
SIMULATION MODELING AND ANALYSIS

Second Edition

Averill M. Law

President

Averill M. Law & Associates

Tucson, Arizona

W. David Kelton

Associate Professor of Operations and Management Science

Curtis L. Carlson School of Management

University of Minnesota

McGraw-Hill, Inc.

New York St. Louis San Francisco Auckland Bogotá Caracas Lisbon
London Madrid Mexico Milan Montreal New Delhi Paris
San Juan Singapore Sydney Tokyo Toronto

CONTENTS

List of Symbols	xvii
Preface to the Second Edition	xix
Preface to the First Edition	xxi

Chapter 1 Basic Simulation Modeling	1
1.1 The Nature of Simulation	1
1.2 Systems, Models, and Simulation	3
1.3 Discrete-Event Simulation	7
1.3.1 Time-Advance Mechanisms	8
1.3.2 Components and Organization of a Discrete-Event Simulation Model	10
1.4 Simulation of a Single-Server Queueing System	13
1.4.1 Problem Statement	13
1.4.2 Intuitive Explanation	19
1.4.3 Program Organization and Logic	29
1.4.4 FORTRAN Program	34
1.4.5 Pascal Program	44
1.4.6 C Program	52
1.4.7 Simulation Output and Discussion	60
1.4.8 Alternative Stopping Rules	62
1.4.9 Determining the Events and Variables	72
1.5 Simulation of an Inventory System	75
1.5.1 Problem Statement	75
1.5.2 Program Organization and Logic	77
1.5.3 FORTRAN Program	82
1.5.4 Pascal Program	89
1.5.5 C Program	96
1.5.6 Simulation Output and Discussion	102
1.6 Distributed Simulation	103
1.7 Steps in a Simulation Study	106
1.8 Other Types of Simulation	109

1.8.1	Continuous Simulation	109
1.8.2	Combined Discrete-Continuous Simulation	112
1.8.3	Monte Carlo Simulation	113
1.9	Advantages, Disadvantages, and Pitfalls of Simulation	114
Appendix 1A: Fixed-Increment Time Advance		116
Appendix 1B: A Primer on Queueing Systems		118
1B.1	Components of a Queueing System	118
1B.2	Notation for Queueing Systems	119
1B.3	Measures of Performance for Queueing Systems	120
Appendix 1C: Notes on the Computers and Compilers Used		122
Problems		123
References		130
Chapter 2	Modeling Complex Systems	133
2.1	Introduction	133
2.2	List Processing in Simulation	134
2.2.1	Approaches to Storing Lists in a Computer	134
2.2.2	Linked Storage Allocation	135
2.3	A Simple Simulation Language, SIMLIB	141
2.4	Single-Server Queueing Simulation with SIMLIB	150
2.4.1	Problem Statement	150
2.4.2	SIMLIB Program	150
2.4.3	Simulation Output and Discussion	156
2.5	Time-Shared Computer Model	157
2.5.1	Problem Statement	158
2.5.2	SIMLIB Program	158
2.5.3	Simulation Output and Discussion	169
2.6	Multiteller Bank with Jockeying	170
2.6.1	Problem Statement	170
2.6.2	SIMLIB Program	171
2.6.3	Simulation Output and Discussion	183
2.7	Job-Shop Model	185
2.7.1	Problem Statement	185
2.7.2	SIMLIB Program	187
2.7.3	Simulation Output and Discussion	199
2.8	Efficient Event-List Manipulation	200
Appendix 2A: FORTRAN Code for SIMLIB		202
Problems		215
References		232
Chapter 3	Simulation Software	234
3.1	Introduction	234
3.2	Comparison of Simulation Languages with General-Purpose Languages	235

