


# A Model of Consumer Choice With Bounded Rationality and Reference Quantity

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## ABSTRACT

The authors consider the economic problem of consumers' demand under the assumption of bounded rationality. They move from the hypothesis that the consumer does not know the amount of the good maximizing the utility but he/she is able to understand, at each time period, if the current consumption is above or below than the optimal choice and decide consequently for the next period. Moreover, they also take into consideration the role of a reference amount of consumption that can influence the consumption choice of the consumer. They show that this assumption led to a failure in the convergence process towards the rational choice because the equilibrium of the model, even when locally stable, is located on an intermediate value between the rational choice and the reference quantity.

## KEYWORDS

Behavioral Economics, Bifurcation Theory, Bounded Rationality, Chaos Theory, Consumer Theory, Dynamical Systems, Prospect Theory, Reference Quantity

## INTRODUCTION

The *Homo Oeconomicus* does not exist. It is a fictitious agent introduced by neoclassic economists to characterize a perfectly rational agent who always maximizes her/his objective function, using all the available information. Nevertheless, economic agents, as time passes, sometimes take decisions that are closer and closer to those that the *Homo Oeconomicus* would take. In some sense, they learn to behave as if they were perfectly rational. In these cases, neoclassical theories are good not only as positive theories but also as descriptive ones. Talking about firms, there can be an evolutive pressure selecting those firms whose decisions are more rational than those of the competitors (Alchian, 1950). This evolutionary explanation is not completely satisfying. It is nowadays well established that economic agents adopt trial-and-error methods to learn the better strategy, and they stop searching when they reach a satisfying output, not necessarily the optimal one. Economic agents are characterized by cognitive limits that have been studied from behavioral economists. They simplify complicated problems and use decisional heuristics to take decisions. The adoption of heuristics and the existence of cognitive biases are among the main features of real decision makers. This is even truer when we

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talk about consumers. For them, the Darwinian argument does not work, because they cannot leave the market, even if their choices are far from optimal.

In particular, one of the main discoveries of behavioral economics is that the decision about how much to consume of a good is not always absolute (that is the amount of good that makes equal the Marginal Rate of Substitution and the prices ratio in the case of two goods) but it can also be relative, in the sense that a consumer may also compare the amount of good he/she is consuming with the amount consumed by a reference group of consumers (friends, colleagues, etc...) in a way such that consumption choices (and not necessarily preferences) become interdependent (Hayakawa & Venieris (1977), Kapteyn et al. (1980), Hayakawa (2000), Janssen & Jager (2001)).

The aim of our work is to understand which role the presence of a reference group may play in the amount and in the possibility of convergence to the equilibrium quantity of good. We also want to find if the amount, high or low, of the reference quantity, can be relevant for the dynamic output of the consumption choice.

This work is organized as follows: in Section 2 we introduce the benchmark model of D'Orlando and Rodano (2006) where a boundedly rational consumer may learn the optimizing strategy under some suitable circumstances. In Section 3 we add a behavioral component to the picture, namely the possibility that an exogenously given reference quantity may influence the consumption decision. We study the position of the equilibrium and its stability properties through a mix of analytical and numerical results. Section 4 concludes and addresses future research work.

## THE ORIGINAL MODEL

D'Orlando and Rodano (2006) consider a consumer endowed with a Cobb-Douglas utility function:

$$U(x, y) = x^\alpha y^{1-\alpha} \quad (1)$$

where  $x$  and  $y$  are two goods, and  $\alpha \in [0, 1]$  measures the preference of the consumer for good  $x$ .

Considering the good  $y$  as *numeraire* and normalizing its price to 1, we have the following budget constraint:

$$px + y = m \quad (2)$$

where  $p$  is the price of the good  $x$  and  $m$  is the consumer's income.

