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A Piecewise Linearization Approach to Non-Convex and Non-Smooth Combined Heat and Power Economic Dispatch

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Abstract- *The important role of electricity generation in the power system is evident and is growing more and more with innovative technologies and requirements. Hence, addressing the combined heat and power economic dispatch (CHPED) as one of the relatively new issues in the power system operation and control is more importance. Since the CHPED problem is a non-smooth, highly non-linear, and non-convex one, it is required to solve it so that an optimal global solution can be achieved. In this paper, by applying the piece-wise linearization approach the CHPED problem is solved so that the problem reformulated to a quadratic optimization problem with linear and quadratic constraints. To demonstrate the applicability of the proposed model, four case studies are implemented in the GAMS software environment and the results compared to the literature.*

Keyword: Combined heat and power economic dispatch, Quadratic optimization, piecewise linearization, Non-convex problem.

NOMENCLATURE

i	Index of power-only units
j	Index of CHP units
k	Index of heat-only units
N_p	Number of power-only units
N_c	Number of CHP units
N_h	Number of heat-only units
$\alpha_i, \beta_i, \gamma_i$	Cost coefficients of *i*th power-only unit
λ_i, ρ_i	Coefficients of *i*th power-only unit for reflecting valve-point loading effect
a_j, b_j, c_j, d_j, e_j, f_j	Cost coefficients of *j*th CHP unit
a_k, b_k, c_k	Cost coefficients of *k*th heat-only unit
$P_i^{PO,\min}$, $P_i^{PO,\max}$	Minimum and maximum power generation boundaries of the *i*th power-only unit
P_d	Total load demand (MW)
B_{im}, B_{ij}, B_{in}	*B* matrixes in the Kron's loss formula
H_d	Total system heat demand (MW)
$H_k^{HO,\min}$, $H_k^{HO,\max}$	Minimum and maximum heat generation boundaries of the heat units
$P_j^{CHP,\min}$, $P_j^{CHP,\max}$	Minimum and maximum power generation boundaries of the CHP units
$H_j^{CHP,\min}$, $H_j^{CHP,\max}$	Minimum and maximum heat generation boundaries of the CHP units
$C_i(P_i^{PO})$	Production cost of the *i*th power-only unit
$C_j(P_j^{CHP})$	Production cost of the *j*th CHP unit
$C_k(P_k^{HO})$	Production cost of the *k*th heat-only unit
P_i^{PO}	Output power of the *i*th power-only unit
P_j^{CHP}	Output power of the *j*th CHP unit
H_k^{HO}	Heat production of the *k*th heat-only unit
H_j^{CHP}	Heat production of the *j*th CHP unit
P_{Losses}	Power losses of transmission lines

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1. INTRODUCTION

1.1. Motivation and aim

Combined heat and power (CHP) is an energy efficient technology that generates electricity and captures the heat that would otherwise be wasted to provide useful thermal energy such as steam or hot water that can be applied for space heating, cooling, domestic hot water and industrial processes. CHP can be located at an individual facility or building, or be a district energy or utility resource. CHP is typically located at facilities where there is a need for both electricity and thermal energy. The common configuration of a CHP is shown in Fig. 1 [1]. The use of CHP units is strongly supported by the regulation in many countries because of the energy efficiency and environmental benefits. Its role in microgrids, energy hubs, and power parks is becoming more and more important in the need to disseminate power production and shorten the distance between locations where energy is converted and used. These technologies show their potentials in urban areas where the concern about environment is higher and a new urbanization of large cities is challenging energy needs [2, 3]. Proper use of energy resources and cost reductions are significant things that have not diminished over time but have become increasingly important due to the energy crisis and rising fuel carrier prices. In most power plants, due to the high losses and low efficiency, as well as high losses of electrical transmission and distribution networks, the use of combined heat and power (CHP) units is highly welcomed.

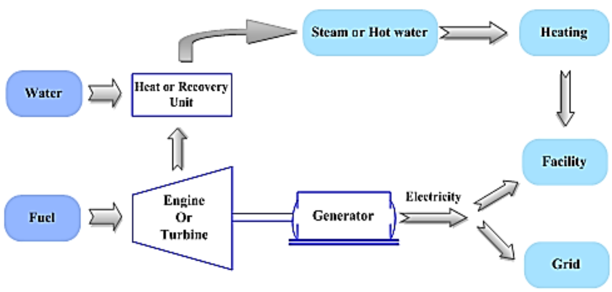

Fig. 1. Common configuration of a CHP

In the usual way, electrical needs of industrial units, commercial, office and residential buildings are provided from the electrical grid, and also their thermal requirements are provided in the same place by conventional methods, including the use of heat boilers, while, according to Fig. 1, using both the heat and power generation system can provide both forms of energy using as a single high-efficiency system. With these explanations, the combined heat and power economic dispatch (CHPED) problem is one of the most important problem in power system operation and control [4]. The aim of this problem is to determine the optimal heat and power of generating units subject to technical constraints so that the cost of generation is minimized [5, 6]. Since the CHPED problem is a non-convex, non-smooth, and highly non-linear, it is important to obtain an optimal global solution so that this solution does not fit into local optimum points. So, this paper aims to convert the CHPED problem to a quadratic optimization problem so that the global optimal solution is achieved. To demonstrate the proposed model, it is applied to four case studies and compared with literature.

1.2. Literature review and contributions

The CHPED problem refers to an optimization procedure in which the objective is the fuel cost minimization subject to all equality and inequality constraints related to power-only unit(s), co-generation (CHP) unit(s), and heat-only unit(s). As reported in the literature, there are three challenges making the CHPED a complex problem [2]:

- Valve-point loading effects related to the power-only units: generally, the conventional fuel cost function of the thermal power generation units is expressed as a quadratic function of active power outputs. Multi-valve steam turbines in large steam turbine generators produce a rippling effect on the input–output characteristic which is known as “valve-point loading effect”. Thus, the generation unit output is not always smooth.
- System power losses: this term arises from power system resistances and is added to the power balance equation. Losses will play a major role when applying these techniques in the dispersed generation which refers to low and medium voltage grids.
- Mutual dependency on the heat and power production related to the CHP units: in co-generation units, the power production capacity of most units depends on the heat generation.

Consequently, the CHPED problem is a highly nonlinear, non-smooth, and non-convex one in which powerful optimization methods are required to solve it, to avoid trapping in local optimum solution and obtain the global system optimum point. So, many methods have been used to solve this problem that category to mathematical and meta-heuristic optimization methods. In recent years, most of the studies focus on solving CHPED problem with mathematical optimization method. In Ref. [7], a two level model has been proposed for solving CHPED problem that the follower level specifies the output of units under Lagrange multipliers and the leader level by a Newton-based iterative process, updates the multipliers and then by applying Kuhn-Tucker conditions the problem is solved. In Ref. [8], the CHPED is divided into two problems so that one states power dispatch and another heat dispatch and then these problems interconnected by power-heat feasible region constraints for CHP units. In mathematical optimization approaches (classical methods) which are proposed for the solution of the CHPED problem, various gradient-based methods have been also used to solve the problem. A two-level Lagrangian relaxation (LR) with the surrogate sub-gradient multiplier updating technique is introduced in Ref. [9]. A mixed integer linear programming (MILP) is presented in Refs. [10, 11]. An algorithm based on benders decomposition (BD) is used in Ref. [2]. Other examples are dual partial-separable programming method [7], and the Lagrange relaxation method [8]. A more recent approach is semi-definite programming (SDP) method, which is proposed in Ref. [13]. Such method gives the optimal solution in the case of convex problems, and SDP relaxation of non-convex problem provides a good computable bound to the optimal value. A numerical method is proposed in Ref. [14], which utilized a direct analytical method to CHPED problem. Another method is reported in Ref. [15], which used the generalized reduced gradient technique with the branch-and-bound (B&B) approach. Most of these numerical approaches give almost the same cost values, but the corresponding computational times and burdens are different. These methods are robust and fast. However, they may get trapped in a local minimum and they are ineffective for non-convex and non-smooth CHPED problem [16]. So, in conjunction with mathematical models for CHPED problem, it should be noted that these models are convex while the nature of CHPED problem is non-convex. So, heuristic and meta-heuristic have been applied to solve the non-convex CHPED problem. In general, it can be said that since the CHPED problem is highly non-linear and non-convex, the main focus in solving this problem is based on heuristic and meta-heuristic optimization methods. So, various intelligent optimization methods, including genetic algorithm (GA) [17-19], real-coded genetic algorithm (RCGA) [20], classic particle swarm optimization (CPSO) [21], particle swarm optimization (PSO) [20], particle swarm optimization with time varying acceleration coefficients (PSO-TVAC) [21], harmony search algorithm (HSA) [22, 23], gravitational search algorithm (GSA) [2], invasive weed optimization algorithm (IWOA) [24], bee colony optimization (BCO) [20], artificial immune system (AIS) [25], group search algorithm (GSA) [26], teaching learning based optimization (TLBO) [27], Oppositional teaching learning based optimization (OTLBO) [27], firefly algorithm (FA) [28], ant colony search algorithm (ACSA) [29], exchange market algorithm (EMA) [30], evolutionary programming approach (EP) [31], Tabu search (TS) [32], differential evaluation (DE) [33], and cultural algorithm (CA) [34], have been applied to successfully solve the CHPED problem. The two main problems of meta-heuristic algorithms are trapping at local optimum points and early convergence to these points [35, 36]. In other words, these algorithms have been based on a random number and an operator for absorbing random numbers in order to select optimum numbers. In fact, in these methods by producing random numbers, the optimal solutions have been found. So, because of the randomized nature of these methods, they may face constraints and problems like trapping in local optimal points and consequently premature convergence and not being able to extract the optimal global solution. In Ref. [37], a mixed integer quadratic programming (MIQP) model for economic load dispatch considering prohibited operating zones has been proposed which the original model was a non-convex problem. This MIQP model can specify of the globally optimal solutions in all cases. In Ref. [38] a two-stage model is proposed to handle the non convexity and non differentiability of valve-point effects in cost functions of power-only units. The proposed model obtains a convex feasible operating region in the first stage using a linear approximation model, and an equivalent formulation is used in the second stage to handle the non convexity term of valve-point effects using the obtained convex feasible operating regions.

