−0.0188	−0.0323	−0.0243	−0.0392
0.05	−0.0161	−0.0268	−0.0143	−0.0271	−0.0151	−0.0277
0.95	0.0148	0.0275	0.0157	0.0253	0.0155	0.0280
0.99	0.0266	0.0383	0.0196	0.0381	0.0217	0.0389

Source: own calculations

The results presented in Tables 7–10 show that, regardless of the period of analysis, investments in silver are about two times riskier than investments in gold. Moreover, the t-Student distribution, as a heavy-tailed in relation to the normal one, more accurately approximates the empirical values of risk measures regardless of the quantile and considered metal. Looking holistically at the entire period, it was observed that regardless of the theoretical distribution and the analysed metal for quantiles 0.95 and 0.99, all risk measures are overestimated. In turn, for quantiles 0.01 and 0.05 – underestimated, respectively. Similar conclusions have been drawn for all subperiods: loss is usually underestimated, while profit is overestimated. The reason may be an uneven distribution of returns for examined metals and skewness of its distributions. Figures 7–9 present graphically the values of estimating risk measures for empirical data.

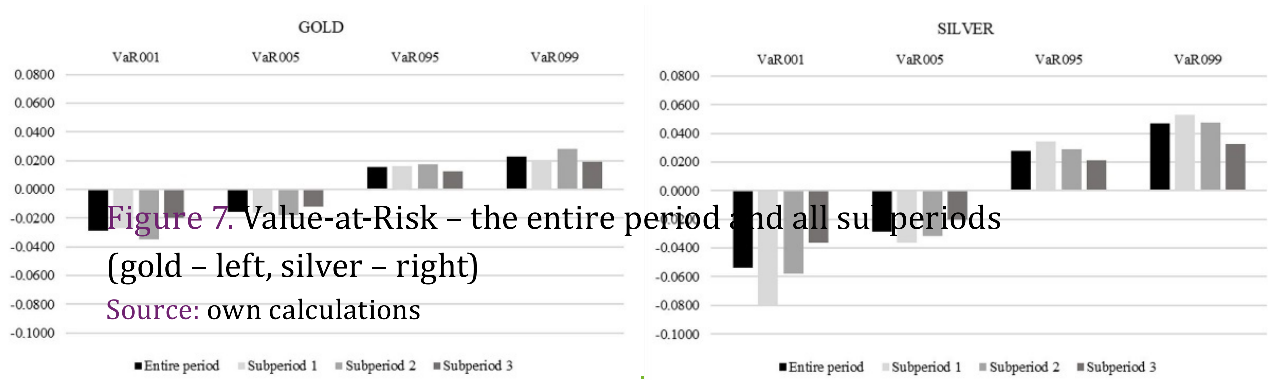


Figure 7. Value-at-Risk – the entire period and all subperiods (gold – left, silver – right)
Source: own calculations

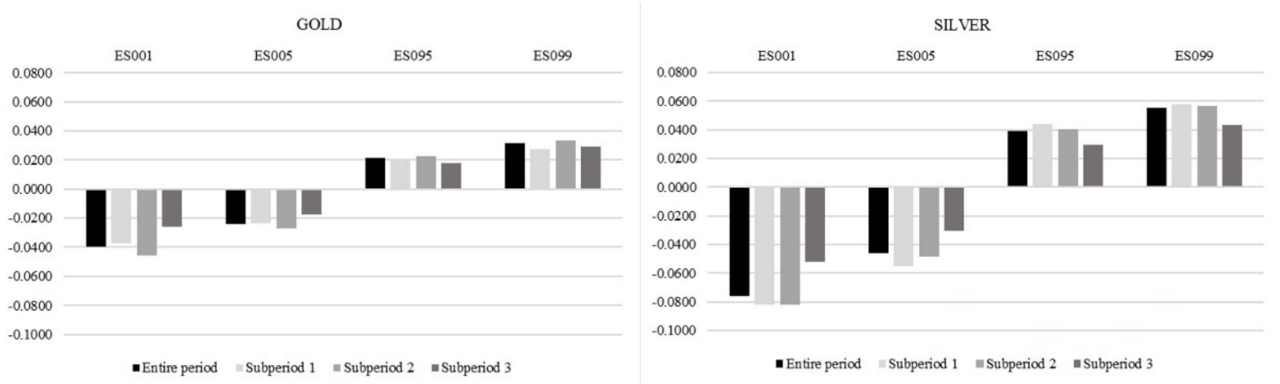


Figure 8. Expected Shortfall – the entire period and all subperiods (gold – left, silver – right)
Source: own calculations

