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Inverse Scattering and Loaded Modified Korteweg-de Vries Equation

Michal Fečkan*
Mathematical Institute of the Slovak Academy of Sciences
Bratislava, Slovakia

Gayrat Urazboev†
Urgench state university
Urgench, Uzbekistan
Institute of Mathematics, Khorezm Branch, Uzbekistan Academy of Sciences
Urgench, Uzbekistan

Iroda Baltaeva‡
Urgench state university
Urgench, Uzbekistan
Khorezm Mamun Academy
Khorezm region, Khiva, Uzbekistan

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Abstract. The Cauchy problem for the loaded modified Korteweg-de Vries equation in the class of "rapidly decreasing" functions is considered in this paper. The main result of this work is a theorem on the evolution of the scattering data of the Dirac operator. Potential of the operator is the solution to the loaded modified Korteweg-de Vries equation. The obtained equalities allow one to apply the method of the inverse scattering transform to solve the Cauchy problem for the loaded modified Korteweg-de Vries equation.

Keywords: loaded modified KdV equation, inverse scattering method, "rapidly decreasing" functions, soliton, evolution of the scattering data.

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Introduction

The study of non-linear waves in the viscoelastic tube is of interest, since system of such tubes is a model of the vessels of the blood circulatory system. Understanding the wave processes in the blood circulatory system can help predict the development of diseases [5].

In arterial mechanics, a widely used model assumes that artery is a thin-walled pre-stressed elastic tube with a variable radius (or with a stenosis) and blood is considered as an ideal fluid [3].

*Michal.Feckan@fmph.uniba.sk https://orcid.org/0000-0002-7385-6737
†gayrat71@mail.ru https://orcid.org/0000-0002-7420-2516
‡iroda-b@mail.ru https://orcid.org/0000-0001-5624-7529

The governing equation that models weakly non-linear waves in such fluid-filled elastic tubes is the modified Korteweg–de Vries equation

$$u_t - 6u^2 u_x + u_{xxx} - h(t) u_x = 0,$$

where t is a scaled coordinate along the axis of the vessel after static deformation characterizing axisymmetric stenosis on the surface of the arterial wall; x is a variable that depends on time and coordinates along the axis of the vessel; $h(t)$ is a form of stenosis and $u(x,t)$ characterizes the average axial velocity of the fluid.

Let us assume that form of stenosis $h(t)$ is proportional to $u(0,t)$ and consider the following loaded modified Korteweg-de Vries equation

$$u_t - 6u^2 u_x + u_{xxx} - \gamma(t) u(0,t) u_x = 0, \tag{1}$$

where $u = u(x,t)$ is unknown real value function ($x \in R,\ \ t \geqslant 0$), and $\gamma(t)$ is arbitrary continuous function. Equation (1) is considered with initial condition

$$u|_{t=0} = u_0(x), \tag{2}$$

where real value function $u_0(x)$ has the following properties:

1. $\int\limits_{-\infty}^{\infty} (1+|x|)\,|u_0(x)|\,dx \ < \infty.$
2. The equation $L(0)\,y \equiv \begin{pmatrix} i\dfrac{d}{dx} & u_0 \\ u_0 & -i\dfrac{d}{dx} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \xi \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \quad x \in R^1$ has N simple eigenvalues and it does not have spectral singularities.

Let us assume that function $u(x,t)$ is sufficiently smooth, tends to its limit rapidly enough when $x \to \pm\,\infty$, and it satisfies the condition

$$\int_{-\infty}^{\infty} \left((1+|x|)\,|u(x,t)| + \sum_{j=1}^{3} \left| \frac{\partial^j u(x,t)}{\partial x^j} \right| \right) dx \ < \infty,\ t \geqslant 0. \tag{3}$$

Note that completed integrability of the modified Korteweg-de Vries (mKdV) equation was established in the class of "rapidly decreasing" functions using the method of the inverse scattering problem [11]. The evolution equations for non-linear waves which differ by small terms from equations soluble by the inverse scattering method (KdV, NSE, mKdV) were considered [4]. A perturbation theory scheme was formulated. It is based on the inverse scattering method. The term "loaded equation" was introduced by A. M. Nakhushev [7]. The most general definition of a loaded equation was given and various loaded equations were classified in detail. Loaded differential equations, the loaded part of which contains only the value of the desired solution at fixed points of the domain were considered [2, 9, 12].

