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PROPERTIES OF A FIRM'S FACTOR DEMANDS, OPTIMAL PRODUCTION CORRESPONDENCE, AND AN ECONOMY'S AGGREGATED SUPPLY/DEMAND

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Abstract. Many theoretically beautiful conclusions of the producer theory were derived on the common assumption that every firm attempts to maximize its profit and minimize its cost, while all firms employ the same methodology in their optimization efforts. By losing up these two behavioral assumptions and by introducing the concept of value-belief systems for individual firms, this paper reestablishes a few well-known results of the producer theory for the general case of not specifying what criteria of priority a firm holds. At the same time, this paper shows by using counterexamples, among others, that generally, (i) except for a specific scenario, the optimal production correspondence does not satisfy the homogeneity of degree zero, and (ii) even when individuals act in their own best self-interests, they may not collectively produce unintended greater social benefits and public goods. In the end, several topics of expected significance are suggested for future research.

1. Introduction

Raiffa [40] points out the fact that because results of the classical game theory hold true mostly on the ground of strict assumptions, that makes it difficult for practitioners to apply these results to real-life situations. For other criticisms of game theory, see, e.g., Abedian *et al.* [2] and Nishino and Tjahjono [37]. Although Raiffa only talks about game theory here and how results developed on such a theory suffer from difficulties in practice, this phenomenon in fact appears in the entire spectrum of business studies in general and economics in particular [43]. For example, Forrest and Liu [11] carefully analyze how various methodologies widely employed in the studies of value creation and capture suffer from deficits of one kind or another so that the truthfulness of consequently established conclusions is subject to the constraint of these deficits. Parallel to such methodological deficits in terms of producing practically useful results, crucially criticized are various commonly imposed behavioral hypotheses in social science and economics [24,36]. Such hypotheses include, in particular, that (i) all decision makers prioritize their available alternatives in the same way in terms of how real numbers are ordered; and (ii) all economic agents aim at maximizing their profits.

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It is evident that in real life, these two behavioral hypotheses are simply not generally true. Firms with different systems of values and beliefs order real numbers differently and employ respectively firm-specific methods to optimize their individual objective functions [10, 17, 44, 46]. For example, there are firms that do not place profit maximization as their primary objective [23, 24]. Instead of maximizing profits for shareholders above all else, an increasing number of firms have also focused their operations on various other purposes, such as:

- Providing opportunities for citizens to succeed through hard work and creativity, while enjoying a life of meaning and dignity (`https://s3.amazonaws.com/brt.org/BRT-StatementonthePurposeofaCorporationOctober2020.pdf`, accessed on January 30, 2021);
- Taking corporate social responsibilities [8];
- Protecting the environment through designing and producing green products [19].

In short, not all economic agents in real life are maximizers or minimizers, as defined conventionally in the literature. Hence, some of the established theoretical results of economics may not apply to such agents [43]. In other words, a firm's system of values and beliefs directly affects how the firm prioritizes its decision choices and how it practically optimizes its objective function [10]. Based on this realization, this paper studies how some of the well-known properties of a firm's factor demands, optimal production correspondence, and an economy's aggregated supply/demand can be extended to the general case of no matter what a system of values and beliefs a firm may embrace, while how some other known results are only true under specific conditions.

Specifically, this paper employs the method of Euclidean spaces to investigate whether or not we can generalize a series of well-known conclusions of the producer theory. The considered known conclusions include, among others, the monotonicity of a firm's conditional factor demands, the homogeneity of a firm's optimal production correspondence, the aggregated supply and aggregated demand of an economy, and the maximization of total productions in an economy.

The contribution this paper makes to the literature is that this work emphasizes the fact that each firm employs its own particular order relation of real numbers, which is defined on the firm's system of values and beliefs. Such a firm-specific order relation naturally forces the firm to adopt its specific method of optimization that reflects how well the stated mission is at least partially materialized. Speaking differently, firms respectively have their particular ways to prioritize their available decision alternatives, when faced with challenges and opportunities. Therefore, they accordingly apply their very individual ways to optimize their objectives. Contrary to this more realistic setting, the literature widely assumes that the ordering of real numbers and the method of optimization are the same across the entire business world, although certain particular details are different from one firm to another.

Because of our emphasis on firm-specific systems of values and beliefs, firm-specific orderings of real numbers, and firm-specific methods of mission optimization, this paper is able to establish results not discovered before. At the same time, it is able to generalize some of the previously established results of the producer theory. More specifically, the marginal contribution this paper makes to the literature consists of showing:

(i) When additional conditions are needed for a desired conclusion to be true, and
(ii) How and when a well-known conclusion holds true only under very specific conditions.

