INSTITUTE OF PHYSICS PUBLISHING WAVES IN RANDOM MEDIA

Waves Random Media **14** (2004) R1–R40 PII: S0959-7174(04)78629-2

TOPICAL REVIEW

A critical survey of approximate scattering wave theories from random rough surfaces

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Received 28 March 2004, in final form 1 July 2004
Published 9 August 2004
Online at stacks.iop.org/WRM/14/R1
doi:10.1088/0959-7174/14/4/R01

Abstract

This review is intended to provide a critical and up-to-date survey of the analytical approximate methods that are encountered in scattering from random rough surfaces. The underlying principles of the different methods are evidenced and the functional form of the corresponding scattering amplitude or cross-section is given. The reader is referred to the original papers in order to obtain the explicit expressions of the coefficients and kernels. We have tried to identify the main strengths and weaknesses of the various theories. We provide synthetic tables of their respective performances, according to a dozen important requirements a valuable method should meet. Both scalar acoustic and vector electromagnetic theories are equally addressed.

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1. Introduction

Wave scattering by rough surfaces is an important issue in diverse areas of science such as measurements in medical, optics, acoustics, geophysics, communications and terrestrial or extraterrestrial remote sensing. Approximate models are still a necessity due to the insurmountable numerical complexity of realistic scattering problems. Even today's machines cannot cope with the enormous amount of computing demanded in the case of rigorous numerical calculations of the most general three-dimensional electromagnetic wave scattering from dielectric or conducting multi-scale surfaces.

This necessity has sprung the development of a myriad of approximate models with more or less careful derivations from first principles such as Maxwell equations. Even alerted users might find themselves lost in the meanders of more than twenty approximate models with no or little guidance on which must be applied under what conditions and for how long it holds until it breaks. The purpose of this review is to help the practitioner find his way among this plethora of models through a critical and, we hope, exhaustive, survey. We attempted to take a census of the existing methods and to classify them in generic families. We adopted a strategy of dissecting all identified scattering models in the literature by providing their functional form along with their general performances from an analytical point of view. Our readers will not be overwhelmed by pages of geometric and arithmetic functions describing the coefficients and kernels of each model. The reader is referred back to the original publications for explicit expression of the kernels. For completeness however, explicit expressions for the two fundamental approximations, first-order Kirchhoff and small perturbation method

(SPM1), are given in the appendix. It is worth mentioning at this point of our contribution that former general reviews can be found in the classical monographs [1–12].

Our review is only concerned with the analytical approximate scattering models. Some other complementary reviews can be found in the recent literature. Saillard and Sentenac [13] reviewed all rigorous numerical techniques for boundary integral methods and large linear systems when short-range interactions dominate. Warnick and Chew [14] listed several numerical approaches on scattering approximations, differential equations and surface integral equation methods. More specific reviews can be found in [15, 16] dealing mainly with two-scale models applied to large rough surfaces such as the ocean surface.

After introducing the notation and normalization adopted for this paper, we present the classical approximate models. Then we present recent scattering theories along with their interconnections before concluding with recapitulating tables.

2. Notation and definitions

A rough surface Σ separates the vacuum (upper medium) from a homogeneous medium (lower medium) with different index (optical or acoustical). We chose the right Cartesian coordinate $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$ system with z-axis directed upward and assume Σ is represented by a Cartesian equation $z = h(\mathbf{r}) = h(x, y)$, where h is the realization of a random stationary process. For an arbitrary vector $\mathbf{a}$, the notation 'a' will refer to its norm and $\hat{\mathbf{a}}$ to its direction. The three-dimensional position vector is $\boldsymbol{R} = (\boldsymbol{r}, z)$. A downward propagating electromagnetic plane wave with wave-vector $\boldsymbol{K}_0 = (\boldsymbol{k}_0, -q_0)$ and wavenumber $K = 2\pi/\lambda$ is incident on the surface and gives rise to up-going scattered wave-vectors in directions $\boldsymbol{K} = (\boldsymbol{k}, q_k)$. The vectors $\boldsymbol{k}_0$ and $\boldsymbol{k}$ are the horizontal components of the incident and scattered waves, respectively, and q_0, q_k are the vertical (positive) components. They are related by the relation $k^2 + q_k^2 = k_0^2 + q_0^2 = K^2$. The vector $\boldsymbol{Q} = \boldsymbol{K} - \boldsymbol{K}_0$ is the so-called momentum transfer [17] and plays an important role in scattering theory. We will denote by $\boldsymbol{Q}_H = \boldsymbol{k} - \boldsymbol{k}_0$ and $Q_z = q_k + q_0$ its horizontal and vertical components, respectively. Equivalently, we introduce $\boldsymbol{W} = \boldsymbol{K} + \boldsymbol{K}_0$ which is inspired from Dashen and Wurmser [18]. Analogously we define its horizontal $\boldsymbol{W}_H = \boldsymbol{k} + \boldsymbol{k}_0$ and vertical $W_z = q_k - q_0$ components.

The scattered field above and far away from the surface is related to the incident one through the scattering operator which reads in dyadic notation:

$$\boldsymbol{E}_s(\boldsymbol{R}) = \mathbb{S}(\boldsymbol{r}, z) \cdot \hat{\boldsymbol{E}}_0 = \int \frac{\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}+\mathrm{i}q_k z}}{q_k} \mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0)\, \mathrm{d}\boldsymbol{k} \cdot \hat{\boldsymbol{E}}_0, \tag{2.1}$$

or equivalently at $R \to \infty$,

$$\boldsymbol{E}_s(\boldsymbol{R}) = 2\pi \frac{\mathrm{e}^{\mathrm{i}KR}}{\mathrm{i}R} \mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0) \cdot \hat{\boldsymbol{E}}_0, \tag{2.2}$$

which is a direct consequence of the Weyl representation of the Green function. Note that all involved field quantities are scalar in the acoustic case. In the electromagnetic case, the scattered field is usually decomposed over a polarization basis (e.g. circular, linear horizontal and vertical, etc). Once expressed in the chosen polarization basis, the tensor $\mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0)$ becomes a 2×2 matrix called the scattering amplitude (SA, $\mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0)$). Our normalization of the scattering amplitude (SA) differs by a trivial geometrical factor from some other conventions. For instance, Voronovich [9] introduces the prefactor $\sqrt{q_0/q_k}$ instead of $1/q_k$ in the defining equation (2.1). The scattering amplitude in our definition and normalization takes the dimension of wavelength (meters).

The scattering amplitude satisfies some fundamental geometrical properties which can be exploited to check the *a priori* correctness of an approximate method. The *reciprocity* expresses the time reversal invariance of the wave in the harmonic regime:

$$\mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0) = \mathbb{S}^T(-\boldsymbol{k}_0, -\boldsymbol{k}), \tag{2.3}$$

where the superscript T stands for the transposed dyad (or matrix). The *shift invariance* refers to the phase-shifting (delays in the time domain) that results from horizontal and vertical translations of the surface:

$$\mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0)|_{h(r-d)+D} = \mathrm{e}^{-\mathrm{i}\boldsymbol{Q}_H \cdot \boldsymbol{d}}\, \mathrm{e}^{-\mathrm{i}Q_z D}\mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0)|_{h(r)}. \tag{2.4}$$

The *tilt invariance* expresses the fact that the scattering amplitude should not depend on the choice of the reference plane and the related coordinate system. Precisely,

$$\mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0)|_{\mathcal{R}(\Sigma)} = \mathbb{S}(\widetilde{\boldsymbol{k}}, \widetilde{\boldsymbol{k}}_0)|_{\Sigma}, \tag{2.5}$$

where $\mathcal{R}$ is any rotation of the surface and the tilted vectors are the horizontal components of the (inversely) rotated wavenumber vectors $\mathcal{R}^{-1}(\boldsymbol{K})$, $\mathcal{R}^{-1}(\boldsymbol{K}_0)$. We insist on the fact that this is tensorial, and not matrix, equality. To express the tilt invariance in terms of scattering matrix, one must intertwine rotation matrices from one local polarization basis to another (see, for instance, equations (21)–(23) of [19]).

The tilt invariance is a very stringent condition and in general an approximate method is only required to be *tilt invariant to first order*, namely:

$$\mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0)|_{h+\boldsymbol{a}\cdot\boldsymbol{r}} = \mathbb{S}(\widetilde{\boldsymbol{k}}, \widetilde{\boldsymbol{k}}_0)|_h + O(a^2), \tag{2.6}$$

where $\widetilde{\boldsymbol{k}} = \boldsymbol{k} + q_k\boldsymbol{a}$ and $\widetilde{\boldsymbol{k}}_0 = \boldsymbol{k} - q_0\boldsymbol{a}$. Given an approximate expression of the scattering amplitude, it is sometimes impossible to verify the last property analytically, and one often contents oneself with checking this in the limit of small roughness.

Some more quantities must be defined to better describe the measurables, such as the scattered power and the ensemble averaged moments in the case of random surfaces. The first-order moment is also called the coherent scattered amplitude and defined as

$$V(\boldsymbol{k}, \boldsymbol{k}_0) = \langle \mathcal{S}(\boldsymbol{k}, \boldsymbol{k}_0) \rangle. \tag{2.7}$$

The incoherent second order or the scattering cross-section of the rough surface is

$$\sigma = \langle |\mathcal{S}(\boldsymbol{k}, \boldsymbol{k}_0)|^2 \rangle - |\langle \mathcal{S}(\boldsymbol{k}, \boldsymbol{k}_0) \rangle|^2. \tag{2.8}$$

The norm and power in the previous equation must be applied to each element of the SA matrix. For distributed targets (infinitely large surfaces of area), a normalized radar cross-section must be used where the previous definition is normalized by the total surface area. This latter definition is conventionally called sigma naught (σ^0) and used in remote sensing of rough surfaces such as those of oceans and soils. For spatially homogeneous random rough surfaces, the shift invariance property imposes the coherent scattered amplitude to vanish everywhere but in the specular direction ($k = k_0$) and the term $V(\boldsymbol{k}, \boldsymbol{k}_0)$ actually contains a factor $\delta(\boldsymbol{k} - \boldsymbol{k}_0)$. For the same reasons, the incoherent cross-section also contains delta-like terms (see the complete discussion in [9]), which is the reason for rather using normalized radar cross-sections.

Very often, the scattering problem is simplified by assuming that the surface is invariant along one space direction. Such a surface will be termed one-dimensional (1D), as opposed to a two-dimensional (2D) surface which truly depends on two space variables. In the following, '1D' and '2D' cases will be referred to as 1D and 2D surfaces, respectively.

3. Small perturbation method

The *small perturbation method (SPM)* or Bragg theory is the oldest and perhaps most popular method in scattering from rough surfaces. It is a perturbative expansion of the scattering amplitude with respect to a small height parameter (in wavelength units). It was first introduced by Rayleigh [20] for sound waves on sinusoidally corrugated surfaces, then adapted by Fano [21] to optical gratings. Rice [22, 23] obtained explicit first- (SPM1) and second-order formulae (SPM2) for 1D conducting surfaces and computed the first-order in horizontal polarization for 1D dielectric rough surfaces. Later Peake [24] derived first-order backscattering cross-sections for both polarizations. Valenzuela completed the calculation at second order for 1D perturbations of levelled [25] and tilted [26] planes. The range of validity of SPM has been studied in detail via numerical simulations, essentially in the 1D acoustic case [27–30] and for Gaussian spectra.

3.1. Definition

The scattering amplitude for SPM is functionally written as a Taylor–Volterra expansion in surface height (h) as

$$\mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0) = \frac{1}{Q_z}\mathbb{B}(\boldsymbol{k}, \boldsymbol{k}_0)\delta(\boldsymbol{Q}_H) - \mathrm{i}\mathbb{B}(\boldsymbol{k}, \boldsymbol{k}_0)h(\boldsymbol{Q}_H) - Q_z \int \mathbb{B}_2(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{\xi})h(\boldsymbol{k} - \boldsymbol{\xi})h(\boldsymbol{\xi} - \boldsymbol{k}_0)\,\mathrm{d}\boldsymbol{\xi} + \cdots, \tag{3.9}$$

where $h(\boldsymbol{\xi})$ is the Fourier transform of the surface elevation. $\mathbb{B}(\boldsymbol{k}, \boldsymbol{k}_0)$ and $\mathbb{B}_2(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{\xi})$ are the first- and second-order coefficients, respectively. For explicit expressions and normalization of these coefficients the reader is referred to the monograph by Voronovich [9, 31] or to our appendices. Our normalization differs slightly from that of Voronovich in that one needs to multiply by $2q_k q_0$ the SPM1 coefficient $\mathbb{B}$ and by $-q_k q_0/Q_z$ the SPM2 coefficient $\mathbb{B}_2$. Please do also pay attention to the fact that the expressions of the kernels depend on the orientation of the vertical axis: some authors (in particular Voronovich) chose to illuminate the surface from below. Transposition to our convention requires formal replacements $q, q' \to -q, -q'$ and $h \to -h$. Both $\mathbb{B}$ and $\mathbb{B}_2$ possess the dimension of wavenumber squared K^2 in our normalization. The expression of the second-order kernel $\mathbb{B}_2$ is not unique, as any transformation of the type $\mathbb{B}_2(\boldsymbol{k}, \boldsymbol{k}_0, \boldsymbol{\xi}) \to \mathbb{B}_2(\boldsymbol{k}, \boldsymbol{k}_0, \xi) + \mathbb{J}(\boldsymbol{k}, \boldsymbol{k}_0, \boldsymbol{k} - \boldsymbol{\xi}) - \mathbb{J}(\boldsymbol{k}, \boldsymbol{k}_0, \boldsymbol{\xi} - \boldsymbol{k}_0)$, where $\mathbb{J}$ is an arbitrary tensor, leaves the second integral unchanged in equation (3.9). Different analytical expressions for this kernel can be obtained, depending on which technique is employed in the derivation, and this is sometimes misleading in the comparison of the different methods (see the discussion of subsection 3.5). We will refer to this non-uniqueness as *gauge arbitrariness*, a phenomenon which turns out to occur for other approximate methods. By *gauge transformation* we will refer in the following to a transformation of one or several integration kernels that leaves the scattering amplitude unchanged. Note that we do not attach any physical meaning to the gauge, and consider it only as a mathematical arbitrariness.