In this paper, a new algorithm based on the linearization to solve CHPED problem is used and different test systems are selected to verify the accuracy and efficiency of the applied method. So, in this paper for considering the CHPED problem mathematically, the CHPED problem is reformulated to a MIQP model by a piece-wise linearization approach, applying convex feasible operating regions as reviewed in Ref. [38], and by using the GAMS software environment the proposed model is easily implemented to show the robustness and efficiency of the proposed model. The main feature of the proposed solving approach is its ability in solving quite large CHPED problems yielding economical benefits with regard to the other tested algorithms reaching a better optimum solution with good convergence characteristics. It should be noted that the contribution in this area derives from the capability of the algorithm in being robust, i.e. always capable of finding a good quality solution without convergence problems and mostly yielding a better optimum which results in economical benefits which is our main performance indicator.

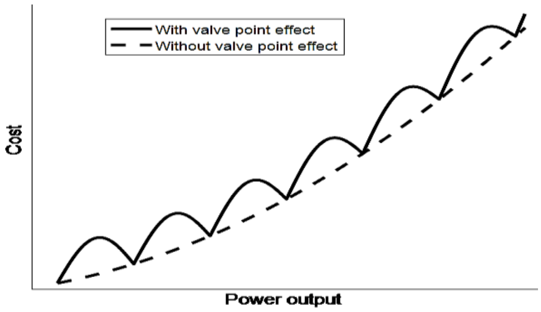

Fig. 2. Comparison of cost function with valve effect and that without valve effect

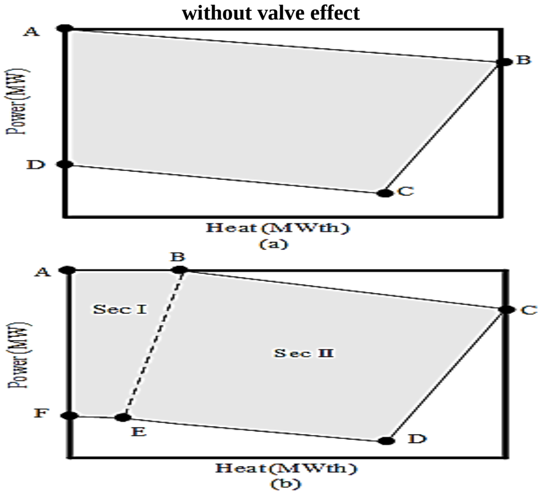

Fig. 3. Heat-power feasible operation region of a CHP unit

1.3. Paper organization

The rest of this paper is organized as follows. The non-linear and non-convex problem is presented in Section 2. Method description for reformulated the CHPED model to a MIQP is presented in section 3. Numerical results are reported and discussed in section 4. Finally, the conclusions are given in section 5.

2. PROBLEM FORMULATION

In this section, the non-convex objective function of the CHPED problem with its technical constraints is presented.

2.1. Objective function

In general, the objective function of CHPED problem is to determine the generating of powe-only (PO) units, the heat-only (HO) units, and CHP units such that the system's generation cost is minimized while the heat and power demands and some technical constraints are satisfied appropriately, as follows:

$$\min \begin{cases} \sum_{i=1}^{N_p} C_i(P_i^{\mathrm{PO}}) + \sum_{j=1}^{N_c} C_j(P_j^{\mathrm{CHP}}, H_j^{\mathrm{CHP}}) + \\ \sum_{k=1}^{N_h} C_k(H_k^{\mathrm{HO}}) \quad (\$/h) \end{cases} \tag{1}$$

where $C_i(P_i^{\mathrm{PO}})$, $C_j(P_j^{\mathrm{CHP}}, H_j^{\mathrm{CHP}})$, and $C_k(H_k^{\mathrm{HO}})$ are cost function of power-only units, CHP units, and heat-only units, respectively. These generation costs of different unit types are define as:

$$C_i(P_i^{\mathrm{PO}}) = \alpha_i (P_i^{\mathrm{PO}})^2 + \beta_i P_i^{\mathrm{PO}} + \gamma_i \quad (\$/h) \tag{2}$$

$$C_j(P_j^{\mathrm{CHP}}, H_j^{\mathrm{CHP}}) = a_j (P_j^{\mathrm{CHP}})^2 + b_j P_j^{\mathrm{CHP}} + c_j + d_j (H_j^{\mathrm{CHP}})^2 + e_j H_j^{\mathrm{CHP}} + f_j P_j^{\mathrm{CHP}} H_j^{\mathrm{CHP}} \quad (\$/h) \tag{3}$$

$$C_k(H_k^{\mathrm{HO}}) = a_k (H_k^{\mathrm{HO}})^2 + b_k H_k^{\mathrm{HO}} + c_k \quad (\$/h) \tag{4}$$

In a practical production unit, valve-point loading effect lead to a ripple in the generation cost. To model the valve-point loading effect more accurately a sinusoidal term is added to the conventional cost function $C_i(P_i^{\mathrm{PO}})$. So, Eq. (5) is applied to take the valve-point loading effect into consideration in the objective function of the conventional thermal units (power-only units) instead of Eq. (2) as follows:

$$C_i(p_i^{\mathrm{PO}}) = \alpha_i (p_i^{\mathrm{PO}})^2 + \beta_i p_i^{\mathrm{PO}} + \gamma_i + \left| \lambda_i \sin(\rho_i (P_i^{\mathrm{PO,min}} - P_i^{\mathrm{PO}})) \right| \quad (\$/h) \tag{5}$$

In Fig. 2, the classic quadratic objective function of a power-only unit (Eq. (2)) and the objective function associated with Eq. (5) (considering valve loading effect) has been shown. In fact, the sinusoidal term is the main cause of non-convexity and non-smoothness of the CHPED problem.

2.2. Constraints

2.2.1. Power and heat balance equation

According to Eq. (6), the total generated power of the CHP units and power-only units must be equal to total system demand plus power losses. It should be noted that, the power losses can be calculated by Kron's loss formula [39] which presented in Eq. (7).

$$\sum_{i=1}^{N_p} p_i^{\mathrm{PO}} + \sum_{j=1}^{N_c} P_j^{\mathrm{CHP}} = P_d + P_{Losses} \tag{6}$$

$$P_{Losses} = \sum_{i=1}^{N_p} \sum_{m=1}^{N_p} P_i^{\mathrm{PO}} B_{im} P_m^{\mathrm{PO}} + \sum_{i=1}^{N_p} \sum_{j=1}^{N_c} P_i^{\mathrm{PO}} B_{ij} P_j^{\mathrm{CHP}} + \sum_{i=1}^{N_c} \sum_{n=1}^{N_c} P_j^{\mathrm{CHP}} B_{in} P_n^{\mathrm{CHP}} \tag{7}$$

In Eq. (7), depending of the B_{ij} matrix, the feasible region for an optimal solution may be convex or non-convex. It should be noted that, if the B_{ij} matrix is a positive semi-definite matrix, this quadratically constrained programming (QCP) model is convex and the optimal global solution is achievable. Otherwise, only locally optimal answers are guaranteed. According to Eq. (8), totally produced heat of CHP units and heat-only units must be equal to total demand heat to balance the heat demand.

$$\sum_{j=1}^{N_c} H_j^{\text{CHP}} + \sum_{k=1}^{N_h} H_k^{\text{HO}} = H_d \quad (8)$$

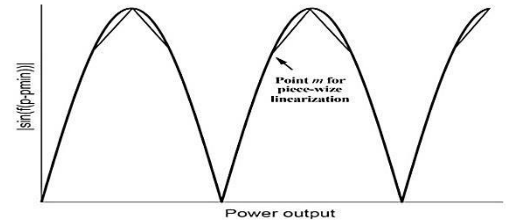

Fig. 4. Piece-wise linearization approach of valve loading effect by four linear segments for a sample unit

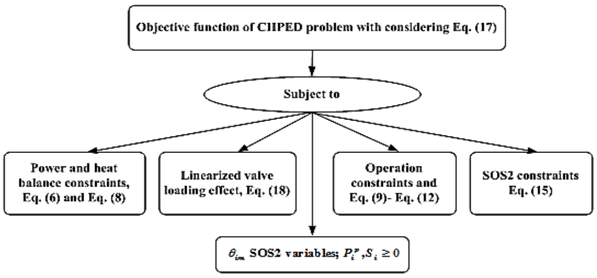

Fig. 5. The flowchart and problem solving procedure of the model

2.2.2. Operational limits

The outputs of heat and electricity units are limited by their own lower and upper boundaries. The output heat and power of CHP units (cogeneration units) must be placed in the feasible operation region as shown in Fig. 3. There are two types of feasible region for CHP units which are shown in Fig. 3. According to Fig. (3-a), the feasible region can be characterized using Eqns. (9)-(13) [40]. The Eqns. (9)-(11) are used to cover under the curve $\overline{\text{AB}}$, upper the curve $\overline{\text{BC}}$, and upper the curve $\overline{\text{CD}}$, respectively. If the unit j is a decommitted then the output power would be zero. Also, according to Eq. (12) and Eq. (13), the power and heat production for a decommitted unit must be set to zero, respectively.

