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Remark about T-duality of Dp-branes

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Abstract: This note is devoted to the analysis of T-duality of Dp-brane when we perform T-duality along directions that are transverse to world-volume of Dp-brane.

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Contents

1 Introduction and summary

It is well known that T-duality is central property of string theory, for review, see for example [1]. Generally, if we consider string sigma model in the background with metric G_{MN} and NSNS two form B_{MN} together with dilation ϕ and this background possesses an isometry along $d-$directions we find that it is equivalent to string sigma model in T-dual background with dual fields $\tilde{G}_{MN}, \tilde{B}_{MN}$ and $\tilde{\phi}$ that are related to the original fields by famous Buscher's rules [2, 3] for generalization to more directions, see for example [4, 5].

It is well known that string theories also contain another higher dimensional objects that transform non-trivially under T-duality. In this note we focus on Dp-branes [6, 7], for more recent review, see [10]. Originally Dp-brane was defined with the open string description where the string embedding coordinates obey $p+1$ Neumann boundary conditions and $9-(p+1)-$Dirrichlet ones at the boundary of the string world-sheet [6].[1] It was also shown by Polchinski that Dp-brane transforms into D(p+1)-brane when T-duality is performed along direction transverse to world-volume of Dp-brane and Dp-brane transforms to D(p-1)-brane in case when T-duality is performed along direction that Dp-brane wraps. In other words Dp-brane transforms with very specific way under T-duality transformations.

On the other hand it is remarkable that many aspects of Dp-brane dynamics can be described by its low energy effective action which is famous Dirac-Born-Infeld action [6]. Then one can ask the question whether this description of Dp-brane dynamics could give correct description of T-duality transformation of Dp-brane. This situation is relatively straightforward when we perform T-duality along directions which Dp-brane wraps. This property is known as covariance of Dp-brane action under T-duality transformations as was previously studied in [8, 9]. We generalize this approach to the T-duality along more longitudinal directions in the next section.

It is important to stress that in order to show full covariance of Dp-brane action with respect to T-duality transformation we should also study how Dp-brane effective action changes when we perform T-duality along transverse directions to its world-volume. The goal of this paper is to perform such an analysis. Our approach is based on previous works

[1] We consider Dp-brane in supersymmetric Type IIA or Type IIB theories where the critical dimension of target space time is 10. Note that $p = 0, 2, 4, 6, 8$ for Type IIA theory while $p = 1, 3, 5, 7, 9$ for Type IIB theory.

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that consider description of N Dp-branes on the circle [12], for review see [11]. It was shown there that such a configuration should be described by infinite number of Dp-branes on $\mathbf{R}$ which is covering space of $\mathbb{S}^1$ when we impose appropriate quotient conditions [11]. Since this description was performed in the context of Matrix theory [13] the low energy effective action describing dynamics of $N-$ Dp-branes was Super Yang-Mills theory (SYM) defined on $p+1$ dimensional world-volume. Then it was shown in [12] that this theory transforms under T-duality into (SYM) defined on $p+2$ dimensional world-volume in T-dual background.

The goal of this paper is to generalize this analysis to the case of full DBI action for Dp-brane in the general background when we study T-duality along transverse directions. It is well known that such a T-duality can be defined when the target space-time fields do not depend on these coordinates explicitly. Since, following previous works, we should consider generalization of DBI action that describes infinite number of Dp-branes in covering space. Such an action is non-abelian generalization of DBI action that was introduced in [14]. Then we follow very nice analysis performed in [15]. Explicitly, we introduce quotient conditions and solve them in the same way as in [15]. We show that non-abelian action for infinite number of Dp-branes transforms to the action for D(p+d)-brane where $d-$ is number of T-dual directions in the T-dual background where T-dual background fields are related to the original one by generalized Buscher's rules [4, 5].

Let us outline our results. We study how Dp-brane transforms under T-duality we consider T-duality either along longitudinal or transverse directions to Dp-brane' world-volume. We show that in the first case it transforms do D(p-d)-brane while in the second one it transforms to D(p+d)-brane when all background fields transform according to generalized Buscher's rules. This fact nicely shows covariance of Dp-brane under T-duality transformations.

Certainly this paper can be extended in many directions. It would be certainly interesting to analyse transverse T-duality transformations in case of the Wess-Zumino term for Dp-brane that determines coupling of Dp-brane to Ramond-Ramond forms. Clearly we should start non-abelian generalization of this term given in [14] when we consider infinite number of Dp-branes on the covering space. This problem is currently under investigation. It would be also nice to analyse non-abelian T-duality on the world-volume of Dp-brane. We hope to return to this problem in future.