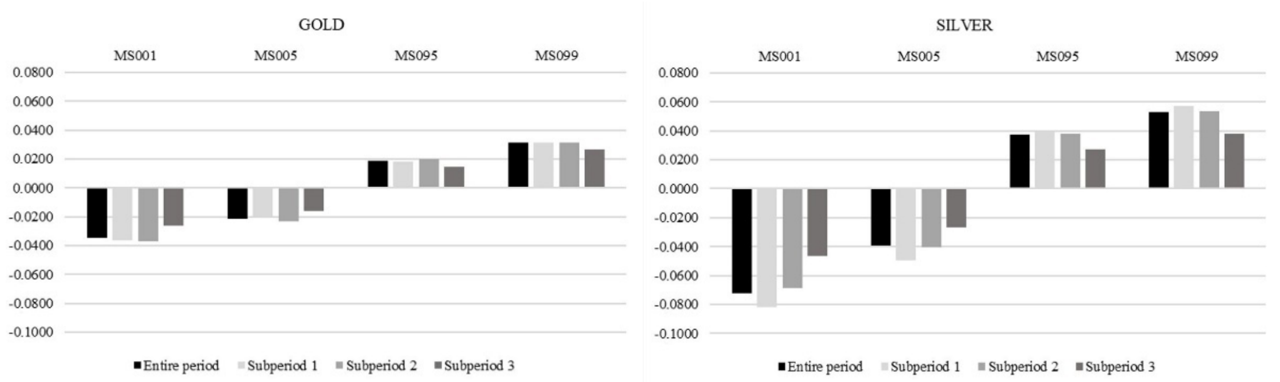


Figure 9. Medias Shortfall – the entire period and all subperiods (gold – left, silver – right)
Source: own calculations

Commenting on results of risk analysis, it was observed that regardless of the type of metal, the risk considered in the sense of loss was higher than risk considered in the sense of profit. The results also show that usually risk estimation for silver returns is higher than for gold, and this difference is about double. In this research, we used two additional risk measures: the Rachev ratio and mGlueVaR. The first one expresses the ratio of the expected profit to the expected loss above the level of the VaR, while the other one assesses the risk considering the subjective attitude towards risk. These measures are described by formulas (4) and (6), while the results are presented in Table 11 and in Figure 10.

Table 11. Rachev ratio and mGlueVaR – gold and silver

Risk measure	Entire period		Subperiod 1		Subperiod 2		Subperiod 3	
	GOLD	**SILVER**	**GOLD**	**SILVER**	**GOLD**	**SILVER**	**GOLD**	**SILVER**
R-ratio 0.95/0.05	0.8781	0.8533	0.8593	0.8054	0.8258	0.8405	1.0239	0.9589
R-ratio 0.99/0.01	0.8002	0.7279	0.7389	0.7020	0.7276	0.6896	1.1162	0.8294
mGlueVaR 0.01/0.05	−0.0358	−0.0687	−0.0340	−0.0754	−0.0412	−0.0737	−0.0238	−0.0468
mGlueVaR 0.99/0.95	0.0292	0.0514	0.0258	0.0543	0.0306	0.0527	0.0262	0.0398