The constrained maximization problem is solved by the quantities:

$$(\bar{x}, \bar{y}) = \left( \frac{\alpha m}{p}, (1 - \alpha) m \right) \quad (3)$$

The consumption of good  $x$  is adjusted from a period to the next one as follows: if the current consumption level is such that the Marginal rate of Substitution (MRS) is larger (resp. lower) than the prices ratio, then the consumer perceives that by increasing (resp. decreasing) the amount of  $x$  it is possible to increase the level of utility. The decisional mechanism is:

$$x' = x + \mu(MRS - p)$$

where:

$$MRS = \frac{\alpha}{1-\alpha} \frac{y}{x} = \frac{\alpha}{1-\alpha} \frac{m-px}{x}$$

so the dynamical system becomes:

$$x' = f(x) = x + \mu \left( \frac{\alpha}{1-\alpha} \frac{m-px}{x} - p \right) = x + \mu \left( \frac{\alpha}{1-\alpha} \frac{m}{x} - \frac{p}{1-\alpha} \right) \quad (4)$$

with  $\mu \geq 0$  as speed of adjustment.

D'Orlando and Rodano show that the nonlinear dynamical system (4) admits as equilibrium the maximizing quantity  $\bar{x}$ , which is locally asymptotically stable provided that:

$$\left| \frac{\mu p^2}{\alpha(\alpha-1)m} \right| < 2 \quad (5)$$

## THE MODEL WITH REFERENCE QUANTITY

In this paper, in order to increase the realism of the model, we add a behavioral component to the demand side. It is well known from the experiments conducted by behavioral economists that consumers, in making their consumption choices, do not only take into consideration their tastes but they may also be influenced by a reference level of consumption (Thaler 1985, Tversky & Kahneman 1991, Kahneman et al. 1991). An example is provided by people belonging to a reference group of people (colleagues, members of the family, friends...) whose consumption of the good can be considered a reference for the consumer who does not want to behave too differently from them. A classic example is the consumption of tobacco or alcohol among adolescents (Varela & Pritchard 2011, Grohmann & Sakha 2019). Even if some of them may not appreciate a high level of consumption of these products, the high consumption of peers may influence their behavior because they do not want to feel excluded by the group. So, their consumption may not reflect only their preference and taste for the product but also the effect of the reference level of consumption.

In order to capture this effect, we add a component to the demand function, such that the consumption of the good is increased (resp. decreased) in the next period if currently it is lower (resp. higher) than the reference one. We obtain the following dynamical system:

$$x' = g(x) = x + \mu \left( \frac{\alpha}{1-\alpha} \frac{m}{x} - \frac{p}{1-\alpha} \right) + h(x_r - x) \quad (6)$$

where parameter  $h \geq 0$  measures how relevant is the role played by the exogenously given reference consumption level  $x_r$ .

### Equilibria

Map (6), for feasible parameters' values, always admits two equilibria:

$$x_{1,2}^* = \frac{1}{2h} \left[ hx_r - \frac{\mu p}{1-\alpha} \pm \sqrt{\left( hx_r - \frac{\mu p}{1-\alpha} \right)^2 + 4 \frac{h\alpha m \mu}{1-\alpha}} \right]$$

We can note that:

1.  $\lim_{h \rightarrow 0} x_{1,2}^* = \frac{\alpha m}{p}$ , that is without the behavioural component we come back to the original model with the rational choice as unique equilibrium.
2.  $x_2^*$  is always negative, so unfeasible.
3. If  $x_r = \frac{\alpha m}{p}$  then  $x_1^* = x_r$ .
4. If  $x_r \neq \frac{\alpha m}{p}$  then:

$$\frac{\alpha m}{p} < x_1^* < x_r \text{ or } x_r < x_1^* < \frac{\alpha m}{p}$$

And in particular, the larger is  $h$ , the closer is the equilibrium  $x_1^*$  to the reference level  $x_r$ .

Point 4 is an important result, which implies that the behavioral component is able to move the equilibrium when the rational choice and the reference level are not coincident. In particular, the equilibrium is located in between these two values as if the consumer were stuck between the choice suggested by its own tastes and the choice of the reference group and he must find some balance. Parameter  $h$  permits to regulate the position of the equilibrium with respect to the two extrema.