The rest of this paper is organized as follows. Section 2 provides the necessary background knowledge, conventions and terminologies in order to make the rest of the presentation self-contained. Section 3 studies the monotonicity of a firm's conditional factor demands and the prices of the firm's products. Section 4 turns attention to the homogeneity of the optimal production correspondence. Section 5 investigates the monotonicity of factor demands in prices of input commodities. Section 6 considers both the aggregated supply and aggregated demand and the maximization problem of total productions of an Economy. Then, this paper is concluded in Section 7 with several important open questions listed for future research.

2. Preparation

To smoothly present the rest of the paper, this section prepares the reader with necessary background information, knowledge and conventions. In particular, the first subsection presents the four natural endowments of a firm and several related properties. The cited example shows the fact that firms' different systems of values and beliefs indeed lead to different outcome, although the optimization problem stays the same. The second subsection lays out all relevant conventions for the rest of the logical reasoning to move smoothly.

2.1. The existence of natural endowments for individual firms

Analogous to the situation of individual persons [30], Forrest, Shao and colleagues [12] and Forrest, Hafezalkotob *et al.* [10] demonstrate that in general each firm makes decisions and conducts its business operations by relying either consciously or unconsciously on its set of four natural endowments – self-awareness, imagination, conscience and free will. Specifically, for each chosen firm, its four natural endowments are defined as follows:

- By self-awareness, it stands for the situation where the firm is aware of its existence as a separate entity from others with its secrets, such as the proprietary understandings of adopted customer value propositions, operational strategies, protected product designs, etc.
- By imagination, it represents the capability that the firm acquires new knowledge, and innovatively imagines what might be the right offer to the market to satisfy an emerging demand. By relying on this endowment, the firm is able to develop processes that can materially introduce the imagined offer(s).
- By conscience, it means such a capability through which the firm can tell which project will be more beneficial than others.

- By free will, it indicates the endowment through which the firm is able to keep promises, and decides on how to keep and to what degree to keep these promises, as made with various business partners.

As is documented in McGrath [33], even though each firm innately has these natural endowments, how well a firm is able to make use of these endowments is determined by its composition, such as its leadership, constraints, organizational culture, etc. [13]. That in fact explains why in each economic sector, some firms do well while others do not seem to matter at all [42].

Through investigating the connotation of the widely assumed rationality [14, 15, 22], Forrest, Shao and colleagues [12] systemically developed the following conclusions:

(i) Each firm possesses a unique system of values and beliefs, which is formulated out of the firm's natural endowments, and echoes the system in its mission statement; and
(ii) When making decisions, a firm optimizes the potential, which is subject to the firm-specific set of constraints, by using its particular system of values and beliefs.

To illustrate the fundamental construct underneath these two conclusions, the following example, cited from [21], [29, p. 136], provides a vivid explanation on how different systems of values and beliefs can and do lead to different optimal solutions.

Example 2.1. Assume that the directed and weighted network in Figure 1 depicts a firm's production line, where node A represents the start of the production, and node C the end. The firm desires to find the weighted path from node A to node C so that the total weight of the path is minimum. Now, we consider the two scenarios that the firm's system of values and beliefs satisfies either (1) the firm orders real-numbers as how they are conventionally ordered, or (2) the firm orders real-numbers by referring to the mod4 function so that for any two integers x and y, $x <_{\text{mod }(4)} y$ if and only if $x(\text{mod}4) < y(\text{mod}4)$.

If case (1) holds true, then $\text{A} \to \text{A}_1 \to \text{B} \to \text{C}_1 \to \text{C}$ is the path the firm looks for. This path has the total weight of 1. And other paths from node A to node C have weights 2, 3, and 4, respectively.

If case (2) holds true, then $\text{A} \to \text{A}_2 \to \text{B} \to \text{C}_2 \to \text{C}$ is the path the firm looks for. The path's weight is equal to $3 + 0 + 0 + 1 = 4 \ (\text{mod } 4) = 0$. In comparison, the weights of other paths have weights 1, 2, or 3, respectively.

Before moving on, let us first notice that the mod4 function or the general mod r function, for any positive real number r (for this generalized case beyond integers, see [10]), actually means periodicity 4 or r . Some of the real-life examples include 12-hour clocks, 7-day weeks, months of various numbers of days. And, more general than these cases are the projects a firm is involved in, where every time a new project is started, the firm also begins a new round of measurements of various economic variables, such as costs, profits, or how well a newly adopted business strategy works.

