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On free vibration of laminated skew sandwich plates: A finite element analysis

https://doi.org/10.1515/nleng-2021-0006
Received Aug 23, 2020; accepted Jan 24, 2021.

Abstract: The present work emphasizes the determination of the fundamental frequency of skew sandwich plates with orthotropic core and laminated facings using different design parameters. Finite elements CQUAD4 and CQUAD8 of MSC/NASTRAN are used for obtaining fundamental frequencies, which are validated against available literature results. The influence of the skew angle, the ratio of the length-to total thickness (a/h) of the sandwich plate, and the ratio of the thickness of the core to face sheet (t_c/t_h) on the fundamental frequency of skew sandwich plates are studied. Also, the influence of parameters such as the number of layers in the face sheet, laminate sequence, and fiber orientation angle on the fundamental frequency of laminated skew sandwich plates have been studied. It is found that the CQUAD8 element yields better results than the CQUAD4 element in the present study. The fundamental frequencies are found to increase with the increasing skew angle. The variation in fundamental frequency is negligible when the number of layers is large in the face sheet.

Keywords: fundamental frequency, non-dimensional frequency parameter, skew sandwich plate, skew angle, antisymmetric laminate, fiber orientation angle

1 Introduction

Skew sandwich plates are now a day frequently used in numerous areas like aeronautical, automobile, civil engineering, and in most structural applications. In skew sandwich plates, the effect of shear deformation is considerably more as compared to laminated composite skew plates, which was the reason behind the widespread applications of such plates. Also skew sandwich plate exhibits less weight, more stiffness, more structural efficiency, and more durability. Much research was made on sandwich plates on the free vibration behavior for more than two decades.

A linear analysis for bending and vibration of sandwich plates was employed for analytical and experimental investigations [1]. Also, refined plate theory was proposed on sandwich plates [7]. The free vibration analysis using higher-order shear deformation theory of sandwich plates [18], laminated composite and sandwich plates [6], skew sandwich plate with laminated composite faces were presented [8]. Free vibrations and buckling of the sandwich panel with a flexible core was investigated using a new improved high-order sandwich panel theory [11]. Free vibration analysis of laminated composite and sandwich plates using trigonometric shear deformation theory was performed [15]. Quasi-3D shear deformation theory was employed for thermo-mechanical bending analysis of functionally graded material (FGM) sandwich plates [35] and buckling and the post-buckling response was recorded from functionally graded carbon nanotube (FG-CNT) - magnesium (Mg) nanocomposite plate with interphase effect [31]. The modified stiffness method was applied to the dynamic analysis of sandwich plates [2]. An experimental modal study was conducted on a cantilever flexible plate underwater due to the hydrodynamic effect [32]. A study dealing with the comparison of free vibration responses obtained from four theories on composite truss core sandwich plates were presented. The natural frequencies of the sandwich plate are calculated by using the classic laminated plate theory, the first-order shear deformation theory, Reddy's third-order shear deformation theory, and a Zig-Zag theory [12]. Various shear deformation theories [13] were considered for the comparison based on the displacement fields [14].

Finite element analysis of composite sandwich plates was carried out based on Mindlin's plate theory [3]. The bending behavior [16] and free vibration response [17] using a four nodded rectangular finite element formulation based on a layer-wise theory, Static analysis [9], and the free vibration response [10] using an improved dis-

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crete Kirchhoff quadrilateral element based on third-order zigzag theory were presented. The p-Ritz method [5] on Skew sandwich plates and a numerical study were made on a sandwich plate to improve the dynamic effects of geometric design variables and material alteration [4]. The vibration parameters of sandwich plates were predicted by a spline finite strip method [19], harmonic quadrature element method [26].

Free vibration analysis of plates and sandwich plates was discussed using C^o iso-parametric finite element model [20], Two new C^0 assumed strain finite element [21], C^0 finite element model [22]. Fundamental flexural frequencies of isotropic and laminated composite skew plates [23], skew sandwich composite plates [27, 28] have been obtained using finite elements. Also, the experimental and finite element studies were carried out on free vibration of isotropic and laminated composite skew plates [24, 25]. The nonlinear static, buckling, and vibration analysis of viscoelastic micro-composite beam reinforced by various distributions of boron nitride nanotube (BNNT) with initial geometrical imperfection by modified strain gradient theory (MSGT) using finite element method (FEM) was presented [33]. A critical review of available literature for the prediction of the behavior of laminated composites and sandwich structures under hygrothermal conditions was carried out [34].

The present research focuses on the free vibration studies on laminated sandwich skew plates with simply supported and clamped boundary conditions. The face sheet consists of a laminated composite reinforced with graphite-epoxy and a heavy core (orthotropic). The key objective is to investigate the influence of the number of layers in the face panel, the ratio [a/h], the ratio [t_c/t_f], the effect of fiber orientation, the effect of the laminate sequence, the effect of boundary conditions, the effect of the skew angle on the sandwich plate's free vibration response. The paper is organized as follows: Firstly, for the free vibration analysis of the sandwich plate, convergence of the results gathered by both CQUAD4 and CQUAD8 elements is evaluated. The validation of the result by the present approach is compared to those available in the literature using converged element density. By implementing the mechanical properties as implemented in [20] for both the orthotropic face sheet (GFRPC) and the orthotropic core (Heavy), computational analysis is finally carried out to describe the effect of various geometric parameters, boundary conditions, and skew angle.

2 Finite element formulation

For thick plates the following equation (1) holds good:

$$\left\{\begin{array}{c} u \\ v \\ w \end{array}\right\} = \left\{\begin{array}{c} u_0 + z\theta_x \\ v_0 + z\theta_y \\ w_0 \end{array}\right\} and \left\{\begin{array}{c} \theta_x \\ \theta_y \end{array}\right\} = \left\{\begin{array}{c} w_{,x} + \phi_x \\ w_{,y} + \phi_y \end{array}\right\} \tag{1}$$

Using five components u, v, w, θ_x, θ_y, the displacement of the plate are fully described where u, v, and w are displacements along Cartesian x, y and z-directions also θ_x (w, x, and φ_x) and θ_y(w, y, and φ_y) are total (bending and shear) rotations about y- and x-axes, respectively, whereas, u_0, v_0, and w_0 are the mid-plane translations along x, y and z directions, respectively. Nodal displacements are used to describe the displacement δ_j at any point within the element by the following equation.

$$\delta_j = N_i \delta_{\mathrm{ij}} \tag{2}$$

Where N_j are isoparametric shape functions [30]. The stiffness matrix of the plate element assumes the form.