Source: own calculations

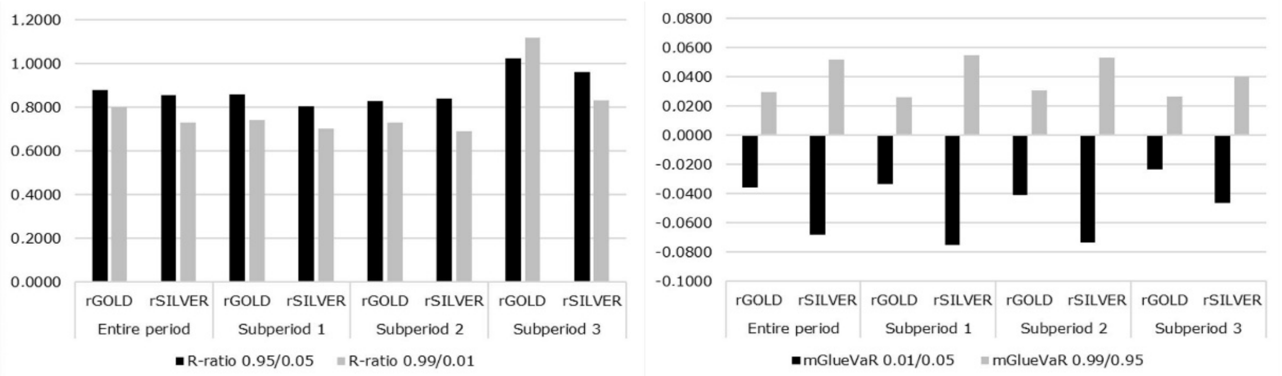


Figure 10. Rachev ratio (left) and mGlueVaR (right) – gold and silver
Source: own calculations

The Rachev ratio was calculated for symmetric pairs of quantiles. The results show that expected profits are lower than expected losses regardless of the analysed period. The difference is visible primarily for silver returns. However, in the third subperiod, it was observed that the Rachev ratio exceeded the value of 1 for gold, which means that in this subperiod the expected profit was higher than the expected loss, for a given pair of quantiles. On the other hand, the mGlueVaR risk measure was calculated under the following assumptions:

– probability of occurrence of very risky event (0.05 and 0.95) and subjective attitude toward risk weighted by 0.25,
– probability of occurrence of extremely risky event (0.01 and 0.99) and subjective attitude toward risk weighted by 0.75.

The results show that regardless of the analysed period, the risk on the average level between a very risky and catastrophic event is higher in the case of losses. In addition, the level of loss is higher for silver.

In the final stage of research, the dispersion in distribution tails was assessed. Formulas 7–8 were used and the coefficient of variation in the tail was additionally calculated. The results are shown in Table 12.

Table 12. Measures of dispersion in the tails of distributions

Risk measure	Entire period		Subperiod 1		Subperiod 2		Subperiod 3	
CTD	**GOLD**	**SILVER**	**GOLD**	**SILVER**	**GOLD**	**SILVER**	**GOLD**	**SILVER**
0.01	0.0123	0.0255	0.0092	0.0023	0.0138	0.0327	0.0046	0.0163
0.05	0.0099	0.0200	0.0081	0.0154	0.0114	0.0227	0.0051	0.0130
0.95	0.0064	0.0103	0.0049	0.0084	0.0063	0.0105	0.0076	0.0093
0.99	0.0070	0.0086	0.0060	0.0038	0.0063	0.0099	0.0096	0.0108
ES	GOLD	SILVER	GOLD	SILVER	GOLD	SILVER	GOLD	SILVER
0.01	−0.0397	−0.0762	−0.0374	−0.0822	−0.0458	−0.0822	−0.0260	−0.0522
0.05	−0.0242	−0.0459	−0.0236	−0.0549	−0.0274	−0.0482	−0.0173	−0.0306
0.95	0.0213	0.0392	0.0203	0.0442	0.0226	0.0405	0.0177	0.0294
0.99	0.0318	0.0555	0.0277	0.0577	0.0333	0.0567	0.0290	0.0433
Coefficient of variation	**GOLD**	**SILVER**	**GOLD**	**SILVER**	**GOLD**	**SILVER**	**GOLD**	**SILVER**
0.01	−31.00%	−33.46%	−24.67%	−2.82%	−30.07%	−39.79%	−17.76%	−31.16%
0.05	−40.85%	−43.51%	−34.21%	−28.07%	−41.53%	−47.16%	−29.25%	−42.51%
0.95	30.14%	26.30%	24.03%	19.05%	27.94%	25.98%	42.78%	31.55%
0.99	22.03%	15.51%	21.66%	6.58%	18.79%	17.41%	33.30%	24.95%

