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Editors

Information Theory

Poincaré Seminar 2018

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Foreword

This book is the eighteenth in a series of Proceedings for the *Séminaire Poincaré*, which is directed towards a broad audience of physicists, mathematicians, philosophers and historians of science.

The goal of the Poincaré Seminar is to provide up-to-date information about general topics of great interest in physics. Both the theoretical and experimental aspects of the topic are covered, generally with some historical background. Inspired by the *Nicolas Bourbaki Seminar* in mathematics, hence nicknamed *"Bourbaphy"*, the Poincaré Seminar is held once or twice a year at the Institut Henri Poincaré in Paris, with written contributions prepared in advance. Particular care is devoted to the pedagogical nature of the presentations, so that they may be accessible to a large audience of scientists.

This new volume of the Poincaré Seminar Series, **Information Theory**, corresponds to the twenty-third such seminar, held on November 17th, 2018, at Institut Henri Poincaré in Paris. Its aim is to provide a thorough description of information theory and some of its most active areas, in particular, its relation to thermodynamics at the nanoscale and the Maxwell demon, and the emergence of quantum computation and of its counterpart, quantum verification.

The first article, entitled *Thermodynamics and Information Theory*, by the theoretical physicists Kirone Mallick and Bertrand Duplantier from the Institut de Physique Théorique at Université Paris-Saclay, begins with a review of the laws of classical thermodynamics and statistical physics with an emphasis on the dynamical properties of equilibrium as exemplified by Brownian fluctuations, a universal characteristic of all complex systems at finite temperature. Then it presents recent advances on systems far from equilibrium, which lie beyond the realm of traditional thermodynamics by continuously exchanging matter, energy, or information with their surroundings. Two major contemporary results are the Gallavotti-Cohen-Evans-Morriss *fluctuation theorem* that characterizes statistical properties of entropy production rate, and Jarzynski's *work relation* that extends the XIXth century maximal work inequality to a precise identity, with far-reaching consequences, particularly in studies of active matter. The fluctuation theorem and the work relations are fingerprints at the macroscopic level of the fundamental microscopic time-reversal invariance, satisfied by most systems in condensed matter physics. The last sections of this chapter relate thermodynamics with information theory, starting from Shannon's definition, and proceeding through a description of Maxwell's paradoxical demon, which acquires and exploits information to violate the Second Law of Thermodynamics. Various attempts to exorcise this demon are sketched, from Leó Szilárd's seminal work, Brillouin's brilliant and bold assumption that Shannon's information contributes to the total entropy to preserve the

Second Law's balance sheet, to Landauer's *principle of information erasure*. This introductory chapter paves the way to the more detailed discussions presented in the next articles.

The second article, entitled *This is IT: A Primer on Shannon's Entropy and Information*, is written by OLIVIER RIOUL, a leading researcher in information theory (IT) and signal processing from Télécom Paris of the Institut Polytechnique de Paris. As an enthusiast of Shannon's life and work (see the wonderful movie by Mark A. Levinson, `http://www.documentarymania.com/player.php?title=The%20Bit%20Player`), he offers an in-depth introductory text from both historical and mathematical viewpoints, which lets the elegance and beauty of the subject matter shine through many aspects of the concepts of information and entropy. Thanks to its thirty-seven concise and mostly independent sections, we can appreciate Shannon's 1948 seminal work, *A Mathematical Theory of Communication* (see `http://bibnum.education.fr/sites/default/files/174-article.pdf`), and its mathematical connections to the manifold notions of entropy. The author carefully defines all basic notions of discrete and continuous entropies and entropy power, explains a fundamental information inequality originally due to Gibbs in relation to the MaxEnt (maximal entropy) principle, on which Statistical Mechanics can be entirely founded as advocated by Jaynes, and goes all the way through the notion of relative entropy (or divergence), Chernov, Fisher, and Kolmogorov informations, and, of course, Shannon's mutual information. Shannon's celebrated first and second coding theorems are all stated with sketchs of proofs. This delightful and mostly complete expository text ends with what is perhaps the most difficult inequality due to Shannon, the so-called *entropy power inequality* (EPI), which states that the entropy power of the sum of independent random variables is no less than the sum of their individual entropy powers. Initially used by Shannon for evaluating the capacity of non-Gaussian channels, the EPI now finds multiple applications in IT and mathematics in general. While Shannon's original proof was incomplete and was corrected ten years later by Stam, the author offers an exposition of his novel and elegant 2017 proof as a nice conclusion to this tutorial.

The third contribution, entitled *Landauer's Bound and Maxwell's Demon*, is due to the leading physicist SERGIO CILIBERTO from the Laboratoire de Physique at École Normale Supérieure de Lyon. It describes recent experimental and theoretical progress made in relation to information theory and thermodynamics. The extraordinary wealth of high-quality experimental data at mesoscopic scales implies a complete overhaul of the traditional entropy and irreversibility paradigms in thermodynamics. Together with collaborators, the author performed in 2012 the first experimental test of the *Landauer principle* of 1961, which predicts that a minimum amount of heat $k_B T \ln 2$ is produced when erasing a bit of information, showing for the first time that this limit can be experimentally reached. The author received several scientific awards for his achievements, notably the Prix Jaffé de l'Académie des Sciences de l'Institut de France in 2018 and the EPS Statistical and Non Linear Physics Prize in 2019.

