

More complex complexity: Exploring the nature
of computational irreducibility across physical,
biological, and human social systems

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1 Abstract

The predictability of many complex systems is limited by computational irreducibility, but we argue that the nature of computational irreducibility varies across physical, biological and human social systems. We suggest that the computational irreducibility of biological and social systems is distinguished from physical systems by functional contingency, biological evolution, and individual variation. In physical systems, computational irreducibility is driven by the interactions, sometimes nonlinear, of many different system components (e.g., particles, atoms, planets). Biological systems can also be computationally irreducible because of nonlinear interactions of a large number of system components (e.g., gene networks, cells, individuals). Biological systems additionally create the probability space into which the system moves: Biological evolution creates new biological attributes, stores this accumulated information in an organism's genetic code, allows for individual genetic and phenotypic variation among interacting agents, and selects for the functionality of these biological attributes in a contextually dependent manner. Human social systems are biological systems that include these same processes, but whose computational irreducibility arises as well from sentience, i.e., the conscious perception of the adjacent possible, that drives social evolution of culture, governance, and technology. Human social systems create their own adjacent possible through the creativity of sentience, and accumulate and store this information culturally, as reflected in the emergence and evolution of, for example, technology. The changing nature of computational irreducibility results in a loss of predictability as one moves from physical to biological to human social systems, but also creates a rich and enchanting range of dynamics.

1.1 keywords

computational irreducibility, complexity, evolution

2 Introduction

Systems change through time—whether the system of interest is a galaxy, a forest ecosystem, a social network, or a circulatory system. This continuous process of change in the system state can be thought of as computation [35]: the state of the system is updated based on its current and past states. In a forest, for example, the growth and recruitment of trees is dependent on the spatial arrangement of trees through processes such as competitive interactions, light shading, and seed dispersal as well as environmental externalities [4]. In some systems, precise predictions of the future state of the system can be made without having to perform the intervening computations. In these systems, prediction is possible because simplified models exist that can be used to bypass the intervening computations intrinsically performed by the system. Astronomical models, for example, can predict the spatial and temporal distribution of sunlight on earth, and describe the past orbital forcing of the climate system [20]. In other systems, like a forest ecosystem, predicting the detailed state of the system is very difficult without allowing the system to update itself on its own characteristic time scale [3]. Systems that require the computation of intervening system states on their characteristic time scale in order to predict future states are computationally irreducible. Computational irreducibility therefore implies the absence of simplifying models that can reproduce future system states without information loss. The dynamics of a system that is computationally irreducible cannot be known without allowing for the evolution of the system on its own time scale. While any process that is computationally irreducible may seem to imply an equivalent degree of unpredictability a priori, we suggest that this is not the case. We argue that the processes that drive computational irreducibility differ across physical, biological and social systems, and that these differences result in some forms of computational irreducibility being 'more irreducible' than others. Computational irreducibility does not imply that predictions are impossible, but that they come at the cost of information loss. In cellular automata, for example, cells can be spatially aggregated into larger units with an associated set of updating rules in a process of coarse-graining [18] [19]. Prediction in some computationally irreducible systems is possible through coarse-graining, but comes at the cost of information loss through spatial and temporal averaging. We suggest, then, that gains in prediction require increasing information loss in physical, biological, and human social systems, and thus some systems are more computationally irreducible than others. We argue that the basis for these differences lie in the different processes operating in physical, biological, and human social systems.

3 Physical systems

Physics has been particularly successful at prediction. Physicists, for example, were able to predict the existence of black holes from a singularity in the equations describing the physical system [31] [32]. Engineers routinely use the laws

of physics to design and build skyscrapers, bridges, airplanes, and to send spacecraft to distant planets, and these efforts are usually successful. We don't mean to imply that physics is axiomatic and its laws universal but rather that, while mathematical representations of physical laws may be only approximate descriptions of underlying physical reality, the approximations can be quite good. The approximate laws of physics seem to be much more useful for prediction than the approximate laws of biological or human social systems.

We argue that physical systems tend to be more predictable than living systems because computational irreducibility in these systems is driven by a smaller set of less complex processes. Computational irreducibility in physical systems largely results from the interactions of particles or objects governed by a fixed set of rules, analogous to simple cellular automata [35]. Physical systems can become computationally irreducible with a relatively small number of interacting objects, e.g., the three body problem [35], and systems with large numbers of interacting components are likely to be computationally irreducible. The evolution of a large volume of a gas, for example, may be computational irreducible even as the gas molecules interact with each other and their surrounding environment according to known physical laws. An approximate, statistical description of the mean state of a gas is still possible, however, without an exact description of the velocities and locations of each molecule: the temperature and pressure of a gas can be described using the ideal gas law. Physical systems that are computationally irreducible can often become predictable from a macro-level perspective due to the averaging of a very large number of separate interactions, albeit with the loss of information. This is analogous to the coarse-graining of cellular automata described earlier.