3.2. The historical method

Historically, the perturbative expansion to an arbitrary order is termed the *Rayleigh method* (also referred to as Rayleigh–Rice or Rayleigh–Fano procedure), which relies on the *Rayleigh hypothesis*. It is assumed that the representation of the electromagnetic field in terms of solely outgoing waves is still valid on the surface, whereas this holds *a priori* only above its maximum excursion. This makes it possible to rewrite the boundary condition for the field

at the surface in terms of scattering amplitude and simplifies considerably the identification of the perturbative orders. This method has been used independently by several authors to derive the second-order perturbative term, which is necessary to account for depolarization effects in the incidence plane [25], the occurrence of surfaces plasmons [32–35] and the shift of Brewster angles [36, 37] in *p*-polarization. More recently, Voronovich [31, 9] derived from this method a compact matrix formulation of SPM2 in the general vectorial dielectric case and Johnson *et al* pushed the calculation to third- [38] and fourth- [39] orders to explore the Brewster effect. The analytical formulae at successive orders become increasingly complicated and dissuasive beyond the second order, and attracted, as a matter of fact, little attention in the literature. An efficient numerical recursive scheme for the perturbed field based on the Rayleigh method was proposed by Bruno and Reitich [40–42]. This allows in principle a computation of the perturbation series at arbitrary order. With the additional use of Padé approximants it is possible to extend the perturbative domain and to gain in accuracy. For the moment the method was numerically tested on gratings only, with satisfactory results. A similar recursive scheme for the evaluation of the perturbation series at arbitrary order was employed by Demir and Johnson [39] with the difference that the recurrence concerns the integration kernels at successive orders and not the scattered field.

3.3. The reduced-Rayleigh equations

An interesting refinement of the Rayleigh method is the so-called method of *reduced-Rayleigh equations*, first proposed in [43]. The equations obtained via the classical Rayleigh method couple the unknown scattering amplitudes of the reflected and transmitted waves that propagate in the two homogeneous media surrounding the surface. Using simple but astute mathematical manipulations, the equations for the scattering amplitude in reflection can be uncoupled, leading to a closed integral equation expression for the latter. The derivation of the perturbative series becomes straightforward. Proceeding this way, Soubret *et al* [44] found a more compact formulation than Johnson *et al* [38] for the third-order scattering amplitude and tailored the technique to the case of a slab with rough boundary. Reduced-Rayleigh equations are also employed in another context to obtain an expansion in permittivity contrast rather than surface elevation [45].

3.4. Formal perturbation methods

The reduced-Rayleigh equations express the scattering amplitude as a solution of an integral equation which is formally similar to the Lippmann–Schwinger equation (i.e. a second-kind integral equation), well-known in potential scattering. This inspired a new formulation of the perturbative problem [46, 47] based on the mathematical formalism of quantum mechanics, which guarantees *a priori* the reciprocity and unitarity of the scattering operator. This approach allows the use of formal perturbation methods based on diagrammatic techniques [48]. This leads to an improved perturbative scheme of the coherent and incoherent scattering amplitudes [49], by relying on Dyson's and Bethe–Salpether equations. This statistical approach is sometimes referred to as *self-energy perturbation theory* [50] and has been intensively exploited to study the backscattering enhancement phenomenon [51, 52], which first appears at the fourth perturbative order in height for the backscattered intensity (and thereby requires the third-order scattering amplitude) or the shift of Brewster angle for 1D surfaces [53].

Another statistical approach based on formal perturbation methods is the so-called *smoothing method*, which was developed primarily in the context of continuous random media and discrete scatterers [54, 55], and which can be applied to any random quantity that satisfies

a second-kind integral equation. The smoothing method consists in decomposing the field into a mean and fluctuating part, and to find a second-kind integral equation for each component, starting from the equation satisfied by the deterministic field. It is formally equivalent to the diagrammatic method as it leads to the same Dyson equation for the mean field [48]. This closed equation implies a complicated kernel (the so-called mass operator), which is formally given by an infinite series, the first term of which is usually retained to find tractable solutions. The resulting approximation for the mean field is known as *first-order smoothing, bi-local* or *Bourret* approximation [56]. This is equivalent to performing the summation of an infinite subseries in the perturbation series of the averaged field. This perturbative approach possesses a larger domain of validity than the conventional one (i.e. the one based on iteration, truncation and averaging of the deterministic perturbation series). The main reason is that only infinite subseries are capable of handling properly secular terms in the perturbation series (see the excellent discussion by Frisch [48] and e.g. the numerical comparisons in [57]). DeSanto [58] investigated in detail the relations between the smoothing and the diagrammatic method for rough surface scattering. He demonstrated that the corresponding mass operators coincide up to second order in surface height, but differ at third order. The smoothing method has been applied to surface scattering by Wentzel [59], Watson and Keller [60, 61], DeSanto [58], Ishimaru [57], Berman and Dacol [62] and Brown [63, 64]. Most of these studies are concerned with the coherent field only. Watson and Keller were the first to work out the statistical cross-section with the smoothing method, in the acoustical case:

$$\sigma^0 = \frac{1}{|1 + R(\boldsymbol{k}_0; \Psi)|^2} |\mathcal{B}(\boldsymbol{k}, \boldsymbol{k}_0)|^2 \Psi(\boldsymbol{k} - \boldsymbol{k}_0), \tag{3.10}$$

where Ψ is the surface power spectrum, $\mathcal{B}$ the SPM1 kernel. The corrective term R is the Fresnel reflection coefficient corresponding to an effective rough surface impedance:

$$Z_s = \int \Psi(\boldsymbol{k} - \boldsymbol{\xi}) \eta(\xi) \, \mathrm{d}\xi, \tag{3.11}$$

for some kernel η. By comparison with the conventional perturbative method, this formula can be seen to bring higher-order corrections to the SPM1 cross-section, which is obtained by setting $R = 0$ in equation (3.10). This expression for the cross-section is, however, not reciprocal. Berman and Dacol combined their 'manifestly reciprocal scattering amplitude' formalism [62] with the smoothing method to derive a similar formula with reciprocal coefficients. The term *smoothing approximation* is also employed to denote the first-term approximation of the intensity operator in Bethe–Salpether equation, that is found by diagrammatic techniques. Ishimaru *et al* [57] recently revisited the result of Watson–Keller for the Dirichlet problem with this approximation and also found a similar formula with a modified, reciprocal, kernel:

$$\sigma^0 = \frac{1}{|1 + R(\boldsymbol{k}_0; \Psi)|^2 \, |1 + R(\boldsymbol{k}; \Psi)|^2} |\mathcal{B}(\boldsymbol{k}, \boldsymbol{k}_0)|^2 \Psi(\boldsymbol{k} - \boldsymbol{k}_0). \tag{3.12}$$

The same result was found using similar smoothing techniques in the Neumann case to correct for anomalous behaviour of SPM1 at grazing angles [65–68]. It can be observed that the kernel η appearing in the surface impedance coincides (up to some trivial factor) with the second-order Bragg kernel for the Dirichlet as well as Neumann case, for reasons that are not clear yet.

3.5. Rigorous derivations

For a long time, there has been a controversy as to the Rayleigh hypothesis leads to the correct perturbative expansion. There is, however, a rigorous way to derive this expansion, that does

not resort to any *a priori* assumption on the field. It is based on the so-called *extinction theorem* (also called *extended boundary condition* or Ewald–Oseen), which is a variant of Green's theorem. The method yields, in principle, the exact height expansion but is much more involved, as witnessed for example by the complexity of the calculations presented in [6]. It is used to reinvestigate the second-order expansion for 1D [69] and 2D conducting [70, 71] surfaces, as well as 1D [43] and 2D dielectric surfaces [72]. It is also employed to derive the intensity at fourth order in height [73, 74] in the scalar Dirichlet case (i.e. hard-pressure release acoustic problem) and for 1D conducting surfaces [29]. The overall conclusion [43, 70, 75, 29] of these studies is that Rayleigh–Fano procedure and the extinction theorem lead to identical expansions in accuracy, even though the higher-order formulae are sometimes difficult to compare as they do not enjoy unique expressions (see the above discussion on gauge arbitrariness). More recently, a general mathematical argument [76, 9] asserted that the Rayleigh hypothesis must yield the correct expansion at arbitrary order (at least for analytic profiles), even though its domain of applicability is very restrained.

Another original approach common to some authors [77–80] consists in expanding the field and its derivatives at the surface in perturbative orders. The resulting expansion is then injected into the boundary conditions, from which effective tangential fields [77] (resp. effective surface currents [78, 79]) are deduced. The radiated field in space is then found by using Kirchhoff–Helmholtz formula [77] (resp. dyadic Green's functions [78, 79]). This procedure can be used for perturbation of non-planar (for instance cylindrical) surfaces.

The main limitation of the SPM is its restricted domain of validity, as it is valid for small RMS height/wavelength ratios. In the limit of large wavelengths, however, this approximation tends to the true solution of the scattering problem. Thus, the SPM constitutes the reference for any approximate method in the low-frequency limit.

4. Kirchhoff approximation

Together with SPM, the *Kirchhoff approximation (KA)* is the oldest and most employed approximate method. It addresses a complete different scattering regime than SPM, since it is valid for large curvature radii or locally smooth surfaces [1]. The KA is also known as the *tangent plane approximation (TPA)* and the *physical optics (PO)* approximation in its high-frequency form. We will employ the first terminology, when necessary, to distinguish the regular (TPA) and high-frequency (PO) form of the Kirchhoff approximation.

4.1. Tangent plane approximation

We refer to the monographs by Ogilvy [8] or by Voronovich [9] for excellent historical reviews on the Kirchhoff approximation. In this model, the field on the surface is assimilated to the field that would be produced by a tangent plane at the same point. Thus, it depends only on the Fresnel reflection coefficient at the local incidence angle. KA is a local approximation, in that the supposed field at a point of a surface does not depend on the surface elsewhere, and thus does not account for multiple scattering. A second obvious limitation is that it does not consider curvature effects. Hence this approximation applies *a priori* to gently undulating surfaces and incidence angles at which shadowing and multiple scattering effects are negligible. The KA relies on the physical intuition of the tangent plane. It can, however, be obtained in a mathematical way. Meecham [81] was the first to recognize the Kirchhoff current as the zeroth iteration in the second-kind integral equation governing the surface current, under scattering of scalar acoustic waves from soft or hard surface or scattering of vector electromagnetic waves from perfectly conducting surfaces.

The KA was first introduced by Brekjovskikh [82, 83] and extended by Isakovich [84] to the statistical case. It was later treated in English in the acoustic [85] and electromagnetic [86] cases but the common reference for the western community is the book by Beckmann and Spizzichino [1], which treats the 1D dielectric and 2D acoustic problems. There have been several vector formulations of the TPA proceeding from the exact Stratton–Chu equations [87–92]. The TPA takes the following functional form:

$$\mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0) = \frac{1}{Q_z} \int \mathbb{K}(\boldsymbol{k}_0; \nabla h)\, \mathrm{e}^{-\mathrm{i}Q_z h(r)}\, \mathrm{e}^{-\mathrm{i}\boldsymbol{Q}_H \cdot \boldsymbol{r}}\, \mathrm{d}\boldsymbol{r}, \tag{4.13}$$

where the (obviously non-reciprocal) kernel is dependent on the local slope as expected. It has the dimension of wavenumber squared K^2 similar to the coefficients of SPM in (3.9).

Rodriguez [17, 93] proposed a different approach to the KA, namely the *momentum transfer expansion (MTE)*. He establishes a perturbative series for the surface current with respect to a novel small parameter Q_H/Q_z, which is the ratio of the horizontal and vertical components of the momentum transfer. KA is recovered at first order while the higher orders bring curvature corrections.

4.2. High-frequency regime

The complicated surface dependence on the surface current via the local Fresnel reflection coefficient makes statistical formulae for the cross-section complicated in addition to a deficiency in the fundamental reciprocity property. However, resorting to some reasonable additional assumptions, Stogryn [91] establishes a simplified statistical formula for the cross-section on Gaussian dielectric surfaces using Taylor expansions about zero of the correlation function and related quantities. In the 1D acoustic case, Dacol [94] also extracts the local dependence on the Fresnel coefficient via Taylor expansions about zero. Kodis [92] establishes simplified formula by using the stationary phase approximation (SPA) before averaging the intensity, while Barrick [95] shows that the result is independent of the order in which the averaging process and the stationary phase approximation are applied. The physical optics (PO) form of KA is also termed the *high-frequency Kirchhoff approximation (KA-HF)*, which simplifies to:

$$\mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0) = \mathbb{K}(\boldsymbol{k}, \boldsymbol{k}_0) \frac{1}{Q_z} \int \mathrm{e}^{-\mathrm{i}Q_z h(r)}\, \mathrm{e}^{-\mathrm{i}\boldsymbol{Q}_H \cdot \boldsymbol{r}}\, \mathrm{d}\boldsymbol{r}, \tag{4.14}$$

where the SA now is reciprocal. Hence, by taking the high-frequency limit of TPA both statistical simplicity and reciprocity property are repaired. In the literature, one might find sometimes a small nuance about PO and KA-HF when shadowing is included or not. We will not make any distinction between KA-HF and PO since shadowing effects are not presently studied and can be added later on. Under perfect conducting conditions, TPA and KA-HF coincide due to the linear dependence on the surface slope which can be integrated by parts without invoking the station phase limit. Hence, some authors linearize the dielectric TPA coefficients in slope instead of invoking the stationary phase limit. This procedure does not yield the correct PO approximation. Explicit expressions for KA-HF coefficients can be found in [9, 91, 96] or in our appendices along with a link to those of SPM1.

In the high-frequency regime (i.e. large wavenumber), the KA amplitude reduces to the probability density function of slopes evaluated at specular points, as was first noted by Barrick [97]. This is called the *specular point theory (SPT)*, alternatively the *geometrical optics (GO)* limit or the *ray-optics* approximation [85, 86, 1, 98, 3]. Its most common form for single reflections is:

$$\sigma^0 = \left| \frac{\mathcal{K}(\boldsymbol{k}, \boldsymbol{k}_0)}{Q_z^2} \right|^2 P\left(-\frac{\boldsymbol{k} - \boldsymbol{k}_0}{q_k + q_0} \right), \tag{4.15}$$

which is expressed in terms of the normalized radar cross-section where P is the probability density function of surface slopes. GO is mainly used for sea surface scattering [99, 95, 100–102] and when convoluted with SPM1 it leads to the well-known *two-scale model (TSM)* [103, 15, 16] (TSM = SPM1 * GO1).

4.3. Curvature corrections

Lynch [104] invokes the variational principle in the acoustical case with the KA as a trial function to improve the tangent-plane approximation. He obtained local curvature corrections to the Kirchhoff surface current. This method is subsequently referred to as *Lynch variational method (LVM)*, and refined and numerically tested in [105, 106]. Another way to take curvature effects into account is the so-called *local parabolic approximation (LPA)*, first introduced by Belobrov and Fuks [107, 108] and subsequently refined in [109, 110]. It is based on a quadratic, rather than linear, approximation of the surface height together with an appropriate representation of the free-space Green function. The SA in both the LVM and LPA enjoys the functional form of a TPA (4.13).

The radius of curvature and the scattering angles are the sole criteria for the validity of KA [82, 16, 3, 104] whenever non-local effects are negligible. For surfaces with Gaussian spectra the key parameter is the correlation length, which must remain much larger than the wavelength [111, 112, 28, 113]. Under multiple scattering conditions, surface shadowing must be included in the KA approximation in order to ensure energy conservation, among other things.