This contribution summarizes in a concise and clear way the basic concepts of *Maxwell's demon* and the *Szilárd engine*, and the resolution in 1982 by C. Bennett of the paradox involved with respect to Clausius' second law. It describes the first experimental implementations of the Maxwell demon, in 2008 with the cooling of atoms in a magnetic trap, in 2010 with the realization of a Szilárd engine with a single microscopic Brownian particle in a fluid in a spiral staircase potential, and in 2015 in electronic circuits. The details of the author's seminal experiment on Landauer's principle, using a colloïdal Brownian particle in a fluid trapped in the double-well potential created by two laser beams, are discussed, as well as the principle's intimate relationship to a generalized form of *Jarzynski's equality*. The survey ends with extensions to the quantum regime, by describing the successful realizations of a quantum Maxwell demon in an NMR set-up in 2016 and in a circuit QED system in 2017, culminating with the recent first experimental verification of the Landauer principle in the quantum case, which used a molecular nano-magnet at a very low temperature.

This volume ends with a perspective on challenges concerning the emerging field of *quantum computation*. It is based on a thoroughly detailed review, entitled *Verification of Quantum Computation: An Overview of Existing Approaches*, by Elham Kashefi, a leading researcher at the Laboratoire d'Informatique de Sorbonne Université and at the School of Informatics at University of Edinburgh, written in collaboration with Alexandru Ghiorghiu, now at the Institute for Theoretical Studies at ETHZ, and with Theodoros Kapourniotis at the Department of Physics of the University of Warwick. This work is reprinted from *Theory of Computing Systems* **63**, 715–808 (2019), https://doi.org/10.1007/s00224-018-9872-3, under the Creative Commons Attribution 4.0 International License (http://creativecommons.org/licenses/by/4.0/).

Over the next five to ten years we will see a state of flux as quantum technologies become part of the mainstream computing landscape. In the meantime we can expect to see quantum computing machines with high variability in terms of architectures and capacities (as we saw when classical computers emerged in the early 1950s). These devices will not be universal in terms of having a simple programming model nor will they be easily applicable to all problems. Adopting and applying such a highly variable and novel technology is both costly and risky for any individual company or research group, as this quantum approach has an acute verification and validation problem: since classical computations cannot scale up to the computational power of quantum mechanics, verifying the correctness of a quantum-mediated computation is challenging; the underlying quantum structure resists classical certification analysis. This contribution covers recently developed techniques to settle these key challenges to make the translation from theory to practice possible. Without this final link the glorious power of quantum technology will not be accessible to us.

This book, by the breadth of topics covered in both the theoretical description and present-day experimental study of the manifold aspects of Information, should be of broad interest to physicists, mathematicians, and philosophers and historians of science. We hope that the continued publication of this series of Proceedings will serve the scientific community, at both the professional and graduate levels.
We thank the COMMISSARIAT À L'ÉNERGIE ATOMIQUE ET AUX ÉNERGIES ALTERNATIVES (Direction de la Recherche Fondamentale), the DANIEL IAGOLNITZER FOUNDATION, the ÉCOLE POLYTECHNIQUE, and the INSTITUT HENRI POINCARÉ for sponsoring the Seminar on which this book is based. Special thanks are due to Chantal DELONGEAS for the preparation of the manuscript.

Saclay & Orsay,
January 2021

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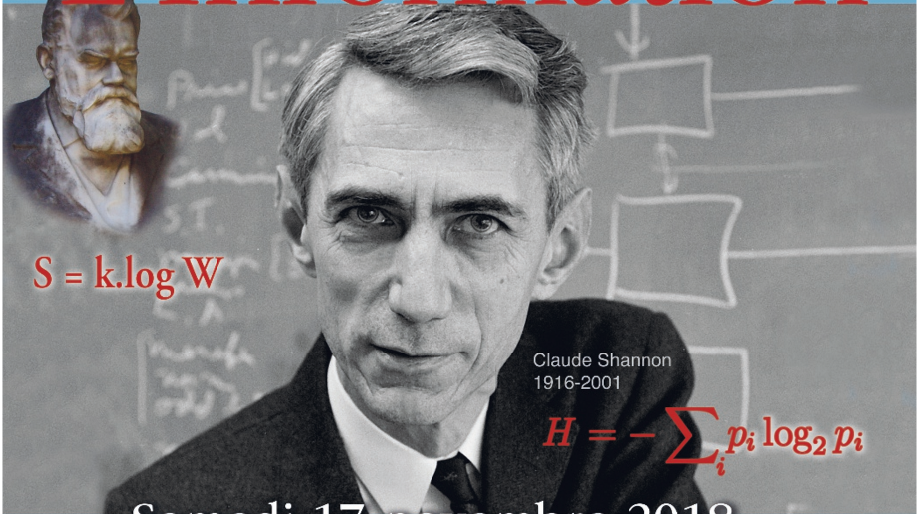
Séminaire Poincaré XXIII

L'Information



Samedi 17 novembre 2018

K. Mallick : Thermodynamique et information • *10h*

O. Rioul : La théorie de l'information sans peine • *11h*

S. Ciliberto : Landauer et le démon de Maxwell • *14h*

E. Kashefi : Quantum Verification • *15h*

C. Salomon : La simulation quantique • *16h*

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