The computational irreducibility of physical systems is related to the Halting Problem in a Universal Turing Machine [34]. A computation is said to be incompressible when the sequential behavior of a computer program cannot be computed in any shorter way than to allow the program to run and note its successive states. The central features of a Turing machine include a head with a set of pre-stated symbols standing for internal states, a tape marked in squares of perhaps infinite length, a pre-stated alphabet of symbols (e.g., 0 and 1) that can be read from and written to the tape by the reading head [10]). Given a set of symbols on the tape, and the reading head over one square with a symbol written on it, the head reads from the tape and, depending upon the symbol, its internal state will not move or move one square to the left or right, erase the symbol on the square below it, write a symbol on that square and go from one internal state to another internal state. Then this set of operations will iterate. Given any initial state of the tape and reading head, the next states of the tape and head can be computed for $1, 2, 3, \dots, N$ finite number of steps ahead. A Turing machine is a subset of classical physics.

We define the computationally irreducibility of physical systems and other Turing-like systems as first order computationally irreducible. This is the simplest mode of computational irreducibility in that the set of rules governing system evolution and the possible states of the particle or node are fixed, e.g., the set of potential states of a cell in a simple automaton, and do not change

as the system itself evolves. We suggest that coarse-graining approaches to prediction would be most effective in systems with first order computational irreducibility, i.e., they would gain greatest predictive capacity with minimal information loss. We argue that biological systems and human social systems have a different set of processes governing system evolution than those found in physical systems and associated with first order computational irreducibility.

4 Biological Systems

Biological systems are computationally irreducible for qualitatively different reasons than physical systems. While the same processes that yield first order computational irreducibility in physical systems also operate in biological systems, i.e., large number of interacting components, the set of rules governing these interactions and the potential states of the system components (e.g., cells in a CA, particles, organisms) evolve along with the overall state of the system. We refer to this as second order computational irreducibility—a more complex computational irreducibility than the first order computational irreducibility. The second order nature of the computational irreducibility of biological systems—meaning that the rules and set of states of fundamental units can evolve—follows from nearly universal attributes of biological systems: i) contingency of the function and selective value of biological attributes on interactions with other organisms and their environment, ii) the creation of new attributes and functions through biological evolution, and iii) individual variability in biological attributes even among organisms of the same species. Note that we use the term ‘biological evolution’ to refer to Darwinian evolution in biological systems as distinguished by ‘system evolution’, which describes changes in the state of a system through time, although biological evolution often leads to system evolution of biological systems.

Functional contingency. Biological attributes have a set of potential functions, and the set of these functions is contextually dependent on interactions with other organisms and the environment. A subset of the functions associated with an attribute may be useful in the current context of an organism and its interactions with other organisms and its environment, while other function of an attribute may be useful in other future (or past) contexts. Feathers in dinosaurs, for example, may have initially functioned in thermal regulation and only later provided additional functionalities that were coopted for flight [6] [33]. The swim bladder, a sac found in some fish that is partly filled with air and partly with water and the ratio of which determines and adjusts neutral buoyancy in the water column, is believed to have arisen from the lungs of lung fish, providing a new functionality to an existing structure [29]. Even the human capacity for reason and logic may have been a new functionality of biological traits with origins in the context of group dynamics of social organisms [26]. Some components of the set of functions of existing biological attributes might have causal consequences that are of selective significance in new environments. Functions of biological structures that are of no selective advantage in the current environ-

ment but that become selectively advantageous in later environments, typically with a new functionality, are referred to as pre-adaptations or exaptations. We assert that the potential functions of biological attributes are both indefinite in number and unorderable, and, importantly, that no algorithm can list them all. We argue that this means that the set of rules governing system evolution changes and contributes to the second order computational irreducibility of biological systems.