This paper is organized as follows. In the next section 2 we introduce Dp-brane action and study T-duality along longitudinal directions. Then in section 3 we consider T-duality along dimensions transverse to Dp-brane world-volume.

2 Longitudinal T-duality

In this section we review T-duality transformation of Dp-brane when we perform T-duality along $d-$ longitudinal directions. Explicitly, let us consider DBI action in the general background with the metric G_{MN}, B_{MN} and dilaton ϕ. This action has the form

$$S = -T_p \int d^{p+1}\xi e^{-\phi}\sqrt{-\det(G_{\alpha\beta}+B_{\alpha\beta}+\lambda F_{\alpha\beta})} \tag{2.1}$$

where

$$\lambda = 2\pi\alpha' , \quad T_p = \frac{2\pi}{\lambda^{(p+1)/2}} , \tag{2.2}$$

where we also defined pull back of G_{MN} and B_{MN} defined as

$$G_{\alpha\beta} = G_{MN}\partial_\alpha x^M \partial_\beta x^N , \quad B_{\alpha\beta} = B_{MN}\partial_\alpha x^M \partial_\beta x^N , \tag{2.3}$$

where $\xi^\alpha, \alpha = 0, 1, \ldots, p$ label world-volume directions of Dp-brane and where $x^M, M = 0, 1, \ldots, 9$ parametrize embedding of DBI action in the target space-time.

Now we would like to perform T-duality along last $d-$directions when we presume that there are directions which Dp-brane wraps. The fact that these directions are longitudinal mean that Dp-brane world-volume coordinates coincide with the target space ones. Explicitly we have

$$x^m = \xi^m , \quad m = 9 - d, \ldots, 9 . \tag{2.4}$$

Then we presume that all world-volume fields do not depend on $\xi^{\hat{\alpha}}$ only where $\hat{\alpha} = 0, 1, \ldots, p - d$. Let us also introduce matrix $E_{MN} = G_{MN} + B_{MN}$. Then we have

$$E_{\alpha\beta} + \lambda F_{\alpha\beta} = \begin{pmatrix} E_{\hat{\alpha}\hat{\beta}} + \lambda F_{\hat{\alpha}\hat{\beta}} & E_{\hat{\alpha}n} + \lambda\partial_{\hat{\alpha}} A_n \\ E_{m\hat{\beta}} - \lambda\partial_{\hat{\beta}} A_m & E_{mn} \end{pmatrix} , \tag{2.5}$$

where $E_{\hat{\alpha}\hat{\beta}} = E_{\mu\nu}\partial_{\hat{\alpha}} x^\mu \partial_{\hat{\beta}} x^\nu , \quad \mu, \nu = 0, 1, \ldots, 9 - d$. Then performing standard manipulation with determinant we obtain

$$\begin{aligned} \det(E_{\alpha\beta} + \lambda F_{\alpha\beta}) &= \det\left(E_{\hat{\alpha}\hat{\beta}} + \lambda F_{\hat{\alpha}\hat{\beta}} - (E_{\hat{\alpha}m} + \lambda\partial_{\hat{\alpha}} A_m)\tilde{E}^{mn}(E_{n\hat{\beta}} - \lambda\partial_{\hat{\beta}} A_n)\right) \det E_{mn} \\ &= \det\left(E_{\hat{\alpha}\hat{\beta}} - E_{\hat{\alpha}m}\tilde{E}^{mn}E_{n\hat{\beta}} + \lambda F_{\hat{\alpha}\hat{\beta}} + \lambda E_{\hat{\alpha}m}\tilde{E}^{mn}\partial_{\hat{\beta}} A_n \right. \\ &\qquad \left. -\lambda\partial_{\hat{\alpha}} A_m \tilde{E}^{mn} E_{n\hat{\beta}} + \lambda^2 \partial_{\hat{\alpha}} A_m \tilde{E}^{mn}\partial_{\hat{\beta}} A_n\right) \det E_{mn} , \end{aligned} \tag{2.6}$$

and where $\tilde{E}^{mn}$ is inverse to E_{mn} in the sense that $\tilde{E}_{mn}E^{nk} = \delta_m^k$. As the next step we define $T-$dual coordinates

