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Allan R. Willms
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A First Course in Complex Analysis

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ABSTRACT

This book introduces complex analysis and is appropriate for a first course in the subject at typically the third-year University level. It introduces the exponential function very early but does so rigorously. It covers the usual topics of functions, differentiation, analyticity, contour integration, the theorems of Cauchy and their many consequences, Taylor and Laurent series, residue theory, the computation of certain improper real integrals, and a brief introduction to conformal mapping. Throughout the text an emphasis is placed on geometric properties of complex numbers and visualization of complex mappings.

KEYWORDS

complex numbers, analytic functions, contour integration, Cauchy theory, Taylor and Laurent series, residues, conformal mapping

In memory of Dan.

Contents

Preface

This text is the product of course notes I have used to teach an introductory course in complex analysis for many of the past 11 years at the University of Guelph. In developing these notes I have of course benefited from many of the texts that are available on this subject. In particular, I mention the classic text by Churchill and Brown [1990] and the texts by Spiegel et al. [2009] and Saff and Snider [2003].

As with any instructor, I have my own preferences in how to order and present material. How these preferences rate compared to anyone else's is left to the judgment of the reader. One of these ordering preferences is the introduction of the exponential function within the first dozen pages of the text. It is my feeling that the utility of the exponential function for both notational convenience and understanding of the geometry of complex multiplication is simply too great to delay its introduction. Although there are other texts that introduce the exponential relatively early, most that do so provide the reader with little justification, often simply letting Euler's formula be the definition of the exponential of an imaginary number. I have taken a different approach. I assume that students will have seen a real Taylor series and in particular the Taylor series for e^x. It is then completely reasonable from their point of view to simply replace the real quantity x with the complex variable z. After stating a theorem (proved in Chapter 6) that a complex series converges if and only if its real and imaginary parts converge, I rigorously show that the series for e^z converges using familiar convergence tests for real series. Euler's formula then emerges as a consequence of this series definition. Although this approach requires dealing with a series before they are discussed generally, it is, in my opinion, a small price to pay for the benefits returned.

Another feature of the text is an emphasis on the geometric properties of complex numbers and the visualization of functions of a complex variable. To achieve this two types of plots are used: mapping plots, showing regions in the complex plane and their images under given maps, and phase plots, showing the argument of the image of each point, and contours of the modulus. As the anticipated primary mode of use of this text is in an electronic format, I have made liberal use of color in these figures.

What background is the student expected to have? A thorough grounding in single-variable differential and integral calculus is assumed, including an understanding of Taylor series. Familiarity with multi-variable differential calculus, primarily for the understanding of partial derivatives is expected. The student would also benefit from being exposed to multi-variable integral calculus, in particular real line integrals and Green's Theorem, although this is not necessary. The ideas of limits and continuity are expected to be understood, and it is assumed the student has had exposure to some real analysis, particularly the notions of open and closed sets, although the main points are summarized at the beginning of Chapter 2.

After working through the material in this text, the student should be able to appreciate the references in the following. Of course a familiarity with the original psalm, available in both the Christian and Jewish Bibles, is necessary.

Psalm 23 on the ℂomplex Plane

The Lord is my analytic function; I shall lack no derivatives.
He makes my modulus submaximal on his domain.
He leads my integral to zero around closed contours.
He remakes my parts harmonic.
He guides me on deformable paths, without changing value.

Even though I walk through the valley of the forest of poles,
I will fear no singularity, for you are with me.
Your radius of convergence, and analytic continuation, they comfort me.
You establish a Laurent series before me, surrounding my enemies.
You anoint my argument by 2π; my residues overflow.

Surely continuity and differentiability will follow me on all of my branches,
And I will dwell on a Riemann surface forever.

Allan R. Willms
December 2021

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Allan R. Willms
December 2021