Biological evolution. Biological evolution is a central process that distinguishes the evolution of the biosphere from other physical systems [24]. Biological systems create and accrue attributes such as new structures, biochemical pathways, gene regulatory networks, etc. through biological evolution. These attributes provide the basis for biological function and exaptations of the previous section. The process of biological evolution is immensely creative and unpredictable, and forms a positive feedback loop that leads to further biological evolution. The evolution of feathers in dinosaurs and their ultimate use in flight resulted in the emergence of a completely new set of ecological niches, and an associated proliferation of species of birds. The emergence of flight in birds, in turn, has allowed for the long range transportation of seeds and organisms to islands and inland water bodies (e.g., [25]), opening even new ecological niches, providing the basis for new functionality of existing biological structure, and for continued evolutionarily development of biological attributes. Seabirds are, for example, responsible for substantial nutrient flows from oceans to terrestrial ecosystems, and their presence or absence can determine whether a landscape is in one of two alternative stable states—a grassland or closed shrubland [9].

Individual variation. Biological systems are distinguished from purely physical systems by individual variation of agents. Individual organisms often differ from other individuals of the same species [7]. Much of this variation is derived from underlying genetic differences and these genetic differences provide the basis for differences in biological attributes, e.g., behaviors, functions, and environmental responses and the raw material for biological evolution. Individual variation within species has been postulated to be a key mechanism driving patterns of and maintaining species diversity in ecological communities [4]. Species phenology, for example, describes the seasonal timing of demographic processes such as flowering in trees (e.g., [27]). Individual variation in response to environmental cues (e.g., day length, temperature) means that some trees will bud out and flower earlier in the spring than others. An earlier phenology could increase the likelihood of seeds colonizing and capturing new available (empty) sites in a forest or, alternatively, increase the risk of being adversely impacted by a late spring frost. The consequences of individual variation, thus, depend on the environmental and ecological context. Individual variability means that the rules for updating a system can vary from individual to individual even if the environmental context is identical. In a cellular automaton, this is analogous to cell to cell variation in the updating rule for a specific cell type, for instance, among different white cells even with identical neighborhoods.

Synthesis. Biological evolution creates attributes of organisms and the biological system creates the context that determines the functionality and utility

of these attributes. Biological evolution led to photosynthesis, and photosynthesis then resulted in abundant free oxygen in the atmosphere (e.g., [30] [21]). Biological attributes that enabled aerobic respiration in the presence of free oxygen were advantageous in this new context. Free oxygen and aerobic respiration, subsequently allowed for a wide array of niches that did not exist before and these niches could be occupied by species with new or pre-adapted functional attributes (e.g., [28]). Biological systems create and modify their own adjacent possible through construction of or extension of biological function or niche space that is immediately adjacent to current niche space. The creation of new biological opportunities allows for the emergence of new organisms, new functionalities, and a new adjacent possible. This process is enormously creative and unpredictable a priori. Biological systems are thus second order computationally irreducible, because the rules for updating and the potential states of the system change as the system evolves. The evolution of the biosphere is non-algorithmic. We claim that no algorithm can pre-state all possible biological attributes, their potential functions, or how these functions might be of selective advantage in potential future environments. The unpredictability of biological systems is thus radically unlike the computational incompressibility of physical systems, the Halting problem on a universal Turing machine or, a fortiori, unlike the irreducibility of cellular automata.

5 Human Social Systems

Human social systems are a specialized case of a biological system with an additional source of computational irreducibility: sentience. We use ‘sentience’ to refer to the state of being conscious, self-aware, and having free will. Humans are sentient beings that are able to perceive their own possibilities within the context of their environment. A person might, for instance, conceive of a network of linked computers that would later become the internet and allow for the world wide web. The creation of the internet and world wide web then provides the basis for other innovations that are dependent on the existence of the internet, e.g. social networking websites, cloud computing, etc. The creation of the internet allowed for the possibility of these subsequent innovations—the internet resulted in a new and expanded adjacent possible. All of these innovations—the internet, social networking websites, and cloud computing—require a person(s) that imagined or perceived the possibility of these innovations in a given context. Similar sequences of creative expansion of the adjacent possible can be found in many contexts outside of technology—from music and visual art to the development of law and systems of governance. Sentience thus acts to create what is possible adjacent to what currently exists in a manner analogous to biological evolution, and this process proceeds in a positive, self-reinforcing feedback loop: Innovation creates the opportunity for more innovation.

Sentience and the perception of possibility distinguish the computational irreducibility of human social systems from physical and other biological systems. The processes that contribute to the computational irreducibility of physical

and biological systems also apply to human social systems, i.e., interactions among many system components, biological evolution, and individual variation, functional contingency. Sentience operates in addition to these processes and sets the computational irreducibility of human social systems apart from these other systems. We thus characterize human social systems as having third order computational irreducibility. Third order computational irreducibility is distinguished by the sentient perception of what is possible in a given context, and drives the evolution of technology, economics, governance, and other components of the human social system. We expect that human social systems will be less predictable than biological or physical systems, meaning that predictive gains from coarse-graining will result in larger information loss than occurs in these other systems.