$$\tilde{x}_m \equiv \lambda A_m . \tag{2.7}$$

Then we can write

$$\begin{aligned} E_{\hat{\alpha}\hat{\beta}} - E_{\hat{\alpha}m}\tilde{E}^{mn}E_{n\hat{\beta}} + \partial_{\hat{\alpha}} x_m \tilde{E}^{mn}\partial_{\hat{\beta}} x^n &= \partial_{\hat{\alpha}} x^\mu (E_{\mu\nu} - E_{\mu m}\tilde{E}^{mn}E_{n\nu})\partial_{\hat{\beta}} x^\nu + \partial_{\hat{\alpha}}\tilde{x}_m \tilde{E}^{mn}\partial_{\hat{\beta}}\tilde{x}_n , \\ E_{\hat{\alpha}m}\tilde{E}^{mn}\partial_{\hat{\beta}}\tilde{x}_n &= \partial_{\hat{\alpha}} x^\mu E_{\mu m}\tilde{E}^{mn}\partial_{\hat{\beta}}\tilde{x}_n , \\ -\partial_{\hat{\alpha}}\tilde{x}_m \tilde{E}^{mn}E_{n\hat{\beta}} &= -\partial_{\hat{\alpha}}\tilde{x}_m \tilde{E}^{mn}E_{n\nu}\partial_{\hat{\beta}} x^\nu , \end{aligned} \tag{2.8}$$

that can be interpreted as an embedding of D(p-d)-brane in T-dual background with the background fields

$$\begin{aligned} &\tilde{E}_{\mu\nu} = E_{\mu\nu} - E_{\mu m}E^{mn}E_{n\nu} , \\ &\tilde{E}_\mu{}^m = E_{\mu n}E^{nm} , \qquad\qquad \tilde{E}^m_{\ \nu} = -E^{mn}E_{n\nu} , \\ &e^{-\tilde{\phi}} = e^{-\phi}\det E_{mn} . \end{aligned} \tag{2.9}$$

Explicitly, the D(p-d)-brane action in T-dual background has the form

$$S = -T_{p-d} \int_0^{\sqrt{\lambda}} d^d\xi \int d^{p-d+1}\xi e^{-\tilde{\phi}} \sqrt{-\det(\tilde{E}_{\hat{\alpha}\hat{\beta}} + \lambda F_{\hat{\alpha}\hat{\beta}})}\,, \tag{2.10}$$

where

$$\tilde{E}_{\hat{\alpha}\hat{\beta}} = \tilde{E}_{\mu\nu}\partial_{\hat{\alpha}}x^\mu\partial_{\hat{\beta}}x^\nu + \tilde{E}_\mu{}^m\partial_{\hat{\alpha}}x^\mu\partial_{\hat{\beta}}\tilde{x}_m + \tilde{E}^m_\nu\partial_{\hat{\alpha}}\tilde{x}_m\partial_{\hat{\beta}}\tilde{x}^\nu + \partial_{\hat{\alpha}}\tilde{x}_m\tilde{E}^{mn}\partial_{\hat{\beta}}\tilde{x}^n\,, \tag{2.11}$$

and where we defined tension for D(p-d)-brane in the form

$$T_{p-d} = T_p \int_0^{\sqrt{\lambda}} d^d\xi = \lambda^{d/2} T_p\,. \tag{2.12}$$

Note that the transformation rules for T-dual fields given in (2.9) coincide with the results derived previously in [4, 5] and which are now derived independently using covariance of Dp-brane under T-duality transformations.

However in order to see consistency of T-duality covariance of Dp-branes we should also consider opposite situation when we consider Dp-brane in general background and perform T-duality along directions that are transverse to the world-volume of Dp-brane.

3 Transverse T-duality

In this section we consider opposite situation when we study Dp-brane in the background that has isometry along $d-$directions in the transverse space to Dp-brane world-volume. The best way how to describe such a Dp-brane in to consider infinite number of Dp-branes on the covering space of torus T^d which is $\mathbf{R}^d$ and impose appropriate quotient conditions. Further, we should also consider appropriate action for N Dp-branes which is famous Myers non-abelian action [14]

$$S = -T_p \mathrm{Str} \int d^{p+1}\xi e^{-\phi} \sqrt{-\det(P[E_{\alpha\beta}] + P[E_{\alpha r}E^{rs}((Q^{-1})^t_s - \delta^t_s)E_{t\beta}] + \lambda F_{\alpha\beta})\det Q^i_j}\,, \tag{3.1}$$

where $i,j,k,l,m,n,r,s,t,\ldots = p+1,\ldots,9$ label directions transverse to the world-volume of N Dp-branes. Note that the location of $N-$ Dp-branes in the transverse space is determined by $N\times N$ Hermitean matrices $\Phi^m, m = p+1,\ldots,9$ and all background fields depend on them so as for example $E_{\alpha\beta}(\Phi)$ and so on. We use convention where Φ^m are Hermitean matrices and field strength $F_{\alpha\beta}$ is defined as