4.4. Non-local corrections

For large surface slopes and elevations the non-local effects can no longer be neglected. A possibility of including multiple-scattering phenomena is to consider further iterations of the surface current equations [81, 114, 115]. However, this results in n-fold integrals that are difficult to evaluate, whose convergence is questionable and which are not adapted to numerical computations especially in terms of ensemble averaged formulae.

Originally, Jin and Lax [116, 96] developed the iterative high-frequency KA even with shadowing effects to second order (KA2-HF). Their model has the general form

$$\begin{aligned}\mathbb{S}(\boldsymbol{k},\boldsymbol{k}_0) = \mathbb{K}_1(\boldsymbol{k},\boldsymbol{k}_0)\int \mathrm{e}^{-\mathrm{i}Q_z h(\boldsymbol{r})}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{Q}_H\cdot\boldsymbol{r}}\,\mathrm{d}\boldsymbol{r}\\ + \iiint \mathbb{K}_2^{+}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})\,\mathrm{e}^{-\mathrm{i}(\boldsymbol{k}-\boldsymbol{\xi})\cdot\boldsymbol{r}_1-\mathrm{i}(q_k+q_\xi)h(\boldsymbol{r}_1)}\,\mathrm{e}^{\mathrm{i}(\boldsymbol{k}_0-\boldsymbol{\xi})\cdot\boldsymbol{r}_2-\mathrm{i}(q_0-q_\xi)h(\boldsymbol{r}_2)}\,\mathrm{d}\boldsymbol{\xi}\,\mathrm{d}\boldsymbol{r}_1\,\mathrm{d}\boldsymbol{r}_2\\ + \iiint \mathbb{K}_2^{-}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})\,\mathrm{e}^{-\mathrm{i}(\boldsymbol{k}-\boldsymbol{\xi})\cdot\boldsymbol{r}_1-\mathrm{i}(q_k-q_\xi)h(\boldsymbol{r}_1)}\,\mathrm{e}^{\mathrm{i}(\boldsymbol{k}_0-\boldsymbol{\xi})\cdot\boldsymbol{r}_2-\mathrm{i}(q_0+q_\xi)h(\boldsymbol{r}_2)}\,\mathrm{d}\boldsymbol{\xi}\,\mathrm{d}\boldsymbol{r}_1\,\mathrm{d}\boldsymbol{r}_2,\end{aligned} \tag{4.16}$$

where the second-order kernel is a combination of the product of two first-order Kirchhoff approximations updated with some shadowing functions. Statistically averaged shadowing corrections, suggested by ray-optics, were introduced by Bass and Fuks [77, 117] and later rediscovered by Wagner [118], Smith [119] and Beckmann [120]. They have been widely used in conjunction with first- [121–127] and second-order KA to elaborate a multiple scattering theory valid for large elevation RMS [116, 96, 128–132]. Second [133] and multiple [134] reflections have also been considered in this framework of Ray-Optics to explain qualitatively phenomena such as the mechanism of backscattering enhancement.

Analytical attempts to include non-local corrections to the KA have been limited to the second-order KA, based on one iteration of the surface current [129, 10, 135–137]. General

non-local models will be detailed in later sections but one could mention that the high-frequency limit of these models is expected to reproduce the second-order geometrical optics (GO2) defined as a correction to the GO1 cross-section by:

$$\sum_{s=\pm 1} \int \mathcal{K}_2^s(\boldsymbol{k}, \boldsymbol{k}_0, \boldsymbol{\xi})\, P\left(-\frac{\boldsymbol{k}-\boldsymbol{\xi}}{q_k + s q_\xi}, -\frac{\boldsymbol{\xi}-\boldsymbol{k}_0}{q_\xi - s q_0}\right) \mathrm{d}\boldsymbol{\xi}, \tag{4.17}$$

where shadowing could be included as an extra weighting function along side the non-local kernel.

For non-local models (4.16), (4.17) based on a second-order KA, there is an ambiguity on the integration domain of the spectral variable. A cut-off over evanescent waves (i.e. $|\xi| > K$) is implicit, even though the spectral integral stems from the Weyl expansion of the Green function and should run over an infinite domain. The reason for doing this is that the occurrence of both signs $\pm \mathrm{i} q_\xi h$ in the integrand phase would lead to exponentially increasing terms for evanescent waves (for which q_ξ is positive imaginary) and cause the integral to diverge. This *ad hoc* assumption seems to compensate for another error introduced in removing the cumbersome absolute values $|h(\boldsymbol{r}_1) - h(\boldsymbol{r}_2)|$ originally present in (4.16).

An important drawback of KA is that it is not consistent with SPM1 in the limit of small roughness. This is the main reason why a variety of so-called unifying methods have been proposed in order to satisfy both the low- and high-frequency limits.

5. Unifying theories

5.1. Meecham–Lysanov method

The *Meecham–Lysanov method (MLM)* is the oldest unifying theory and dates back to the fifties. It was suggested independently by Meecham [138] and Lysanov [139], for the Dirichlet case. The unknown surface current at the surface is sought via its first-kind (i.e. single-layer potential) integral equation. The Weyl representation is used for the corresponding Green function, together with a key 'small-slope' assumption: the distance between two points on the surface is approximated by the horizontal lag. This turns the surface integral into a convolution that can be Fourier inverted and provides an approximate solution for the surface current. The resulting expression for the far-field scattering amplitude is a triple integral (two space and one frequency variables) with a simple kernel:

$$\mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0) = \iiint \phi(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{\xi})\, \mathrm{e}^{-\mathrm{i}(\boldsymbol{k}-\boldsymbol{\xi})\cdot\boldsymbol{r}_1 - \mathrm{i} q_k h(\boldsymbol{r}_1)} \mathrm{e}^{\mathrm{i}(\boldsymbol{k}_0-\boldsymbol{\xi})\cdot\boldsymbol{r}_2 - \mathrm{i} q_0 h(\boldsymbol{r}_2)}\, \mathrm{d}\boldsymbol{\xi}\, \mathrm{d}\boldsymbol{r}_1\, \mathrm{d}\boldsymbol{r}_2. \tag{5.18}$$

The KA and SPM limits are recovered [9] and non-local effects are taken into account, owing to the two-fold space integration. Yarovoy [140] generalized the method to the 2D pseudo-dielectric case (polarization effects are ignored since the field is assumed scalar), using also an integral equation of the first kind for the surface fields. However, the inversion procedure that is used in the Meecham–Lysanov method to estimate the surface current seems not adaptable to the vectorial case, since the corresponding integral operator is no longer invertible (essentially because it implies a projection on the tangential direction). As a matter of fact, we are not aware of any vectorial formulation of the method. Recently, however, a purely numerical approximate method inspired from Meecham–Lysanov was proposed, the so-called *small slope integral equation (SSIE)* [141]. Starting from the magnetic field integral equation, it makes the same approximation on the Green function as Meecham–Lysanov. This transforms the integral operator implying the unknown surface field into a convolution operator, which can be implemented efficiently via FFT. This numerical method reaches in principle the same accuracy as a Meecham–Lysanov procedure and is much faster than

a rigorous computation. The lack of analytical expression, however, prevents the use of statistical formulae.

5.2. Phase-perturbation method

The *phase-perturbation method (PPM)* originates from an idea by Shen and Maradudin [142], but was developed by Winebrenner [143, 144] for the acoustic case. It relies on an Ansatz first proposed by Lynch [104] and Rytov *et al* [7]: the surface current on the rough surface is written as a multiplicative (unknown) phase correction to that of a reference plane. It is then this phase, rather than the field itself, that is sought in the form of a perturbative series. The corresponding expansion is found by imposing consistency with SPM at all orders in the small roughness limit. The PPM functional form is

$$\mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0) = \mathbb{B}(\boldsymbol{k}, \boldsymbol{k}_0)\frac{1}{Q_z}\int \mathrm{e}^{-\mathrm{i}Q_z\Phi(\boldsymbol{k},\boldsymbol{k}_0;[h])}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{Q}_H\cdot\boldsymbol{r}}\mathrm{e}^{-\mathrm{i}Q_z h(\boldsymbol{r})}\,\mathrm{d}\boldsymbol{r}, \tag{5.19}$$

where the functional Φ is to first order in surface height

$$\Phi(\boldsymbol{k}, \boldsymbol{k}_0; [h]) = \int \beta(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{\xi})h(\boldsymbol{\xi})\mathrm{e}^{\mathrm{i}\boldsymbol{\xi}\cdot\boldsymbol{r}}\,\mathrm{d}\boldsymbol{\xi} + \cdots. \tag{5.20}$$

This method has been tested, discussed and compared with other approximations by several authors [145–151, 9], essentially in the 1D Dirichlet case. To first order, this method was shown [145] to recover KA and SPM in the appropriate limits, and to possess a wider validity domain than the standard approximations. It is also appropriate to establish statistical formulae via the use of cumulants [143]. Contrary to both SPM and KA, PPM fails to satisfy reciprocity. A reciprocal variant of PPM [152] sharing the same properties was, however, later proposed in the statistical case.

5.3. Small-slope approximation

The *small-slope approximation (SSA)* was proposed by Voronovich [153, 31, 9, 154] as a unifying theory that could reconcile SPM and KA. It starts from a structure ansatz for the scattering amplitude, that makes it formally similar to a KA integral with an unknown multiplicative correction of the integrand. This guarantees the geometrical properties that a scattering amplitude should satisfy *a priori*. By construction, the SSA respects the proper shifting that follows from horizontal and vertical translations of the surface (in that it is already superior to SPM), as well as reciprocity. The unknown correction to a reference plane is sought in a functional Taylor expansion of roughness. This is done by requiring consistency with SPM at all orders in the limit of small roughness, together with some intuitive (as it turned out later [155, 156]) choice of gauge to remove certain arbitrariness in the identification of coefficients. In practice, the expansion is performed at the lowest two orders only, for higher terms become too involved. The functional form of SSA at second order (SSA2) is

$$\mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0) = \frac{1}{Q_z}\int \mathrm{e}^{-\mathrm{i}Q_z h(\boldsymbol{r})}\mathrm{e}^{-\mathrm{i}\boldsymbol{Q}_H\cdot\boldsymbol{r}}\left(\mathbb{B}(\boldsymbol{k}, \boldsymbol{k}_0) - \mathrm{i}Q_z\int \mathbb{M}(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{\xi})h(\boldsymbol{\xi})\mathrm{e}^{\mathrm{i}\boldsymbol{\xi}\cdot\boldsymbol{r}}\mathrm{d}\boldsymbol{\xi}\right)\mathrm{d}\boldsymbol{r}, \tag{5.21}$$

where

$$\mathbb{M}(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{\xi}) = \tfrac{1}{2}(\mathbb{B}_2(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{k} - \boldsymbol{\xi}) + \mathbb{B}_2(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{k}_0 + \boldsymbol{\xi}) - \mathbb{B}(\boldsymbol{k}, \boldsymbol{k}_0)). \tag{5.22}$$

The first integral taken alone in (5.21) is the first-order SSA (SSA1). Here $\mathbb{B}$ and $\mathbb{B}_2$ are the first- and second-order kernels of SPM, respectively.

Contrary to most perturbative approaches, the expansion is not performed with respect to a well-determined small parameter, but is realized as a functional series of surface roughness.

Dimensional arguments have led Voronovich to identify the successive iterations as orders in roughness slopes (whence the terminology). However, further discussions and numerical tests have clearly demonstrated that the expansion cannot be recast in powers of a single parameter [157–159]. SSA at first- (SSA1) and second- (SSA2) order has been abundantly discussed and tested against rigorous methods, essentially in the 1D case [30, 160–162]. Its qualitative predictions for scattering on anisotropic sea spectra have been studied [163, 38, 164], but only recently was it compared with 2D rigorous numerical [158] and experimental [165, 166] results. It turns out from these experiments that SSA1 considerably extends the validity domain of SPM, but remains outperformed by KA for large roughness in the domain where this last method is known to be accurate. SSA2, however, seems to be remarkably accurate and possesses some desirable properties than SSA1 does not display: it reduces exactly (in the conducting 2D case) or approximatively (in the dielectric 2D case) to KA in the high-frequency limit [167], accounts properly for the large-scale tilting effects [19] and can be incorporated advantageously in a two-scale model [168]. The main limitation of SSA2 is its numerical complexity. It involves a double integral (one space and one frequency variable) with an oscillating complex-valued kernel that exhibits branch cuts and singularities. For this reason SSA1, which is given by a single integral as KA-HF, remains mostly employed, and 2D numerical implementation of SSA2 was achieved only recently [169, 166] in the statistical case.

SSA2 can be transformed to produce a manifestly reciprocal version of PPM, including the dielectric vectorial case. This can be done easily by noting that the functional (5.21) is obtained from the PPM functional (5.19) by mere linearization of the exponential. This was first suggested by Berman and Dacol [62] in the 1D Dirichlet case. The same observation can be used to produce a statistically tractable version of SSA2, by making the reverse transformation (that is turning SSA2 to a PPM functional form), as was done for instance in [165].

Several authors have tried rigorous mathematical approaches to SSA. Tatarskii [155, 170] developed the *quasi-slope expansion (QSE)* and McDaniel [171] the *extended small-slope approximation (ESSA)* in the acoustic case. They proceed from the extinction theorem or the Meecham–Lysanov method, respectively. Then, they seek the surface single potential in a functional Taylor series of height and find recursively the coefficients. They resort to a small-slope condition at some point to render the formulae tractable (actually at lowest-two orders). Some *a priori* assumptions, such as reciprocity at every order, and gauge arbitrariness are relaxed, but the obtained approximations are close variants to SSA. ESSA does not reduce to SPM2 due to the presence of an asymmetric kernel in addition to that of SSA2. Another attempt to derive SSA from first principles was made by Elfouhaily *et al* [157, 172] in the 2D conducting case. Inspired by an earlier work of Holliday [173] in the backscattering case, they proceeded through iterations of the surface current equation together with refined small-slope approximations on the phase of the integrand. They obtained the scattering field in a Kirchhoff functional form, which was only later [159] recognized to coincide with SSA1.

Even though SSA2 requires a double integration, it does not take spatial non-local effects into account (this can be seen in the high-frequency limit where it reduces to the mere first order KA or GO1). To remedy this limitation, an improvement of SSA was proposed by Voronovich himself [174], namely the *non-local small-slope approximation, NLSSA*. Its ansatz is identical to that of Meecham–Lysanov method (MLM). This method is in principle superior to SSA, but is difficult to use in practice. It implies a triple oscillating integral which is unsuited to numerical applications. The only numerical simulations we are aware of have been performed in the 1D Dirichlet case for specific spectra [175–178]. More recently, Elfouhaily *et al* [179, 180] also adopted the MLM and NLSSA ansatz for the *non-local curvature approximation (NLCA)*.