$$[K]_e = \int_{A_e} [B]^T [D] [B] \, dA \tag{3}$$

Where,

$$\{\varepsilon\} = [B] \{\delta\} \ldots \tag{4}$$

$\{\varepsilon\}$ being the strain vector, and $\{\delta\}$ the nodal displacement vector. [B] is the strain-displacement matrix, and [D] is the stiffness matrix given below.

$$[D] = \begin{bmatrix} A_{\mathrm{ij}} & B_{\mathrm{ij}} & 0 \\ B_{\mathrm{ij}} & \mathrm{D_{ij}} & 0 \\ 0 & 0 & \mathrm{A_{lm}} \end{bmatrix} \tag{5}$$

Where,

$$A_{ij}, B_{ij}, D_{ij} = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} (Q_{ij})^k \; (1, z, z^2) dz, i, j = 1, 2, 6 \ldots \tag{6a}$$

And

$$A_{km} = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \kappa (Q_{lm})^k \; dz, l, m = 4, 5, \kappa = 5/6 \ldots. \tag{6b}$$

Here, Q_{ij} is the element of off-axis stress-strain relations. ${Q_{ij}}^k$ relates stresses and strains in a k^{th} layer by the relation ${\sigma_i}^k$ = ${Qij}^k$ ${\epsilon_j}^k$. i, j =1,2,6. Here σ_1, σ_2, and σ_6 denote σ_x, σ_y and τ_{xy} respectively and ϵ_1, ϵ_2, ϵ_6 denote ϵ_x, ϵ_y, y_{xy}

respectively. Whereas $\sigma_l{}^k = Q_{lm}{}^k\, \epsilon_m{}^k$ where l,m= 4,5 and κ is the shear correction factor taken as 0.8334. The mass matrix of the plate element is given by

$$[M]_e = \int_{A_e} [N]^T [\rho] [N] \, dA \tag{7}$$

$[\rho]$ being the density matrix functions.

The integration in every case is carried out over the area of the plate element. Generally, a 3-point Gauss quadrature is adopted to compute the bending stiffness of the elements, whereas 2-point integration is applied to calculate the shear stiffness, mass matrix, and element force vector. The governing equations, without damping being accounted for free vibration is

$$M\ddot{x} + Kx = 0 \tag{8}$$

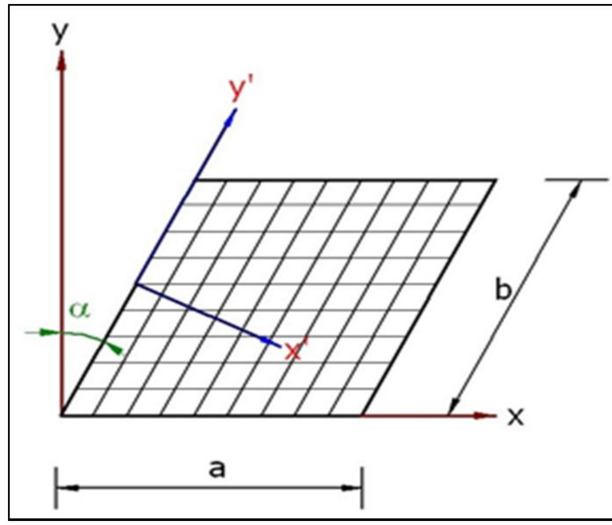


Figure 2: Finite element mesh model of flat skew sandwich plate

3 Convergence and validation

3.1 Convergence

The geometrical representation of the sandwich plate is as shown in Figure 1. The skewed sandwich plate with global and local coordinate systems is as shown in Figure 2. The displacement boundary conditions cannot be applied directly, due to the inclination of displacements to the skew edges. To overcome this, a local coordinate system (x′, y′) normal and tangential to the skew edges is preferred.

A total number of elements in the plate model is optimized to get exact and consistent values. Consequently, it is essential to analyze the convergence of the values. The convergence was made on simply supported and clamped skew sandwich plates using CQUAD4 (four-node plate element) and CQUAD8 (eight-node isoparametric curved shell element) elements of MSC/NASTRAN. Skew sandwich plates with varying aspect ratio, length to thickness ratio, and the ratio of a thickness of core to facing for skew angles 0°, 15°, 30°, and 45° using both the elements are evaluated. The converged detailed results are conveyed in Table 1. The material properties used are, for face sheets E= 68.948 GPa, G= 25.924 GPa, ν=0:33, ρ=2768.0 kg/m^3 and core G_{23}= 0.05171 GPa, G_{13}=0.13445 GPa, ρ=121.83 kg/m^3 [29].

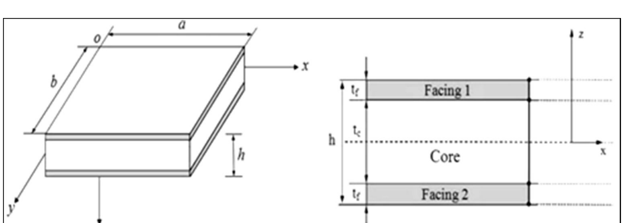


Figure 1: Geometry details of sandwich plate (0° skew angle)

3.2 Validation

Validation of the results from the elements used in the present study is made by matching up the values for the natural frequency found in the present study to the available literature values. The comparison is shown in Table 2 and 3, for clamped and simply supported boundary conditions respectively of a skew sandwich plate in Hz. The material constants employed are similar to those used in [19]. The values found in the study are in good harmony with the literature results. Also for simply supported sandwich skew plates, the material constants are referred to as in [8].

Non-dimensional frequency parameter (K_f) of simply supported five-layered symmetric laminated composite skew sandwich plates with orthotropic core was determined by using the formula $\mathbf{K}_f = \mathbf{100}\omega\mathbf{a}\sqrt{(\rho/\mathbf{E}_1)_f}$. The validation results for simply supported boundary conditions are shown in Table 6. The material properties employed for the study were as mentioned in [22].

From Table 1 to 4 it is observed that the CQUAD8 element gives accurate and converged results as then the CQUAD4 element. From now CQUAD8 is adopted in further work.

Table 1: Convergence study for fundamental natural frequencies (Hz) of simply supported skew sandwich plates (a/b=1,a/h=10, tc/tf =10).