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha + i[A_\alpha, A_\beta]\,, \tag{3.2}$$

where A_α is $N\times N$ Hermitian matrix corresponding to non-abelian gauge field. Finally, $P[E_{\alpha\beta}]$ is a pull-back of the background $E_{MN}(\Phi)$ defined as

$$P[E]_{\alpha\beta} = E_{\alpha\beta} + D_\alpha\Phi^r E_{r\beta} + E_{\alpha r}D_\beta\Phi^r + D_\alpha\Phi^r E_{rs}D_\beta\Phi^s\,, \tag{3.3}$$

where $D_\alpha\Phi^m$ is covariant derivative

$$D_\alpha\Phi^m = \partial_\alpha\Phi^m + i[A_\alpha, \Phi^m]\,. \tag{3.4}$$

Note that non-abelian action for N Dp-branes is implicitly defined in the static gauge where world-volume coordinates ξ^α coincide with the target space ones x^α. Finally Str means symmetrized trace and in order to describe infinite number of Dp-branes we should divide the action (3.1) by the infinite order the quotient group $\mathbf{Z}^d$.

Further, $Q^i{}_j$ is defined as

$$Q^i{}_j = \delta^i{}_j + i\lambda^{-1}[\Phi^i, \Phi^k]E_{kj} \tag{3.5}$$

and $(Q^{-1})^j{}_k$ is its inverse in the sense

$$Q^i{}_j(Q^{-1})^j{}_k = \delta^i_k \ . \tag{3.6}$$

Finally, $P[E_{\alpha r}E^{rs}((Q^{-1})^t_s - \delta^t_s)E_{t\beta}]$ is defined as

$$\begin{aligned} &P[E_{\alpha r}E^{rs}((Q^{-1})^t_s - \delta^t_s)E_{t\beta}] \\ &= E_{\alpha r}E^{rs}((Q^{-1})^t_s - \delta^t_s)E_{t\beta} + D_\alpha\Phi^m E_{mr}E^{rs}((Q^{-1})^t_s - \delta^t_s)E_{t\beta} \\ &\quad + E_{\alpha r}E^{rs}((Q^{-1})^t_s - \delta^t_s)E_{tk}D_\beta\Phi^k + D_\alpha\Phi^m E_{mr}E^{rs}((Q^{-1})^t_s - \delta^t_s)E_{tn}D_\beta\Phi^n \ , \end{aligned} \tag{3.7}$$

where E^{rs} is matrix inverse to E_{mr} defined as

$$E_{mr}E^{rs} = \delta^s_m \ . \tag{3.8}$$

In order to implement T-duality along d transverse directions we follow analysis performed in [15] which we generalize to the case of non-linear non-abelian action (3.1). Let us presume that the background fields do not depend on x^A coordinates, where $A = p+1, \dots, p+d$ and that these coordinates are periodic with period $\sqrt{2\pi\lambda}$. This is natural if we recognize that all geometrical properties of the background are encoded in the field E_{MN}. Then we consider an infinite number of Dp-branes on compact space with coordinates x^A when we impose following quotient conditions

$$\begin{aligned} &\mathcal{U}_B^{-1}\Phi^A\mathcal{U}_B = \delta^A_B\sqrt{\lambda} + \Phi^A \ , \\ &\mathcal{U}_B^{-1}\Phi^{i'}\mathcal{U}_B = \Phi^{i'} \ , \quad i' = p+d+1, \dots, 9 \ . \end{aligned} \tag{3.9}$$

Let us presume that solution of the quotient equation corresponds to operators $\mathcal{U}_A$ that commute

$$[\mathcal{U}_A, \mathcal{U}_B] = 0 \ . \tag{3.10}$$

In order to solve (3.9) it is natural to introduce an auxiliary Hilbert space on which Φ^A and $\mathcal{U}_B$ act. The simplest way is to introduce Hilbert space of auxiliary functions living on $d-$dimensional torus taking value in $\mathbf{C}^d$. Then we take $\mathcal{U}_A$ as generators of the functions on $d-$dimensional torus

$$\mathcal{U}_A = e^{i\lambda^{-1/2}\sigma_A} \tag{3.11}$$

where σ_A are coordinates on the covering space of torus. Then Φ^A has to be equal to

$$\Phi^A = -i\lambda(\partial^A - iA^A(\sigma_A)) \tag{3.12}$$

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since then

$$\mathcal{U}_B^{-1}\Phi^A\mathcal{U}_B=\lambda^{1/2}\delta^A_B+\Phi^A \ . \tag{3.13}$$