The goal of this paper is to study the integration of the loaded mKdV equation in the class of "rapidly decreasing" functions in terms of inverse scattering problem.

1. Uniqueness of the solution

In this part we use the method given in [6].

Theorem 1. *If problem* (1)–(2) *has solution then it is unique.*

Proof. Let $v(x,t)$ be another solution of (1)–(2). Let us introduce $w(x,t) = (u(x,t) - v(x,t))_x$. Then we obtain

$$
\begin{aligned}
w_t &= 6[(u-v)(u_x^2+v_x^2) + (u+v)(u_x+v_x)w] + \\
&+ 3[(u-v)(u+v)(u+v)_{xx} + (u^2+v^2)w_x] - w_{xxx} + \\
&+ \frac{\gamma(t)}{2}\left((u(0,t)+v(0,t)w_x + (u(0,t)-v(0,t)(u+v)_{xx}\right).
\end{aligned} \tag{4}
$$

Multiplying (4) by ω and integrating with respect to x over $(-\infty,\infty)$, we have

$$
\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int_{-\infty}^{\infty} w^2dx = 6\int_{-\infty}^{\infty}(u-v)(u_x^2+v_x^2)wdx + 6\int_{-\infty}^{\infty}(u+v)(u_x+v_x)w^2dx + \\
&+ 3\int {}_{-\infty}^{\infty}(u-v)(u+v)(u+v)_{xx}wdx + 3\int_{-\infty}^{\infty}(u^2+v^2)w_xwdx - \int_{-\infty}^{\infty} w_{xxx}wdx + \\
&+ \frac{\gamma(t)}{2}(u(0,t)-v(0,t))\int_{-\infty}^{\infty}(u+v)_{xx}wdx + \frac{\gamma(t)}{2}(u(0,t)+v(0,t))\int_{-\infty}^{\infty} w_xwdx.
\end{aligned} \tag{5}
$$

Let us denote $\max(u_x^2+v_x^2)$ by m, $\max|(u+v)(u_x+v_x)|$ by n, $\max|(u+v)(u+v)_{xx}|$ by k, $\max\left|(u^2+v^2)_x\right|$ by l, $\max|u(0,t)+v(0,t)|$ by p and $\max|(u+v)_{xx}|$ by q. Using the Cauchy–Schwarz inequality we obtain from (5) the following inequality

$$
\begin{aligned}
\frac{d}{2dt}\int_{-\infty}^{\infty} w^2dx &\leqslant 6m\sqrt{\int_{-\infty}^{\infty}(u-v)^2dx}\sqrt{\int_{-\infty}^{\infty} w^2dx} + 6n\int_{-\infty}^{\infty} w^2dx + \\
&+3k\sqrt{\int_{-\infty}^{\infty}(u-v)^2dx}\sqrt{\int_{-\infty}^{\infty} w^2dx} + + \frac{3l}{2}\int_{-\infty}^{\infty} w^2dx + \frac{1}{2}\int_{-\infty}^{\infty}(w_x^2)_xdx + \\
&+\frac{q\gamma(t)}{2}\max|u-v|\sqrt{\int_{-\infty}^{\infty} w^2dx} + + \frac{\gamma(t)}{2}p\sqrt{\int_{-\infty}^{\infty} w_x^2dx}\sqrt{\int_{-\infty}^{\infty} w^2dx}.
\end{aligned}
$$