Element Density	Element Type	S-S-S-S				C-C-C-C			
		Skew Angle (α)				Skew Angle (α)			
		0^0	15^0	30^0	45^0	0^0	15^0	30^0	45^0
Present (10 x 10)	CQUAD 4	2493.991	2570.462	2827.037	3369.724	3017.562	3081.522	3300.511	3779.848
	CQUAD 8	**2519.097**	**2596.492**	**2856.251**	**3405.989**	**3052.652**	**3117.397**	**3339.141**	**3824.811**
Present (14 x 14)	CQUAD 4	2507.189	2584.146	2842.358	3388.675	3036.291	3100.641	3320.982	3803.530
	CQUAD 8	**2520.018**	**2597.446**	**2857.271**	**3407.130**	**3054.261**	**3119.014**	**3340.760**	**3826.509**
Present (18 x 18)	CQUAD 4	2512.631	2589.787	2848.699	3396.465	3044.039	3108.550	3329.487	3813.304
	CQUAD 8	**2520.395**	**2597.836**	**2857.720**	**3407.607**	**3054.925**	**3119.680**	**3341.466**	**3827.207**
Present (22 x 22)	CQUAD 4	2515.387	2592.644	2851.877	3400.404	3047.969	3112.561	3333.756	3818.257
	CQUAD 8	**2520.586**	**2598.034**	**2857.901**	**3407.851**	**3055.261**	**3120.017**	**3341.777**	**3827.562**
Present (26 x 26)	CQUAD 4	2516.973	2594.288	2853.718	3402.667	3050.232	3114.871	3336.230	3821.107
	CQUAD 8	**2520.696**	**2598.147**	**2858.040**	**3407.993**	**3055.455**	**3120.211**	**3341.975**	**3827.767**
Present (30 x 30)	CQUAD 4	2517.968	2595.319	2854.872	3404.086	3051.653	3116.322	3337.783	3822.894
	CQUAD 8	**2520.765**	**2598.218**	**2858.119**	**3408.082**	**3055.577**	**3120.333**	**3342.099**	**3827.896**
Present (34 x 34)	CQUAD 4	2518.634	2596.009	2855.666	3405.033	3052.603	3117.291	3338.852	3824.090
	CQUAD 8	**2520.811**	**2598.266**	**2858.193**	**3408.143**	**3055.658**	**3120.415**	**3342.213**	**3827.982**
Present (38 x 38)	CQUAD 4	2519.100	2596.492	2856.207	3405.698	3053.270	3117.971	3339.581	3824.927
	CQUAD 8	**2520.843**	**2598.299**	**2858.230**	**3408.186**	**3055.715**	**3120.472**	**3342.271**	**3828.043**

Table 2: Fundamental frequencies (Hz) of clamped laminated composite sandwich plates with orthotropic core.

Layup Sequence	Authors	Mode				
		1	2	3	4	5
$30^0/30^0/30^0$ /C/ $30^0/30^0/30^0$	Yuan [19]	708.0000	1153.0000	1423.0000	1629.0000	1999.0000
	LEE (1966) [3]	707.0000	1150.0000	1424.0000	1627.0000	1990.0000
	Kanematsu (1988) [1]	720.0000	1181.0000	1463.0000	1683.0000	2074.0000
	Present CQUAD4	**762.9000***	**1240.6000***	**1527.7000***	**1753.7000***	**2131.0000***
	Present CQUAD8	**763.6000***	**1241.9000***	**1529.9000***	**1756.7000***	**2135.0000***
$0^0/0^0/0^0$ /C/ $0^0/0^0/0^0$	Yuan [19]	692.3000	1191.0000	1366.0000	1720.0000	1954.0000
	LEE (1966) [3]	691.0000	1200.0000	1353.0000	1715.0000	1997.0000
	Kanematsu (1988) [1]	701.0000	1215.0000	1401.0000	1768.0000	2017.0000
	Present CQUAD4	**746.8000***	**1296.8000***	**1454.0000***	**1846.1000***	**2128.9000***
	Present CQUAD8	**747.3000***	**1298.3000***	**1455.5000***	**1850.6000***	**2132.3000***
$30^0/-30^0/30^0$ /C/ $30^0/-30^0/30^0$	Yuan [19]	559.1000	1001.0000	1088.0000	1484.0000	1615.0000
	LEE (1966) [3]	558.0000	997.0000	1090.0000	1478.0000	1604.0000
	Kanematsu (1988) [1]	567.0000	1024.0000	1115.0000	1528.0000	1670.0000
	Present CQUAD4	**630.7000***	**1124.0000***	**1226.8000***	**1662.4000***	**1803.8000***
	Present CQUAD8	**631.5000***	**1125.8000***	**1228.9000***	**1667.6000***	**1807.5000***
$0^0/90^0/0^0$ /C/ $0^0/90^0/0^0$	Yuan [19]	628.3000	1011.0000	1273.0000	1521.0000	1604.0000
	LEE (1966) [3]	628.0000	1007.0000	1272.0000	1517.0000	1593.0000
	Kanematsu (1988) [1]	637.0000	1032.0000	1313.0000	1568.0000	1658.0000
	Present CQUAD4	**709.4000***	**1137.0000***	**1433.0000***	**1708.9000***	**1794.0000***
	Present CQUAD8	**709.7000***	**1138.1000***	**1434.3000***	**1712.3000***	**1796.2000***

*Present values obtained excluding kinetic energy of adhesive layer.

4 Results and discussion

The present numerical study considers a variety of parameters, such as aspect ratio, a ratio of length to thickness of sandwich plates, ration thickness of face sheet to thickness of the core, skew angle, and boundary conditions of the sandwich skew plates. The results from the numerical methods are obtained by adopting material properties for further study hereafter as for Face sheet, E_1=206.84 GPa, E_3=5.1711 GPa, G_{12}=5.1711 GPa,ν_{12}=0:25, and ρ=1603.1kg/m^3 and core G_{13}=0.11721 GPa, G_{23}=0.24132 GPa and ρ=2351.2 kg/m^3 [20].

Table 3: Fundamental frequencies (Hz) of simply supported laminated composite skew sandwich plates with orthotropic core.

Layup Sequence	Authours	Skew Angle (α)			
		0^0	15^0	30^0	45^0
$0^0/90^0$ /C/ $0^0/90^0$	Ibrahim [2]	152.6000	-	-	-
	Yuan and Dawe [19]	152.5800	-	-	-
	Ajay Kumar Garg [20]	166.3086	177.6942	217.7630	310.6456
		152.2992	161.7182	194.3770	267.3398
	Voyiadjis [8]	150.9120	161.1690	195.8480	269.5720
	Present CQUAD4	**152.3300**	**163.0240**	**198.8113**	**277.4106**
	Present CQUAD8	**152.3602**	**163.0580**	**198.8503**	**277.4685**
$0^0/90^0$ /C/ $90^0/0^0$	Ibrahim [2]	146.0000	-	-	-
	Yuan and Dawe [19]	145.9900	-	-	-
	Ajay Kumar Garg [20]	159.8275	170.7568	209.3430	299.3778
		146.5089	155.5495	186.9801	257.5617
	Voyiadjis [8]	145.0002	155.1070	188.7120	260.1220
	Present CQUAD4	**145.6081**	**155.8880**	**190.2696**	**265.4776**
	Present CQUAD8	**145.6373**	**155.9213**	**190.3088**	**265.5364**
$90^0/0^0$ /C/ $0^0/90^0$	Ibrahim [2]	159.3000	-	-	-
	Yuan and Dawe [19]	159.3000	-	-	-
	Ajay Kumar Garg [20]	172.7237	184.5342	225.9660	321.4230
		158.0954	167.8775	201.7029	276.9311
	Voyiadjis [8]	156.6980	161.1840	202.9450	278.6170
	Present CQUAD4	**158.9292**	**170.0157**	**207.1453**	**289.0373**
	Present CQUAD8	**158.9602**	**170.0502**	**207.1840**	**289.0932**

Table 4: Non dimensional frequency parameter (K_f) of simply supported laminated composite skew sandwich plates with orthotropic core.