Using these results we can now proceed to write corresponding action in T-dual background. As the first step we perform following manipulation with the determinant in the action (3.1)

$$\begin{aligned}&\det(P[E_{\alpha\beta}]+P[E_{\alpha r}E^{rs}((Q^{-1})^t_s-\delta^t_s)E_{t\beta}]+\lambda F_{\alpha\beta})\det Q^i_j\\ &\quad=\det\begin{pmatrix}P[E]_{\alpha\beta}-P[E_{\alpha r}E^{rs}E_{s\beta}]+\lambda F_{\alpha\beta} & \mathbf{A}_\alpha{}^n\\ \mathbf{B}^m_\beta & Q^{mn}\end{pmatrix}\det E_{mn}\,,\end{aligned} \tag{3.14}$$

where $Q^{ij}=E^{ij}+i\lambda^{-1}[\Phi^i,\Phi^j]$ and where

$$\mathbf{A}_\alpha{}^n=D_\alpha\Phi^kE_{kr}E^{rn}+E_{\alpha r}E^{rn}\,,\quad \mathbf{B}^m_\beta=-E^{mk}E_{k\beta}-D_\beta\Phi^m \ . \tag{3.15}$$

First of all we observe that

$$P[E_{\alpha\beta}-E_{\alpha r}E^{rs}E_{s\beta}]=E_{\alpha\beta}-E_{\alpha r}E^{rs}E_{s\beta} \ . \tag{3.16}$$

To proceed further we use the fact that $D_\beta\Phi^A$ acting on arbitrary function $f(\sigma)$ defined on the space labelled by σ_A is equal to

$$D_\alpha\Phi^Af=\lambda(\partial_\alpha\Phi^A+i[A_\alpha,\Phi^A])f=\lambda(\partial_\alpha A^A-\partial^AA_\alpha)f\equiv\lambda F_\alpha{}^Af \tag{3.17}$$

and hence we can identify $D_\alpha\Phi^A$ with $\lambda F_\alpha{}^A$. Using this identification we obtain

$$\begin{aligned}&\mathbf{A}_\alpha{}^A=\lambda F_\alpha{}^A+E_{\alpha r}E^{rA}\,, \qquad\qquad \mathbf{A}_\alpha{}^{i'}=D_\alpha\Phi^{i'}+E_{\alpha r}E^{ri'} \ .\\ &\mathbf{B}^A_\beta=-E^{AB}E_{B\beta}-E^{Ai'}E_{i'\beta}-\lambda F_\beta{}^A\,,\\ &\mathbf{B}^{i'}{}_\beta=-E^{i'j'}E_{j'\beta}-E^{i'A}E_{A\beta}-\partial_\beta\Phi^{i'}\end{aligned} \tag{3.18}$$

and finally

$$\begin{aligned}&Q^{AB}=E^{AB}+i\lambda^{-1}[\Phi^A,\Phi^B]=E^{AB}+\lambda F^{AB}\,,\\ &Q^{Ai'}=E^{Ai}+\lambda\partial^A\Phi^{i'}\,,\qquad Q^{i'B}=E^{i'B}-\partial^B\Phi^{i'}\,,\qquad Q^{i'j'}=E^{i'j'}\,,\end{aligned} \tag{3.19}$$

where we used (3.12) so that

$$[\Phi^A,\Phi^B]=-i\lambda^2F^{AB}\,,\quad [\Phi^A,\Phi^{i'}]=-i\lambda\partial^A\Phi^{i'} \ . \tag{3.20}$$

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Now we return to the first determinant in (3.14) and rewrite it to the form