Source: own calculations

The results show a higher level of volatility in the left tail of the distribution. Moreover, in the left tail (the area of loss), the greater dispersion of extreme returns for silver was observed, while in the right tail – for gold. The result can be explained by the skewness and a thicker left tail of empirical distributions for analysed metals.

5. Remarks and conclusions

The paper attempts to assess the risk of volatility observed in returns for gold and silver. The analysis covers the research period from January 2010 to May 2018. The analysis of gold and silver returns shows the relationship with some selected market indices, primarily with the USD index. Factor analysis helps to identify two uncorrelated factors describing changes of gold and silver returns. For both metals, factor 1 (created by all industrial indices) and factor 2 (created by USD index) exhibit a positive impact. Different effects of these revealed factors are pointed out. Factor 2 has almost a two times stronger impact on returns changes than factor 1 for gold, whereas for silver the impact of these two factors is similar.

Three subperiods of price levels were determined. The empirical distributions of returns within these subperiods are leptokurtic and skewed indicating the existence of outliers. Risk analysis has shown that, regardless of the level of quantile, investments in silver are almost two times riskier than investments in gold. While estimating risk measures, the empirical distribution and theoretical (normal and t-Student) distributions were used. For a given quantile levels, it was observed that the volatility of returns for silver seems to produce higher risk than for gold, regardless of the measure of risk. The normal distribution usually underestimates the level of risk. Moreover, it has been shown that the expected values of risk beyond VaR in the area of profits and losses are not symmetrical – losses are greater than profits.

In summary, we can confirm that the volatility of returns of gold and silver depends on the level of unobserved factors: the industrial factor and USD factor. It has been found that these factors determine the volatility of returns at a different level. Returns of gold behave more stable than those of silver, hence risk assessments for this metal are lower. However, from the investment point of view, lower risk usually means lower profits, so the investor's attitude toward risk should be considered when selecting an investment asset. It has been observed that in the case of risk, investments in gold and silver are different due to the level of risk. Silver is almost two times riskier than gold, regardless of the quantile risk measure and the level of quantile. The results may be helpful in building strategies for hedging investment portfolios, especially in the times of economic crisis.

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Czynniki zmienności stóp zwrotu i analiza ryzyka z wykorzystaniem miar kwantylowych na rynku złota i srebra

Streszczenie: Rynki alternatywne, do których zalicza się rynek towarowy, stanowią doskonałe miejsce na zabezpieczanie lokowanych środków inwestycyjnych w sytuacji pogarszającej się koniunktury na rynkach finansowych. Szczególne zainteresowanie kierowane jest w stronę inwestycji w metale szlachetne, takie jak złoto czy srebro. Oba metale mają szerokie zastosowanie w przemyśle, jednakże dodatkowo ich wartość ukryta powiązana jest ze stosunkiem człowieka do tego rodzaju kruszców. Od dawien dawna złoto i srebro kojarzone były z majątkiem i bogactwem. Przedmiotem pracy jest identyfikacja ukrytych czynników, mogących wpływać na zmienność stóp zwrotu złota i srebra oraz ocena ryzyka inwestycji w te metale szlachetne. Zidentyfikowano dwa istotne czynniki oraz dokonano porównania ocen ryzyka za pomocą miar kwantylowych. Wyniki wskazują na istotny wpływ zidentyfikowanych czynników na zmienność stóp zwrotu złota i srebra. Zaobserwowano także różnice w poziomie ryzyka inwestycji dla tych dwóch metali.

Słowa kluczowe: ryzyko, złoto, srebro, metale szlachetne, kwantylowe miary ryzyka

JEL: G01, G11, G31

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