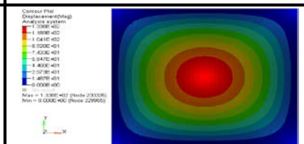
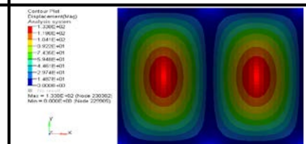
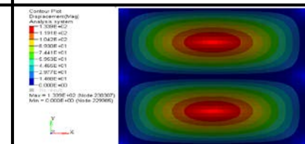
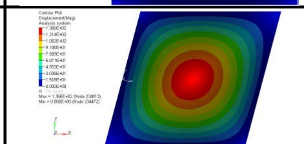
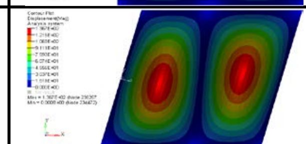
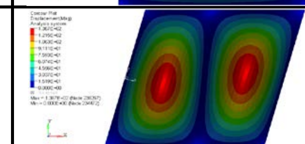
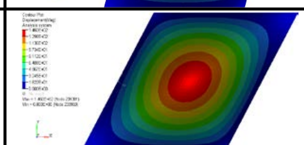
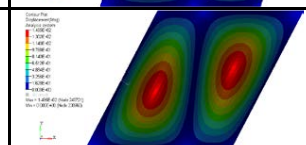
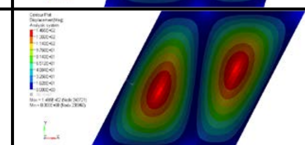
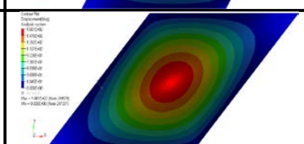
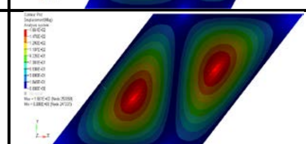
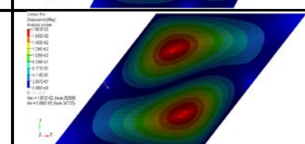
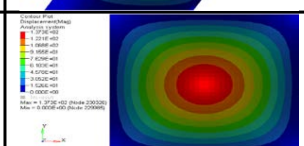
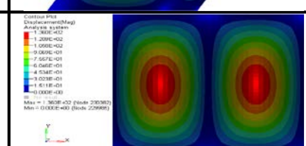
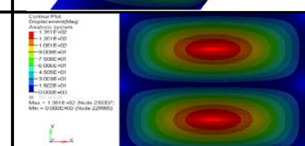
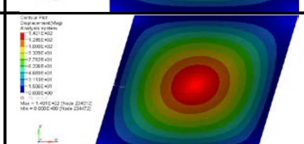
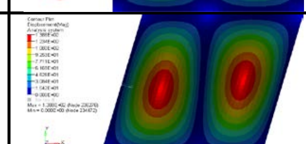
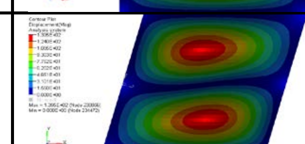
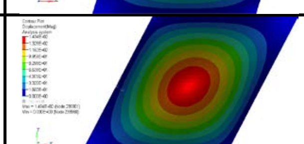
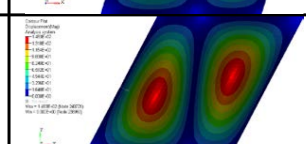
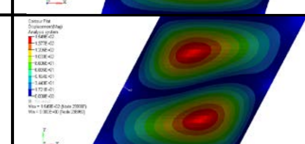
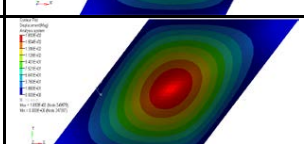
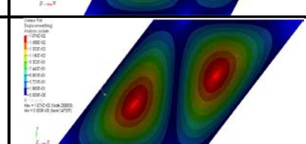
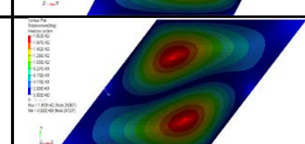
AUTHOURS		Skew Angle (α)			
		0^0	15^0	30^0	45^0
Kulkarni and Kapuria [10]	ZIGT FE	9.8130	-	-	-
		9.8200	-	-	-
		9.8240	-	-	-
	3D ZIGT	9.8281	-	-	-
	ZIGT	9.8300	-	-	-
Chakrabarti and Sheikh [7]		10.0510	-	-	-
		10.0520	-	-	-
		10.0530	-	-	-
Wang [5]	p-Ritz	10.5550	-	-	-
Kulkarni and Kapuria [10]	TOT	12.0880	-	-	-
Chalak and Chakrabarti [22]	HOZIGT	9.8365	10.2467	11.6056	14.4349
Present CQUAD4		**9.2704**	**9.5546**	**10.5191**	**12.5896**
Present CQUAD8		**9.2768**	**9.5612**	**10.5266**	**12.5988**

4.1 Study on the effect of number of layers

The effect of the number of layers on the fundamental frequency is assessed and results are graphically presented in Figure 3 and 4 in non-dimensional form K_f as well as the mode shapes in Table 5. The aspect ratio kept constant to 1, skew angle, and the number of layers in the face sheet is varied for all sides simply supported and clamped edge condition. The following observations were made from the results,

- An initial increase in the layers increases the stiffness of the plate, later the added layers do not contribute to the sandwich plate's vibration response. Adding the number of layers in the face sheet allows the sandwich skew plate to accumulate in its weight. The largest impact is the core thickness that takes the majority of

Table 5: Mode shapes of anti symmetric 5 layer (0°/90°/C/0°/90°) skew sandwich plates.

