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Rajan Chattamvelli and Ramalingam Shanmugam

ISBN: 978-3-031-79409-4 paperback
ISBN: 978-3-031-79410-0 ebook
ISBN: 978-3-031-79411-7 hardcover

DOI 10.1007/978-3-031-79410-0

A Publication in the Springer series
SYNTHESIS LECTURES ON ENGINEERING

Lecture #37
Series ISSN
Print 1939-5221 Electronic 1939-523X

Generating Functions in Engineering and the Applied Sciences

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SYNTHESIS LECTURES ON ENGINEERING #37

ABSTRACT

This is an introductory book on generating functions (GFs) and their applications. It discusses commonly encountered generating functions in engineering and applied sciences, such as ordinary generating functions (OGF), exponential generating functions (EGF), probability generating functions (PGF), etc. Some new GFs like Pochhammer generating functions for both rising and falling factorials are introduced in Chapter 2. Two novel GFs called "mean deviation generating function" (MDGF) and "survival function generating function" (SFGF), are introduced in Chapter 3. The mean deviation of a variety of discrete distributions are derived using the MDGF. The last chapter discusses a large number of applications in various disciplines including algebra, analysis of algorithms, polymer chemistry, combinatorics, graph theory, number theory, reliability, epidemiology, bio-informatics, genetics, management, economics, and statistics.

Some background knowledge on GFs is often assumed for courses in analysis of algorithms, advanced data structures, digital signal processing (DSP), graph theory, etc. These are usually provided by either a course on "discrete mathematics" or "introduction to combinatorics." But, GFs are also used in automata theory, bio-informatics, differential equations, DSP, number theory, physical chemistry, reliability engineering, stochastic processes, and so on. Students of these courses may not have exposure to discrete mathematics or combinatorics. This book is written in such a way that even those who do not have prior knowledge can easily follow through the chapters, and apply the lessons learned in their respective disciplines. The purpose is to give a broad exposure to commonly used techniques of combinatorial mathematics, highlighting applications in a variety of disciplines.

Any suggestions for improvement will be highly appreciated. Please send your comments to `shanmugam@txstate.edu`, and they will be incorporated in subsequent revisions.

KEYWORDS

algebra, analysis of algorithms, bio-informatics, CDF generating functions, combinatorics, cumulants, difference equations, discrete mathematics, economics, epidemiology, finance, genetics, graph theory, management, mean deviation generating function, moments, number theory, Pochhammer generating functions, polymer chemistry, power series, recurrence relations, reliability engineering, special numbers, statistics, strided sequences, survival function, truncated distributions.

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Glossary of Terms

Term	Meaning
CDFGF	Cumulative Distribution Function GF
CGF	Cumulant Generating Function
ChF	Characteristic Function
CMGF	Central Moment Generating Function
EGF	Exponential Generating Function
FCGF	Factorial Cumulant Generating Function
FMGF	Factorial Moment Generating Function
FPGF	Falling Pochhammer Generating Function
GF	Generating Function
HCW	Health Care Worker
IGF	Information Generating Function
MDGF	Mean Deviation Generating Function
MGF	Moment Generating Function
OGF	Ordinary Generating Function
PGF	Probability Generating Function
PMF	Probability Mass Function
RoC	Radius of Convergence
RPGF	Rising Pochhammer Generating Function
SF	Survival Function
SFGF	Survival Function Generating Function
SIR	Susceptible, Infected, Recovered model
SIS	Susceptible, Infected, Susceptible model