5.4. Operator expansion method

The *operator expansion method (OEM)* originated from hydrodynamics [181–183] and was first adapted to rough surface scattering by Milder for the 1D acoustic problem [184]. It was subsequently extended to the 2D acoustic [185] case and to the conducting [186] and dielectric vectorial problems [187]. The principle of OEM is to express the scattered field as a Green–Helmholtz surface integral involving the so-called Dirichlet-to-Neumann Operator (DtN). The DtN relates the value of the field at the surface to the value of its normal derivative, one of which (Dirichlet or Neumann problem) or a combination of which (dielectric case), is prescribed by the boundary condition. The DtN is explicit for plane waves and can be sought in a functional series of height, whose successive orders are determined recursively. It is only the DtN, and not the field itself, that is expressed perturbatively and thus it is only a partial height expansion. This makes the validity domain of OEM much wider than SPM. In the 1D acoustic case [184], OEM was shown to reproduce analytically the SPM and KA expressions in the appropriate limits. The functional form of OEM is

$$\mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0) = \int \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}-\mathrm{i}q_k h(\boldsymbol{r})}\mathbb{N}(\boldsymbol{k}, \boldsymbol{k}_0; [h])\,\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}-\mathrm{i}q_0 h(\boldsymbol{r})}\,\mathrm{d}\boldsymbol{r}, \tag{5.23}$$

where the kernel operator $\mathbb{N}(\boldsymbol{k}, \boldsymbol{k}_0; [h])$ is formally expanded in powers of surface elevation h. In the acoustical case, the OEM kernel $\mathbb{N}$ does not depend on the scattering geometry and thus the scattering amplitude is manifestly reciprocal. This is not the case under general dielectric conditions.

The OEM iterations can be formally written as a succession of Fourier multiplicators, which can be efficiently implemented by FFT. At the first level (actually zeroth order for the DtN), the scattering amplitude is already a non-local triple integral coinciding with the Meecham–Lysanov method (at least in the Dirichlet case). The OEM is very well adapted to the scalar problem but the vectorial formulation remains obscure and has been in fact of little use. Numerical experiments have been performed for the 1D [184, 188] and 2D [185] Dirichlet cases and for the vectorial case [186, 187]. For 1D surfaces with Gaussian and Pierson–Moskowitz spectra [188], the OEM at first order appears superior to both KA and SPM, and to extend their added domains of validity. For sinusoidal gratings [184, 186, 187] it was also shown to remain accurate for RMS heights that go well beyond the perturbative domain and in any case superior to SSA1. Statistical formulae are given in the 1D acoustic case and for the coherent field only [189].

5.5. Tilt-invariant approximation

An important property that an approximate method should satisfy to cope with large scales is the so-called tilt invariance, as was recalled in the first section. It is the ability to treat correctly perturbations of a slightly tilted plane. While this is not satisfied by SSA1 and was only checked recently for SSA2 [19], Charnotskii and Tatarskii [190] constructed a method that would primarily account for the tilting effect, namely the *tilt-invariant approximation (TIA)*, which they presented for the Dirichlet problem only. They chose to write the surface current as a multiplicative unknown correction to the Kirchhoff current, which is exact for a tilted plane. The unknown function is sought, as usual, in a perturbative expansion that would be consistent with SPM. The lowest order of the expansion leads back to KA, while SPM1 is

recovered with the first two terms:

$$\mathbb{S}(\boldsymbol{k},\boldsymbol{k}_0)=\frac{\mathbb{K}(\boldsymbol{k},\boldsymbol{k}_0)}{Q_z}\int \mathrm{e}^{-\mathrm{i}Q_z h(\boldsymbol{r})}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{Q}_H\cdot\boldsymbol{r}}\,\mathrm{d}\boldsymbol{r}-\mathrm{i}\iint \mathbb{T}_1(\boldsymbol{k}_0;\boldsymbol{\xi})h(\boldsymbol{\xi})\,\mathrm{e}^{-\mathrm{i}Q_z h(\boldsymbol{r})}\,\mathrm{e}^{-\mathrm{i}(\boldsymbol{Q}_H-\boldsymbol{\xi})\cdot\boldsymbol{r}}\,\mathrm{d}\boldsymbol{\xi}\,\mathrm{d}\boldsymbol{r}$$
$$+\iiint \mathbb{T}_2(\boldsymbol{k}_0;\boldsymbol{\xi}_1,\boldsymbol{\xi}_2)h(\boldsymbol{\xi}_1)h(\boldsymbol{\xi}_2)\,\mathrm{e}^{-\mathrm{i}Q_z h(\boldsymbol{r})}\,\mathrm{e}^{-\mathrm{i}(\boldsymbol{Q}_H-\boldsymbol{\xi}_1-\boldsymbol{\xi}_2)\cdot\boldsymbol{r}}\,\mathrm{d}\boldsymbol{\xi}_1\,\mathrm{d}\boldsymbol{\xi}_2\,\mathrm{d}\boldsymbol{r}. \quad (5.24)$$

TIA is however not reciprocal in its correction to KA. Note that the first two integrals possess the functional form of SSA2. Elfouhaily *et al* [159] actually demonstrated that one could operate a simple gauge function change on SSA2 which transforms it into TIA decomposition. The functional form of TIA motivated the finding of a new general model for dielectric surfaces which is termed the *local curvature approximation LCA* [191, 179]. LCA can be seen as a generalization of TIA to the dielectric case. The issue of gauge arbitrariness and possible connection between models will be discussed in more detail in a later section.

5.6. Local weight approximation

In a series of papers [18, 192, 193], Dashen and Wurmser developed a novel formalism for scattering from rough surfaces, which is intermediate between variational and perturbative methods. They investigated the change of scattering amplitude resulting from a infinitesimal variation of the rough surface itself (and not of a reference plane) and deduced some interesting properties for the latter. In the 2D acoustic and conducting case, they showed that, to first order in curvature, the scattering amplitude can be written as a TPA integral with a local kernel, which is the solution of a differential equation that is solved explicitly. They consequently obtained an expression of the same complexity as the TPA, that takes into account curvature effects. LWA functional form is

$$\mathbb{S}(\boldsymbol{k},\boldsymbol{k}_0)=\frac{1}{Q_z}\int \mathbb{G}(\boldsymbol{k},\boldsymbol{k}_0;-Q_z\boldsymbol{\nabla}h)\,\mathrm{e}^{-\mathrm{i}Q_z h(\boldsymbol{r})}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{Q}_H\cdot\boldsymbol{r}}\,\mathrm{d}\boldsymbol{r}. \quad (5.25)$$

In addition, this approximation recovers the SPM and KA limits, is reciprocal and tilt-invariant. The authors did not give a name to their new method, but it was referred to by Milder [184] and subsequently others as *local weight approximation (LWA)*. The main limitation of LWA is its inapplicability to the dielectric case, since the differential equation for the local kernel becomes inextricable (due to the complex slope dependence of the Fresnel coefficients). Albeit powerful, this approach remained relatively unknown, perhaps because it has been developed by theoretical physicists outside the radar community.

5.7. Weighted curvature approximation

Recently, Elfouhaily *et al* [191, 179] derived a model inspired from Dashen and Wurmser ansatz, which they named the *weighted curvature approximation (WCA)*. They arrived at a scattering amplitude which is a correction to SSA1 that possesses the structure of LWA, namely a Kirchhoff integral with a local unknown kernel, and construct such a kernel with all desirable properties: reciprocity, compatibility with SPM1 and KA limits and shift- and tilt invariance. This kernel includes curvature effects since it is quadratic in its lowest order when expanded about zero. It is based on a combination of SPM1 and KA kernels, evaluated at local angles. WCA is written as

$$\mathbb{S}(\boldsymbol{k},\boldsymbol{k}_0)=\frac{1}{Q_z}\int \{\mathbb{B}(\boldsymbol{k},\boldsymbol{k}_0)-\mathbb{T}(\boldsymbol{k},\boldsymbol{k}_0;-Q_z\boldsymbol{\nabla}h)\}\,\mathrm{e}^{-\mathrm{i}Q_z h(\boldsymbol{r})}\mathrm{e}^{-\mathrm{i}\boldsymbol{Q}_H\cdot\boldsymbol{r}}\,\mathrm{d}\boldsymbol{r}, \quad (5.26)$$

where

$$\mathbb{T}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi})=\mathbb{B}\left(\frac{\boldsymbol{k}+\boldsymbol{k}_0+\boldsymbol{\xi}}{2},\frac{\boldsymbol{k}+\boldsymbol{k}_0-\boldsymbol{\xi}}{2}\right)-\mathbb{K}\left(\frac{\boldsymbol{k}+\boldsymbol{k}_0+\boldsymbol{\xi}}{2},\frac{\boldsymbol{k}+\boldsymbol{k}_0-\boldsymbol{\xi}}{2}\right). \quad (5.27)$$

Contrary to LWA, WCA enjoys a dielectric statistical formulation. The method was tested on surfaces with Gaussian spectrum [194] and was shown to improve both KA and SSA1 in some range of moderate roughness and in co-polarization. It is unable, however, to predict a correct cross-polarization in the incidence plane. This is not surprising since the WCA is a local model.

5.8. Wiener–Hermite approach

Nakayama (acoustic case) and Eftimiu (electromagnetic case) have employed a probabilistic approach that we will refer to as *Wiener–Hermite approach (WHA)* to the scattering problem. They consider the scattered field as a nonlinear stochastic functional of the random surface, that can be expanded on a basis of orthogonal processes. This is done via a functional Wiener–Hermite expansion. The unknown deterministic coefficients are then found either by imposing consistency with SPM [195–200] or via the surface current equation [201–203]. Eftimiu later refined this approach with an ansatz similar to TIA: the surface current is written as an amplitude correction to the Kirchhoff current and sought in a functional Wiener–Hermite expansion. This last method is termed *phased Wiener–Hermite approach* by his author [204, 205]. The SPM1 limit is recovered at first order, but not the KA limit. The diffuse scattering cross-section of WHA can be written as follows:

$$\sigma^0(\boldsymbol{k}, \boldsymbol{k}_0) = \mathrm{e}^{-Q_z^2\rho(0)} \left\{ \delta_{\boldsymbol{Q}_H} \mathcal{F}_0(\boldsymbol{k}, \boldsymbol{k}_0) + \int \mathcal{F}_1(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{\xi}) \Psi(\boldsymbol{\xi})\, \mathrm{d}\boldsymbol{\xi} + \iint \mathcal{F}_2(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{\xi}_1, \boldsymbol{\xi}_2) \Psi(\boldsymbol{\xi}_1) \Psi(\boldsymbol{\xi}_2) \mathrm{d}\boldsymbol{\xi}_1 \mathrm{d}\boldsymbol{\xi}_2 \right\}, \tag{5.28}$$

where $\rho(\boldsymbol{r})$ and $\Psi(\boldsymbol{\xi})$ are the surface autocorrelation function and the surface power spectrum, respectively. WHA is structurally equivalent to SPM in its statistical formulation.

WHA is in principle well adapted to the statistical case as it is readily available in ensemble averaged form, but in practice the analytical derivation of the Wiener–Hermite functional expansion and the resulting expressions for the surface currents become inextricably complicated beyond the first order.

5.9. Unified perturbation expansion

In the *unified perturbation expansion (UPE)*, Rodriquez *et al* [206–208] pursued the former idea of the momentum transfer expansion (MTE) but change the small parameter from $\boldsymbol{Q}_H / Q_z$ to $W_z h(\boldsymbol{r})$. SPM and KA are recovered with the lowest two orders of the corresponding perturbative expansion, and SPM2 with the third order. UPE in our notation becomes

$$\mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0) = \frac{\mathbb{K}(\boldsymbol{k}, \boldsymbol{k}_0)}{Q_z} \delta_{\boldsymbol{Q}_H} - \mathrm{i} \iint \mathbb{F}_1(\boldsymbol{k}_0; \boldsymbol{\xi}) h(\boldsymbol{\xi})\, \mathrm{e}^{-\mathrm{i}Q_z h(\boldsymbol{r})}\, \mathrm{e}^{-\mathrm{i}(\boldsymbol{Q}_H - \boldsymbol{\xi})\cdot\boldsymbol{r}}\, \mathrm{d}\boldsymbol{\xi}\, \mathrm{d}\boldsymbol{r} + \iiint \mathbb{F}_2(\boldsymbol{k}_0; \boldsymbol{\xi}_1, \boldsymbol{\xi}_2) h(\boldsymbol{\xi}_1) h(\boldsymbol{\xi}_2)\, \mathrm{e}^{-\mathrm{i}Q_z h(\boldsymbol{r})}\, \mathrm{e}^{-\mathrm{i}(\boldsymbol{Q}_H - \boldsymbol{\xi}_1 - \boldsymbol{\xi}_2)\cdot\boldsymbol{r}}\, \mathrm{d}\boldsymbol{\xi}_1\, \mathrm{d}\boldsymbol{\xi}_2\, \mathrm{d}\boldsymbol{r}. \tag{5.29}$$

This model is not reciprocal by construction since the expanded current is a function of the incident wave alone. Another limitation of UPE is that it is available for perfect conducting surfaces only.

5.10. Full-wave approach

The *full-wave approach (FWA)* has been introduced and developed over many years by Bahar in the 1D case [209–214], then modified to include local slope dependence and extended

to the 2D case [215–218]. The surface is seen as the lower boundary of a semi-infinite waveguide and the scattered field is decomposed over a basis of local modes, with unknown coefficients. These local spectral components are shown to satisfy a differential equation of propagation along the lateral direction, the so-called telegraphist equation for cable and waveguide transmissions. Under further approximations such as a small-slope assumption, the equation can be solved iteratively by the method of subsequent approximations. In the 1D Dirichlet case, the first iteration was recognized by Voronovich [9] to coincide with SSA1. As early as 1980, Bahar [219] showed a version of his model fully compatible with SSA1 as we know it today.