Boundary Condition	Skew Angle	Mode Shapes [NL=4]		
		1	2	3
S-S-S-S	0^0			
	15^0			
	30^0			
	45^0			
C-C-C-C	0^0			
	15^0			
	30^0			
	45^0			

shear stress. The K_f initially increases up to 4 layers, as the number of layers of the face sheet increased and after this, the shift is constant or insignificant.

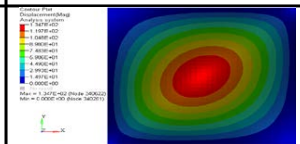
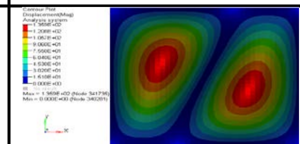
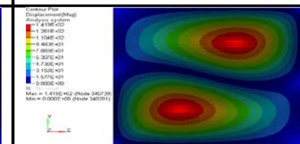
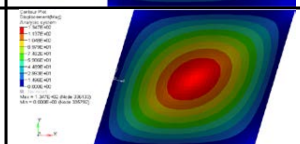
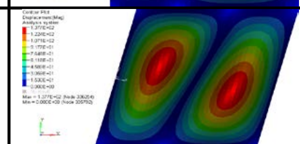
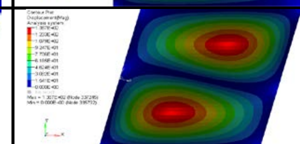
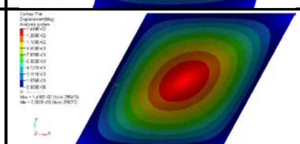
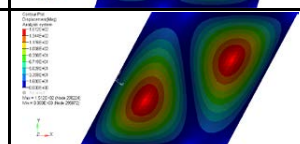
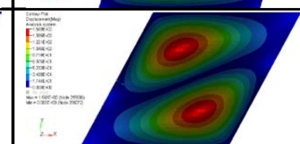
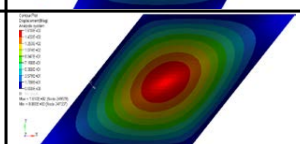
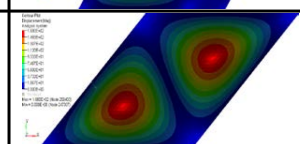
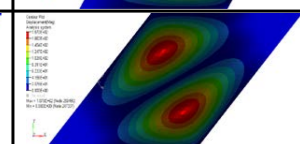
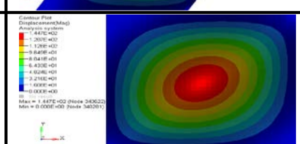
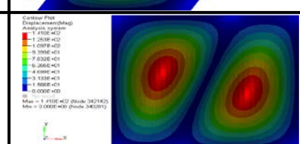
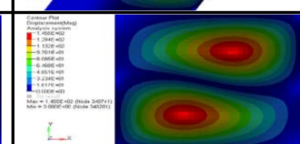
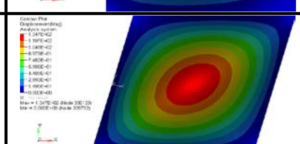
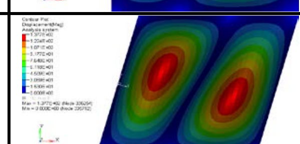
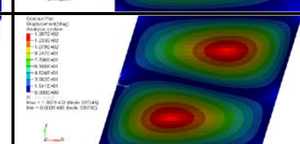
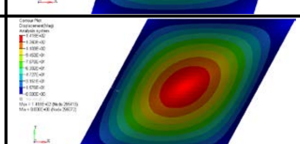
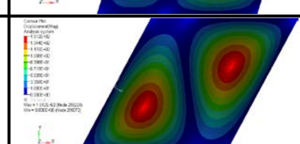
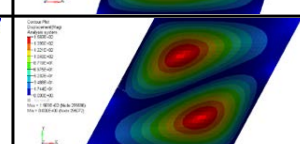
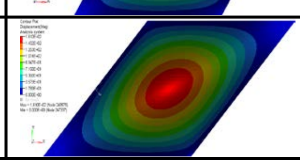
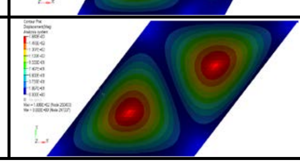
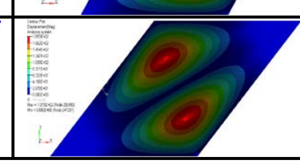
- The clamped condition has no degree of freedom free to rotate or oscillate in the plate element. This makes the plate stiffer compared to the simply supported one. Because of this, the value of K_f is higher for all sides' clamped condition than all sides simply supported.
- With the skew angle of the sandwich skew plates is increased, the value of K_f is found increasing in all cases of the parametric study.

4.2 Effect of ratio of t_c/t_f

Aspect ratio and a/h ratio kept constant as 1 and 10 respectively, only the ratio t_c/t_f is varied. The results are obtained for antisymmetric cross-ply, 5 layers simply supported and clamped boundary conditions for different skew angles. The K_f values are graphically presented in Figure 5 and 6. From the graph, the following observations are drawn.

Core Thickness, which takes the most of shear stress, is the key influencer for the vibration response of the sandwich skew plate. With the ratio of t_c/t_f is increased, the

Table 6: Mode shapes of symmetric 3 layer (θ°/C/θ°) skew sandwich plates.