$$
\begin{aligned}
&\det\begin{pmatrix} P[E]_{\alpha\beta}-P[E_{\alpha r}E^{rs}E_{s\beta}]+\lambda F_{\alpha\beta} & \mathbf{A}_\alpha{}^n \\ \mathbf{B}^m{}_\beta & Q^{mn}\end{pmatrix} \\
&\quad =\det\begin{pmatrix} E_{\alpha\beta}-E_{\alpha r}E^{rs}E_{st}+\lambda F_{\alpha\beta} & \mathbf{A}_\alpha{}^B & \mathbf{A}_\alpha{}^{j'} \\ \mathbf{B}^A{}_\beta & Q^{AB} & Q^{Aj'} \\ \mathbf{B}^{i'}{}_\beta & Q^{i'B} & Q^{i'j'}\end{pmatrix} \\
&\quad =\det\begin{pmatrix} E_{\alpha\beta}-E_{\alpha r}E^{rs}E_{s\beta}+\lambda F_{\alpha\beta} & \mathbf{A}_\alpha{}^B & \mathbf{A}_\alpha{}^{j'} \\ \mathbf{B}^A{}_\beta-Q^{Ak'}(Q^{-1})_{k'i'}\mathbf{B}^{i'}{}_\beta & Q^{AB}-Q^{Ak'}(Q^{-1})_{k'i'}Q^{i'B} & 0 \\ \mathbf{B}^{i'}{}_\beta & Q^{i'B} & Q^{i'j'}\end{pmatrix} \\
&\quad =\det\begin{pmatrix} E_{\alpha\beta}-E_{\alpha r}E^{rs}E_{s\beta}-\mathbf{A}_\alpha{}^{i'}(Q^{-1})_{i'j'}\mathbf{B}^{j'}{}_\beta+\lambda F_{\alpha\beta} & \mathbf{A}_\alpha{}^B-\mathbf{A}_\alpha{}^{j'}(Q^{-1})_{i'j'}Q^{j'B} & 0 \\ \mathbf{B}^A{}_\beta-Q^{Ak'}(Q^{-1})_{k'i'}\mathbf{B}^{i'}{}_\beta & Q^{AB}-Q^{Ak'}(Q^{-1})_{k'i'}Q^{i'B} & 0 \\ \mathbf{B}^{i'}{}_\beta & Q^{i'B} & Q^{i'j'}\end{pmatrix} \\
&\quad \equiv\det\begin{pmatrix} \mathbf{D}_{\alpha\beta} & \mathbf{D}_\alpha{}^B & 0 \\ \mathbf{D}^A{}_\beta & \mathbf{D}^{AB} & 0 \\ \mathbf{B}^{i'}{}_B & Q^{i'B} & Q^{i'j'}\end{pmatrix}
\end{aligned}
\tag{3.21}
$$

Since $Q^{i'j'}=E^{i'j'}$ it is clear that the matrix inverse $(Q^{-1})_{i'j'}$ is equal to $(Q^{-1})_{i'j'}=\tilde{E}_{i'j'}$ where

$$
\tilde{E}_{i'k'}E^{k'l'}=\delta_{i'}^{l'}\,. \tag{3.22}
$$

Now we explicitly calculate components of the matrix $\mathbf{D}$ as

$$
\begin{aligned}
\mathbf{D}_{\alpha\beta}&=E_{\alpha\beta}-E_{\alpha r}E^{rs}E_{s\beta}+\partial_\alpha\Phi^{\ i'}\tilde{E}_{i'j'}E^{j'r}E_{r\beta}+\partial_\alpha\Phi^{i'}\tilde{E}_{i'j'}\partial_\beta\Phi^{j'} \\
&\quad +E_{\alpha r}E^{ri'}\tilde{E}_{i'j'}E^{j'k}E_{k\beta}+E_{\alpha r}E^{ri'}\tilde{E}_{i'j'}\partial_\beta\Phi^{j'}+\lambda F_{\alpha\beta} \\
&=E_{\alpha\beta}-E_{\alpha A}(E^{AB}-E^{Ai'}\tilde{E}_{i'j'}E^{j'B})E_{B\beta} \\
&\quad +\partial_\alpha\Phi^{\ i'}\tilde{E}_{i'j'}E^{j'r}E_{r\beta}+\partial_\alpha\Phi^{i'}\tilde{E}_{i'j'}\partial_\beta\Phi^{j'} \\
&\quad +E_{\alpha r}E^{ri'}\tilde{E}_{i'j'}\partial_\beta\Phi^{j'}+\lambda F_{\alpha\beta}\,.
\end{aligned}
\tag{3.23}
$$

To proceed further we observe that

$$
(E^{AB}-E^{Ai'}\tilde{E}_{i'j'}E^{j'B})E_{BC}=\delta^A_C y \tag{3.24}
$$

and hence we can identify expression in the bracket with the matrix inverse $\tilde{E}^{AB}$ to E_{AB} so that $\tilde{E}^{AB}E_{BC}=\delta^A_C$. Further, let us consider following expression

$$
E_{i'j'}-E_{i'A}\tilde{E}^{AB}E_{Bj'} \tag{3.25}
$$

and multiply it with $E^{j'k'}$. Then, after some calculations, we get

$$
(E_{i'j'}-E_{i'A}\tilde{E}^{AB}E_{Bj'})E^{j'k'}=\delta_{i'}^{k'} \tag{3.26}
$$