In the context of human social systems, the adjacent possible is related to the concept of affordances. Affordances are the attributes of an object or environment that allow an action to be performed [12]. Affordances are action possibilities that humans perceive as, for example, the many potential uses of a screw driver (e.g., turning a screw, opening a can, puncturing a tire). Affordances are in many ways analogous to the process of biological evolution ‘discovering’ the function of attributes of organisms in the context of an organism’s environment. The relationships between humans and their environment can thus lead to perceived possibilities, actions, and cognition, and is dynamic, reciprocal, and contextual [23]. While our discussion has focused on individuals and consciousness, human social systems operate across hierarchical levels of structure. Social systems include individuals, small groups of people, more expansive social organizations and institutions, and networks of organizations [22]. Each of these levels of social organization contributes to the computational irreducibility of social systems, but the sentience of individuals—and the inherit variability among individuals—is the defining process that distinguishes human social systems from other purely biological systems. The computational irreducibility that stems from sentience is compounded by the interactions between and among the other components of social systems. The agency of an individual person can affect higher, levels of social organization (e.g., through leadership and contagion of beliefs), but social groups and organizations also impact the actions and identities of individuals. These feedbacks and linkages between individuals and groups have likely been made stronger and more fluid with the advent of social media, and are central to understanding and predicting trajectories of human social systems. Lastly, the role of culture in accumulating and transferring information among individuals is a central feature of human social systems that is akin to information storage in the genetic code in biological systems. *Challenges.* Designing algorithms for essentially non-algorithmic problems has been problematic since the onslaught of Turing-complete machines (e.g., [8]). In human social systems, the problem of framing affordances has not yet been programmed. Whether it is programmable or not is a question that is central to the field of computational complexity, artificial intelligence and robotics. Agent based models (ABMs) have opened up new vistas of scientific discovery to simulate decision-making by heterogeneous agents in artificial soci-

eties (e.g. [13]), but there are significant limits to the algorithmic approach for simulating both creative decision making by intelligent agents in rapidly shifting environments and social dynamics, a problem that was even acknowledged by Turing [10]. Fundamental assumptions that are engrained in each algorithm about the behavioral rules, creative decision-making, learning, treatment of uncertainty and so forth, constrain the modeling of emergence, self-organization and adaptation in complex social systems. Different types of algorithms and Turing-complete machines such as agent based models, genetic algorithms and artificial neural networks, have opened up new vistas for modeling creative decision making in finite, discrete, computational steps (e.g., [17] [16] [14]). Human social systems with heterogeneous agents with the capability for creative decision making in rapidly shifting social environments may significantly limit the potential for algorithms to model and predict the trajectory of these systems. Our understanding of emergence, self-organization and adaptation in complex systems populated by sentient agents that undertake creative decision-making is limited by algorithms.

6 Conclusions

The limits to predictive capacity imposed by computational irreducibility is increasingly important as we confront complex and interlinked problems that incorporate natural and human social systems. Predicting the trajectory of earth's climate system, for example, is an important but difficult problem because it incorporates human social, biological, and physical systems. Computational irreducibility is an inevitable feature of complex systems, but we argue that not all forms of computational irreducibility are equivalent. The underlying processes that lead to computational irreducibility and the potential for gains in predictive capacity vary across physical, biological, and social systems. Physical systems have the simplest kind of computational irreducibility, which we define as first order computational irreducibility, in which neither the set of potential states nor the rules for updating the states change as the system evolves. The potential for system prediction is likely to be the greatest with first order computational irreducibility but with the loss of information. Biological systems have a more intransigent computational irreducibility because the potential system states and updating rules change as the system evolves. Functional contingency, biological evolution, and individual variation are three underlying processes that lead to this second order computational irreducibility of biological systems. Humans perceive and create their own adjacent possible and this sentience leads to human social systems being characterized by third order computational irreducibility. The increasingly difficult forms of computational irreducibility across physical, biological, and human social systems, and the low predictive capacity found in these living systems is offset by their remarkably rich, diverse, and creative dynamics. Although we argue that ultimately, the evolution of the biosphere is non-algorithmic, there is much to be learned in the pursuit of the frontier of first, second and third order compu-

tational irreducibility, and this will challenge computational modelers to reach the outer limits of computational irreducibility.

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