Here it was taken into account that w and its derivatives tend to zero as $x \to \pm\,\infty$. There are constants m_1, $k_1 > 0$ such that [8]

$$
\sqrt{\int_{-\infty}^{\infty}(u-v)^2dx} \leqslant m_1\sqrt{\int_{-\infty}^{\infty}(u-v)_x^2dx}, \qquad \max|u-v| \leqslant k_1\sqrt{\int_{-\infty}^{\infty}(u-v)_x^2dx}.
$$

Then we derive

$$
\begin{aligned}
\frac{d}{2dt}\int_{-\infty}^{\infty} w^2dx &\leqslant 6mm_1\sqrt{\int_{-\infty}^{\infty} w^2dx}\sqrt{\int_{-\infty}^{\infty} w^2dx} + 6n\int_{-\infty}^{\infty} w^2dx + \\
&+3km_1\sqrt{\int_{-\infty}^{\infty} w^2dx}\sqrt{\int_{-\infty}^{\infty} w^2dx} + \frac{3l}{2}\int_{-\infty}^{\infty} w^2dx + \\
&+\frac{qk_1\gamma(t)}{2}\sqrt{\int_{-\infty}^{\infty} w^2dx}\sqrt{\int_{-\infty}^{\infty} w^2dx}.
\end{aligned}
$$

Let us denote $\int\limits_{-\infty}^{\infty} \omega^2dx$ by $E(t)$ and $(12mm_1 + 12n + 6km_1 + 3l + qk_1\gamma(t))$ by $C(t)$,

$$
\frac{dE(t)}{dt} \leqslant C(t)E(t).
$$

This differential inequality yields

$$E(t) \leqslant E(0) \exp \int_0^t C(s) ds,$$

which implies that if $E(0) = 0$ then $E(t) = 0$ and thereby

$$w(x,t) = (u(x,t) - v(x,t))_x = 0$$

,

$$u(x,t) - v(x,t) = C.$$

Assuming $t = 0$, we obtain $C = 0$. Theorem 1 is proved. □

2. Scattering problem

Let us consider the following system of equation

$$Lv \equiv \begin{pmatrix} i\dfrac{d}{dx} & u \\ u & -i\dfrac{d}{dx} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \xi \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}, \quad -\infty < x < \infty, \tag{6}$$

with real value function $u(x)$ that satisfies the condition of "rapid decrease"

$$\int_{-\infty}^{\infty} (1 + |x|) \, |u(x)| \, dx < \infty. \tag{7}$$

** The present section contains well known information on the direct and inverse scattering problem for problem (6)–(7) that is required for further consideration [1]. Condition (7) implies that system of equation (6) has the Jost solutions $\varphi(x,\xi)$ and $\psi(x,\xi)$ with asymptotic relations

$$\begin{matrix} \varphi \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp(-i\xi x), \\ \bar{\varphi} \sim \begin{pmatrix} 0 \\ -1 \end{pmatrix} \exp(i\xi x), \end{matrix} \quad as \ \ x \to -\infty, \tag{8}$$

$$\begin{matrix} \psi \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix} \exp(i\xi x), \\ \bar{\psi} \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp(-i\xi x), \end{matrix} \quad as \ \ x \to \infty. \tag{9}$$

For real ξ, pairs $\{\varphi, \bar{\varphi}\}$ and $\{\psi, \bar{\psi}\}$ are pairs of linearly independent solutions of equation (6). Therefore,

$$\begin{cases} \varphi = a(\xi)\bar{\psi} + b(\xi)\psi, \\ \bar{\varphi} = -\bar{a}(\xi)\psi + \bar{b}(\xi)\bar{\psi}. \end{cases} \tag{10}$$

The following equality holds

$$a(\xi) = W\{\varphi, \psi\} \equiv \varphi_1\psi_2 - \varphi_2\psi_1 \tag{11}$$

and for all real ξ

$$a(\xi)\bar{a}(\xi)+b(\xi)\bar{b}(\xi)=1.$$

Function $a(k)$ admits analytic continuation into the upper half-plane $\operatorname{Im} k>0$. In $\operatorname{Im} k \geqslant 0$ function $a(k)$ has asymptotic behavior $a(\xi)=1+O\left(\frac{1}{|\xi|}\right)$. Function $a(k)$ can have a finite number of zeroes ξ_k, $k=1,2,\ldots,N$ in the upper half-plane $\operatorname{Im} k>0$. Zeros ξ_k of function $a(k)$ correspond to the eigenvalues of operator L in the upper half-plane. Let us note that operator L can have spectral singularities which are in the continuous spectrum.