Ever since FWA was introduced by Bahar in early seventies, it has been continuously evolving at a rate of more than one paper a year on average, and it is therefore difficult to accomplish a thorough review of this model in a short paragraph. The functional form of FWA can be written as:

$$\begin{aligned}\mathbb{S}(\boldsymbol{k},\boldsymbol{k}_0) = \int & D_1(\boldsymbol{k},\boldsymbol{k}_0;-Q_z\nabla h)[\mathrm{e}^{-\mathrm{i}Q_z h(\boldsymbol{r})}-1]\,\mathrm{e}^{-\mathrm{i}\boldsymbol{Q}_H\cdot\boldsymbol{r}}\,\mathrm{d}\boldsymbol{r}\\ &+\iiint \mathbb{D}_2^{+}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi},[\nabla h])\,\mathrm{e}^{-\mathrm{i}(\boldsymbol{k}-\boldsymbol{\xi})\cdot\boldsymbol{r}_1-\mathrm{i}(q_k+q_\xi)h(\boldsymbol{r}_1)}\\ &\times\mathrm{e}^{\mathrm{i}(\boldsymbol{k}_0-\boldsymbol{\xi})\cdot\boldsymbol{r}_2-\mathrm{i}(q_0-q_\xi)h(\boldsymbol{r}_2)}\,\mathrm{d}\boldsymbol{\xi}\,\mathrm{d}\boldsymbol{r}_1\,\mathrm{d}\boldsymbol{r}_2\\ &+\iiint \mathbb{D}_2^{-}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi},[\nabla h])\,\mathrm{e}^{-\mathrm{i}(\boldsymbol{k}-\boldsymbol{\xi})\cdot\boldsymbol{r}_1-\mathrm{i}(q_k-q_\xi)h(\boldsymbol{r}_1)}\\ &\times\mathrm{e}^{\mathrm{i}(\boldsymbol{k}_0-\boldsymbol{\xi})\cdot\boldsymbol{r}_2-\mathrm{i}(q_0+q_\xi)h(\boldsymbol{r}_2)}\,\mathrm{d}\boldsymbol{\xi}\,\mathrm{d}\boldsymbol{r}_1\,\mathrm{d}\boldsymbol{r}_2,\end{aligned}\tag{5.30}$$

which encompasses all published, local and nonlocal, forms of FWA over the years. For the local form we selected equation (2.12) in a recent revision of FWA [220]. The nonlocal form comes from equation (1) in [221] with some adaptation to our notation.

The first integral in (5.30) is similar to SSA1 but with the kernel expressed in a local frame of reference. This kernel enjoys the properties

$$\mathbb{D}_1(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{k}-\boldsymbol{k}_0)=\mathbb{K}(\boldsymbol{k},\boldsymbol{k}_0)\tag{5.31a}$$

$$\mathbb{D}_1(\boldsymbol{k},\boldsymbol{k}_0;0)=\mathbb{B}(\boldsymbol{k},\boldsymbol{k}_0).\tag{5.31b}$$

The first property ensures the high-frequency limit or the Kirchhoff approximation (KA). The second property yields the correct low-frequency limit SPM1 by relying too heavily on the (-1) term between square brackets in (5.30). This (-1) term corresponds to the diffraction field G_D in equation 2.12 [220]. Indeed, if the (-1) term is dropped, FWA will tend to KA for both low and high frequencies (see Thorsos and Winebrenner [222]). Keeping this (-1) will cause a side effect by contaminating the ensemble averaged radar cross section even away from specular (see Thompson and Chapman [223]). In this paper, it was also demonstrated that FWA breaks even for a tilted planar surface which jeopardise its tilt invariance property. It is obvious that the shift invariance property is also not satisfied due to the same (-1) term.

The nonlocal kernels $\mathbb{D}_2^s$ in the other multiple integrals (5.30) are merely expressible as the product of two first-order coefficients $\mathbb{D}_1$. The additional integrals are recent heuristic modifications of FWA to account for multiple scattering [135, 224, 221].

The FWA is essentially and originally a 1D method, as it relies on the telegraphist equations, and the attempts that have been made by the author to extend it to higher dimensions call on additional simplifying assumptions which are not always mathematically justified. In an excellent critical review on the FWA, Collin [225, 226] provided a clean and self-contained derivation of the 2D case and made comparisons with other approximate methods. FWA is

shown to reproduce SPM in the small-roughness limit, but the KA limit is only recovered via the half-sum of the co-polarization coefficients. FWA has been constantly improved and refined by his author and some 2D comparisons with experimental data are made [227], where it is found in better agreement with data than KA and SPM. However, our subjective opinion is that FWA is analytically as well as numerically very involved while the resulting gain in accuracy over other unifying methods is not established. Moreover, the long series of mutually referring papers and the successions of different versions of FWA that have been developed by Bahar over 30 years render its understanding and numerical implementation difficult to even advised users.

Two other scattering models were derived in a similar manner to FWA. The first is the *local spectral expansion method (LSEM)* developed by García-Valenzuela and Collin [228, 229] for 1D surfaces but for both conducting and dielectric cases. Their first-order model is formed by the sum of two single integrals. The first integral is the high-frequency Kirchhoff while the second is an original form where $W_z = q_k - q_0$ replaces the standard vertical component of the momentum transfer $Q_z = q_k + q_0$. The second-order functional form is similar to that of SSA2 [31] and it helps the model retrieve the low-frequency SPM2 limit. The expression of the total LSEM is:

$$\mathbb{S}(k, k_0) = \frac{\mathbb{K}(k, k_0)}{Q_z} \int \mathrm{e}^{-\mathrm{i}Q_z h(x)} \mathrm{e}^{-\mathrm{i}Q_H x}\, \mathrm{d}x + \frac{\mathbb{B}(k_{,k_0}) - \mathbb{K}(k, k_0)}{W_z} \int (\mathrm{e}^{-\mathrm{i}W_z h(x)} - 1)\, \mathrm{e}^{-\mathrm{i}Q_H x}\, \mathrm{d}x$$
$$- \mathrm{i} \iint \mathbb{F}(k_0; \xi) h(\xi) (\mathrm{e}^{-\mathrm{i}q_k h(x)} - \mathrm{e}^{\mathrm{i}q_k h(x)})\, \mathrm{e}^{-\mathrm{i}(Q_H - \xi)x} \mathrm{d}\xi\, \mathrm{d}x, \tag{5.32}$$

where both SPM1 and SPM2 are reached. The Kirchhoff limit is, however, not formally reached away from specular or backscatter where $W_z = 0$. LSEM fails also some other fundamental properties such as reciprocity, shift and tilt invariances.

The other method based on a modification of FWA derivations is the *correction current method (CCM)* by Schwering *et al* [230]. The model has a single integral form similar to the tangent plan approximation. The kernel of CCM is formed in its constant part by that of SPM1 in the spirit of SSA1 and FWA. The variable part of the kernel is, however, original and depends simply on a quadratic form of the surface slope. CCM is hence very structurally similar to WCA [179] with the inclusion of less orders of surface slope. CCM exists only for 1D perfect conducting surfaces and reads:

$$\mathbb{S}(k, k_0) = \frac{\mathbb{B}(k, k_0)}{Q_z} \int [1 + \alpha h'^2(x)]\, \mathrm{e}^{-\mathrm{i}Q_z h(x)}\, \mathrm{e}^{-\mathrm{i}Q_H x}\, \mathrm{d}x. \tag{5.33}$$

CCM does not reach the Kirchhoff approximation or the physical optics under the high-frequency limit. However, the KA scattering amplitude is recovered through the half-sum of Dirichlet and Neumann scattering amplitudes. CCM does, however, reproduce the SPM1 low-frequency limit and stays reciprocal and simple to implement.

5.11. Improved Green's function methods

Another variety of unifying theories can be grouped under the common denomination of *improved Green's function methods (IGFM)*. It is based on an improved choice of the Green function in the Green–Helmholtz surface integral expressing the scattered field. Kodis [231, 232] and Krill and Andreo [233] combine the variational principle with the use of half-space Green functions. Berman and Perkins [234] invoked the half-space Green function in conjunction with the Kirchhoff current in the scalar case, a procedure which was extended by Shaw and Dougan to the 2D conducting [235] and dielectric [236] cases. They obtain a

hybrid formula for the scattering amplitude, which is written as the sum of a KA term and an SPM-like term:

$$\mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0) = \mathbb{K}(\boldsymbol{k}, \boldsymbol{k}_0)\frac{1}{Q_z}\int \mathrm{e}^{-\mathrm{i}Q_z h(\boldsymbol{r})}\mathrm{e}^{-\mathrm{i}\boldsymbol{Q}_H\cdot\boldsymbol{r}}\,\mathrm{d}\boldsymbol{r} - \mathrm{i}[\mathbb{B}(\boldsymbol{k}, \boldsymbol{k}_0) - \mathbb{K}(\boldsymbol{k}, \boldsymbol{k}_0)]h(\boldsymbol{Q}_H). \tag{5.34}$$

This expression is compatible with both SPM and KA in the appropriate limits to within a small bias in the high-frequency limit due to the corrective term in $h(\boldsymbol{Q}_H)$. This model does not preserve the required shift and tilt invariance properties.

5.12. Volumetric methods

There is a category of approximate methods which we call volumetric because they consider the case of a rough interface between two homogeneous media as a particular space distribution of permittivity. The most famous is the *Born approximation (BA)* which dates back to the early works of quantum mechanics (e.g. [237]) and was first applied to general homogeneous rough surfaces in the context of x-ray and neutron scattering [238]. The BA refers to a family of methods in which the unknown field satisfies a Lippmann–Schwinger equation that is essentially an integral equation of second kind, with a 'potential' term and a Green's function in the integrand. This is the usual representation of solution of the Schrödinger equation, but the electric field satisfying Maxwell's equations can also be put in this form, using the dyadic Green functions and the permittivity contrast as potential. The solution can then formally be expressed as an iteration series (the so-called Born- or Neumann series). The zeroth iteration is the free field of the reference problem. The Born approximation corresponds to the first iteration. In the context of scattering by rough surfaces, the incoherent scattering amplitude takes the form

$$\mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0) = \mathbb{U}(\boldsymbol{k}, \boldsymbol{k}_0)\int \mathrm{e}^{-\mathrm{i}\boldsymbol{Q}_H\cdot\boldsymbol{r}}\mathrm{e}^{-\mathrm{i}Q_z h(\boldsymbol{r})}\,\mathrm{d}\boldsymbol{r}. \tag{5.35}$$

This expression is structurally identical to SSA1 and KA-HF, but coincides with neither of them. A consequence is that it does not recover SPM1 in the limit of small heights. For small contrast, however, the kernel becomes identical to that of SSA1. Precisely, $\mathbb{U}$ can be obtained from the SSA1/SPM1 kernel $\mathbb{B}$ by making the replacements $q_0', q' \to q, q_0$ and $\epsilon q \to q$. The Born approximation is valid when the permittivity contrast between the two media is small, regardless of the roughness. Therefore the BA is mostly used in x-ray surface scattering, a regime in which the contrasts are extremely weak. The main weakness of the BA is that it does not take properly into account the reflection by the surface and therefore fails as one approaches the so-called critical angle, at which total external reflection occurs [239, 240]. Another (related) shortcoming of the method is its inconsistency with SPM1 in the limit of small heights. A great improvement of BA in this respect can be obtained by choosing a more adapted reference problem, namely the flat surface between two media. In that case the free-space Green function is replaced by the half-space Green function, that depends explicitly on the Fresnel reflection coefficient at the plane interface . This is the so-called distorted-wave Born approximation (DWBA), a method which takes again its roots in quantum mechanics [237] and was developed in the eighties for x-ray scattering by rough surfaces [239–241]. The incoherent scattering amplitude obtained by retaining the first-iteration takes the following form:

$$\mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0) = \sum_{s,s_0=\pm 1} \mathbb{U}_{s,s_0}(\boldsymbol{k}, \boldsymbol{k}_0)\int \mathrm{e}^{-\mathrm{i}\boldsymbol{Q}_H\cdot\boldsymbol{r}}\,\mathrm{e}^{\mathrm{i}(sq+s_0q_0)h(\boldsymbol{r})}\,\mathrm{d}\boldsymbol{r}, \tag{5.36}$$

where the sum runs over all possible signs s, s_0 (four possibilities) and hence induces different kernels. This method is known to provide accurate results as long as $\sqrt{\Delta\epsilon}\,Q_z h < 1$, where

$\Delta\epsilon$ is the contrast of permittivity. The analytical expression (5.36) can be further expanded in height to obtain SPM1 (it was in fact already known [242] that a perturbative height expansion based on a Lippmann–Schwinger equation with half-space Green's function is consistent with SPM1). The advantage of a perturbative approach based on volumetric method is the possibility to treat non-homogeneous media. With a further iteration on the integral equation, the small-perturbation expansion up to second order in height can be recovered for rough deposits on top of homogeneous substrates with different permittivity [243] or even heterogeneous rough deposits on layered substrates [244].

The *mean-field theory (MFT)* [245, 246] is a volumetric and statistical approach to rough surface scattering inspired from DWBA. The roughness interface is seen as a permittivity fluctuation in the strip limited by the maximum excursions of the surface. A mean dielectric permittivity is obtained for each elevation by averaging over the horizontal direction. Sampling over different elevations, a reference problem is defined as a multi-layer medium whose Green's function can be computed numerically. The DWBA is then applied to this reference problem and an ensemble average is performed to obtain the first two statistical moments of the electric field. This leads readily to the mean diffuse intensity and the corresponding expression depends in a simple manner on the reference field and Green's function and the surface height-correlation function. The method is semi-analytical inasmuch as the multi-layer Green function has to be first computed numerically. The corresponding cross-section is of the form,

$$\sigma^0 = \int \mathcal{U}(\boldsymbol{k}, \boldsymbol{k}_0, z'; \rho_0)\mathcal{U}^*(\boldsymbol{k}, \boldsymbol{k}_0, z''; \rho_0)\,\mathrm{d}z'\,\mathrm{d}z'' \int \mathrm{e}^{-\mathrm{i}\boldsymbol{Q}_H\cdot\boldsymbol{r}}\mathcal{L}(\boldsymbol{r}, z', z''; \rho)\,\mathrm{d}\boldsymbol{r}, \tag{5.37}$$

where $\mathcal{L}$ is an explicit functional with respect to the surface autocorrelation function $\rho(\boldsymbol{r})$ and $\rho_0 = \rho(0)$. $\mathcal{U}$ is related to the multi-layer field and Green's function and depends only on the surface via its RMS height. The key parameter for the validity of the method is the permittivity contrast, which must remain moderate ($\Delta\epsilon < 2$). Contrary to practically all surfacic approaches, the MFT becomes more accurate at small correlation lengths, at a given level of (moderate but non-perturbative) RMS roughness and permittivity. It is thus able to cope with surface parameters that do not fall in the (known or supposed) validity domain of any surfacic method, typically for correlation length and RMS height that are one third of wavelength. The reason is that fast permittivity fluctuations are in favour of the homogenization process that is done at each elevation. This has been confirmed by numerical [245, 246] as well as experimental [247] tests. The MFT has also been numerically shown in excellent agreement with SPM1 for small roughness and moderate permittivity. Its main limitations are its restriction to small and moderate roughness and the inability to predict cross-polarization in the incidence plane.