To illustrate why different systems of values and beliefs order real numbers differently, we only need to look at two incomes of different amounts – \$30 K and

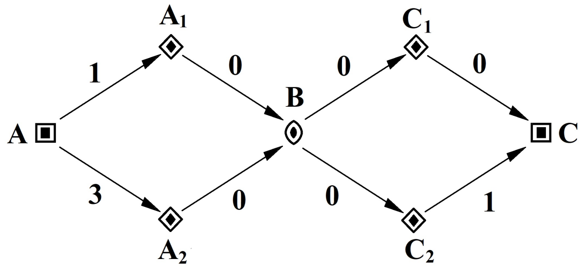

Figure 1. How systems of values and beliefs lead to varied solutions of minimization

\$3 million. If no system of values and beliefs is involved, people would most likely order these two amounts as \$30 K $<$ \$3 million. However, if the information behind these two figures indicates that the income of \$30 K is from a lawful employment, while the income of \$3 million is from participating in the robbery of a bank, then systems of values and beliefs will play their roles in the ordering of \$30 K and \$3 million. For instance, a lot of people will have \$30 K $>$ \$3 million. Similar to this example, other ones can be constructed readily when related background knowledge, such as environmental protection [31], corporate social responsibilities [8], and others becomes known to the decision maker. In other words, for each given system of values and beliefs, a particular ordering of real numbers is specified.

Based on the previous discussions, Example 2.1 indicates that even when all aspects of a decision-making scenario might look the same, such as the objective function, the setup of the production line, etc., the eventual optimal decision is dictated by the decision-maker's system of values and beliefs. In particular, the firm's system of values and beliefs determines how it prioritizes available potentials and how the eventual optimal solution is specifically derived. In theory, each prioritization of available potentials can be seen as a particular way of how real numbers are ordered. This takeaway of the previous example analytically confirms Mises's [35, p. 244] statement that "the value judgements a man pronounces about another man's satisfaction do not assert anything about this other man's satisfaction. They only assert what condition of this other man better satisfies the man who pronounces the judgement." Speaking in the language of neoclassical economics and in the context of this research, the economist proclaims a firm's condition that better satisfies the economist. This end stands for a reason for why the economist experiences uncertainties or takes risks when he draws conclusions and makes claims, if the firm pursues after its own mission as driven by its values and beliefs. That is in fact mostly the case in real life [43], instead of what the economist believes and expects the firm ought to do to achieve its best.

In the rest of this paper, assume that corresponding to its unique system of values and beliefs, each firm orders the real numbers within the domain D of its decision-making activities in a unique way. Let $\leq_F$ represent the firm-specific order relation of real numbers.

2.2. The firm and its representation in a Euclidean space

For the sake of convenience of communication, discussions in this paper address a randomly selected business entity, known as the firm. When available times and delivery locations are different, a same type underlying commodity will be seen as different commodities. Assume that there is a total of ℓ many commodities, all of which can be exchanged in the marketplace and are linearly ordered. To simplify our analysis, no interests, no discounts and no exchange rates of money are considered.

As commonly done in economic analysis [39], assume that the linearly ordered totality of all commodities is written as a vector $c = (c_1, c_2, \ldots, c_\ell)$, where the components respectively stand for the amounts of the commodities $1, 2, \ldots,$ and ℓ the firm needs for production as inputs and outputs the firm produces. Hence, for each such vector of commodities, most of the components are equal to 0. To distinguish the amounts of inputs and those of outputs, we use negative numbers for the former and positive numbers for the latter.

Let $\mathbb{R}$ be the set of all real numbers, $\mathbb{R}_-$ the set of all negative real numbers, and $\mathbb{R}_+$ the set of all positive real numbers. For any given set X and any natural number n, let X^n be the Cartesian product of n copies of X. Symbolically, we have

$$X^n = \{(x_1, x_2, \ldots, x_n) : \ x_i \in X, i = 1, 2, \ldots, n \} .$$

So, each vector $c = (c_1, c_2, \ldots, c_\ell)$ of commodities is an element in $\mathbb{R}^\ell$. If $p \in \mathbb{R}^\ell_+$ is a vector of the prices of the commodities, known as a price system, then the dot product $p \cdot c = \sum_{h=1}^{\ell} p_h c_h$ provides the overall cash flow of the firm, where the subscript h stands for a commodity that takes values from 1 to ℓ.