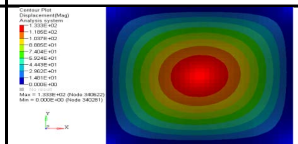
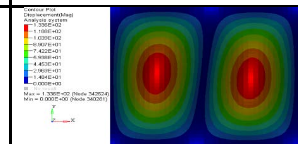
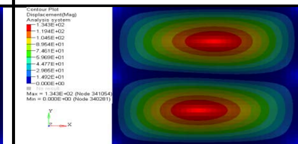
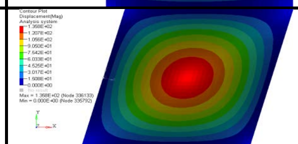
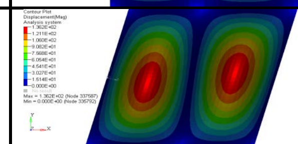
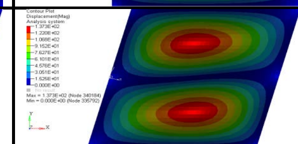
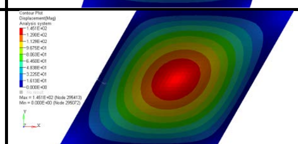
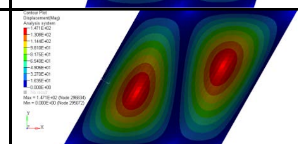
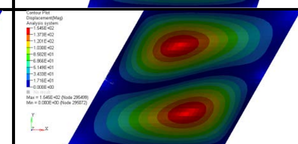
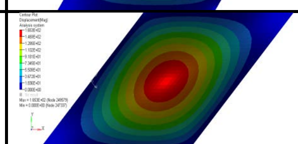
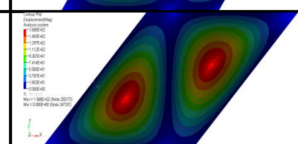
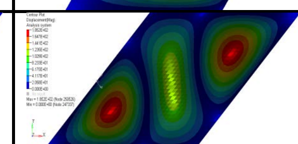
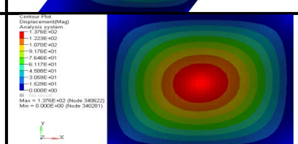
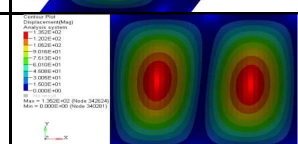
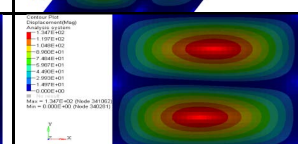
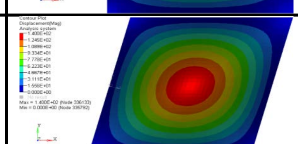
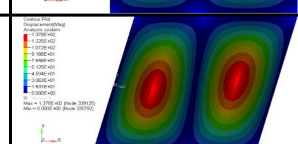
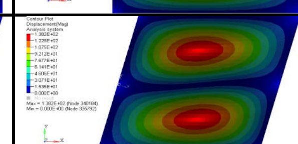
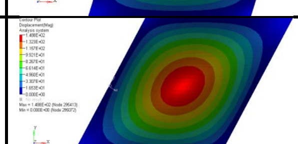
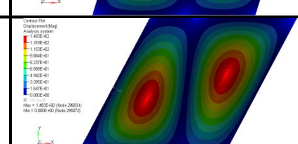
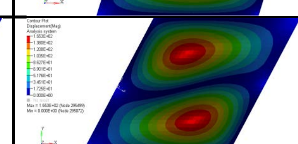
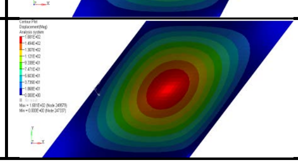
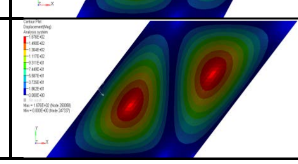
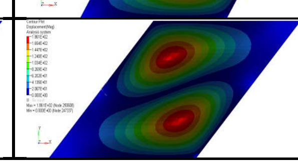
Boundary Condition	Skew Angle	Mode Shapes [Fiber Angle=50^0] 1	2	3
S-S-S-S	0^0			
	15^0			
	30^0			
	45^0			
C-C-C-C	0^0			
	15^0			
	30^0			
	45^0			

core thickness will also increase relative to the face sheet thickness. The higher the core thickness, the sandwich skew will become less stiff, and the K_f value for a given skew angle will be greatly decreased.

4.3 Effect of ratio of a/h

Aspect ratio and t_c/t_f ratio kept constant as 1 and 10 respectively, only the ratio a/h is varied. The results are obtained for antisymmetric cross-ply, 5 layers simply supported and clamped boundary conditions for different skew angles. The K_f values are graphically presented in Figure 7 and 8. From the graph, the following observations are drawn. A potential influencer is the core thickness compared to face sheet thickness. It is inappropriate to add more layers to the face sheet rather than vary the core thickness. The length of the sandwich plate kept constant only variable is the total thickness of the sandwich skew plate. When the ratio of a/h is increased, the K_f value decreases considerably for a given skew angle.

Table 7: Mode shapes of symmetric 5 layer ($\theta^\circ/-\theta^\circ/C/-\theta^\circ/\theta^\circ$) skew sandwich plates.

Boundary Condition	Skew Angle	Mode Shapes [Fiber Angle=50^0]		
		1	2	3
S-S-S-S	0^0			
	15^0			
	30^0			
	45^0			
C-C-C-C	0^0			
	15^0			
	30^0			
	45^0			

4.4 Effect of laminate sequence

A symmetric angle ply laminated skew sandwich plate is considered. Aspect ratio 1, a/h=10, and t_c/t_f =10 kept constant, only skew angle and fiber angle are varied for the study.

4.5 Symmetric three layer angle ply skew sandwich plates

The results are obtained for the symmetric 3 layers simply supported and clamped boundary conditions. The K_f values are graphically presented in Figure 9 and 10 also the mode shapes in Table 6. From the graph, the following observations are drawn. For the 0° skew angle, the K_f increases as an increase in the value of fiber angle. As the fiber angle is increased for skew angle15°, 30°, and 45°, the value of K_f initially decreases and then increases.

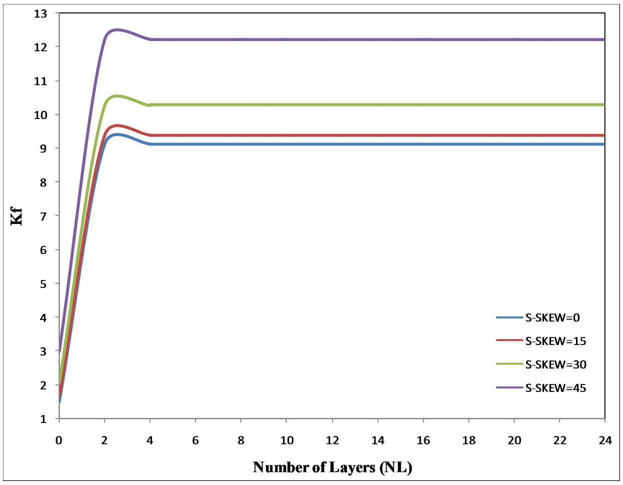


Figure 3: K_f values for laminated simply supported antisymmetric cross-ply (0°/90°/C/0°/90°) skew sandwich plates

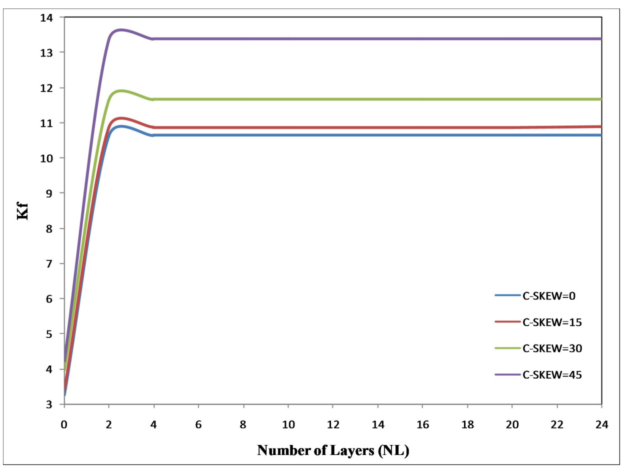


Figure 4: K_f values for laminated clamped antisymmetric (0°/90°/C/0°/90°) cross-ply skew sandwich plates.