$$P_j^{\text{CHP}} - P_{j,A}^{\text{CHP}} - \frac{P_{j,A}^{\text{CHP}} - P_{j,B}^{\text{CHP}}}{H_{j,A}^{\text{CHP}} - H_{j,B}^{\text{CHP}}}(H_j^{\text{CHP}} - H_{j,A}^{\text{CHP}}) \le 0 \quad (9)$$

$$P_j^{\text{CHP}} - P_{j,B}^{\text{CHP}} - \frac{P_{j,B}^{\text{CHP}} - P_{j,C}^{\text{CHP}}}{H_{j,B}^{\text{CHP}} - H_{j,C}^{\text{CHP}}}(H_j^{\text{CHP}} - H_{j,B}^{\text{CHP}}) \ge 0 \quad (10)$$

$$P_j^{\text{CHP}} - P_{j,C}^{\text{CHP}} - \frac{P_{j,C}^{\text{CHP}} - P_{j,D}^{\text{CHP}}}{H_{j,C}^{\text{CHP}} - H_{j,D}^{\text{CHP}}}(H_j^{\text{CHP}} - H_{j,C}^{\text{CHP}}) \ge 0 \quad (11)$$

$$0 \le P_j^{\text{CHP}} \le P_{j,A}^{\text{CHP}} \quad (12)$$

$$0 \le H_j^{\text{CHP}} \le H_{j,B}^{\text{CHP}} \quad (13)$$

$$P_j^{\text{CHP}} - P_{j,B}^{\text{CHP}} - \frac{P_{j,B}^{\text{CHP}} - P_{j,C}^{\text{CHP}}}{H_{j,B}^{\text{CHP}} - H_{j,C}^{\text{CHP}}}(H_j^{\text{CHP}} - H_{j,B}^{\text{CHP}}) \le 0 \quad (14)$$

$$P_j^{\text{CHP}} - P_{j,C}^{\text{CHP}} - \frac{P_{j,C}^{\text{CHP}} - P_{j,D}^{\text{CHP}}}{H_{j,C}^{\text{CHP}} - H_{j,D}^{\text{CHP}}}(H_j^{\text{CHP}} - H_{j,C}^{\text{CHP}}) \ge 0 \quad (15)$$

$$P_j^{\text{CHP}} - P_{j,E}^{\text{CHP}} - \frac{P_{j,E}^{\text{CHP}} - P_{j,F}^{\text{CHP}}}{H_{j,E}^{\text{CHP}} - H_{j,F}^{\text{CHP}}}(H_j^{\text{CHP}} - H_{j,E}^{\text{CHP}}) \ge -(1 - X_1) \times M \quad (16)$$

$$P_j^{\text{CHP}} - P_{j,D}^{\text{CHP}} - \frac{P_{j,D}^{\text{CHP}} - P_{j,E}^{\text{CHP}}}{H_{j,D}^{\text{CHP}} - H_{j,E}^{\text{CHP}}}(H_j^{\text{CHP}} - H_{j,D}^{\text{CHP}}) \ge -(1 - X_2) \times M \quad (17)$$

$$H_j^{\text{CHP}} - H_{j,E}^{\text{CHP}} \ge -(1 - X_2) \times M \quad (18)$$

$$H_j^{\text{CHP}} - H_{j,E}^{\text{CHP}} \le (1 - X_1) \times M \quad (19)$$

$$X_1 + X_2 = 1 \quad (20)$$

$$0 \le P_j^{\text{CHP}} \le P_{j,A}^{\text{CHP}} \quad (21)$$

$$0 \le H_j^{\text{CHP}} \le H_{j,B}^{\text{CHP}} \quad (22)$$

According to Fig. (3-b), the feasible region which is a non-convex area, can be divided into two convex area as Sec I and Sec II. This non-convex are is handled by implementing binary variables X_1 and X_2. According to Fig. (3-b), the feasible region can be characterized using Eqns. (14)-(22) [40]. The Eqns. (14)-(17) are used to cover under the curve $\overline{\text{BC}}$, upper the curve $\overline{\text{CD}}$, upper the curve $\overline{\text{EF}}$, and upper the curve $\overline{\text{DE}}$, respectively. In Eqns. (18) - (20), $X_1 = 1$ or $X_2 = 1$ means that the CHP unit operates in the first or second convex section of feasible region. Also, according to Eq. (21) and Eq. (22), the power and heat production for a decommitted unit must be set to zero, respectively. It should be noted that M is a sufficient large number, and indices A, B, C, D, E and F are marginal points of the feasible region. Operational restricts of production limits of heat-only units as Eq. (23), power-only units as Eq. (24), and capacity limits of CHP units as Eq. (25) and Eq. (26) represent the inequality constraints of the CHPED problem.

$$H_k^{\text{HO,min}} \le H_k^{\text{HO}} \le H_k^{\text{HO,max}} \quad k = 1, 2, ..., N_h \quad (23)$$

$$P_i^{\text{PO,min}} \le P_i^{\text{PO}} \le P_i^{\text{PO,max}} \quad i = 1, 2, ..., N_p \quad (24)$$

$$P_j^{\text{CHP,min}}(H_j^c) \le P_j^{\text{CHP}}(H_j^{\text{CHP}}) \le P_j^{\text{CHP,max}}(H_j^{\text{CHP}}) \quad \forall\, j = 1, 2, ..., N_c \quad (25)$$

$$H_j^{\text{CHP,min}}(P_j^{\text{CHP}}) \le H_j^{\text{CHP}}(P_j^c) \le H_j^{\text{CHP,max}}(P_j^{\text{CHP}}) \quad \forall\, j = 1, 2, ..., N_c \quad (26)$$

3. A PIECEWISE LINEARIZATION APPROACH TO THE PROBLEM

As mentioned, the sinusoidal term (valve loading effects) in Eq. (5) is the main cause of non-convexity and non-smoothness of the CHPED problem. The piece-wise linearization approach for consecutive half-sine cycles is the main core of this the main sinusoidal curve close valve points for a paper. In this approach, the quadratic term remains unchanged. In Fig. 4, the piece-wise linearization approach for a sample unit is applied. As seen, the segments of the linearization narrowly stray from 4-linear segment approximation. In order to reduce the deviation between the piece-wise linear segments near the maximum and the original curve, it is better to increase the number of segments which is illustrated in Fig. 4.

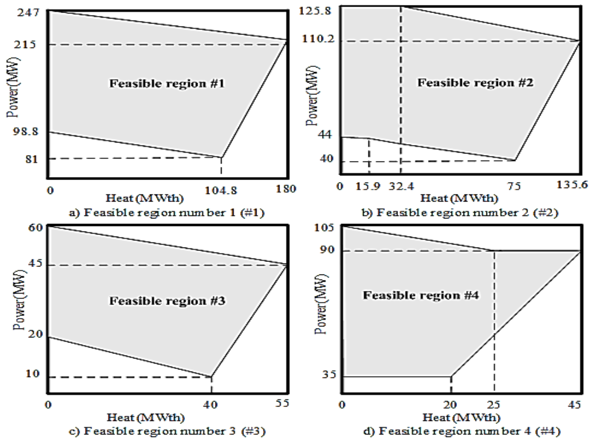

Fig. 6. Feasible regions of CHP units [2]

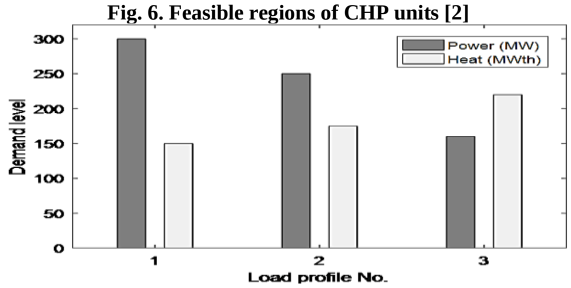

Fig. 7. Different load demands for case study 2 [23]

The number of points for linear approximation for a half-sine cycle is one of the most significant cases. Because of the sinusoidal half-wave symmetry, the number of points selected for a linear approximation should be an odd number to maintain a sinusoidal waveform similar to that original curve. Therefore, the number of points for linearization which denoted as N_L should be an even number to approximate a unit half-sine curve. The Points that are used in a sinusoidal half-wave are indicated by the *m* index and the total number of points applied for linear approximation following sequential half-wave cycles is NP . The last points always associated with the maximum power for each power-only units, i.e, $PK_{i,NT} = P_i^{\text{PO,max}}$. By applying the Eq. (27) and Eq. (28), each point associated to output power and sinusoidal function values is calculated.

$$PK_{im} = \begin{cases} \dfrac{\pi(m-1)}{\rho_i NL} + P_i^{\text{PO,min}} & \forall i = 1,2,...,n;\ m = 1,2,...,NP-1 \\ P_i^{\text{PO,max}} & \forall i = 1,2,...,n;\ m = NP \end{cases} \quad (27)$$

$$CS_{im} = \left|\sin(\rho_i (PK_{im} - P_i^{\text{PO,min}}))\right| \quad \forall i = 1,2,...,n; m = 1,2,...,NP \quad (28)$$

In other words, PK_{im} and CS_{im} parameters have been applied to the *QCP* model. Then the variable θ_{im} is corresponds to a point (i, m) which in *GAMS* software environment these variables (θ_{im}) are set as variables such that at most two variables within a group may have non-zero values and the two non-zero values are adjacent (which express as *SOS2* variables) [41]. The feature of these variables (*SOS2*) is that commonly applied in mathematical optimization with severable programming [42] and it is obvious that the summation of these variables must be equal to one in piece-wise linearization approximation as a general approach (Eq. (29)).

$$\sum_{m=1}^{NP} \theta_{im} = 1 \qquad \forall i = 1,2,...,n \quad (29)$$

So, the output power of each power-only unit is as follows:

$$P_i^{\text{PO}} = \sum_{m=1}^{NP} PK_{im}\theta_{im} \qquad \forall i = 1,2,...,n \quad (30)$$

The objective function corresponding to power-only units is as follow:

$$C_i(P_i^{\text{PO}}) = \alpha_i (P_i^{\text{PO}})^2 + \beta_i P_i^{\text{PO}} + \gamma_i + \lambda_i S_i \quad (31)$$

where:

$$S_i = \sum_{m=1}^{NP} CS_{im}\theta_{im} \qquad \forall i = 1,2,...,n \quad (32)$$

The flowchart and problem-solving procedure are presented in Fig. 5. Now, the model is convex and smooth and can be solved to global optimality.