To maintain its viability, the firm chooses a plan of action, written as $y = (y_1, y_2, \ldots, y_\ell) \in \mathbb{R}^\ell$, that specifies the quantity of each commodity it either consumes for its livelihood or offers to satisfy some market demands. So, the price or return of action y is given by $p \cdot y = \sum_{h=1}^{\ell} p_h y_h$. Within its boundary conditions, the firm optimizes its return by choosing such a plan that best reflects its system of values and beliefs [12], as stated in its specific mission [33]. Let Y be the set of all feasible production plans of the firm. Without causing confusion, each $y \in Y$ will also be known as a production.

By sorting through what has been discussed, one notes that two binary relations $\leq$ and $\leq_F$ have been involved here. The first one is defined on Y such that $x, y \in Y$, $x \leq y$ if and only if $x_h \leq y_h$, for each $h = 1, 2, \ldots, \ \ell$. The second is the firm-specific ordering $\leq_F$ of real numbers. Evidently, we did not assume that each firm is rational; that is, $\leq$ is not assumed to satisfy the conditions of completeness, transitivity and reflexivity, as assumed by Mas-Collel *et al.* [32] for the preference relation of a consumer on his set of all possible consumptions.

Since $\leq_F$ represents firm F's specific criteria of priorities defined on the real-number domain D of decision-making activities, when no confusion appears, assume that $\leq_F$ satisfies: (i) transitivity (for $x, y, z \in D$, if $x \leq_F y$ and $y \leq_F z$, then $x \leq_F z$); (ii) reflexivity (for $x \in D$, $x \leq_F x$); and (iii) anti-symmetry (for different $x, y \in D$, $x \leq_F y$ and $y \leq_F x$ cannot hold true at the same time). In short, conditions (i) – (iii) are not equivalent to the assumption that the firm considered in this paper is rational for the research economist who asserts conditions that achieve his optimal possibility, as so phrased in the language of Mises [35].

Given two sets U and W, $f : U \to W$ is known as a partial function from U into W, if there are $u^1,\ u^2 \in U$ such that $f\left(u^1\right) \in W$ is a well-defined element, while $f\left(u^2\right)$ is not defined. In this case, the domain of f, denoted by $domain(f)$, is not equal to U. Without causing confusion, f will be simply known as a function from U into W. If for each $u \in domain(f)$, $f(u)$ is a non-empty subset of W, then f is known as a set-valued function from U into W.

3. Monotonicity of the conditional factor demands and product prices of the firm

For each production $y \in Y \subseteq \mathbb{R}^\ell$, let

$$y^{in} = \left(y_{h_1^{in}}, y_{h_2^{in}}, \dots, y_{h_t^{in}}\right) \in \mathbb{R}^t_- \quad \text{and} \quad y^{out} = \left(y_{h_1^{out}}, y_{h_2^{out}}, \dots, y_{h_s^{out}}\right) \in \mathbb{R}^s_+$$

be respectively the sub-vector of the quantities of all the corresponding commodity inputs $h_1^{in},\ h_2^{in}, \dots, h_t^{in}$, and that of all commodity outputs $h_1^{out}, h_2^{out}, \dots, h_s^{out}$. That is, what is implicitly meant is that in both y^{in} and y^{out} no zero components appear so that $h_1^{in} < h_2^{in} < \cdots < h_t^{in}$ and $h_1^{out} < h_2^{out} < \cdots < h_s^{out}$, and

$$y_{h_j^{in}} < 0 \quad \text{and} \quad y_{h_k^{out}} > 0, \quad j = 1, 2, \dots, t;\ k = 1, 2, \dots, s. \tag{3.1}$$

Correspondingly, to distinguish prices of commodity inputs and outputs, for any given price system $p \in \mathbb{R}^\ell_+$, we write $p^{in} \in \mathbb{R}^t$ for the price system of all inputs in y^{in} and $p^{out} \in \mathbb{R}^s$ for the corresponding price system of the outputs in y^{out}. Hence, the production function f for the firm is defined as follows: for any $y \in Y$, $f\left(y^{in}\right) = y^{out}$. And the firm's cost minimization problem can be written as follows, assuming that the firm is a price taker. For a given price system $p \in \mathbb{R}^\ell_+$,

$$\min\nolimits^F_{y \in Y} p^{in} \cdot y^{in}, \quad \text{s.t.} \quad f\left(y^{in}\right) \geq q, \tag{3.2}$$

where q is a given vector of some commodity quantities, representing the market demand for these commodities. The constraint in equation (3.2) implies that the set of all commodities $\left\{h_1^{out}, h_2^{out}, \dots, h_{s_1}^{out}\right\}$ contained in q is a subset of the set of all the commodities $\left\{k_1^{out}, k_2^{out}, \dots, k_{s_2}^{out}\right\}$ that appear in $f\left(y^{in}\right) = y^{out}$. Without loss of generality, we assume that these two sets are the same, that is,

$$\left\{h_1^{out}, h_2^{out}, \dots, h_{s_1}^{out}\right\} = \left\{k_1^{out}, k_2^{out}, \dots, k_{s_2}^{out}\right\}, \tag{3.3}$$

because producing additional products beyond what are listed in q requires at least an increased amount of labor input.