4.6 Symmetric five layer angle ply skew sandwich plates

The results are obtained for the symmetric 5 layers simply supported and clamped boundary conditions. The K_f values are graphically presented in Figure 11 and 12, and mode shapes in Table 7. From the graph, the following observations are drawn. As the fiber orientation angle increases, the K_f value increases and reaches a maximum value or symmetric about 52.5° then decreases for simply supported and 50° for clamped boundary conditions.

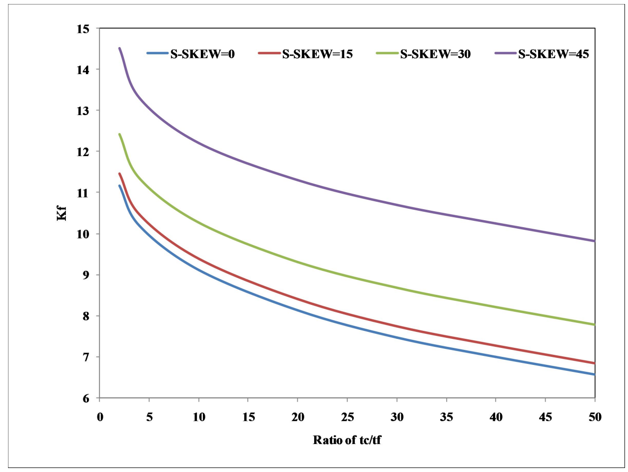


Figure 5: K_f v/s values of t_c/t_f ratio for laminated simply supported antisymmetric cross-ply (0°/90°/C/0°/90°) skew sandwich plates

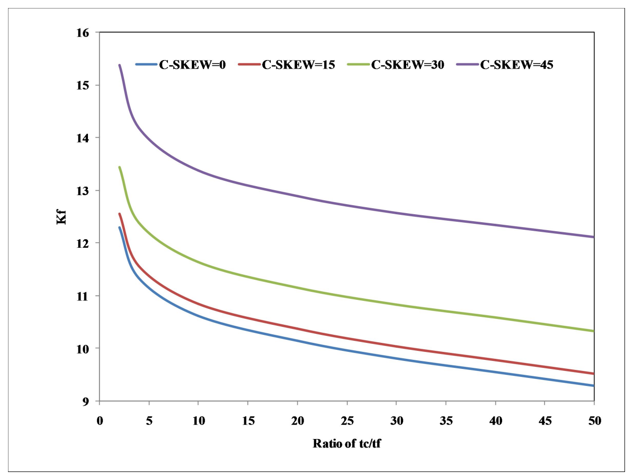


Figure 6: K_f v/s values of t_c/t_f ratio for laminated clamped antisymmetric cross-ply (0°/90°/C/0°/90°) skew sandwich plates

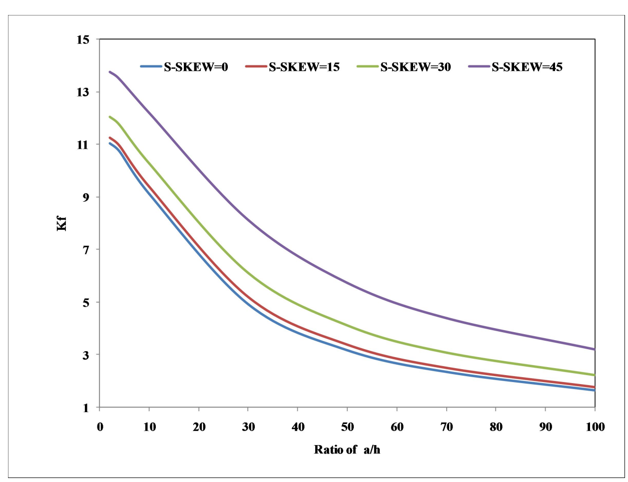


Figure 7: K_f v/s values of a/h ratio for laminated simply supported antisymmetric cross-ply (0°/90°/C/0°/90°) skew sandwich plates

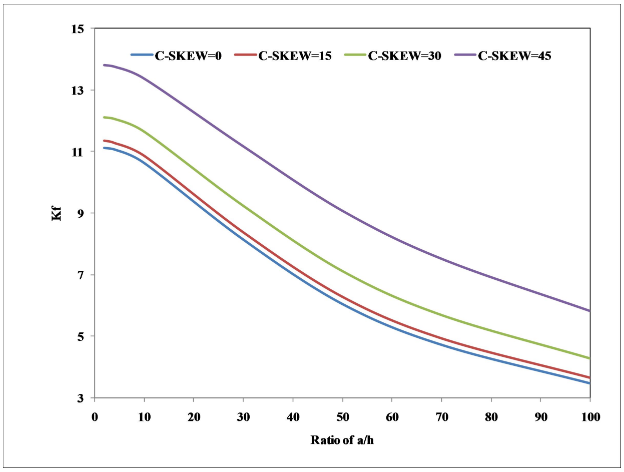


Figure 8: K_f v/s values of a/h ratio for laminated clamped antisymmetric cross-ply (0°/90°/C/0°/90°) skew sandwich plates

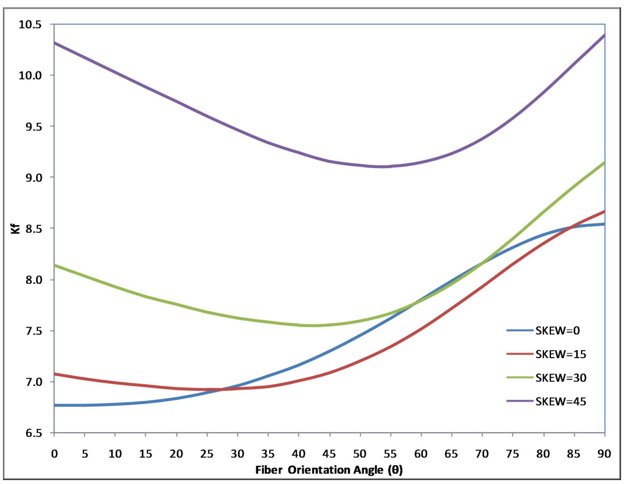


Figure 9: K_f values for laminated simply supported symmetric ($\theta°$/C/$\theta°$) angle-ply skew sandwich plates.