### Stability of the Feasible Equilibrium

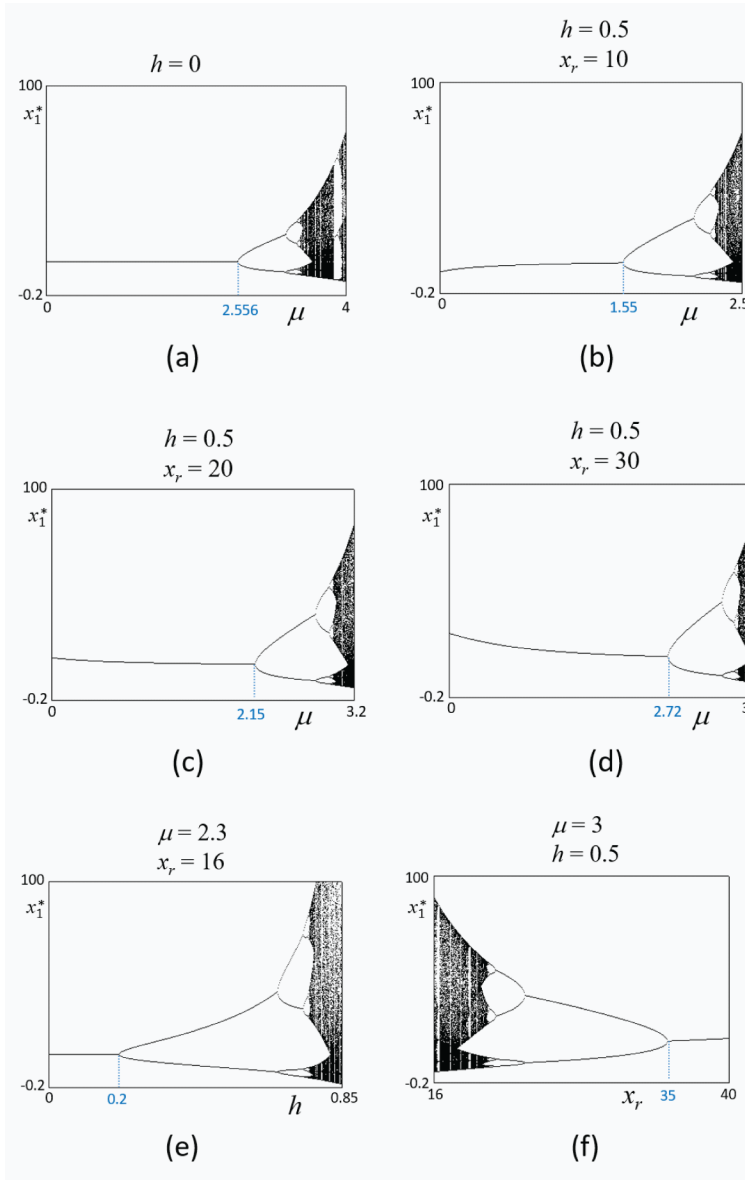
It is not easy to analytically study the stability of the feasible equilibrium  $x_1^*$ . Nevertheless, we can numerically compare the stability region in the parameters' space of the original model with respect to the behavioral one.

We move from a benchmark set of parameters:  $\alpha = 0.2$ ,  $p = 10$  and  $m = 800$ . The rational choice in this case is  $\bar{x} = 16$ . We know that by increasing the reactivity parameter  $\mu$  the equilibrium loses stability via flip bifurcation triggering a cascade of period-doubling bifurcations until chaos is reached. We want to investigate the role played by the behavioral parameters  $h$  and  $x_r$ .

Let us look at the bifurcation diagrams shown in Figure 1.