We suppose that operator L does not have spectral singularities and zeros of function $a(k)$ are simple:

$$\varphi(x,\xi_k)=C_k\psi(x,\xi_k),\ k=1,2,\ldots,N. \tag{12}$$

The set $\left\{r^+(\xi)\equiv\dfrac{b(\xi)}{a(\xi)},\ \xi_k,C_k,\ k=1,2,3,\ldots,N\right\}$ is called scattering data for system of equations (6).

The following representation for the solution $\psi(x,\xi)$ is valid

$$\psi=\begin{pmatrix}0\\1\end{pmatrix}e^{i\xi x}+\int_x^\infty \mathbf{K}\ (x,s)\,e^{i\xi\ s}ds, \tag{13}$$

where $\mathbf{K}\ (x,s)=\begin{pmatrix}K_1(x,s)\\K_2(x,s)\end{pmatrix}$ does not depend on variable ξ and it is related to the potential function $u(x)$ as follows

$$u(x)=2iK_1(x,x)\,. \tag{14}$$

The components of kernel $K(x,y)$ for $y>x$ are solutions of the following Gelfand-Levitan-Marchenko (GLM) system of equation

$$\begin{cases} K_2(x,y)+\displaystyle\int_x^\infty K_1(x,s)F(s+y)ds=0,\\ -K_1(x,y)+F(x+y)+\displaystyle\int_x^\infty K_2(x,s)F(s+y)ds=0,\end{cases} \tag{15}$$

where $F(x)=\dfrac{1}{2\pi}\displaystyle\int_{-\infty}^\infty \frac{b(\xi)}{a(\xi)}e^{i\xi x}d\xi-i\sum_{j=1}^N C_je^{i\xi_j x}$.

3. Evolution of scattering data

It is easy to verify that functions

$$h_n(x)=\frac{\frac{d}{d\xi}\left(g(x,\xi)-B_nf(x,\xi)\right)\Big|_{\xi=\xi_n}}{\dot{a}(\xi_n)},\qquad n=1,2,3,\ldots,N \tag{16}$$

are solutions of the system of equations $Ly=\xi_ny$. Using (11) for $\operatorname{Im}\xi>0$, we define the following asymptotic relations

$$\psi\sim a(\xi)\begin{pmatrix}0\\1\end{pmatrix}\exp(i\xi x)\quad as\quad x\to-\infty,$$
$$\varphi\sim a(\xi)\begin{pmatrix}1\\0\end{pmatrix}\exp(-i\xi x)\quad as\quad x\to\infty.$$

Using these asymptotic relations, we obtain asymptotic relations for solutions $h_n(x)$

$$\begin{aligned} h_n(x) &\sim -C_n \begin{pmatrix} 0 \\ 1 \end{pmatrix} \exp(i\xi_n x) \quad as \quad x \to -\infty, \\ h_n(x) &\sim \begin{pmatrix} 0 \\ 1 \end{pmatrix} \exp(-i\xi_n x) \quad as \quad x \to \infty, \end{aligned} \tag{17}$$

and

$$W\{\varphi_n, h_n\} \equiv \varphi_{n1} h_{n2} - \varphi_{n2} h_{n1} = -C_n, \quad n = 1, 2, 3, \ldots, N,$$

where $\varphi_n = \varphi(x, \xi_n)$.