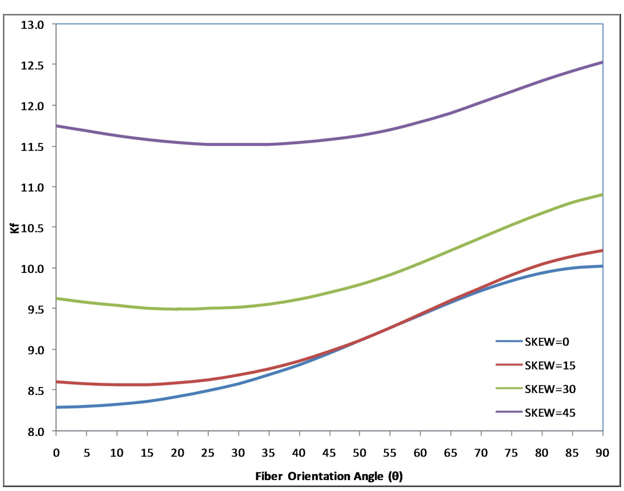


Figure 10: K_f values for laminated clamped symmetric ($\theta°$/C/$\theta°$) angle-ply skew sandwich plates

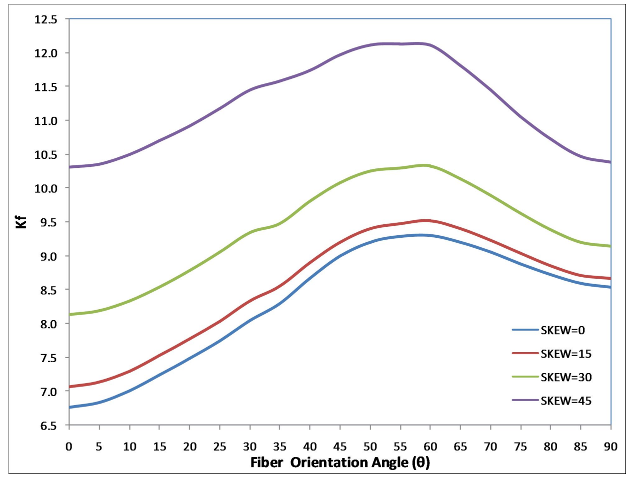


Figure 11: K_f values for laminated simply supported symmetric ($\theta°$/$-\theta°$/C/$-\theta°$/$\theta°$) angle-ply skew sandwich plates.

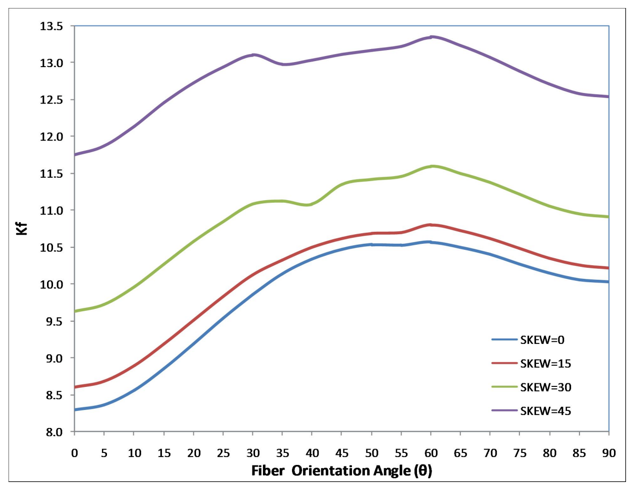


Figure 12: K_f values for laminated clamped symmetric ($\theta°$/$-\theta°$/C/$-\theta°$/$\theta°$) angle-ply skew sandwich plates.

5 Conclusion

Sandwich skew plates exhibit excellent high stiffness to weight ratio as compared to other laminated structures. The material properties at the interface of the face sheet and core components create complexities to accurately evaluate the mechanics of the sandwich skew plates by the analytical method. The finite element method (FEM) provides the flexibility in designing the structure and recording the response of the skew sandwich plate effortlessly. The present analysis uses CQUAD4 and CQAUD8 elements to evaluate the vibration response of the skew sandwich plate. A convergence study is performed by imposing simply supported and clamped boundary edge conditions. Results obtained by the present method are validated with those available in the literature. Aspect ratio, skew angle, the thickness of face sheet and core, number of layers in the face sheet, edge conditions, etc are considered in evaluating vibration response of skew sandwich plates. Concluding remarks are made after performing numerical analysis as:

- Both CQUAD4 and CQUAD8 elements have good agreement with the available literature results. But CQUAD8 element yields more converged, accurate results since the element has 8 nodes while CQUAD4 has 4 nodes.
- The number of layers in the face sheet, when increased, the K_f initially increases up to 4 layers due to the initial increase in the stiffness of the face sheet, after that the change is constant or negligible.
- When increasing the core thickness (increasing t_c/t_f and a/h ratios) an increase in total plate thickness, the stiffness of the plate decreases, K_f value decreases considerably for a given skew angle. Higher core thickness does not contribute to stiffness and vibration response of the skew sandwich plate.
- While the skew angle is increased, the side length shortens. This leads to an increase in stiffness of the skew sandwich plate. Because of which the increased value of K_f is observed for a given ratio of $t_c/t_f$and a/h.
- Considerable influence is observed while studying fiber orientation on the sandwich skew plate for vibration response. For 3 layers and 5 layers symmetrically laminated composite sandwich plate, the value of K_f initially decreases then increases. A similar variation can be seen [5] for both simply supported and clamped boundary conditions.
- The value of K_f is higher for all side clamped condition than all sides simply supported. In the clamped edge condition, the plate becomes stiffer than simply supported edge condition.

Acknowledgments: The authors would like to thank the Management and Principal of GM Institute of Technology, Davangere, Karnataka, for the kind encouragement and constant support provided.

Funding information: The authors state no funding involved.

Author contributions: All authors have accepted responsibility for the entire content of this manuscript and approved its submission.

Conflict of interest: The authors state no conflict of interest.

Abbreviations:

a - length of the sandwich plate (mm)
b - width of the sandwich plate (mm)
t_c **-** core thickness (mm)
t_f **-** face sheet thickness (mm)
h - total thickness of the sandwich plate (mm)
E - Young's modulus (GPa)
G_{ij} **-** rigidity modulus (GPa)
ν **-** Poisons' ratio
ρ **-** density (kg/m^3)
S-S-S-S - all sides simply supported
C-C-C-C - all sides clamped
C - core
α **-** skew angle in degree
ω **-** circular frequency (rad/s)

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