5.13. Integral equation method

In the late 1980s, the integral equation method (IEM) model was developed to bridge the gap between the SPM and PO models [248]. Initially an IEM version covering the scattering from perfectly conducting surfaces was published [249, 250]. Subsequently the IEM model was extended to include the case of scattering from a dielectric rough interface [251, 10]. IEM is essentially a second iteration of the iterative Kirchhoff approximation [114, 96]. It provides an iterative solution of the pair of integral equations for the tangential components of the electric and magnetic fields, at the dielectric interface, developed by Poggio and Miller [252]. The expressions of the tangential surface fields are sought as the superposition of the Kirchhoff surface fields and corrective terms originated by the *complementary* surface currents. Hence,

IEM has this functional form:

$$\mathbb{S}(\boldsymbol{k},\boldsymbol{k}_0) = \int F_1(\boldsymbol{k},\boldsymbol{k}_0;-Q_z\nabla h)\,\mathrm{e}^{-\mathrm{i}Q_z h(\boldsymbol{r})}\mathrm{e}^{-\mathrm{i}\boldsymbol{Q}_H\cdot\boldsymbol{r}}\,\mathrm{d}\boldsymbol{r} + \iiint F_2^{+}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi},[h])\,\mathrm{e}^{-\mathrm{i}(\boldsymbol{k}-\boldsymbol{\xi})\cdot\boldsymbol{r}_1-\mathrm{i}(q_k+q_\xi)h(\boldsymbol{r}_1)}\,\mathrm{e}^{\mathrm{i}(\boldsymbol{k}_0-\boldsymbol{\xi})\cdot\boldsymbol{r}_2-\mathrm{i}(q_0-q_\xi)h(\boldsymbol{r}_2)}\,\mathrm{d}\boldsymbol{\xi}\,\mathrm{d}\boldsymbol{r}_1\,\mathrm{d}\boldsymbol{r}_2 + \iiint F_2^{-}(\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{\xi},[h])\,\mathrm{e}^{-\mathrm{i}(\boldsymbol{k}-\boldsymbol{\xi})\cdot\boldsymbol{r}_1-\mathrm{i}(q_k-q_\xi)h(\boldsymbol{r}_1)}\,\mathrm{e}^{\mathrm{i}(\boldsymbol{k}_0-\boldsymbol{\xi})\cdot\boldsymbol{r}_2-\mathrm{i}(q_0+q_\xi)h(\boldsymbol{r}_2)}\,\mathrm{d}\boldsymbol{\xi}\,\mathrm{d}\boldsymbol{r}_1\,\mathrm{d}\boldsymbol{r}_2. \tag{5.38}$$

One drawback of this approach is that the conditions for the convergence of the iterative series are not known *a priori*. For instance, the parameter which should be considered as small for this approach depends on the statistics of the rough surface under study. To obtain the scattering coefficient expressions of 1D rough surfaces in a relatively *simple* form several additional approximations are made in the IEM model (for a critical review see [136]). In the low-frequency region (i.e. $Ks < 2$, K being the impinging wavenumber and s the surface RMS heights), the scattering coefficient consists of single- and multiple-scattering contributions. The single-scattering term is a mere Kirchhoff approximation, whereas the multiple-scattering terms are given as three-fold bidimensional integrals (quite demanding in terms of numerical computation). Some of the assumptions made in the IEM development have subsequently been recognized as simplistic by the same original authors. This is the case of the spectral representation of the Green function (as well as of its gradient) where one of the phase terms was arbitrarily dropped leading to wrong results for the computation of cross-polarized components (i.e. multiple-scattering contributions). In addition, for slightly rough surfaces, the IEM fails to reproduce SPM in the general bistatic geometry [157]. An *improved* version of IEM was released in 1997 [253], then a further corrected version was published in 2000 [254]. The expressions of co-polarized scattering coefficient (i.e. single-scattering contribution) as well as the general IEM expressions (including multiple-scattering terms) have been continuously amended until recently [136, 255, 256]. Extensions of the IEM model to take into account multi-scale roughness statistics have also been suggested [257, 258]. The IEM accuracy should deteriorate at large incidence angle and for small radius of curvature of the rough surface. The IEM predictions of co-polarized backscattering coefficient rough surfaces with moderate slopes and heights have been widely investigated through numerical and experimental studies (see for instance [10, 259, 260]), where reasonable agreement was found.

6. Connections between functional forms

6.1. Local and non-local models

In the course of this review we several times employed the terminology 'local' and 'non-local' for the different models. Although these concepts are rather intuitive in terms of scattering process, it is difficult to give a general mathematical definition. We propose to term a model local if the corresponding scattering amplitude can be written as a *single* space integral with a kernel that depends only on the surface and a finite number of derivatives (in general zero, one or two) at a given point. The generic form of a local model is:

$$\mathbb{S}(\boldsymbol{k},\boldsymbol{k}_0) = \int \mathbb{F}[\boldsymbol{k},\boldsymbol{k}_0;\boldsymbol{r};h(\boldsymbol{r});(\nabla h)(\boldsymbol{r});\ldots;(\nabla^n h)(\boldsymbol{r})]\,\mathrm{d}\boldsymbol{r} \tag{6.39}$$

SPM1, TPA or WCA are examples of local models. A model will be referred to as *non-local* of degree (p, q) if the associated SA is a functional in $h(\boldsymbol{r})$ and $h(\boldsymbol{\xi})$ that requires at least $p+q$ integrations, with $p \geqslant 0$ spatial and $q \geqslant 1$ spectral variables. The generic form for $\mathbb{S}(\boldsymbol{k}, \boldsymbol{k}_0)$ is

$$\int \mathbb{F}[\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{r}_1; \ldots; \boldsymbol{r}_p; \boldsymbol{\xi}_1; \ldots; \boldsymbol{\xi}_q; h(\boldsymbol{r}_1); \ldots; h(\boldsymbol{r}_p); h(\boldsymbol{\xi}_1); \ldots; h(\boldsymbol{\xi}_q)]\, \mathrm{d}\boldsymbol{r}_1 \ldots \mathrm{d}\boldsymbol{r}_p \mathrm{d}\boldsymbol{\xi}_1 \ldots \mathrm{d}\boldsymbol{\xi}_q. \tag{6.40}$$

Note that possible derivatives of h can be absorbed in the Fourier integrals. The dependence of the kernel on the Fourier variables $\boldsymbol{\xi}$ must not be solely polynomial, otherwise the integral can be reduced to a local form. For instance, SPM2, PPM, MLM and TIA are non-local $(0, 1)$, $(1, 1)$, $(2, 1)$ and $(1, 2)$, respectively. Whenever a model has a purely statistical formulation (such as GO), we will apply the same terminology to its scattering cross-section.

The relevant question one might ask now is whether there are any possible connections between these models. What is the transformation that reduces a non-local model to a local one? Positive answers to these questions clarify most models and help understand not only the functional forms but also the corresponding kernels. Indeed, most previously listed models share roughly the same functional form but with major differences in their coefficients or kernels. Building bridges across models could also be seen as a criterion of cross-validation of kernels in use. Connection between models can be established in two ways: by a gauge transformation (which preserves the degree of non-locality) or by reduction of spatial integrals.

6.2. Gauge transformation

The main difference between SSA (5.21) and TIA or LCA (5.24) is that the first single integral is multiplied by the SPM1 kernel instead of the Kirchhoff one. This minor difference has a fundamental repercussion on the second-order kernel in the double integral. It also indicates that there is some arbitrariness in the separation of the scattering amplitude in the sum of a single and double integrals. The possibility of gauge arbitrariness was already discussed by several authors including [9, 155, 171, 191, 179]. In this prospect, we provide the reader with a general gauge function which transforms SSA2 (5.21) to TIA/LCA-form (5.24) (discarding the third integral). This gauge function changes the first- and second-orders SSA according to

$$\mathbb{B}(\boldsymbol{k}, \boldsymbol{k}_0) \Rightarrow \mathbb{K}(\boldsymbol{k}, \boldsymbol{k}_0), \tag{6.41a}$$

$$\mathbb{M}(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{\xi}) \Rightarrow \mathbb{M}(\boldsymbol{k}, \boldsymbol{k}_0; \boldsymbol{\xi}) - [\mathbb{B}(\boldsymbol{k}, \boldsymbol{k}_0) - \mathbb{K}(\boldsymbol{k}, \boldsymbol{k}_0)]\, \frac{\boldsymbol{W}_H}{W_z} \cdot \frac{\boldsymbol{\xi}}{Q_z}. \tag{6.41b}$$

This transformed second-order kernel does not necessarily coincide with that of LCA or TIA. However, the gauge transformation makes the functional forms agree.

6.3. Reduction of non-local models

Two major families of non-local models can be identified in our previous extensive list of functional forms. The first is based on the Meecham–Lysanov form (5.18). The second family regroups all iterative Kirchhoff models as in (4.16). While it seems difficult to establish links between these families, it is possible to compare their respective transforms after a reduction process from a double spatial integral to a single one. This reduction process is not trivial but can be outlined succinctly as follows. Let us employ the triple integral in (4.16) as a general form which includes MLM (5.18) if the $s = \pm\text{sign}$ is set to zero

and the kernel to ϕ. We first start by defining the change of variable inspired from [171], $(\boldsymbol{r}_1, \boldsymbol{r}_2) \Rightarrow (\boldsymbol{u}_1 + \alpha\boldsymbol{u}_2, \boldsymbol{u}_1 - (1-\alpha)\boldsymbol{u}_2)$ where α is an arbitrary positive parameter ($0 \leqslant \alpha \leqslant 1$). This change of variable translates the fact that the mathematical limit of the non-local model when $\boldsymbol{r}_1$ tends to $\boldsymbol{r}_2$ depends on how the two vectors approach each other. This arbitrariness in taking the limit is reduced in our derivation to the sole arbitrary parameter α. The reduction process yields a constant coefficient and a kernel which will be placed in the single and double integrals as in (5.21), respectively. These reduced coefficients are

$$\widetilde{\mathbb{B}}(\boldsymbol{k}, \boldsymbol{k}_0) = \mathbb{K}_2^s(\boldsymbol{k}, \boldsymbol{k}_0, \boldsymbol{k}_\alpha), \tag{6.42a}$$

$$\begin{aligned}\widetilde{\mathbb{M}}(\boldsymbol{k}, \boldsymbol{k}_0, \boldsymbol{\xi}) = {} & \left(q_k + sq_{\xi_\alpha}\right)\mathbb{K}_2^s(\boldsymbol{k}, \boldsymbol{k}_0, \boldsymbol{\xi}_\alpha) - (q_k + q_0)\mathbb{K}_2^s(\boldsymbol{k}, \boldsymbol{k}_0, \boldsymbol{k}_\alpha) \\ & + \left(q_0 - sq_{\xi+\xi_\alpha}\right)\mathbb{K}_2^s(\boldsymbol{k}, \boldsymbol{k}_0, \boldsymbol{\xi} + \boldsymbol{\xi}_\alpha),\end{aligned} \tag{6.42b}$$

where $\boldsymbol{k}_\alpha = \alpha\boldsymbol{k} + (1-\alpha)\boldsymbol{k}_0$ and $\boldsymbol{\xi}_\alpha = \boldsymbol{k}_\alpha - \alpha\boldsymbol{\xi}$. These transformations can be applied with particular choices of α. All the previously listed non-local models can be reduced to the SSA (5.21) and LCA (5.24) functional forms by setting α to 0 or 1 and q_0/Q_z, respectively. A perfect counter example of the SSA-LCA decomposition is when one takes $\alpha = 1/2$ in the reduction of the non-local models. In this case, the reduced model preserves all the limits reached by its non-local parent even though Kirchhoff and SMP1 kernels are not readily apparent in the analytical decomposition.

7. Synthetic tables

To summarize and possibly complete all the properties of the different methods that are discussed in the core of this review, we propose a set of synthetic tables, which we hope retain and outline the most important features.

Table 1 recalls the abbreviations that are in use for the different methods. Any acronym of the form $XXXN$ will refer to the method XXX at Nth order (for instance GO1, SPM2, SSA1, etc).

In table 2, we tried to identify the main principles underlying the construction of the approximate methods. We listed the models according to the number of properties in common, starting from the most basic ones (lower numbers). This corresponds to a 'first-filled column' or lexicographic order. For practically all the methods, the starting point is either a closed (first- or second-kind) integral equation for the surface field and/or its derivative (which we regroup under the vague denomination of surface currents), or some requirement for the scattering amplitude (such as e.g. matching the boundary conditions, as in the Rayleigh–Rice procedure, or satisfying several geometrical properties, as in the SSA). The exception is the family of volumetric methods, which start from volume integral equations for the electric field. Then we inventoried the most usual recipes that are suited to derive a surface current or a scattering amplitude. There are a limited number of them:guessing a structure for the unknown field/amplitude and operating a perturbative expansion or requiring consistency with SPM in the small height limit, iterating the integral equations, choosing an appropriate reference Green's function, estimating oscillatory integral by means of the stationary phase approximation.

Table 3 presents the list of properties that are satisfied by the different methods. The most discriminating criterion in our opinion is that the method be not restricted to a particular non-realistic case (1D or conducting surfaces) and be able to treat the most general dielectric vectorial problem. Then, there are several *a priori* properties that a scattering amplitude should satisfy, as mentioned in section 2: reciprocity, shift invariance, tilt invariance. Satisfying

Table 1. List of acronyms (alphabetical order) together with two main references for the corresponding method.

Acronym	Full name	Main references
BA	Born approximation	[238, 240]
CCM	Correction current method	[230]
DWBA	Distorted-wave born approximation	[239, 240]
ESSA	Extended small-slope approximation	[171]
FWA	Full wave approach	[209, 227]
GO	Geometrical optics	[85, 86]
KA	Kirchhoff approximation	[1, 3]
TPA	Tangent-plane approximation	[88, 89]
KA-HF	High-frequency kirchhoff approximation	[91, 92]
IEM	Integral equation method	[10, 136]
IGFM	Improved Green function method	[234, 236]
LPA	Local parabolic approximation	[107, 110]
LSEM	Local spectral expansion method	[228, 229]
LVM	Lynch variational method	[104, 106]
LCA	Local curvature approximation	[159, 191]
LWA	Local weight approximation	[192, 193]
MFT	Mean-field theory	[245, 246]
MLM	Meecham–Lysanov method	[138, 139]
MTE	Momentum transfer expansion	[17, 93]
NLCA	Non-local curvature approximation	[179, 180]
NLSSA	Non-local small-slope approximation	[174]
OEM	Operator expansion method	[186, 187]
PO	Physical optics	[91, 92]
PPM	Phase perturbation method	[143, 144]
QSE	Quasi slope expansion	[155, 170]
SPM	Small perturbation method	[22, 23]
SSA	Small-slope approximation	[9, 31]
TIA	Tilt-invariant approximation	[190]
TPA	Tangent plane approximation	[88, 89]
TSM	Two-scale model	[3, 16]
UPE	Unified perturbation expansion	[206, 208]
WCA	Weighted curvature approximation	[179, 194]
WHA	Wiener–Hermite approach	[195, 201]

these abstract properties does not ensure that the method will be systematically accurate in various roughness domains, but it is a universal and systematic criterion to see whether an approximation can be a good candidate for a given type of surface. For example, a difference between two scattering amplitudes at reciprocal angles sets a bound for the maximal accuracy that can be reached by such method, a method that does not satisfy the shift invariance (such as SPM) has no chance to hold beyond the perturbative regime (small roughness), a method that does not possess the tilt-invariance property will not be able to cope with surfaces with large scales, etc. A further important test is that the method satisfies the fundamental high- and low-frequency limit, namely GO1 and SPM1. This ensures that the method be at least as good as the latter. Checking in addition that the method is not restricted to small height or large correlation lengths indicates an actual extension of validity domain with respect to the SPM1 and GO1. Finally, an important criterion from a practical point of view is the existence of statistical formulae for the cross-section and an efficient numerical implementation (which

Table 2. Main principles of the different methods, in the order of 'first' filled columns. A '×' in a row indicates that the method uses the corresponding rule.