4. NUMERICAL RESULTS

In this paper, to show the efficiency and effectiveness of the proposed CHPED problem, it is applied to four case studies. In all case studies optimal results (optimal heat and power productions in MWth and MW, respectively) are expressed in related tables. Also, to show the capability of the proposed CHPED problem, the problem compared with different methods presented in the literature. It is noteworthy that all optimization methods presented in this section ensure that all constraints are satisfied. For all case studies, the feasible regions of CHP units are shown in Fig. 6 [2].

4.1. Test case 1

This case study includes 3 CHP units, one power-only unit, and one heat-only unit and also network losses are considered negligible. The data of this case study can be found in Table 1 [2]. Three levels of heat and power demand for this case study is shown in Fig. 7. Table 2 shows the capability of GAMS software to find the optimal global solution to meta-heuristic (GA, HSA, CPSO, GSA, and TVAC-PSO) methods for different load levels. As shown in Table 2, the results of the model by *GAMS* software are better than other meta-heuristic methods. It should be noted the *NLP* model in this case study has been solved by *BARON* solver in *GAMS* software environment.

4.2. Test case 2

This case study includes two CHP units, four power-only units, and one heat only unit. The data of this case study can be found in Table 3. In this case study, the system losses and valve-point loading effect are taken into account. The total heat and power demand are 600 MW and 150 MWth, respectively. Network losses coefficients are given as Eq. (33). The obtained results have been shown in Table 4 and compared with the obtained results introduced by meta-heuristic methods such as PSO [31], CPSO [21], TVAC-PSO [21], RCGA [20], BCO [20], EP [31], and DE [31].

$$B = \begin{bmatrix} 49 & 14 & 15 & 15 & 20 & 25 \\ 14 & 45 & 16 & 20 & 18 & 19 \\ 15 & 16 & 39 & 10 & 12 & 15 \\ 15 & 20 & 10 & 40 & 14 & 11 \\ 20 & 18 & 12 & 14 & 35 & 17 \\ 25 & 19 & 15 & 11 & 17 & 39 \end{bmatrix} \times 10^{-7} \quad (33)$$

As illustrated in Table (4), the obtained loss by GSA and PSO-TVAC algorithm is 0.7128 and 0.7329 MW, respectively, less than ten times the other optimization methods. However, based on the examination these obtained results are invalid.

Table 1. System data of test case 1

Unit No.	Cost function	Capacity limits or feasible region number
1	$C_1(P_1^{PO}) = 0.000115(P_1^{PO})^3 + 0.00172(P_1^{PO})^2 + 7.6997P_1^{PO} + 254.8863$	$35 \le P_1^{PO} \le 135$ MW
2	$C_2(P_2^{CHP}, H_2^{CHP}) = 0.0435(P_2^{CHP})^2 + 36P_2^{CHP} + 0.027(H_2^{CHP})^2 + 0.6H_2^{CHP} + 0.011P_2^{CHP}, H_2^{CHP} + 1250$	#2*
3	$C_3(P_3^{CHP}, H_3^{CHP}) = 0.1035(P_3^{CHP})^2 + 34.5P_3^{CHP} + 0.025(H_3^{CHP})^2 + 2.203H_3^{CHP} + 0.051P_3^{CHP}, H_3^{CHP} + 2650$	#3*
4	$C_3(P_4^{CHP}, H_4^{CHP}) = 0.072(P_4^{CHP})^2 + 20P_4^{CHP} + 0.02(H_4^{CHP})^2 + 2.34H_4^{CHP} + 0.04P_4^{CHP}, H_4^{CHP} + 1565$	#4*
5	$C_5(H_5^{HO}) = 0.038(H_5^{HO})^2 + 2.0109H_5^{HO} + 950$	$0 \le H_5^{HO} \le 60$ MWth

* See Fig. 6

Table 2. Optimal results for test case 1

Demand		Method	Unit 1	Unit 2		Unit 3		Unit 4		Unit 5	C ($)
P	H		P_1^{PO}	P_2^{CHP}	H_2^{CHP}	P_3^{CHP}	H_3^{CHP}	P_4^{CHP}	H_4^{CHP}	H_5^{HO}	
300	150	GA [23]	135.0000	70.8100	80.5400	10.8400	39.8100	83.2800	0.0000	29.6400	13779.5000
		HSA [23]	134.7400	48.2000	81.0900	16.2300	23.9200	100.8500	6.2900	38.7000	13723.2000
		CPSO [21]	135.0000	40.7309	64.4003	19.2728	26.4119	105.0000	0.0000	59.1955	13692.5212
		TVAC-PSO [21]	135.0000	41.4019	73.3562	18.5981	37.4295	105.0000	0.0000	39.2143	13672.8892
		GAMS- This paper	135.0000	40.7689	73.59553	19.2311	36.77661	105.0000	0.0000	39.62785	13672.83413
250	175	GA [23]	119.2200	45.1200	78.9400	15.8200	22.6300	69.8900	18.4000	54.9900	12327.3700
		HSA [23]	134.6700	52.9900	85.6900	10.1100	39.7300	52.2300	4.1800	45.4000	12284.4500
		CPSO [21]	135.0000	40.3446	70.9318	10.0506	39.9918	64.6060	4.0773	60.0000	12132.8579
		TVAC-PSO [21]	135.0000	40.0118	74.8263	10.0391	39.8443	64.9491	16.1867	44.1428	12117.3895
		GSA [2]	135.0000	39.9998	74.9844	10.0000	40.0000	64.9807	17.8939	42.1095	12117.3700
		GAMS- This paper	135.000	40.0000	75.0000	10.0000	40.0000	65.0000	14.05948	45.94052	12117.17012
160	220	GA [23]	37.9800	76.3900	106.0000	10.4100	38.3700	35.0300	15.8400	59.9700	11837.4000
		HSA [23]	41.4100	66.6100	97.7300	10.5900	40.2300	41.3900	22.8300	59.2100	11810.8800
		CPSO [21]	35.5972	57.3554	89.9767	10.0070	40.0025	57.0587	30.0232	60.0000	11781.3690
		GAMS- This paper	42.18183	64.6699	96.29624	10.0000	40.0000	43.14827	23.70376	60.0000	11759.00968

Table 3. System data of test case 2

Unit No.	Cost function	Capacity limits or feasible region number
1	$C_1(P_1^{PO}) = 0.008(P_1^{PO})^2 + 2P_1^{PO} + 25 + \left\|100\sin(0.042(10 - P_1^{PO}))\right\|$	$10 \le P_1^{PO} \le 75$ MW
2	$C_2(P_2^{PO}) = 0.003(P_2^{PO})^2 + 1.8P_2^{PO} + 60 + \left\|140\sin(0.04(20 - P_2^{PO}))\right\|$	$20 \le P_2^{PO} \le 125$ MW
3	$C_3(P_3) = 0.0012(P_3^{PO})^2 + 2.1P_3^{PO} + 100 + \left\|160\sin(0.038(30 - P_3^{PO}))\right\|$	$30 \le P_3^{PO} \le 175$ MW
4	$C_4(P_4^{PO}) = 0.001(P_4^{PO})^2 + 2P_4^{PO} + 120 + \left\|180\sin(0.037(40 - P_4^{PO}))\right\|$	$40 \le P_4^{PO} \le 250$ MW
5	$C_5(P_5^{CHP}, H_5^{CHP}) = 0.0345(P_5^{CHP})^2 + 14.5P_5^{CHP} + 0.03(H_5^{CHP})^2 + 4.2H_5^{CHP} + 0.031P_5^{CHP}H_5^{CHP} + 2650$	#1*

Table 3. (Continued)

6	$C_6(P_6^{\text{CHP}}, H_6^{\text{CHP}}) = 0.0435(P_6^{\text{CHP}})^2 + 36P_6^{\text{CHP}} + 0.027(H_6^{\text{CHP}})^2 + 0.6H_6^{\text{CHP}} + 0.011P_6^{\text{CHP}}H_6^{\text{CHP}} + 1250$	#2*
7	$C_7(H_7^{\text{HO}}) = 0.038(H_7^{\text{HO}})^2 + 2.0109H_7^{\text{HO}} + 950$	$0 \le H_7^{\text{HO}} \le 60$ MWth

* See Fig. (6)