Figure 1(a) shows a bifurcation diagram of the model without the behavioral component. In our set of parameters, the steady state becomes unstable at a value of the reactivity parameter close to 2.556. In Figure (b) we have inserted the behavioral parameter ( $h = 0.5$ ) and a reference quantity lower than the rational choice. In this case, the flip bifurcation of the steady state occurs at a lower value of  $\mu$  (close to 1.55). Figures 1(c) and (d) are obtained with higher reference quantities (20 and 30 respectively). We can see that the higher is the reference quantity, the more stable is the equilibrium. For  $x_r = 2.72$  the bifurcation value of the speed of adjustment is even higher than the corresponding value without behavioral component. To deepen the role of the two parameters we have computed two more bifurcation diagrams. In Figure 1(e) we have detected the destabilizing role of parameter  $h$  by fixing  $\mu = 2.3$  and  $x_r$  equal to the rational choice (i.e. 16). So looking at this graph we can

Figure 1. Bifurcation diagrams



draw a first conclusion: the more the consumer is influenced by a reference level of consumption, the more difficult is to converge to the equilibrium. The interpretation of this result is quite intuitive. In fact in the behavioral model, the consumer is between two fires (at least when the reference quantity is not equal to the rational choice). On the hand there are his tastes and on the other there is the reference level. To find a balance is more difficult in this case than when there are only the tastes and the only problem is not being too reactive.

Figure 1(f), obtained by keeping fixed  $\mu = 3$  and  $h = 0.5$ , is more puzzling because it proves that the larger is the reference level of consumption, the more larger should be the reactivity to get the flip bifurcation of the equilibrium. A possible interpretation is the following: when there is a large

discrepancy between the rational choice and the reference consumption level, it is easier for the consumer to leave aside his tastes and focus almost exclusively on the feeling of not being too different with respect to peers.

## **CONCLUSION AND FUTURE RESEARCH**

We have studied a model of consumption choice of a boundedly rational consumer, who does not know its optimal quantity and, at the same time, is influenced by the consumption of a reference group.

We proved that in this case, as time passes, the choice may converge to a quantity that does not only reflect his/her tastes but also the will of being inserted in the reference group. In this case it is not possible to infer the preferences of consumers for a good only by looking at the consumption choices. They also reflect the interdependence with other consumers. Moreover, if this behavioural feature is accentuated (by increasing the parameter  $h$ ), convergence to an equilibrium may fail, and consumption choices may vary erratically. The consumer may not learn the equilibrium quantity representing a compromise between his tastes and the consumption of a reference group. At the same time, if the discrepancy between the consumption driven by tastes and the one driven by the others is quite large, convergence may be facilitated. These results confirm that the effects of behavioural features are not easily predictable. Finally, if the speed of adjustment ( $\mu$ ) is too high, also in this case the convergence process may fail.

This work can be extended in several directions. We can make endogenous the reference quantity and see what happens to this variable if it is influenced by the consumption of the consumer we consider. At the same time we can refine the behavioral component of the model by considering loss aversion, so by introducing an asymmetry in the way the consumer reacts with respect to what is classified as a loss or as a gain. The model can also be extended by considering more than one consumer.

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## APPENDIX

### Position of the Equilibrium in the Behavioural Model

We consider the behavioural model formalized in map (6).

If we assume that  $x_r > \frac{\alpha m}{p}$  then we can write  $x_r = \frac{\alpha m}{p}(1 + \tau)$ , with  $\tau > 0$ . We have that:

$$g(x_r) = x_r - \mu \frac{p}{(1 - \alpha)} \frac{\tau}{(1 + \tau)} < x_r$$

while:

$$g\left(\frac{\alpha m}{p}\right) = \frac{\alpha m}{p} + h\left(x_r - \frac{\alpha m}{p}\right) > \frac{\alpha m}{p}$$

So, for the intermediate value theorem, there exists a point  $\frac{\alpha m}{p} < x^* < x_r$  such that:

$$g(x^*) = x^*$$

and this is the equilibrium of map (6).

Similarly, if we assume that  $x_r < \frac{\alpha m}{p}$  then we can write  $x_r = \frac{\alpha m}{p}(1 - \tau)$ , with  $\tau > 0$ . We have that:

$$g(x_r) = x_r + \mu \frac{p}{(1 - \alpha)} \frac{\tau}{(1 + \tau)} > x_r$$

while:

$$g\left(\frac{\alpha m}{p}\right) = \frac{\alpha m}{p} + h\left(x_r - \frac{\alpha m}{p}\right) < \frac{\alpha m}{p}$$

So, for the intermediate value theorem, there exists a point  $x_r < x^* < \frac{\alpha m}{p}$  such that:

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