3.3	Classification of Simulation Software	236
3.3.1	Simulation Languages vs. Simulators	236
3.3.2	Modeling Approaches	237
3.3.3	Common Modeling Elements	240
3.4	Desirable Software Features	240
3.4.1	General Features	240
3.4.2	Animation	241
3.4.3	Statistical Capabilities	242
3.4.4	Customer Support	243
3.4.5	Output Reports	243
3.5	GPSS	243
3.5.1	GPSS/H	244
3.5.2	Simulation of the $M/M/1$ Queue	244
3.5.3	GPSS/PC	248
3.6	SIMAN/Cinema	248
3.6.1	Simulation of the $M/M/1$ Queue	249
3.7	SIMSCRIPT II.5	252
3.7.1	Simulation of the $M/M/1$ Queue	254
3.8	SLAM II and Related Software	258
3.8.1	Simulation of the $M/M/1$ Queue	259
3.9	Comparison of Simulation Languages	263
3.10	Additional Simulation Software	265
	References	266
Chapter 4	Review of Basic Probability and Statistics	267
4.1	Introduction	267
4.2	Random Variables and Their Properties	268
4.3	Simulation Output Data and Stochastic Processes	279
4.4	Estimation of Means, Variances, and Correlations	282
4.5	Confidence Intervals and Hypothesis Tests for the Mean	286
4.6	The Strong Law of Large Numbers	292
4.7	The Danger of Replacing a Probability Distribution by Its Mean	292
	Appendix 4A: Comments on Covariance-Stationary Processes	293
	Problems	294
	References	297
Chapter 5	Building Valid and Credible Simulation Models	298
5.1	Introduction and Definitions	298
5.2	Some Principles of Valid Simulation Modeling	300
5.3	Verification of Simulation Computer Programs	302
5.4	General Perspectives on Validation	306

5.5	A Three-Step Approach for Developing Valid and Credible Simulation Models	307
5.5.1	Develop a Model with High Face Validity	308
5.5.2	Test the Assumptions of the Model Empirically	310
5.5.3	Determine How Representative the Simulation Output Data Are	311
5.6	Statistical Procedures for Comparing Real-World Observations and Simulation Output Data	314
5.6.1	Inspection Approach	315
5.6.2	Confidence-Interval Approach Based on Independent Data	319
5.6.3	Time-Series Approaches	321
	Problems	322
	References	323
Chapter 6	Selecting Input Probability Distributions	325
6.1	Introduction	325
6.2	Useful Probability Distributions	329
6.2.1	Parameterization of Continuous Distributions	329
6.2.2	Continuous Distributions	329
6.2.3	Discrete Distributions	343
6.2.4	Empirical Distributions	350
6.3	Techniques for Assessing Sample Independence	353
6.4	Activity I: Hypothesizing Families of Distributions	356
6.4.1	Summary Statistics	358
6.4.2	Histograms and Line Graphs	360
6.4.3	Quantile Summaries and Box Plots	363
6.5	Activity II: Estimation of Parameters	367
6.6	Activity III: Determining How Representative the Fitted Distributions Are	372
6.6.1	Heuristic Procedures	372
6.6.2	Goodness-of-Fit Tests	380
6.7	An Extended Example	394
6.8	Shifted and Truncated Distributions	400
6.9	Selecting a Distribution in the Absence of Data	403
6.10	Models of Arrival Processes	405
6.10.1	Poisson Processes	405
6.10.2	Nonstationary Poisson Processes	406
6.10.3	Batch Arrivals	409
6.11	Assessing the Homogeneity of Different Data Sets	409
	Appendix 6A: Tables of MLEs for the Gamma and Beta Distributions	411
	Problems	413
	References	417
Chapter 7	Random-Number Generators	420
7.1	Introduction	420

7.2	Linear Congruential Generators	424
7.2.1	Mixed Generators	427
7.2.2	Multiplicative Generators	428
7.3	Other Kinds of Generators	431
7.3.1	More General Congruences	432
7.3.2	Composite Generators	433
7.3.3	Tausworthe and Related Generators	434
7.4	Testing Random-Number Generators	436
7.4.1	Empirical Tests	436
7.4.2	Theoretical Tests	442
7.4.3	Some General Observations on Testing	447
7.5	Random-Number Generation on Microcomputers	447
7.6	Generators Used by Simulation Languages	448
Appendix 7A: Portable Computer Codes		449
7A.1	FORTRAN	449
7A.2	Pascal	451
7A.3	C	454
7A.4	Obtaining Initial Seeds for the Streams	456
Problems		457
References		459
Chapter 8	Generating Random Variates	462
8.1	Introduction	462
8.2	General Approaches to Generating Random Variates	465
8.2.1	Inverse Transform	465
8.2.2	Composition	474
8.2.3	Convolution	477
8.2.4	Acceptance-Rejection	478
8.2.5	Special Properties	484
8.3	Generating Continuous Random Variates	485
8.3.1	Uniform	485
8.3.2	Exponential	486
8.3.3	<i>m</i> -Erlang	486
8.3.4	Gamma	487
8.3.5	Weibull	490
8.3.6	Normal	490
8.3.7	Lognormal	492
8.3.8	Beta	492
8.3.9	Pearson Type V	493
8.3.10	Pearson Type VI	494
8.3.11	Triangular	494
8.3.12	Empirical Distributions	494
8.4	Generating Discrete Random Variates	496
8.4.1	Bernoulli	496
8.4.2	Discrete Uniform	497
8.4.3	Arbitrary Discrete Distribution	497