Table 4. Optimal results for test case 2

Output	CPSO [21]	TVAC-PSO [21]	RCGA [20]	BCO [20]	EP [31]	DE [31]	PSO [20]	GSA [2]	Proposed
P_1	75.0000	47.3383	74.6834	43.9457	61.3610	44.2118	18.4626	48.7638	52.6847
P_2	112.3800	98.5398	97.9578	98.5888	95.1205	98.5383	124.2602	98.7469	98.5398
P_3	30.0000	112.6735	167.2308	112.9320	99.9427	122.6913	112.7794	112.0000	112.6734
P_4	250.0000	209.8158	124.9079	209.7719	208.7319	209.7741	209.8158	208.5113	209.8158
P_5	93.2701	92.3718	98.8008	98.8000	98.8000	98.8217	98.8140	92.6909	93.8341
P_6	40.1585	40.0000	44.0001	44.0000	44.0000	44.0000	44.0107	40.0000	40.0000
H_5	32.5655	37.8467	58.0965	12.0974	18.0713	12.5379	57.9236	35.9704	29.2420
H_6	72.6738	74.9999	32.4116	78.0236	77.5548	78.3481	32.7603	75.0000	75.0000
H_7	44.7606	37.1532	59.4919	59.8790	54.3739	59.1139	59.3161	39.0000	45.7579
P_{Loss}	0.8086[a]	0.7392[a]	7.5808	8.0384	7.9561	8.0372	8.1427	0.7128[a]	7.5479
Total power	600.8086[a]	600.7392[a]	607.5808	608.038	607.9561	608.0372	608.1427	601.43[a]	607.5479
Total heat	150.09[a]	150	150	150	150	149.9999	150	149.97	150
Total cost ($)	10325.3339[a]	10100.3164[a]	10667	10317	10390	10317	10613	9912.6928[a]	10111.0732

[a] Invalid response

Table 5. System data of test case 3

Unit No.	Cost function	Capacity limits or feasible region number
1	$C_1(P_1^{\text{PO}}) = 0.00028(P_1^{\text{PO}})^2 + 8.1P_1^{\text{PO}} + 550 + \lvert 300\sin(0.035(-P_1^{\text{PO}}))\rvert$	$0 \le P_1^{\text{PO}} \le 680$ MW
2	$C_2(P_2^{\text{PO}}) = 0.00056(P_2^{\text{PO}})^2 + 8.1P_2^{\text{PO}} + 309 + \lvert 200\sin(0.042(-P_2^{\text{PO}}))\rvert$	$0 \le P_2^{\text{PO}} \le 360$ MW
3	$C_3(P_3^{\text{PO}}) = 0.00056(P_3^{\text{PO}})^2 + 8.1P_3^{\text{PO}} + 309 + \lvert 200\sin(0.042(-P_3^{\text{PO}}))\rvert$	$0 \le P_3^{\text{PO}} \le 360$ MW
4	$C_4(P_4^{\text{PO}}) = 0.00324(P_4^{\text{PO}})^2 + 7.74P_4^{\text{PO}} + 240 + \lvert 150\sin(0.063(60 - P_4^{\text{PO}}))\rvert$	$60 \le P_4^{\text{PO}} \le 180$ MW
5	$C_5(P_5^{\text{PO}}) = 0.00324(P_5^{\text{PO}})^2 + 7.74P_5^{\text{PO}} + 240 + \lvert 150\sin(0.063(60 - P_5^{\text{PO}}))\rvert$	$60 \le P_5^{\text{PO}} \le 180$ MW
6	$C_6(P_6^{\text{PO}}) = 0.00324(P_6^{\text{PO}})^2 + 7.74P_6^{\text{PO}} + 240 + \lvert 150\sin(0.063(60 - P_6^{\text{PO}}))\rvert$	$60 \le P_6^{\text{PO}} \le 180$ MW
7	$C_7(P_7^{\text{PO}}) = 0.00324(P_7^{\text{PO}})^2 + 7.74P_7^{\text{PO}} + 240 + \lvert 150\sin(0.063(60 - P_7^{\text{PO}}))\rvert$	$60 \le P_7^{\text{PO}} \le 180$ MW
8	$C_8(P_8^{\text{PO}}) = 0.00324(P_8^{\text{PO}})^2 + 7.74P_8^{\text{PO}} + 240 + \lvert 150\sin(0.063(60 - P_8^{\text{PO}}))\rvert$	$60 \le P_8^{\text{PO}} \le 180$ MW
9	$C_9(P_9^{\text{PO}}) = 0.00324(P_9^{\text{PO}})^2 + 7.74P_9^{\text{PO}} + 240 + \lvert 150\sin(0.063(60 - P_9^{\text{PO}}))\rvert$	$60 \le P_9^{\text{PO}} \le 180$ MW
10	$C_{10}(P_{10}^{\text{PO}}) = 0.00284(P_{10}^{\text{PO}})^2 + 8.6P_{10}^{\text{PO}} + 126 + \lvert 100\sin(0.084(40 - P_{10}^{\text{PO}}))\rvert$	$40 \le P_{10}^{\text{PO}} \le 120$ MW
11	$C_{11}(P_{11}^{\text{PO}}) = 0.00284(P_{11}^{\text{PO}})^2 + 8.6P_{11}^{\text{PO}} + 126 + \lvert 100\sin(0.084(40 - P_{11}^{\text{PO}}))\rvert$	$40 \le P_{11}^{\text{PO}} \le 120$ MW
12	$C_{12}(P_{12}^{\text{PO}}) = 0.00284(P_{12}^{\text{PO}})^2 + 8.6P_{12}^{\text{PO}} + 126 + \lvert 100\sin(0.084(55 - P_{12}^{\text{PO}}))\rvert$	$55 \le P_{12}^{\text{PO}} \le 120$ MW
13	$C_{13}(P_{13}^{\text{PO}}) = 0.00284(P_{13}^{\text{PO}})^2 + 8.6P_{13}^{\text{PO}} + 126 + \lvert 100\sin(0.084(55 - P_{13}^{\text{PO}}))\rvert$	$55 \le P_{13}^{\text{PO}} \le 120$ MW
14	$C_{14}(P_{14}^{\text{CHP}}, H_{14}^{\text{CHP}}) = 0.0345(P_{14}^{\text{CHP}})^2 + 14.5P_{14}^{\text{CHP}} + 0.03(H_{14}^{\text{CHP}})^2 + 4.2H_{14}^{\text{CHP}} + 0.031P_{14}^{\text{CHP}}H_{14}^{\text{CHP}} + 2650$	#1*
15	$C_{15}(P_{15}^{\text{CHP}}, H_{15}^{\text{CHP}}) = 0.0435(P_{15}^{\text{CHP}})^2 + 36P_{15}^{\text{CHP}} + 0.026(H_{15}^{\text{CHP}})^2 + 0.6H_{15}^{\text{CHP}} + 0.011P_{15}^{\text{CHP}}H_{15}^{\text{CHP}} + 1250$	#2*
16	$C_{16}(P_{16}^{\text{CHP}}, H_{16}^{\text{CHP}}) = 0.0345(P_{16}^{\text{CHP}})^2 + 14.5P_{16}^{\text{CHP}} + 0.03(H_{16}^{\text{CHP}})^2 + 4.2H_{16}^{\text{CHP}} + 0.031P_{16}^{\text{CHP}}H_{16}^{\text{CHP}} + 2650$	#1*
17	$C_{17}(P_{17}^{\text{CHP}}, H_{17}^{\text{CHP}}) = 0.0435(P_{17}^{\text{CHP}})^2 + 36P_{17}^{\text{CHP}} + 0.026(H_{17}^{\text{CHP}})^2 + 0.6H_{17}^{\text{CHP}} + 0.011P_{17}^{\text{CHP}}H_{17}^{\text{CHP}} + 1250$	#2*
18	$C_{18}(P_{18}^{\text{CHP}}, H_{18}^{\text{CHP}}) = 0.1035(P_{18}^{\text{CHP}})^2 + 34.5P_{18}^{\text{CHP}} + 0.025(H_{18}^{\text{CHP}})^2 + 2.203H_{18}^{\text{CHP}} + 0.051P_{18}^{\text{CHP}}H_{18}^{\text{CHP}} + 2650$	#3*
19	$C_{19}(P_{19}^{\text{CHP}}, H_{19}^{\text{CHP}}) = 0.072(P_{19}^{\text{CHP}})^2 + 20P_{19}^{\text{CHP}} + 0.02(H_{19}^{\text{CHP}})^2 + 2.34H_{19}^{\text{CHP}} + 0.04P_{19}^{\text{CHP}}H_{19}^{\text{CHP}} + 1565$	#4*
20	$C_{20}(H_{20}^{\text{HO}}) = 0.038(H_{20}^{\text{HO}})^2 + 2.0109H_{20}^{\text{HO}} + 950$	$0 \le H_{20}^{\text{HO}} \le 2695.2$ MWth
21	$C_{21}(H_{21}^{\text{HO}}) = 0.038(H_{21}^{\text{HO}})^2 + 2.0109H_{21}^{\text{HO}} + 950$	$0 \le H_{21}^{\text{HO}} \le 60$ MWth

Table 5. (Continued)

22	$C_{22}(H_{22}^{\text{HO}}) = 0.038(H_{22}^{\text{HO}})^2 + 2.0109H_{22}^{\text{HO}} + 950$	$0 \le H_{22}^{\text{HO}} \le 60$ MWth
23	$C_{23}(H_{23}^{\text{HO}}) = 0.052(H_{23}^{\text{HO}})^2 + 3.0651H_{23}^{\text{HO}} + 480$	$0 \le H_{23}^{\text{HO}} \le 120$ MWth
24	$C_{24}(H_{24}^{\text{HO}}) = 0.052(H_{24}^{\text{HO}})^2 + 3.0651H_{24}^{\text{HO}} + 480$	$0 \le H_{24}^{\text{HO}} \le 120$ MWth