Method	1	2	3	4	5	6	7	8	9
SPM (Rigorous)	×	–	×	×	×	–	–	×	–
PPM	×	–	×	×	×	–	–	–	–
UPE	×	–	×	×	–	–	×	–	–
FWA	×	–	×	×	–	–	×	–	–
LSEM	×	–	×	×	–	–	×	–	–
CCM	×	–	×	×	–	–	×	–	–
TIA	×	–	×	×	–	–	–	×	–
WHA	×	–	×	×	–	–	–	×	–
OEM	×	–	×	×	–	–	–	–	–
LVM	×	–	×	–	–	–	×	–	–
LPA	×	–	×	–	–	–	×	–	–
MTE	×	–	×	–	–	–	–	×	–
TPA	×	–	×	–	–	–	–	–	–
MLM	×	–	–	×	–	×	–	×	–
QSE	×	–	–	×	–	–	–	×	–
ESSA	×	–	–	×	–	–	–	×	–
IGFM	×	–	–	–	–	×	–	–	–
IEM	×	–	–	–	–	–	×	–	–
SPM (Rayleigh)	–	×	×	×	×	–	×	–	–
TSM	–	×	×	×	×	–	×	–	×
NLSSA	–	×	×	×	×	–	×	–	–
SSA	–	×	×	×	×	–	–	–	–
NLCA	–	×	×	–	×	–	–	–	×
LCA	–	×	×	–	×	–	–	–	×
WCA	–	×	×	–	×	–	–	–	×
LWA	–	×	×	–	–	–	–	–	×
KA–HF (PO)	–	×	–	–	–	–	–	–	×
GO (SPT)	–	×	–	–	–	–	–	–	×
DWBA	–	–	–	–	–	×	×	–	–
MFT	–	–	–	–	–	×	×	–	–
BA	–	–	–	–	–	–	×	–	–

1. Proceeds via estimation of a surface current.
2. Works directly on the scattering amplitude.
3. Makes a structure Ansatz for the surface current or the scattering amplitude.
4. Uses a functional expansion of some quantity in surface height.
5. Requires consistency with SPM for the identification of some unknown series.
6. Uses a particular choice of Green's function.
7. Uses second-kind Fredholm integral equation.
8. Uses first-kind Fredholm integral equation.
9. Uses the stationary phase approximation.

should be fast, stable and easy). Note that this notion of numerical efficiency is quite subjective and we chose to remain vague in this statement. By 'fast' we mean essentially an analytical expression that involves a single integral when it comes to local model, and the extensive use of FFT or a small number of integrations (given by the degree of non-locality) when it comes to a non-local model. By 'stable' we mean that the integration procedure is not complicated by the occurrence of a singular, oscillating or diverging kernel, which would require heavy

Table 3. Properties of the local models.

Property	1	2	3a	3b	3c	4	5	6	7	8a	8b
SPM1	■	■	☒	∅	∅	■	■	☒	∅	■	∅
TPA	■	■	∅	■	■	■	☒	∅	■	∅	☒
KA-HF	■	■	☒	■	■	■	■	∅	■	∅	☒
GO1	■	■	☒	–	☒	■	■	∅	■	∅	■
LVM	∅	∅	∅	☒	∅	☒	☒	∅	■	∅	☒
LPA	∅	∅	☒	☒	☒	☒	☒	☒	☒	?	■
MTE	∅	■	∅	☒	∅	☒	☒	∅	■	∅	☒
SSA1	■	■	■	■	∅	■	■	■	■	■	∅
IGFM	■	■	■	∅	∅	■	☒	☒	■	☒	□
LWA	∅	■	■	■	■	■	☒	☒	■	■	■
WCA	■	■	■	■	■	■	☒	☒	■	■	■
BA	∅	■	☒	☒	∅	■	■	■	■	∅	∅
DWBA	∅	■	☒	∅	∅	■	■	■	■	☒	∅
MFT	∅	■	☒	∅	∅	☒	■	■	■	☒	∅
CCM	∅	∅	☒	☒	∅	☒	☒	■	■	☒	∅

1. All types of surfaces (dielectric, conducting, acoustic).
2. Full two-dimensional surfaces.
3a. Reciprocal (manifestly).
3b. Shift invariant (to arbitrary order in shift).
3c. Tilt invariant (at least to first order in tilt).
4. Numerically fast and stable while easy to implement.
5. Statistical formulae already available or easily derivable.
6. Not restricted to large correlation lengths.
7. Not restricted to small surface height.
8a. SPM1 limit.
8b. GO1 limit.

The significance of the symbols is as follows:
■ = satisfied by construction
☒ = satisfied upon inspection, □ = satisfied under special conditions (specular, backscattering, conducting case, etc)
? = unknown or not tested.
∅ = not satisfied
– = irrelevant.

over-sampling or specific quadrature rules. By easy, we mean that there exists an explicit, readable and ready-to-implement formula for the scattering cross-section.

In table 4, we added some extra suitable properties for a non-local model: to be able to predict a correct cross-polarization in the incidence plane (a requirement that no local model is able to meet) and to satisfy the second-order fundamental high- and low-frequency limit SPM2 and GO2. We also introduced the degree of non-locality in the last column in table 4.

For both tables 3 and 4, we adopted the following diagrammatic rule to read them: any 'positive' feature of a method is accounted for by a box (empty, crossed or filled). We make a distinction between properties that are inherent to the model (satisfied 'by construction') and those which are a suitable but not necessary outcome (satisfied 'upon inspection'). We left an empty square for the models that satisfy a property only partially or under special conditions (specular, backscattering, conducting case, etc). Once again, we insist on the fact that a method will not be more accurate in general than another one just because it has more boxes in the table. These desirable properties serve as indicators of the 'good' candidates to perform well across the regimes.

Table 4. Properties of the non-local models.

Property	1	2	3a	3b	3c	4	5	6	7	8a	8b	9a	9b	10	11
SPM2	■	■	☒	∅	∅	■	☒	■	∅	–	∅	■	–	■	(0, 1)
GO2	■	■	☒	–	☒	■	■	∅	■	∅	–	∅	■	☒	(0, 1)
WHA	■	■	?	–	∅	☒	■	■	∅	☒	∅	□	∅	□	(0, 2)
PPM	∅	■	∅	☒	∅	∅	☒	■	☒	■	☒	∅	∅	–	(1, 1)
SSA2	■	■	■	■	☒	∅	☒	■	■	■	□	■	∅	■	(1, 1)
QSE	∅	■	∅	■	☒	∅	☒	■	■	☒	☒	☒	∅	–	(1, 1)
ESSA	∅	∅	☒	☒	∅	∅	☒	■	■	■	□	∅	∅	–	(1, 1)
LSEM	■	∅	∅	∅	∅	∅	☒	■	☒	☒	∅	☒	∅	–	(1, 1)
LCA	■	■	■	■	■	∅	∅	■	■	■	■	∅	∅	∅	(1, 1)
UPE	∅	■	∅	☒	∅	∅	☒	■	■	☒	☒	☒	∅	☒	(1, 2)
TIA	∅	■	∅	■	■	∅	∅	☒	■	☒	■	☒	∅	–	(1, 2)
KA2–HF	■	■	■	☒	☒	☒	☒	∅	■	∅	–	∅	☒	☒	(2, 1)
MLM	∅	■	■	■	∅	☒	∅	☒	■	☒	☒	∅	∅	–	(2, 1)
NLSSA	■	■	■	■	∅	□	∅	■	□	■	∅	■	∅	■	(2, 1)
OEM1	■	■	□	■	∅	□	∅	☒	■	■	☒	∅	∅	∅	(2, 1)
FWA	■	■	■	∅	∅	∅	☒	☒	■	☒	☒	∅	?	∅	(2, 1)
IEM	■	■	∅	■	∅	∅	☒	☒	■	☒	■	∅	☒	∅	(2, 1)
OEM2	■	■	□	■	?	□	∅	☒	☒	■	☒	■	?	☒	(3, 2)

1. All types of surfaces (dielectric, conducting, acoustic).
2. Full two-dimensional surfaces.
3a. Reciprocal (manifestly).
3b. Shift invariant (to arbitrary order in shift).
3c. Tilt invariant (at least to first order in tilt).
4. Numerically fast and stable while easy to implement.
5. Statistical formulae already available or easily derivable.
6. Not restricted to large correlation lengths.
7. Not restricted to small surface height.
8a. SPM1 limit.
8b. GO1 limit.
9a. SPM2 limit.
9b. GO2 limit.
10. Can predict correct cross-polarization in the plane of incidence.
11. Degree of non-locality (p, q).

The significance of the symbols is as follows:
■ = satisfied by construction
☒ = satisfied upon inspection
□ = satisfied under special conditions (specular, backscattering, conducting case, etc)
? = unknown or not tested
∅ = not satisfied
– = irrelevant.
We classified the models according to their degree of non-locality.

8. Conclusion

We provided a review of approximate scattering wave theories from random surfaces in a unified notation that put the emphasis on the functional form of the scattering amplitude. We classified the models in three families, which follow approximatively the historical evolution: the low-frequency models (SPM and variants), the high-frequency approximation (Kirchhoff

and variants) and the so-called unified methods, which aim at bridging the gap between the former two. We tried to outline the main principles of the methods, the list of which is recapitulated in table 2. We attempted to evaluate the different methods according to a dozen of general criteria which we believe are fundamental. The different performances are synthesized in tables 3 and 4. We made a distinction between local and non-local models, for which we proposed a precise definition. Some horizontal and vertical links between non-local models are developed in order to explain their apparent differences. At the present time, there does not seem to be a universal method that is to be preferred systematically. All the methods present a compromise between versatility, simplicity, numerical efficiency, accuracy and robustness, with a different weighting in these various fields. At a first glance at the tables, it appears obvious that no approximate model has fulfilled all listed criteria. Moreover, most models did not even satisfy half of the requirements. One should also mention that a fair comparison can only be accomplished between models that address the same scattering problem (acoustic, conducting, dielectric, 1D, 2D, etc). There is still room for improvement in the development of approximate scattering methods, and we hope the check-list of performances we proposed in tables 3 and 4 will help future researchers and users.

Appendix A. Recapitulation of our vector notation

Incident and scattered wavenumbers and their related variables are defined as follows:

$$\boldsymbol{K}_0 = \boldsymbol{k}_0 - q_0\hat{\boldsymbol{z}} \tag{A.1a}$$

$$\boldsymbol{K} = \boldsymbol{k} + q_k\hat{\boldsymbol{z}} \tag{A.1b}$$

$$q_k = \sqrt{K^2 - \boldsymbol{k}\cdot\boldsymbol{k}} \tag{A.1c}$$

$$q_0 = \sqrt{K^2 - \boldsymbol{k}_0\cdot\boldsymbol{k}_0} \tag{A.1d}$$

$$\boldsymbol{K}_0^2 = \boldsymbol{K}^2 = K^2 \tag{A.1e}$$

$$\boldsymbol{Q} = \boldsymbol{K} - \boldsymbol{K}_0 \tag{A.1f}$$

$$\boldsymbol{W} = \boldsymbol{K} + \boldsymbol{K}_0 \tag{A.1g}$$

$$\boldsymbol{Q}_H = \boldsymbol{k} - \boldsymbol{k}_0 \tag{A.1h}$$

$$\boldsymbol{W}_H = \boldsymbol{k} + \boldsymbol{k}_0 \tag{A.1i}$$

$$Q_z = q_k + q_0 \tag{A.1j}$$

$$W_z = q_k - q_0. \tag{A.1k}$$

More definitions and normalization are given in section 2.

Appendix B. Neumann and Dirichlet boundary conditions

From [9, 192], one can write the first-order small perturbation method (SPM1) $\mathcal{B}$ and high-frequency Kirchhoff $\mathcal{K}$ coefficients for both the Neumann ($\mathcal{N}$) and Dirichlet ($\mathcal{D}$) boundary conditions as

$$\mathcal{B}_{\mathcal{N}}(\boldsymbol{k},\boldsymbol{k}_0) = 2(K^2 - \boldsymbol{k}\cdot\boldsymbol{k}_0), \qquad \mathcal{B}_{\mathcal{D}}(\boldsymbol{k},\boldsymbol{k}_0) = -2q_kq_0 \tag{B.1}$$

$$\mathcal{K}_{\mathcal{N}}(\boldsymbol{k},\boldsymbol{k}_0) = -\mathcal{K}_{\mathcal{D}}(\boldsymbol{k},\boldsymbol{k}_0) = K^2 - \boldsymbol{k}\cdot\boldsymbol{k}_0 + q_kq_0. \tag{B.2}$$

One can easily note the following relationship between $\mathcal{B}_{\mathcal{N}D}$ and $\mathcal{K}_{\mathcal{N}D}$:

$$\mathcal{K}_{\mathcal{N}D}(\boldsymbol{k}, \boldsymbol{k}_0) = \mathcal{B}_{\mathcal{N}D}\left(\frac{\boldsymbol{W}}{2}, \frac{\boldsymbol{W}}{2}\right). \tag{B.3}$$

This relationship can be explained by changing the notation to dyadic form with three-dimensional vectors. The first-order small perturbation method (SPM1) coefficients become (see [192])

$$\mathcal{B}_{\mathcal{N}}(\boldsymbol{W}; \boldsymbol{Q}) = \tfrac{1}{2}(\boldsymbol{W} + \boldsymbol{Q}) \cdot (\hat{\boldsymbol{z}}\hat{\boldsymbol{z}} - 2\hat{\boldsymbol{Q}}\hat{\boldsymbol{Q}}) \cdot (\boldsymbol{W} - \boldsymbol{Q}) \tag{B.4a}$$

$$\mathcal{B}_{\mathcal{D}}(\boldsymbol{W}; \boldsymbol{Q}) = \tfrac{1}{2}(\boldsymbol{W} + \boldsymbol{Q}) \cdot \hat{\boldsymbol{z}}\hat{\boldsymbol{z}} \cdot (\boldsymbol{W} - \boldsymbol{Q}) \tag{B.4b}$$

while the Kirchhoff ones are

$$\mathcal{K}_{\mathcal{N}}(\boldsymbol{W}; \boldsymbol{Q}) = -\mathcal{K}_{\mathcal{D}}(\boldsymbol{W}; \boldsymbol{Q}) = \frac{Q^2}{2}. \tag{B.5}$$

The Kirchhoff coefficients are derivable from those of SPM1 by operating these simple substitutions

$$\hat{\boldsymbol{z}} \Rightarrow \hat{\boldsymbol{Q}} \tag{B.6a}$$

$$\boldsymbol{k}, \boldsymbol{k}_0 \Rightarrow \frac{\boldsymbol{W}}{2} \tag{B.6b}$$

$$q_k, q_0 \Rightarrow \frac{Q}{2}. \tag{B.6c}$$

Hence,

$$\mathcal{K}_{\mathcal{N}D}(\boldsymbol{W}; \boldsymbol{Q}) = \mathcal{B}_{\mathcal{N}D}(\boldsymbol{W}; \boldsymbol{Q})|_{\hat{\boldsymbol{z}} \Rightarrow \hat{\boldsymbol{Q}}} \tag{B.7}$$

which merely expresses a change in the frame of reference.