Let function $u(x,t)$ in (6) be a solution of the mKdV equation

$$u_t - 6u^2 u_x + u_{xxx} = G(x,t), \tag{18}$$

where function $G(x,t)$ is sufficiently smooth and $G(x,t) = o(1)$ when $x \to \pm\infty$, $t \geqslant 0$. Equation (18) is considered with initial condition (2). According to [10], the following Theorem is valid.

Theorem 2. *If function $u(x,t)$ is a solution of equation* (18) *in the class of functions* (3) *then the scattering data of system* (6) *with function $u(x,t)$ depend on t as follows*

$$\frac{dr^+}{dt} = 8i\xi^3\, r^+ - \frac{i}{a^2} \int_{-\infty}^{\infty} G(\varphi_1^2 + \varphi_2^2) dx, \quad \operatorname{Im} \xi = 0,$$

$$\frac{dC_n}{dt} = \left(8i\xi_n^3 - i \int_{-\infty}^{\infty} G\left(h_{n1}\psi_{n1} + h_{n2}\psi_{n2}\right) dx \right) C_n,$$

$$\frac{d\xi_n}{dt} = \frac{\int\limits_{-\infty}^{\infty} G(\varphi_{n1}^2 + \varphi_{n2}^2) dx}{2 \int\limits_{-\infty}^{\infty} \varphi_{n1}\varphi_{n2} dx}, \quad n = 1,\ 2,\ \ldots,\ N.$$

Here $\varphi_n(x,t)$ are normalized eigenfunctions which correspond to the eigenvalue ξ_n of system of equations (6).

Let us apply the result of Theorem 2 when

$$G(x,t) = \gamma(t) u(0,t) u_x.$$

According to Theorem 2, we have the following representation

$$\frac{dr^+}{dt} = 8i\xi^3\, r^+ - \frac{i\gamma(t)u(0,t)}{a^2} \int_{-\infty}^{\infty} u_x(\varphi_1^2 + \varphi_2^2) dx, \quad \operatorname{Im} k = 0.$$

By virtue of system of equations (6) and asymptotic relations (9), we have

$$\begin{aligned} \int_{-\infty}^{\infty} u_x(\varphi_1^2 + \varphi_2^2) dx &= -2 \int_{-\infty}^{\infty} \left[\varphi'_{1x} \left(i\varphi'_{2x} + \xi\varphi_2 \right) + \varphi'_{2x} \left(-i\varphi'_{1x} + \xi\varphi_1 \right) \right] dx = \\ &= -2 \int_{-\infty}^{\infty} \xi (\varphi_1 \cdot \varphi_2)'_x dx = -2\xi\, a(\xi) b(\xi). \end{aligned}$$

Consequently, for $\operatorname{Im} k = 0$ we obtain

$$\frac{dr^+}{dt} = \left(8i\xi^3 + 2i\xi\, \gamma(t) u(0,t) \right)\, r^+. \tag{19}$$

From relations

$$\frac{d\xi_n}{dt} = \frac{\gamma(t)u(0,t)\int\limits_{-\infty}^{\infty} u_x(\varphi_{n1}^2 + \varphi_{n2}^2)dx}{2\int\limits_{-\infty}^{\infty} \varphi_{n1}\varphi_{n2}dx}, \qquad n = 1,2,3,\dots,N,$$

and

$$\int_{-\infty}^{\infty} u_x(\varphi_{n1}^2 + \varphi_{n2}^2)dx == -2\int_{-\infty}^{\infty} \left[\varphi'_{n1}\left(i\varphi'_{n2} + \xi_n\varphi_{n2}\right) + \varphi'_{n2}\left(-i\varphi'_{n1} + \xi_n\varphi_{n1}\right)\right]dx =$$
$$= -2\xi_n\left(\varphi_{n1}\cdot\varphi_{n2}\right)\big|_{-\infty}^{\infty} = 0,$$

we have

$$\frac{d\xi_n}{dt} = 0\,, \qquad n = 1,2,3,\dots,N. \tag{20}$$

Using system of equations (6) and asymptotic relations (17), we have

$$\int_{-\infty}^{\infty} u_x\left(h_{n1}\psi_{n1} + h_{n2}\psi_{n2}\right)dx = -\xi_n\int_{-\infty}^{\infty}\left[\left(h_{n1}\psi_{n2}\right)'_x + \left(h_{n2}\psi_{n1}\right)'_x\right]dx = -\xi_n.$$

