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Rajan Chattamvelli and Ramalingam Shanmugam

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Continuous Distributions in Engineering and the Applied Sciences – Part II

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SYNTHESIS LECTURES ON MATHEMATICS AND STATISTICS #43



ABSTRACT

This is the second part of our book on continuous statistical distributions. It covers inverse-Gaussian, Birnbaum-Saunders, Pareto, Laplace, central χ^2 , T, F, Weibull, Rayleigh, Maxwell, and extreme value distributions. Important properties of these distribution are documented, and most common practical applications are discussed. This book can be used as a reference material for graduate courses in engineering statistics, mathematical statistics, and econometrics. Professionals and practitioners working in various fields will also find some of the chapters to be useful.

Although an extensive literature exists on each of these distributions, we were forced to limit the size of each chapter and the number of references given at the end due to the publishing plan of this book that limits its size. Nevertheless, we gratefully acknowledge the contribution of all those authors whose names have been left out.

Some knowledge in introductory algebra and college calculus is assumed throughout the book. Integration is extensively used in several chapters, and many results discussed in Part I (Chapters 1 to 9) of our book are used in this volume.

Chapter 10 is on Inverse Gaussian distribution and its extensions. The Birnbaum-Saunders distribution and its extensions along with applications in actuarial sciences is discussed in Chapter 11. Chapter 12 discusses Pareto distribution and its extensions. The Laplace distribution and its applications in navigational errors is discussed in the next chapter. This is followed by central chi-squared distribution and its applications in statistical inference, bioinformatics and genomics. Chapter 15 discusses Student's T distribution, its extensions and applications in statistical inference. The F distribution and its applications in statistical inference appears next. Chapter 17 is on Weibull distribution and its applications in geology and reliability engineering. Next two chapters are on Rayleigh and Maxwell distributions and its applications in communications, wind energy modeling, kinetic gas theory, nuclear and thermal engineering, and physical chemistry. The last chapter is on Gumbel distribution, its applications in the law of rare exceedances.

Suggestions for improvement are welcome. Please send them to rajan.chattamvelli@vit.ac.in.

KEYWORDS

Birnbaum-Saunders distribution, chi-squared distribution, electronics, Gumbel distribution, Laplace distribution, Maxwell distribution, mean deviation, Pareto distribution, Rayleigh distribution, size-biased distributions, survival function, truncated distributions, Weibull distribution

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Preface

Continuous distributions are encountered in several engineering fields. They are used either to model a continuous variate (like time, temperature, pressure, amount of rainfall) or to approximate a discrete variate. This book (in two parts) introduces the most common continuous univariate distributions. A comprehensive treatment requires entire volumes by itself, as the literature on these distributions are extensive and ever-increasing. Hence, only the most important results that are of practical interest to engineers and researchers in various fields are included. Professionals working in some fields usually encounter only a few of the distributions for which probability mass function (PMF), probability density function (PDF), cumulative distribution function (CDF), survival function (SF) (complement of CDF), or hazard rate are needed for special variable values. These appear in respective chapters.

Rajan Chattamvelli and Ramalingam Shanmugam
July 2021

Glossary of Terms

Term	Meaning
ALD	Asymmetric Laplace Distribution
ANOR	Analysis Of Reciprocals
ANOVA	Analysis Of Variance
BSD	Birnbaum–Saunders Distribution
BSR	Birnbaum–Saunders Regression
ChF	Characteristic Function
CFAR	Constant False Alarm Rate
CLD	Classical Laplace Distribution
CST	Classical Student's T distribution
CT	Contingency Table
CV	Coefficient of Variation
DFR	Decreasing Failure Rate
DLD	Double Lomax Distribution
DoF	Degrees of Freedom
DSP	Digital Signal Processing
EVD	Extreme-Value Distribution
FSTD	Folded Student's T Distribution
GF	Generating Function
GPD	Generalized Pareto Distribution
HF	Hazard Function
IBF	Incomplete Beta Function
IDF	Inverse Distribution Function
IFR	Increasing Failure Rate
IGD	Inverse Gaussian Distribution
IID	Independently and Identically Distributed
INID	Independent, Not Identically Distributed
IRD	Inverse Rayleigh Distribution
IWD	Inverse Weibull Distribution
LBM	Linear Brownian motion
LBS	Log-Birnbaum–Saunders Distribution
LDL	Lower Detection Limit
LLD	Log-Laplace Distribution

xxii GLOSSARY OF TERMS

LND	Log-Normal Distribution
LRT	Likelihood Ratio Tests
LST	Log-Student's T Distribution
MBD	Maxwell-Boltzman Distribution
MD	Mean Deviation
MGF	Moment Generating Function
MLE	Maximum Likelihood Estimate
MoM	Method of Moments Estimate
MRI	Magnetic Resonance Imaging
MRL	Mean Residual Life
MVUE	Minimum Variance Unbiased Estimate
NCT	Noncentral T distribution
PDF	Probability Density Function
PFR	Percentage Failure Rate
PMD	Population Mahalanobis Distance
PMF	Probability Mass Function
RFM	Rayleigh Fading Model
RIG	Reciprocal Inverse Gaussian Distribution
SED	Standard Exponential Distribution
SF	Survival Function
SGD	Standard Gumbel Distribution
SkLD	Skew Laplace Distribution
SLD	Standard Laplace Distribution
SNR	Signal to Noise Ratio
SPRT	Sequential Probability Ratio Tests
SRD	Standard Rayleigh Distribution
STF	Student T Filters
TTF	Time To Failure
UAV	Unmanned Aerial Vehicles
UDL	Upper Detection Limit
WPP	Weibull Probability Plot