* See Fig. (6)

Table 6. Optimal results for test case 3

Output	CPSO [21]	GSO [43]	TVAC-PSO [21]	GSA [2]	IGSO [43]	TLBO [27]	OTLB [27]	GWO [44]	EMA [30]	Proposed
P_1^{PO}	680.0000	627.7455	538.5587	538.5150	628.1520	628.3240	538.5656	538.5840	628.3171	610.387
P_2^{PO}	0.0000	76.2285	224.4608	224.4727	299.4778	227.3588	299.2123	299.3426	299.1859	306.145
P_3^{PO}	0.0000	299.5794	224.4608	224.4611	154.5535	225.9347	299.1220	299.3423	299.1624	306.145
P_4^{PO}	180.0000	159.4386	109.8666	109.8666	60.8460	110.3721	109.992	109.9653	109.8665	108.387
P_5^{PO}	180.0000	61.2378	109.8666	109.8666	103.8538	110.2461	109.9545	109.9653	109.8665	108.387
P_6^{PO}	180.0000	60.0000	109.8666	109.8666	110.0552	160.1761	110.4042	109.9653	109.8665	108.387
P_7^{PO}	180.0000	157.1503	109.8666	109.8666	159.0773	108.3552	109.8045	109.9653	60.0000	108.387
P_8^{PO}	180.0000	107.2654	109.8666	109.8666	109.8258	110.5379	109.6862	109.9653	109.8665	108.387
P_9^{PO}	180.0000	110.1816	109.8666	109.8666	159.9920	110.5672	109.8992	109.9653	109.8665	108.387
P_{10}^{PO}	50.5304	113.9894	77.5210	77.5210	41.103	75.7562	77.3992	77.6223	40.0000	40.0000
P_{11}^{PO}	50.5304	79.7755	77.5210	77.5210	77.7055	41.8698	77.8364	77.6223	77.0195	40.0000
P_{12}^{PO}	55.0000	91.1668	120.0000	120.0000	94.9768	92.4789	55.2225	55.0000	55.0000	55.0000
P_{13}^{PO}	55.0000	115.6511	120.0000	120.0000	55.7143	57.5140	55.0861	55.0000	55.0000	55.0000
P_{14}^{CHP}	117.4854	84.3133	88.3514	92.5632	83.9536	82.5628	81.7524	83.4650	81.0000	81.0000
P_{15}^{CHP}	45.9281	40.0000	40.5611	40.0050	40.0000	41.4891	41.7615	40.0000	40.0000	40.0000
P_{16}^{CHP}	117.4854	81.1796	88.3514	84.4916	85.7133	84.7710	82.2730	82.7732	81.0000	81.0000
P_{17}^{CHP}	45.9281	40.0000	40.5611	40.0079	40.0000	40.5874	40.5599	40.0000	40.0000	40.0000
P_{18}^{CHP}	10.0013	10.0000	10.0245	10.0000	10.0000	10.0010	10.0002	10.0000	10.0000	10.0000
P_{19}^{CHP}	42.1109	35.0970	40.4288	41.1998	35.0000	31.0978	31.4679	31.4568	35.0000	35.0000
H_{14}^{CHP}	125.2754	106.6588	108.9256	111.2790	106.4569	105.6717	105.2219	106.0991	104.8002	104.800
H_{15}^{CHP}	80.1175	74.9980	75.4844	74.9980	74.9980	76.2843	76.5205	75.0000	75.0000	75.0000
H_{16}^{CHP}	125.2754	104.9002	108.9256	106.7495	107.4073	106.9125	105.5142	105.7890	104.8002	104.800
H_{17}^{CHP}	80.1174	74.9980	75.4840	74.9978	74.9980	75.5061	75.4833	75.0000	75.0000	75.0000
H_{18}^{CHP}	40.0005	40.0000	40.0104	40.0000	40.0000	39.9986	39.9999	40.0000	40.0000	40.0000
H_{19}^{CHP}	23.2322	19.7385	22.4676	22.8181	20.0000	18.2205	18.3944	18.3782	20.0000	20.0000
H_{20}^{HO}	415.9815	469.3368	458.7020	458.8811	466.2575	468.2278	468.9043	469.7337	470.3996	470.400
H_{21}^{HO}	60.0000	60.0000	60.0000	60.0000	60.0000	59.9867	59.9994	60.0000	60.0000	60.0000
H_{22}^{HO}	60.0000	60.0000	60.0000	60.0000	60.0000	59.9814	59.9999	60.0000	60.0000	60.0000
H_{23}^{HO}	120.0000	119.6511	120.0000	120.0000	120.0000	119.9854	119.9854	120.0000	120.0000	120.0000
H_{24}^{HO}	120.0000	119.7176	120.0000	120.0000	119.8823	119.6030	119.9768	120.0000	120.0000	120.0000
Total cost ($)	59736.2635	58225.7450	58122.7460	58121.8640	58049.0197	58006.99	57856.26	57846.84	57825.4792	57776.663

Table 7. System data of test case 4

Unit No.	Cost function	Capacity limits or feasible region number
1	$C_1(P_1^{\text{PO}}) = 0.00028(P_1^{\text{PO}})^2 + 8.1P_1^{\text{PO}} + 550 + \lvert 300\sin(0.035(-P_1^{\text{PO}}))\rvert$	$0 \le P_1^{\text{PO}} \le 680$ MW
2	$C_2(P_2^{\text{PO}}) = 0.00056(P_2^{\text{PO}})^2 + 8.1P_2^{\text{PO}} + 309 + \lvert 200\sin(0.042(-P_2^{\text{PO}}))\rvert$	$0 \le P_2^{\text{PO}} \le 360$ MW
3	$C_3(P_3^{\text{PO}}) = 0.00056(P_3^{\text{PO}})^2 + 8.1P_3^{\text{PO}}{}_3 + 309 + \lvert 200\sin(0.042(-P_3^{\text{PO}}))\rvert$	$0 \le P_3^{\text{PO}} \le 360$ MW
4	$C_4(P_4^{\text{PO}}) = 0.00324(P_4^{\text{PO}})^2 + 7.74P_4^{\text{PO}} + 240 + \lvert 150\sin(0.063(60 - P_4^{\text{PO}}))\rvert$	$60 \le P_4^{\text{PO}} \le 180$ MW

Table 7. (Continued)