Taking into account the last expression, we obtain

$$\frac{dC_n}{dt} = \left(8i\xi_n^3 + i\gamma(t)u(0,t)\xi_n\right)C_n, \quad n = 1,2,3,\dots,N. \tag{21}$$

Considering relations (19), (20) and (21), we arrive to the following theorem.

Theorem 3. *If function $u(x,t)$ is a solution of problem (1)–(3) then the scattering data of system of equations (6) with function $u(x,t)$ depend on t as follows*

$$\frac{dr^+}{dt} = \left(8i\xi^3 + 2i\xi\,\gamma(t)u(0,t)\right)\, r^+ \text{ for } \operatorname{Im}\xi = 0,$$
$$\frac{dC_n}{dt} = \left(8i\xi_n^3 + i\gamma(t)u(0,t)\xi_n\right)C_n,$$
$$\frac{d\xi_n}{dt} = 0\,, \quad n = 1,2,3,\dots,N.$$

The obtained relations completely determine the evolution of the scattering data for system of equations (6). It allows us to find the solution of problem (1)–(3) by using the method of inverse scattering problem.

Example. Let us consider the following Cauchy problem:

$$u_t - 6u^2u_x + u_{xxx} - \gamma(t)u(0,t)u_x = 0,$$

$$u\,|_{t=0} = \frac{2}{\operatorname{sh} 2x},$$

where $\gamma(t) = -\dfrac{1}{4\sqrt{1+t^2}}\operatorname{sh}(-8t + \operatorname{arcsh} t)$.

To find the general solution of this problem we use the method of inverse scattering problem. First of all, let us find a solution of the direct problem for the following system of equations

$$L(0)y \equiv \begin{pmatrix} i\dfrac{d}{dx} & u_0 \\ u_0 & -i\dfrac{d}{dx} \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \xi\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}.$$

In this case, we have the following scattering data

$$r^{+}(0)=0, \quad N=1, \;\; \xi(0)=i, \;\; C_1=2.$$

According to Theorem 3, we find the evolution of scattering data depending on t :

$$r^{+}(\xi,t)=0, \quad \xi_1(t)=i, \;\; C_1(t)=2\exp\,\delta(t),$$

where

$$\delta(t)=8t-\int_0^t \gamma(\tau)u(0,\tau)dx.$$

Then using this scattering data, we find a solution of inverse scattering problem . Solving the GLM system of equations with $F(x)=-2i\exp\left(-x+\delta(t)\right)$, we obtain

$$K_1(x,y)=-\frac{2i\exp(-x-y+\delta(t))}{1-\exp(-4x+2\delta(t))}\,.$$

Applying equality (14), we obtain

$$u(x,t)=\frac{2}{\operatorname{sh}\left(2x-8t+\int\limits_0^t \gamma(\tau)u(0,\tau)d\tau\right)}\,.$$

Putting $x=0$ and introducing $f(t)=\int\limits_0^t \gamma(\tau)u(0,\tau)dx$, we obtain the following Cauchy problem

$$\begin{cases} \dfrac{f'(t)}{\gamma(t)}=\dfrac{2}{\operatorname{sh}(f(t)-8t)}\,, \\ f(0)=0. \end{cases}$$

Solving this problem with $\gamma(t)=-\dfrac{1}{4\sqrt{1+t^2}}\operatorname{sh}(-8t+\operatorname{arcsh}t)$, we have

$$f(t)=\operatorname{arcsh}t.$$

As a result, the solution of problem under consideration is expressed as follows

$$u(x,t)=\frac{2}{\operatorname{sh}(2x-8t+\operatorname{arcsh}t)}\,.$$

It is well known that solution of the modified Korteweg-de Vries equation

$$u_t-6u^2u_x+u_{xxx}=0,$$

that satisfies the same initial condition has the form

$$u(x,t)=\frac{2}{\operatorname{sh}(2x-8t)}.$$

The difference between solutions of the loaded modified Korteweg-de Vries equation and the modified Korteweg-de Vries equation is shown in Fig. 1.