so that we can identify expression in the bracket with matrix $\tilde{E}_{i'j'}$

$$
\tilde{E}_{i'j'}=E_{i'j'}-E_{i'A}\tilde{E}^{AB}E_{Bj'}\,. \tag{3.27}
$$

Using these results we obtain following useful expressions

$$E^{Ai'}\tilde{E}_{i'j'}=-\tilde{E}^{AB}E_{Bj'}\,,\quad \tilde{E}_{i'j'}E^{j'B}=-E_{i'C}\tilde{E}^{CB} \tag{3.28}$$

and

$$\tilde{E}_{i'j'}E^{j'r}E_{r\beta}=E_{i'\beta}-E_{i'A}\tilde{E}^{AB}E_{B\beta}\,. \tag{3.29}$$

Using (3.28) and (3.29) we get

$$\begin{aligned}\mathbf{D}_{\alpha\beta}=&E_{\alpha\beta}-E_{\alpha A}\tilde{E}^{AB}E_{B\beta}+\partial_\alpha\Phi^{i'}(E_{i'\beta}-E_{i'A}\tilde{E}^{AB}E_{B\beta})\\&+(E_\alpha-E_{\alpha A}\tilde{E}^{AB}E_{Bj'})\partial_\beta\Phi^{j'}+\partial_\alpha\Phi^{i'}(E_{i'j'}-E_{i'A}\tilde{E}^{AB}E_{Bj'})\partial_\beta\Phi^{j'}\end{aligned} \tag{3.30}$$

and

$$\begin{aligned}\mathbf{D}^{AB}=&\tilde{E}^{AB}+\lambda F^{AB}+\partial^A\Phi^{i'}(E_{i'j'}-E_{i'A}\tilde{E}^{AB}E_{Bj'})\partial^B\Phi^{j'}\\&-\tilde{E}^{AC}E_{Cj'}\partial^B\Phi^{j'}+\partial^A\Phi^{i'}E_{i'C}\tilde{E}^{CB}\,.\end{aligned} \tag{3.31}$$

In the same way we obtain

$$\begin{aligned}\mathbf{D}_\alpha^{\ B}=&\lambda F_\alpha^{\ B}+E_{\alpha A}\tilde{E}^{AB}+\partial_\alpha\Phi^{i'}E_{i'C}\tilde{E}^{CB}\\&+\partial_\alpha\Phi^{i'}(E_{i'j'}-E_{i'A}\tilde{E}^{AB}E_{Bj'})\partial^B\Phi^{j'}+(E_{\alpha i'}-E_{\alpha C}\tilde{E}^{CD}E_{Di'})\partial^B\Phi^{i'}\,,\end{aligned} \tag{3.32}$$

and also

$$\begin{aligned}\mathbf{D}^A_{\ \beta}=&-\tilde{E}^{AB}E_{B\beta}-\lambda F_\beta^{\ B}-\tilde{E}^{AB}E_{Bj'}\partial_\beta\Phi^{j'}+\\&+\partial^A\Phi^{i'}(E_{i'\beta}-E_{i'B}\tilde{E}^{BC}E_{B\beta})+\partial^A\Phi^{i'}\tilde{E}_{i'j'}\partial_\beta\Phi^{j'}\,.\end{aligned} \tag{3.33}$$

Finally we consider following combinations of determinants that appear under square root in the action (3.1)

$$\begin{aligned}\det E_{mn}\det E^{i'j'}=&\det(E_{i'j'}-E_{i'A}\tilde{E}^{AB}E_{Bj'})\det E_{AB}\det E^{i'j'}\\=&\det\tilde{E}_{i'j'}\det E^{i'j'}\det E_{AB}=\det E_{AB}\,.\end{aligned} \tag{3.34}$$

Collecting these terms together we obtain final form of D(p+d)-brane action in T-dual background in the form

$$S=-\frac{T_p}{\lambda^{d/2}}\int d^{p+1}\xi d^d\sigma e^{-\tilde{\phi}}\sqrt{-\det\mathbf{D}}\,, \tag{3.35}$$

where we also used the relation between trace over infinite dimensional matrices and integration over coordinates σ