5	$C_5(P_5^{PO}) = 0.00324(P_5^{PO})^2 + 7.74P_5^{PO} + 240 + \left\|150\sin(0.063(60 - P_5^{PO}))\right\|$	$60 \le P_5^{PO} \le 180$ MW
6	$C_6(P_6^{PO}) = 0.00324(P_6^{PO})^2 + 7.74P_6^{PO} + 240 + \left\|150\sin(0.063(60 - P_6^{PO}))\right\|$	$60 \le P_6^{PO} \le 180$ MW
7	$C_7(P_7^{PO}) = 0.00324(P_7^{PO})^2 + 7.74P_7^{PO} + 240 + \left\|150\sin(0.063(60 - P_7^{PO}))\right\|$	$60 \le P_7^{PO} \le 180$ MW
8	$C_8(P_8^{PO}) = 0.00324(P_8^{PO})^2 + 7.74P_8^{PO} + 240 + \left\|150\sin(0.063(60 - P_8^{PO}))\right\|$	$60 \le P_8^{PO} \le 180$ MW
9	$C_9(P_9^{PO}) = 0.00324(P_9^{PO})^2 + 7.74P_9^{PO} + 240 + \left\|150\sin(0.063(60 - P_9^{PO}))\right\|$	$60 \le P_9^{PO} \le 180$ MW
10	$C_{10}(P_{10}^{PO}) = 0.00284(P_{10}^{PO})^2 + 8.6P_{10}^{PO} + 126 + \left\|100\sin(0.084(40 - P_{10}^{PO}))\right\|$	$40 \le P_{10}^{PO} \le 120$ MW
11	$C_{11}(P_{11}^{PO}) = 0.00284(P_{11}^{PO})^2 + 8.6P_{11}^{PO} + 126 + \left\|100\sin(0.084(40 - P_{11}^{PO}))\right\|$	$40 \le P_{11}^{PO} \le 120$ MW
12	$C_{12}(P_{12}^{PO}) = 0.00284(P_{12}^{PO})^2 + 8.6P_{12}^{PO} + 126 + \left\|100\sin(0.084(55 - P_{12}^{PO}))\right\|$	$55 \le P_{12}^{PO} \le 120$ MW
13	$C_{13}(P_{13}^{PO}) = 0.00284(P_{13}^{PO})^2 + 8.6P_{13}^{PO} + 126 + \left\|100\sin(0.084(55 - P_{13}^{PO}))\right\|$	$55 \le P_{13}^{PO} \le 120$ MW
14	$C_{14}(P_{14}^{PO}) = 0.00028(P_{14}^{PO})^2 + 8.1P_{14}^{PO} + 550 + \left\|300\sin(0.035(-P_{14}^{PO}))\right\|$	$0 \le P_{14}^{PO} \le 680$ MW
15	$C_{15}(P_{15}^{PO}) = 0.00056(P_{15}^{PO})^2 + 8.1P_{15}^{PO} + 309 + \left\|200\sin(0.042(-P_{15}^{PO}))\right\|$	$0 \le P_{15}^{PO} \le 360$ MW
16	$C_3(P_{16}^{PO}) = 0.00056(P_{16}^{PO})^2 + 8.1P_{16}^{PO} + 309 + \left\|200\sin(0.042(-P_{16}^{PO}))\right\|$	$0 \le P_{16}^{PO} \le 360$ MW
17	$C_{17}(P_{17}^{PO}) = 0.00324(P_{17}^{PO})^2 + 7.74P_{17}^{PO} + 240 + \left\|150\sin(0.063(60 - P_{17}^{PO}))\right\|$	$60 \le P_{17}^{PO} \le 180$ MW
18	$C_{18}(P_{18}^{PO}) = 0.00324(P_{18}^{PO})^2 + 7.74P_{18}^{PO} + 240 + \left\|150\sin(0.063(60 - P_{18}^{PO}))\right\|$	$60 \le P_{18}^{PO} \le 180$ MW
19	$C_{19}(P_{19}) = 0.00324(P_{19}^{PO})^2 + 7.74P_{19}^{PO} + 240 + \left\|150\sin(0.063(60 - P_{19}^{PO}))\right\|$	$60 \le P_{19}^{PO} \le 180$ MW
20	$C_{20}(P_{20}) = 0.00324(P_{20}^{PO})^2 + 7.74P_{20}^{PO} + 240 + \left\|150\sin(0.063(60 - P_{20}^{PO}))\right\|$	$60 \le P_{20}^{PO} \le 180$ MW
21	$C_{21}(P_{21}) = 0.00324(P_{21}^{PO})^2 + 7.74P_{21}^{PO} + 240 + \left\|150\sin(0.063(60 - P_{21}^{PO}))\right\|$	$60 \le P_{21}^{PO} \le 180$ MW
22	$C_{22}(P_{22}) = 0.00324(P_{22}^{PO})^2 + 7.74P_{22}^{PO} + 240 + \left\|150\sin(0.063(60 - P_{22}^{PO}))\right\|$	$60 \le P_{22}^{PO} \le 180$ MW
23	$C_{23}(P_{23}) = 0.00284(P_{23}^{PO})^2 + 8.6P_{23}^{PO} + 126 + \left\|100\sin(0.084(40 - P_{23}^{PO}))\right\|$	$40 \le P_{23}^{PO} \le 120$ MW
24	$C_{24}(P_{24}) = 0.00284(P_{24}^{PO})^2 + 8.6P_{24}^{PO} + 126 + \left\|100\sin(0.084(40 - P_{24}^{PO}))\right\|$	$40 \le P_{24}^{PO} \le 120$ MW
25	$C_{24}(P_{25}) = 0.00284(P_{25}^{PO})^2 + 8.6P_{25}^{PO} + 126 + \left\|100\sin(0.084(55 - P_{25}^{PO}))\right\|$	$55 \le P_{25}^{PO} \le 120$ MW
26	$C_{26}(P_{26}) = 0.00284(P_{26}^{PO})^2 + 8.6P_{26}^{PO} + 126 + \left\|100\sin(0.084(55 - P_{26}^{PO}))\right\|$	$55 \le P_{26}^{PO} \le 120$ MW
27	$C_{27}(P_{27}^{CHP}, H_{27}^{CHP}) = 0.0345(P_{27}^{CHP})^2 + 14.5P_{27}^{CHP} + 0.03(H_{27}^{CHP})^2 + 4.2H_{27}^{CHP} + 0.031P_{27}^{CHP}H_{27}^{CHP} + 2650$	#1*
28	$C_{28}(P_{28}^{CHP}, H_{28}^{CHP}) = 0.0435(P_{28}^{CHP})^2 + 36P_{28}^{CHP} + 0.026(H_{28}^{CHP})^2 + 0.6H_{28}^{CHP} + 0.011P_{28}^{CHP}H_{28}^{CHP} + 1250$	#2*
29	$C_{29}(P_{29}^{CHP}, H_{29}^{CHP}) = 0.0345(P_{29}^{CHP})^2 + 14.5P_{29}^{CHP} + 0.03(H_{29}^{CHP})^2 + 4.2H_{29}^{CHP} + 0.031P_{29}^{CHP}H_{29}^{CHP} + 2650$	#1*
30	$C_{30}(P_{30}^{CHP}, H_{30}^{CHP}) = 0.0435(P_{30}^{CHP})^2 + 36P_{30}^{CHP} + 0.026(H_{30}^{CHP})^2 + 0.6H_{30}^{CHP} + 0.011P_{30}^{CHP}H_{30}^{CHP} + 1250$	#2*
31	$C_{31}(P_{31}^{CHP}, H_{31}^{CHP}) = 0.1035(P_{31}^{CHP})^2 + 34.5P_{31}^{CHP} + 0.025(H_{31}^{CHP})^2 + 2.203H_{31}^{CHP} + 0.051P_{31}^{CHP}H_{31}^{CHP} + 2650$	#3*
32	$C_{32}(P_{32}^{CHP}, H_{32}^{CHP}) = 0.072(P_{32}^{CHP})^2 + 20P_{32}^{CHP} + 0.02(H_{32}^{CHP})^2 + 2.34H_{32}^{CHP} + 0.04P_{32}^{CHP}H_{32}^{CHP} + 1565$	#4*
33	$C_{33}(P_{33}^{CHP}, H_{33}^{CHP}) = 0.0345(P_{33}^{CHP})^2 + 14.5P_{33}^{CHP} + 0.03(H_{33}^{CHP})^2 + 4.2H_{33}^{CHP} + 0.031P_{33}^{CHP}H_{33}^{CHP} + 2650$	#1*
34	$C_{34}(P_{34}^{CHP}, H_{34}^{CHP}) = 0.0435(P_{34}^{CHP})^2 + 36P_{34}^{CHP} + 0.026(H_{34}^{CHP})^2 + 0.6H_{34}^{CHP} + 0.011P_{34}^{CHP}H_{34}^{CHP} + 1250$	#2*
35	$C_{35}(P_{35}^{CHP}, H_{35}^{CHP}) = 0.0345(P_{35}^{CHP})^2 + 14.5P_{35}^{CHP} + 0.03(H_{35}^{CHP})^2 + 4.2H_{35}^{CHP} + 0.031P_{35}^{CHP}H_{35}^{CHP} + 2650$	#1*
36	$C_{36}(P_{36}^{CHP}, H_{36}^{CHP}) = 0.0435(P_{36}^{CHP})^2 + 36P_{36}^{CHP} + 0.026(H_{36}^{CHP})^2 + 0.6H_{36}^{CHP} + 0.011P_{36}^{CHP}H_{36}^{CHP} + 1250$	#2*
37	$C_{37}(P_{37}^{CHP}, H_{37}^{CHP}) = 0.1035(P_{37}^{CHP})^2 + 34.5P_{37}^{CHP} + 0.025(H_{37}^{CHP})^2 + 2.203H_{37}^{CHP} + 0.051P_{37}^{CHP}H_{37}^{CHP} + 2650$	#3*
38	$C_{38}(P_{38}^{CHP}, H_{38}^{CHP}) = 0.072(P_{38}^{CHP})^2 + 20P_{38}^{CHP} + 0.02(H_{38}^{CHP})^2 + 2.34H_{38}^{CHP} + 0.04P_{38}^{CHP}H_{38}^{CHP} + 1565$	#4*
39	$C_{39}(H_{39}^{HO}) = 0.038(H_{39}^{HO})^2 + 2.0109H_{39}^{HO} + 950$	$0 \le H_{39}^{HO} \le 2695.2$ MWth
40	$C_{40}(H_{40}^{HO}) = 0.038(H_{40}^{HO})^2 + 2.0109H_{40}^{HO} + 950$	$0 \le H_{40}^{HO} \le 60$ MWth
41	$C_{41}(H_{41}^{HO}) = 0.038(H_{41}^{HO})^2 + 2.0109H_{41}^{HO} + 950$	$0 \le H_{41}^{HO} \le 60$ MWth
42	$C_{42}(H_{42}^{HO}) = 0.052(H_{42}^{HO})^2 + 3.0651H_{42}^{HO} + 480$	$0 \le H_{42}^{HO} \le 120$ MWth
43	$C_{43}(H_{43}^{HO}) = 0.052(H_{43}^{HO})^2 + 3.0651H_{43}^{HO} + 480$	$0 \le H_{43}^{HO} \le 120$ MWth

Table 7. (Continued)

44	$C_{44}(H_{44}^{HO}) = 0.038(H_{44}^{HO})^2 + 2.0109H_{44}^{HO} + 950$	$0 \le H_{44}^{HO} \le 2695.2$ MWth
45	$C_{45}(H_{45}^{HO}) = 0.038(H_{45}^{HO})^2 + 2.0109H_{45}^{HO} + 950$	$0 \le H_{45}^{HO} \le 60$ MWth
46	$C_{46}(H_{46}^{HO}) = 0.038(H_{46}^{HO})^2 + 2.0109H_{46}^{HO} + 950$	$0 \le H_{46}^{HO} \le 60$ MWth
47	$C_{47}(H_{47}^{HO}) = 0.052(H_{47}^{HO})^2 + 3.0651H_{47}^{HO} + 480$	$0 \le H_{47}^{HO} \le 120$ MWth
48	$C_{48}(H_{48}^{HO}) = 0.052(H_{48}^{HO})^2 + 3.0651H_{48}^{HO} + 480$	$0 \le H_{48}^{HO} \le 120$ MWth