8.4.4	Binomial	502
8.4.5	Geometric	502
8.4.6	Negative Binomial	502
8.4.7	Poisson	503
8.5	Generating Correlated Random Variates	504
8.5.1	Using Conditional Distributions	504
8.5.2	Multivariate Normal and Multivariate Lognormal	505
8.5.3	Correlated Gamma Random Variates	506
8.6	Generating Arrival Processes	507
8.6.1	Poisson Processes	507
8.6.2	Nonstationary Poisson Processes	507
8.6.3	Batch Arrivals	510
Appendix 8A: Validity of the Acceptance-Rejection Method		512
Appendix 8B: Setup for the Alias Method		513
Problems		514
References		518
Chapter 9	Output Data Analysis for a Single System	522
9.1	Introduction	522
9.2	Transient and Steady-State Behavior of a Stochastic Process	525
9.3	Types of Simulations with Regard to Output Analysis	527
9.4	Statistical Analysis for Terminating Simulations	532
9.4.1	Estimating Means	532
9.4.2	Estimating Other Measures of Performance	540
9.4.3	Choosing Initial Conditions	543
9.5	Statistical Analysis for Steady-State Parameters	544
9.5.1	The Problem of the Initial Transient	545
9.5.2	Replication/Deletion Approach for Means	551
9.5.3	Other Approaches for Means	553
9.5.4	Estimating Other Measures of Performance	564
9.6	Statistical Analysis for Steady-State Cycle Parameters	565
9.7	Multiple Measures of Performance	568
9.8	Time Plots of Important Variables	572
Appendix 9A: Ratios of Expectations and Jackknife Estimators		572
Problems		575
References		579
Chapter 10	Comparing Alternative System Configurations	582
10.1	Introduction	582
10.2	Confidence Intervals for the Difference between Performance Measures of Two Systems	586
10.2.1	A Paired- <i>t</i> Confidence Interval	587

10.2.2	A Modified Two-Sample- <i>t</i> Confidence Interval	588
10.2.3	Contrasting the Two Methods	589
10.2.4	Comparisons Based on Steady-State Measures of Performance	590
10.3	Confidence Intervals for Comparing More Than Two Systems	591
10.3.1	Comparisons with a Standard	592
10.3.2	All Pairwise Comparisons	594
10.4	Ranking and Selection	595
10.4.1	Selecting the Best of <i>k</i> Systems	596
10.4.2	Selecting a Subset of Size <i>m</i> Containing the Best of <i>k</i> Systems	598
10.4.3	Selecting the <i>m</i> Best of <i>k</i> Systems	600
10.4.4	Additional Problems and Methods	601
Appendix 10A: Validity of the Selection Procedures		604
Appendix 10B: Constants for the Selection Procedures		606
Problems		607
References		609
Chapter 11 Variance-Reduction Techniques		612
11.1	Introduction	612
11.2	Common Random Numbers	613
11.2.1	Rationale	613
11.2.2	Applicability	615
11.2.3	Synchronization	617
11.2.4	Some Examples	620
11.3	Antithetic Variates	628
11.4	Control Variates	634
11.5	Indirect Estimation	641
11.6	Conditioning	644
	Problems	648
	References	652
Chapter 12 Experimental Design and Optimization		656
12.1	Introduction	656
12.2	2^k Factorial Designs	659
12.3	Coping with Many Factors	670
12.3.1	2^{k-p} Fractional Factorial Designs	670
12.3.2	Factor-Screening Strategies	677
12.4	Response Surfaces and Metamodels	679
12.5	Gradient Estimation	689
	Problems	691
	References	693
Chapter 13 Simulation of Manufacturing Systems		696
13.1	Introduction	696
13.2	Objectives of Simulation in Manufacturing	697

13.3	Simulation Software for Manufacturing Applications	699
13.4	Modeling System Randomness	703
13.4.1	Sources of Randomness	703
13.4.2	Machine Downtimes	705
13.5	An Extended Example	713
13.5.1	Problem Description and Simulation Results	713
13.5.2	Statistical Calculations	723
13.6	A Simulation Case Study of a Metal-Parts Manufacturing Facility	725
13.6.1	Description of the System	725
13.6.2	Overall Objectives and Issues to Be Investigated	726
13.6.3	Development of the Model	726
13.6.4	Model Verification and Validation	727
13.6.5	Results of the Simulation Experiments	729
13.6.6	Conclusions and Benefits	732
	Problems	733
	References	735
	Appendix	737
	INDEXES	741
	Author Index	743
	Subject Index	749