$$\mathrm{Tr}=\frac{1}{\lambda^{d/2}}\int d^d\sigma\ , \tag{3.36}$$

and where the matrix $\mathbf{D}$ has following components

$$\begin{aligned}\mathbf{D}_{\alpha\beta}=&\tilde{E}_{\alpha\beta}+\partial_\alpha\Phi^{i'}\tilde{E}_{i\beta}+\tilde{E}_{\alpha j'}\partial_\beta\Phi^{j'}+\partial_\alpha\Phi^{i'}\tilde{E}_{i'j'}\partial_\beta\Phi^{j'}+\lambda F_{\alpha\beta}\,,\\\mathbf{D}_\alpha^{\ B}=&\tilde{E}_\alpha^{\ B}+\partial_\alpha\Phi^{i'}\tilde{E}_{i'}^{\ B}+\tilde{E}_{\alpha j'}\partial^B\Phi^{j'}+\partial_\alpha\Phi^{i'}\tilde{E}_{i'j'}\partial^B\Phi^{j'}+\lambda F_\alpha^{\ B}\,,\\\mathbf{D}^A_{\ \beta}=&\tilde{E}^A_{\ \beta}+\tilde{E}^A_{\ j'}\partial_\beta\Phi^{j'}+\partial^A\Phi^{i'}\tilde{E}_{i'\beta}+\partial^A\Phi^{i'}\tilde{E}_{i'j'}\partial_\beta\Phi^{j'}-\lambda F_\beta^{\ A}\,,\\\mathbf{D}^{AB}=&\tilde{E}^{AB}+\partial^A\Phi^{i'}\tilde{E}_{i'j'}\partial^B\Phi^{j'}+\tilde{E}^A_{\ j'}\partial^B\Phi^{j'}+\partial^A\Phi^{i'}\tilde{E}_{i'}^{\ B}+\lambda F^{AB}\ ,\end{aligned} \tag{3.37}$$

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where we have following components of the background matrix $\tilde{E}$

$$
\begin{aligned}
&\tilde{E}_{\alpha\beta} = E_{\alpha\beta} - E_{\alpha A}\tilde{E}^{AB}E_{B\beta}\,, \quad && \tilde{E}_{i'\beta} = E_{i'\beta} - E_{i'A}\tilde{E}^{AB}E_{B\beta}\,, && \\
&\tilde{E}_{\alpha j'} = E_{\alpha j'} - E_{\alpha A}\tilde{E}^{AB}E_{Bj'}\,, \quad && \tilde{E}_{i'j'} = E_{i'j'} - E_{i'A}\tilde{E}^{AB}E_{Bj'}\,, && \\
&\tilde{E}_{\alpha}{}^{B} = E_{\alpha A}\tilde{E}^{AB}\,, && \tilde{E}_{i'}{}^{B} = E_{i'C}\tilde{E}^{CB}\,, && \tilde{E}_{i'j'} = E_{i'j'} - E_{i'A}\tilde{E}^{AB}E_{Bj'}\,, \\
&\tilde{E}_{\alpha i'} = E_{\alpha i'} - E_{\alpha C}\tilde{E}^{CD}E_{Di'}\,, && \tilde{E}^{A}{}_{\beta} = -\tilde{E}^{AB}E_{B\beta}\,, && \\
&\tilde{E}_{i'j'} = E_{i'j'} - E_{i'A}\tilde{E}^{AB}\tilde{E}_{Bj'}\,, && \tilde{E}^{A}{}_{j'} = -\tilde{E}^{AC}E_{Cj'}\,, && \tilde{E}_{i'}{}^{B} = E_{i'C}\tilde{E}^{CB}\,,
\end{aligned}
\tag{3.38}
$$

and where

$$
e^{-\tilde{\phi}} = e^{-\phi}\sqrt{-\det E_{AB}}\,. \tag{3.39}
$$

It is important to stress that resulting D(p+d)-brane propagates in T-dual background where the T-dual background is given by Buscher's rules that are given in equations in (2.11). More explicitly, note that we perform T-duality along directions labelled by $A, B, \dots$ where $A = p+1, \dots, p+d$ that should be identify with $m, n, \dots$ given in the section 2. In the same way $\alpha, \beta = 0, \dots, p$ and $i', j' = p+d+1, \dots, 9$ should be identified with μ, ν again given in section 2. Explicitly, if we denote $\mu, \nu, \dots = (\alpha, \beta, i', j', \dots)$ and $A, B, C, \dots, = m, n, k, \dots$, then we can rewrite (3.38) into the form

$$
\tilde{E}_{\mu\nu} = E_{\mu\nu} - E_{\mu m}\tilde{E}^{mn}E_{n\nu}\,, \quad \tilde{E}_{\mu}{}^{n} = E_{\mu m}\tilde{E}^{mn}\,, \quad \tilde{E}^{m}_{\ \nu} = -\tilde{E}^{mn}E_{n\nu}\,, \tag{3.40}
$$

which exactly match generalized T-duality rules given in (2.11) and hence fully proves covariance of Dp-brane action under T-duality transformations.

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