Appendix C. Boundary conditions of an interface between two fluid half-spaces

The first medium has a density ρ_1 and a speed of sound c_1. In the second medium the density and the celerity are ρ_2 and c_2, respectively. The wavenumbers in each medium are defined as $K_1 = \frac{\omega}{c_1}$ and $K_2 = \frac{\omega}{c_2}$ where $\omega = 2\pi f$ and f is the frequency of the sound wave.

For the first-order small perturbation method SPM1 [9], one obtains

$$\mathcal{B}(\boldsymbol{k}, \boldsymbol{k}_0) = 2q_k q_0 \frac{\rho_2^2\left(K_1^2 - \boldsymbol{k} \cdot \boldsymbol{k}_0\right) - \rho_2\rho_1\left(K_2^2 - q_0' q_k' - \boldsymbol{k} \cdot \boldsymbol{k}_0\right) - \rho_1^2 q_0' q_k'}{(\rho_2 q_0 + \rho_1 q_0')(\rho_2 q_k + \rho_1 q_k')}, \tag{C.1}$$

where

$$q_*' = \sqrt{K_2^2 - \boldsymbol{k}_* \cdot \boldsymbol{k}_*}. \tag{C.2}$$

The Kirchhoff coefficient is

$$\mathcal{K}(\boldsymbol{k}, \boldsymbol{k}_0) = \mathcal{B}\left(\frac{\boldsymbol{W}}{2}, \frac{\boldsymbol{W}}{2}\right) = \frac{Q^2}{2}\mathcal{R}\left(\frac{\boldsymbol{W}}{2}\right), \tag{C.3}$$

where $\mathcal{R}(\boldsymbol{k})$ is the standard Fresnel coefficient,

$$\mathcal{R}(\boldsymbol{k}) = \frac{\rho_2 q_k - \rho_1 q_k'}{\rho_2 q_k + \rho_1 q_k'}. \tag{C.4}$$

We will see in the remainder of the appendix that the relationship in (C.3) holds even for the dielectric electromagnetic case but in dyadic form.

Appendix D. Perfectly conducting boundary conditions

In the electromagnetic case the polarization matrices are usually expressed in the canonical basis of vertical/horizontal incident and scattered polarization vectors:

$$\hat{\boldsymbol{V}}_i = \frac{k_0\hat{\boldsymbol{z}} + q_0\hat{\boldsymbol{k}}_0}{K} \qquad \text{and} \qquad \hat{\boldsymbol{H}}_i = \hat{\boldsymbol{z}} \times \hat{\boldsymbol{k}}_0 \tag{D.1a}$$

$$\hat{\boldsymbol{V}}_s = \frac{k\hat{\boldsymbol{z}} - q_k\hat{\boldsymbol{k}}}{K} \qquad \text{and} \qquad \hat{\boldsymbol{H}}_s = \hat{\boldsymbol{z}} \times \hat{\boldsymbol{k}}. \tag{D.1b}$$

The different matrices are then read using the convention

$$\begin{pmatrix} \hat{\boldsymbol{V}}_i \to \hat{\boldsymbol{V}}_s & \hat{\boldsymbol{H}}_i \to \hat{\boldsymbol{V}}_s \\ \hat{\boldsymbol{V}}_i \to \hat{\boldsymbol{H}}_s & \hat{\boldsymbol{H}}_i \to \hat{\boldsymbol{H}}_s \end{pmatrix}. \tag{D.2}$$

The SPM1 polarization matrix for the perfectly conducting boundary conditions is

$$\mathcal{B}_\infty(\boldsymbol{k}, \boldsymbol{k}_0) = 2\begin{pmatrix} K^2\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{k}}_0 - kk_0 & Kq_0(\hat{\boldsymbol{k}}_0 \times \hat{\boldsymbol{k}})\cdot\hat{z} \\ Kq_k(\hat{\boldsymbol{k}}_0 \times \hat{\boldsymbol{k}})\cdot\hat{z} & -q_kq_0\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{k}}_0 \end{pmatrix}. \tag{D.3}$$

The Kirchhoff matrix is

$$\mathcal{K}_\infty(\boldsymbol{k}, \boldsymbol{k}_0) = \begin{pmatrix} [(K^2 + q_kq_0)\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{k}}_0 - kk_0] & K(q_k + q_0)(\hat{\boldsymbol{k}}_0 \times \hat{\boldsymbol{k}})\cdot\hat{z} \\ K(q_k + q_0)(\hat{\boldsymbol{k}}_0 \times \hat{\boldsymbol{k}})\cdot\hat{z} & -[(K^2 + q_kq_0)\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{k}}_0 - kk_0] \end{pmatrix}. \tag{D.4}$$

Property (B.7), valid in the scalar case, does not directly carry over to the polarization matrices:

$$\mathcal{K}_\infty(\boldsymbol{k}, \boldsymbol{k}_0) \neq \mathcal{B}_\infty\left(\frac{\boldsymbol{W}}{2}, \frac{\boldsymbol{W}}{2}\right)\bigg|_{\hat{\boldsymbol{z}}\Rightarrow\hat{\boldsymbol{Q}}}. \tag{D.5}$$

The reason for this is that some polarization mixing will not be accounted for in the matrix form. One needs to rewrite these coefficients in dyadic form before applying any transformation on the frame of reference.

The dyadic form of SPM1, see equation C9 in [192], is

$$\begin{aligned}\mathbb{B}_\infty(\boldsymbol{W}; \boldsymbol{Q}) = {} & \tfrac{1}{2}(\boldsymbol{W} + \boldsymbol{Q})\cdot\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}\cdot(\boldsymbol{W} - \boldsymbol{Q})\mathbb{I} \\ & + \hat{\boldsymbol{z}}\hat{\boldsymbol{z}}\cdot(\boldsymbol{QQ} - \boldsymbol{WW}) + (\boldsymbol{QQ} - \boldsymbol{WW})\cdot\hat{\boldsymbol{z}}\hat{\boldsymbol{z}} - Q^2\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}.\end{aligned} \tag{D.6}$$

It can be shown that

$$\mathbb{K}_\infty(\boldsymbol{W}; \boldsymbol{Q}) = \mathbb{B}_\infty(\boldsymbol{W}; \boldsymbol{Q})|_{\hat{\boldsymbol{z}}\Rightarrow\hat{\boldsymbol{Q}}} = \frac{Q^2}{2}(2\hat{\boldsymbol{Q}}\hat{\boldsymbol{Q}} - \mathbb{I}) \tag{D.7}$$

which is a compact dyadic form of the Kirchhoff coefficient under perfect conducting conditions.

Appendix E. Fully dielectric boundary conditions

The SPM1 coefficients for dielectric boundary conditions are taken from [244]

$$\mathcal{B}_{VV}(\boldsymbol{k}, \boldsymbol{k}_0) = \frac{\epsilon - 1}{2}\left([1 - R_\perp(k)][1 - R_\perp(k_0)]q_kq_0\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{k}}_0 - [1 + R_\perp(k)][1 + R_\perp(k_0)]\frac{kk_0}{\epsilon}\right) \tag{E.1a}$$

$$\mathcal{B}_{VH}(\boldsymbol{k}, \boldsymbol{k}_0) = -\frac{\epsilon - 1}{2}Kq_0[1 + R_\parallel(k)][1 - R_\perp(k_0)]\hat{\boldsymbol{z}}\cdot(\hat{\boldsymbol{k}} \times \hat{\boldsymbol{k}}_0) \tag{E.1b}$$

$$\mathcal{B}_{HV}(\boldsymbol{k},\boldsymbol{k}_0) = -\frac{\epsilon-1}{2}Kq_k[1-R_\perp(k)][1+R_\parallel(k_0)]\hat{\boldsymbol{z}}\cdot(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{k}}_0) \tag{E.1c}$$

$$\mathcal{B}_{HH}(\boldsymbol{k},\boldsymbol{k}_0) = -\frac{\epsilon-1}{2}K^2[1+R_\parallel(k)][1+R_\parallel(k_0)]\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{k}}_0, \tag{E.1d}$$

where the standard Fresnel coefficients are defined as

$$R_\perp(k) = \frac{\epsilon q_k - q_k'}{\epsilon q_k + q_k'} \tag{E.2a}$$

$$R_\parallel(k) = \frac{q_k - q_k'}{q_k + q_k'}. \tag{E.2b}$$

The primed variables are generically given by

$$q_*' = \sqrt{\epsilon K^2 - \boldsymbol{k}_*\cdot\boldsymbol{k}_*}. \tag{E.3}$$

The SPM1 coefficients are readily available in dyadic form [244]

$$\mathbb{B}(\boldsymbol{k},\boldsymbol{k}_0) = -\frac{\epsilon-1}{2}K^2\left[\mathbb{I}-\hat{\boldsymbol{K}}\hat{\boldsymbol{K}}+\mathbb{R}(\boldsymbol{k})\right]\cdot\mathbb{T}(\boldsymbol{k}_0). \tag{E.4}$$

The reflected $\mathbb{R}$ and transmitted $\mathbb{T}$ dyadic operators are defined by

$$\mathbb{R}(\boldsymbol{k}) = R_\perp(\boldsymbol{k})\hat{\boldsymbol{p}}_1^+(\boldsymbol{k})\hat{\boldsymbol{p}}_1^-(\boldsymbol{k}) + R_\parallel(\boldsymbol{k})\hat{\boldsymbol{p}}_2^+(\boldsymbol{k})\hat{\boldsymbol{p}}_2^-(\boldsymbol{k}) \tag{E.5a}$$

$$\mathbb{T}(\boldsymbol{k}_0) = \frac{1}{\sqrt{\epsilon}}[1+R_\perp(\boldsymbol{k}_0)]\hat{\boldsymbol{p}}_1^-(\boldsymbol{k}_0)\hat{\boldsymbol{p}}_1^{-\prime}(\boldsymbol{k}_0) + [1+R_\parallel(\boldsymbol{k}_0)]\hat{\boldsymbol{p}}_2^-(\boldsymbol{k}_0)\hat{\boldsymbol{p}}_2^{-\prime}(\boldsymbol{k}_0). \tag{E.5b}$$

Here the polarization vectors which are the basis for the dyads are again the vertical and horizontal states, taken in the lower and upper medium:

$$\hat{\boldsymbol{p}}_1^\pm(\boldsymbol{k}) = \frac{k\hat{\boldsymbol{z}}\mp q_k\hat{\boldsymbol{k}}}{K} \qquad \text{and} \qquad \hat{\boldsymbol{p}}_2^\pm(\boldsymbol{k}) = \hat{\boldsymbol{z}}\times\hat{\boldsymbol{k}} \tag{E.6a}$$

$$\hat{\boldsymbol{p}}_1^{\pm'}(\boldsymbol{k}_0) = \frac{k_0\hat{\boldsymbol{z}}\mp q_0'\hat{\boldsymbol{k}}_0}{K\sqrt{\epsilon}} \qquad \text{and} \qquad \hat{\boldsymbol{p}}_2^{\pm'}(\boldsymbol{k}_0) = \hat{\boldsymbol{z}}\times\hat{\boldsymbol{k}}_0. \tag{E.6b}$$

The expression for $\mathbb{B}(\boldsymbol{k},\boldsymbol{k}_0)$ in (E.4) is reciprocal but not manifest. In order to generate a fully symmetric $\mathbb{B}(\boldsymbol{k},\boldsymbol{k}_0)$ one can rewrite (E.4) as

$$\mathbb{B}(\boldsymbol{k},\boldsymbol{k}_0) = -\frac{\epsilon-1}{2}K^2[\mathbb{I}-\hat{\boldsymbol{K}}\hat{\boldsymbol{K}}+\mathbb{R}(\boldsymbol{k})]\cdot\mathbb{A}\cdot[\mathbb{R}(\boldsymbol{k}_0)-\hat{\boldsymbol{K}}_0\hat{\boldsymbol{K}}_0+\mathbb{I}], \tag{E.7}$$

where $\mathbb{A}$ is

$$\mathbb{A} = \mathbb{I} + \left(\frac{1}{\epsilon}-1\right)\hat{\boldsymbol{z}}\hat{\boldsymbol{z}} \tag{E.8}$$

which is termed the pseudo-horizontal projector since under perfect conductivity ($\epsilon \mapsto -\mathrm{i}\infty$) this projector coincides with the standard horizontal one ($\mathbb{A}_\infty = \mathbb{I} - \hat{\boldsymbol{z}}\hat{\boldsymbol{z}}$).

It can be demonstrated that the dielectric Kirchhoff polarization coefficients are related to those of SPM1 by

$$\mathbb{K}(\boldsymbol{k},\boldsymbol{k}_0) = \mathbb{B}\left(\frac{\boldsymbol{W}}{2},\frac{\boldsymbol{W}}{2}\right)\Big|_{\hat{\boldsymbol{z}}\Rightarrow\hat{\boldsymbol{Q}}} = \frac{Q^2}{2}\mathbb{R}\left(\frac{\boldsymbol{W}}{2}\right)\Big|_{\hat{\boldsymbol{z}}\Rightarrow\hat{\boldsymbol{Q}}}. \tag{E.9}$$

This leads to

$$\mathbb{K}(\boldsymbol{k}, \boldsymbol{k}_0) = \frac{Q^2}{2} \frac{R_\perp(W/2)\hat{\boldsymbol{K}}\hat{\boldsymbol{K}}_0 + R_\parallel(W/2)(\hat{\boldsymbol{K}} \times \hat{\boldsymbol{K}}_0)(\hat{\boldsymbol{K}} \times \hat{\boldsymbol{K}}_0)}{(\hat{\boldsymbol{K}} \times \hat{\boldsymbol{K}}_0)^2} \tag{E.10}$$

which is consistent with that given by equation 10b in Jin and Lax [96].

This link between Kirchhoff and SPM1 is unveiled here mainly due a change of frame of reference owing to a combination of our normalization along with the use of the dyadic notation.

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