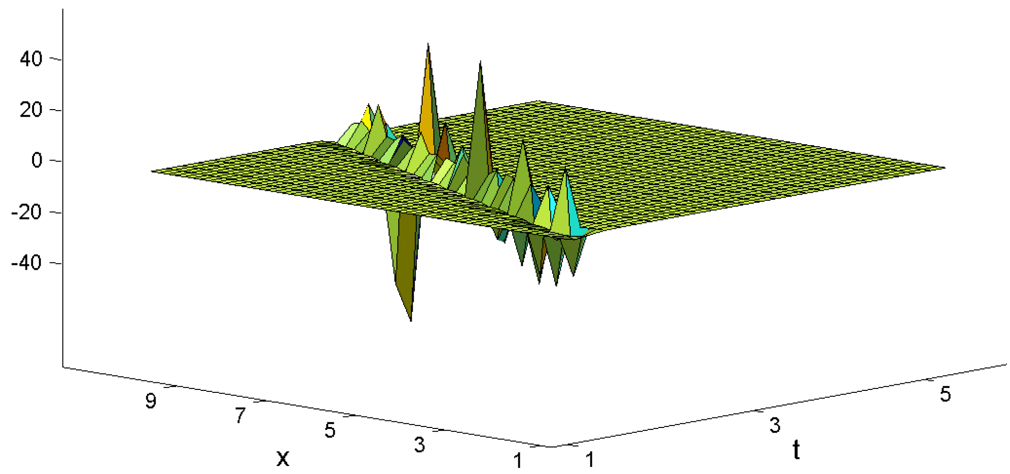

Fig. 1

Conclusion

The method of inverse scattering problem can be used to obtain solutions of the Cauchy problem for the loaded modified Korteweg-de Vries equations in the class of "rapidly decreasing" functions. Function $u(0,t)$ that appears in equations of Theorem 3 is unknown in contrast to function $u(x,0)$. If the scattering data is used to find potential $u(x,t)$ then function $u(0,t)$ is included in the solution. Therefore, we have a functional equation relating $u(x,t)$ to $u(x,0)$ which is reduced to the Cauchy problem for an ordinary differential equation of the first order. For some $\gamma(t)$ the Cauchy problem for ODE can be solved exactly and we obtain a solution of the Cauchy problem for the loaded mKdV equation.

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Метод обратной задачи рассеяния и нагруженное модифицированное уравнение Кортевега-де Фриза

Михал Фечкан
Математический институт Словацкой академии наук
Братислава, Словакия

Гайрат Уразбоев
Ургенчский государственный университет
Ургенч, Узбекистан
Институт математики Хорезмского отделения Академии наук Узбекистана
Ургенч, Узбекистан

Ирода Балтаева
Ургенчский государственный университет
Ургенч, Узбекистан
Хорезмская Академия Мамуна
Хорезмская область, Хива, Узбекистан

Аннотация. В данной статье мы рассматриваем задачу Коши для нагруженного модифицированного уравнения Кортевега-де Фриза в классе «быстроубывающих» функций. Основной результат настоящей работы представляет собой теорему об эволюции данных рассеяния оператора Дирака, потенциал которого является решением нагруженного модифицированного уравнения Кортевега-де Фриза. Полученные равенства позволяют применить метод обратной задачи рассеяния для решения задачи Коши для нагруженного модифицированного уравнения Кортевега-де Фриза.

Ключевые слова: нагруженное модифицированное уравнение КдФ, метод обратной задачи рассеяния, "быстроубывающие" функции, солитонное решение, эволюция данных рассеяния.