Table 8. Optimal results for test case 4

Output	CPSO [21]	TVAC-PSO [21]	GSA [2]	TLBO [27]	OTLBO [27]	EMA [30]	Proposed
P_1^{PO}	359.0392	538.5587	359.8656	538.5693	628.3199	628.3166	638.191
P_2^{PO}	74.5831	75.1340	227.2336	225.3021	225.3313	299.1692	320.046
P_3^{PO}	74.5831	75.1340	152.7852	229.9473	223.9653	224.3017	320.046
P_4^{PO}	139.3803	140.6146	160.4948	159.1352	159.8516	109.8618	110.790
P_5^{PO}	139.3803	140.6146	109.5165	160.0561	109.9150	109.8665	110.790
P_6^{PO}	139.3803	140.6146	159.3399	109.7821	159.7795	109.8415	110.790
P_7^{PO}	139.3803	140.6146	162.1068	159.6609	109.8946	109.8663	110.790
P_8^{PO}	139.3803	140.6146	109.5873	159.6492	109.9321	109.8583	110.790
P_9^{PO}	139.3803	140.6146	158.9737	109.9660	159.9569	109.8665	110.790
P_{10}^{PO}	74.7998	112.1998	113.3540	40.3726	40.8970	40.0000	40.0000
P_{11}^{PO}	74.7998	112.1998	114.9745	77.5821	41.3115	40.0000	40.0000
P_{12}^{PO}	74.7998	74.7999	55.3445	92.2489	55.1748	55.0000	55.0000
P_{13}^{PO}	74.7998	74.7999	120.0000	55.1755	92.4003	55.0000	55.0000
P_{14}^{PO}	679.8810	269.2794	361.9144	448.6854	448.8359	628.3185	638.191
P_{15}^{PO}	148.6585	299.1993	223.9861	149.4238	225.7871	298.6422	320.046
P_{16}^{PO}	148.6585	299.1993	241.2574	224.7173	75.4600	299.0560	180.000
P_{17}^{PO}	139.0809	140.3973	159.8437	109.9355	160.1192	109.8685	110.790
P_{18}^{PO}	139.0809	140.3973	159.0831	159.9052	110.3532	109.8667	110.790
P_{19}^{PO}	139.0809	140.3973	110.7603	159.7255	159.8190	159.7331	110.790
P_{20}^{PO}	139.0809	140.3973	108.1711	159.7820	159.7765	109.8386	110.790
P_{21}^{PO}	139.0809	140.3973	165.0457	60.0777	159.7370	109.8667	110.790
P_{22}^{PO}	139.0809	140.3973	160.8239	110.0689	160.1751	109.8613	110.790
P_{23}^{PO}	74.7998	74.7998	98.8179	77.6818	40.1140	40.0000	40.0000
P_{24}^{PO}	74.7998	74.7998	83.8242	40.2707	40.3042	40.0000	40.0000
P_{25}^{PO}	112.1993	112.1997	55.0000	92.4108	92.4149	55.0000	55.0000
P_{26}^{PO}	112.1993	112.1997	120.0000	55.0956	92.5012	55.0000	55.0000
P_{27}^{CHP}	92.8423	86.9119	91.2279	81.4882	85.9857	81.0000	81.0000
P_{28}^{CHP}	98.7199	56.1027	39.9998	44.5478	98.5005	40.0000	40.0000
P_{29}^{CHP}	92.8423	86.9119	81.0027	81.0560	81.7197	81.0000	81.0000
P_{30}^{CHP}	98.7199	56.1027	40.0072	91.6819	48.9055	40.0000	40.0000
P_{31}^{CHP}	10.0002	10.0031	10.0000	10.5480	10.0832	10.0000	10.0000
P_{32}^{CHP}	56.7153	35.0000	35.0000	52.7180	39.3110	35.0000	35.0000
P_{33}^{CHP}	109.1877	95.4799	81.0020	82.1522	82.0236	81.0000	81.0000
P_{34}^{CHP}	65.6006	54.9235	41.9658	52.0606	40.1105	40.0000	40.0000
P_{35}^{CHP}	109.1877	95.4799	81.0020	82.7394	81.3039	81.0000	81.0000
P_{36}^{CHP}	65.6006	54.9235	46.6684	45.7398	45.6700	40.0000	40.0000
P_{37}^{CHP}	10.6158	23.4981	10.0000	10.0075	13.8709	10.0000	10.0000
P_{38}^{CHP}	60.5994	54.0882	90.0000	30.0332	30.3881	35.0000	35.0000
H_{27}^{CHP}	111.4458	108.1177	110.5296	105.0678	107.5951	104.8002	104.800
H_{28}^{CHP}	125.6898	88.9006	74.9844	78.9162	125.4997	75.0000	75.000
H_{29}^{CHP}	111.4458	108.1177	104.7869	104.8270	105.1942	104.8002	104.800

Table 8. (Continued)

H_{30}^{HO}	125.6898	88.9006	74.8787	119.6006	82.6853	75.0000	75.0000
H_{31}^{HO}	40.0001	40.0013	40.0000	40.2345	40.0346	40.0000	40.0000
H_{32}^{CHP}	29.8706	20.0000	19.0125	28.0508	21.9568	20.0000	20.0000
H_{33}^{CHP}	120.6188	112.9260	104.7912	105.4339	105.3622	104.8002	104.800
H_{34}^{CHP}	97.0997	87.8827	76.6806	85.4086	75.0938	75.0000	75.0000
H_{35}^{CHP}	120.6188	112.9260	104.7912	105.7694	104.9667	104.8002	104.800
H_{36}^{CHP}	97.0997	87.8827	80.7377	79.9447	79.8936	75.0000	75.0000
H_{37}^{CHP}	40.2639	45.7849	40.0000	40.0001	41.6554	40.0000	40.0000
H_{38}^{CHP}	31.6361	28.6765	45.0000	17.7401	17.9018	20.0000	20.0000
H_{39}^{HO}	357.9456	433.9113	488.8361	394.6160	445.0937	470.3802	470.400
H_{40}^{HO}	59.9916	60.0000	60.0000	59.9300	59.9967	60.0000	60.0000
H_{41}^{HO}	59.9916	60.0000	60.0000	59.9578	59.9974	60.0000	60.0000
H_{42}^{HO}	120.0000	120.0000	120.0000	118.5797	119.8834	120.0000	120.0000
H_{43}^{HO}	120.0000	120.0000	120.0000	118.3425	119.5231	120.0000	120.0000
H_{44}^{HO}	370.6214	415.9741	60.0000	480.6566	428.7605	470.4190	470.4000
H_{45}^{HO}	59.9999	60.0000	60.0000	59.9346	59.9957	60.0000	60.0000
H_{46}^{HO}	59.9999	60.0000	120.0000	59.9810	59.9638	60.0000	60.0000
H_{47}^{HO}	119.9856	119.9989	120.0000	117.8207	119.5025	120.0000	120.0000
H_{48}^{HO}	119.9856	119.9989	415.0132	119.1898	119.4440	120.0000	120.0000
Algorithm	119708.8818	117824.8956	117266.6810	116739.3640	116579.23	115611.8447	115563.220

4.3. Test case 3

This test case includes six CHP units, thirteen power-only units, and five heat-only units. Heat and power demand are 1250 MWth and 2350 MW, respectively. In Table 5, related data for this system are given. It is understandable that this test case has many locally optimal solutions. Therefore, finding an optimal solution for this case study can be as a complicated benchmark in evolutionary algorithm studies. The results have been shown in Table 6 and compared with the obtained results introduced by meta-heuristic methods such as CPSO, TVAC-PSO, TLBO, OTLBO, GSO, IGSO, GSA, GWO, and EMA. However, the proposed method can successfully extract the optimal solution by cost of $57776.663 that is less than CPSO, GSO, TVAC-PSO, GSA, IGSO, TLBO, OTLBO, GWO, and EMA by $59736.2635, $58225.7450, $58122.7460, $58121.8640, $58049.0197, $58006.99, $57856.26, $57846.84, and $57825.4792, respectively. The results indicate that great superiority of the proposed method over the other well-behaved meta-heuristic optimization methods.

4.4. Test case 4

The case study 4 is also a large system. This test case includes twelve CHP units, twenty-six power-only units, and ten heat-only units. Heat and power demands are 2500 MWth and 4700 MW, respectively. Related data of this test case is given in Table 7. In this test case, units 1-26 are power-only units, 27-28 are CHP units and 39-48 are heat only units. The obtained results have been shown in Table 8 and compared with the obtained results introduced by meta-heuristic methods such as CPSO, TVAC-PSO, GSA, TLBO, OTLBO, and EMA. However, the proposed method can successfully extract the optimal solution by the cost of $115563.22 that is less than CPSO, TVAC-PSO, GSA, TLBO, OTLBO, and EMA by $119708.8818, $117824.8956, $117266.6810, $116739.3640, $116579.2390, and $115611.8447, respectively. According to the results, the great superiority of the proposed method over the other well-behaved meta-heuristic optimization methods is illustrated.

5. CONCLUSION

This paper introduces the piece-wise linearization to solve the combined heat and power economic dispatch problem because the valve point loading effect makes the problem is highly non-linear, non-convex and non-smooth. To investigate the effectiveness and robustness of the proposed model and its solution methodology, the model was applied to four different CHPED problems with fuel convex (non-convex) cost . The main conclusions from numerical results are as follows:

- The solution and implementation of the proposed model are straight via the use of standard modeling tools, such as the GAMS software environment.

- The obtained results by the proposed approach in term of the quality solution is compared with different meta-heuristic and heuristic optimization methods.
- The results illustrate that the proposed method indicates the potential to solve large-scale, highly non-linear, non-convex, and non-smooth combined heat and power economic dispatch problem.

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