In equation (3.2), the total cost of production y is minimized in terms of Firm $F's$ specific system of values and beliefs. Corresponding to this minimization, in neoclassic economics, there is such a long-standing convention that one of a firm' objectives is to minimize its cost [45]. In reality, however, there are business firms that do not truly place cost minimization as one of their primary objectives. For example, a group of powerful US chief executives recently gave away the idea for firms to maximize profits (respectively, minimize costs) for shareholders above all else (`https://opportunity.businessroundtable.org/`

`ourcommitment/`, accessed on January 30, 2021). The organization issues that "Americans deserve an economy that allows each person to succeed through hard work and creativity and to lead to a life of meaning and dignity" and "we commit to deliver value to all of them, for the future success of our companies, our communities, and our country" (`https://s3.amazonaws.com/brt.org/BRT-StatementonthePurposeofaCorporationOctober2020.pdf`, accessed on January 30, 2021). The statement clearly shows that these executives run their business to best fit their values and beliefs, as defined by their systems of values and beliefs. Once again, this example supports the notion that how a firm behaves is dictated by its system of values and beliefs.

The existence of such firms that maximize their missions instead of profits only naturally leads to the following question: Can such firms successfully coexist with those that do? To this end, empirical evidence suggests that consumers in a buyer's market have shown increasing levels of consideration towards socially responsible companies [20]; and employees, market competitions and governments pressured downstream companies to distribute and sell socially responsible goods [26]. American Express's support for the Statue of Liberty in 1983 well demonstrates this end [1]. In particular, during September to December 1983, the company donated 1 cent to the Restoration of the Statue of Liberty fund for every usage of the card, and \$1 for each new American Express card account opened. The fund collected over \$1.7 million during the time period, while the usage of the card went up 28% in just the first month when compared to the previous year, and new card applications increased 45%. Cone/Roper research reveals [34] that over 70% of survey respondents are more likely to choose firms that participate in public service when faced with the same goods in terms of quality and price, and more than 50% are willing to pay additional for their products and services.

Let $Z = \left\{z : \text{there is } y \in Y \text{ such that } z = y^{in} \text{ and } f(z) \geq q\right\}$. Assume that the objective function in equation (3.2) has the following solution:

$$c^F(p,q) =_F \min\nolimits^F_{z\in Z} p^{in} \cdot z, \quad \text{for} \quad p \in \mathbb{R}^\ell_+ \tag{3.4}$$

which stands for the minimum cost that is needed for producing the demanded outputs q. As for other symbols, the components of p^{in} are determined accordingly by those of $z \in Z$. In other words, for any $z, z' \in Z$, if $z = \left(z_{h_1^{in}}, z_{h_2^{in}}, \ldots, z_{h_{t_1}^{in}}\right)$ and $z' = \left(z'_{h'^{in}_1}, z'_{h'^{in}_2}, \ldots, z'_{h'^{in}_{t_2}}\right)$, in the expressions $p^{in} \cdot z$ and $p^{in} \cdot z'$, the corresponding p^{in}'s are given respectively as follows:

$$\left(p_{h_1^{in}}, p_{h_2^{in}}, \ldots, p_{h_{t_1}^{in}}\right) \quad \text{and} \quad \left(p_{h'^{in}_1}, p_{h'^{in}_2}, \ldots, p_{h'^{in}_{t_2}}\right).$$

For a given price system $p \in \mathbb{R}^\ell_+$ and a market demand vector $q \in \mathbb{R}^s_+$, for some $s < \ell$, define the set of conditional factor demands as follows:

$$\xi^F(p,q) = \left\{z \in Z : p^{in} \cdot z =_F \overset{F}{\min_{z' \in Z}} p^{in} \cdot z'\right\}.$$

Each element of this set is conditional on the desired level of outputs q, as reflected in the definition of Z. In other words, ξ^F maps each price system p of commodities to the subset $\xi^F(p,q) \subseteq Z$ of all cost-minimizing commodity inputs of productions, if $\xi^F(p,q) \neq \varnothing$.

Proposition 3.1. *For each conditional factor demand* $z\left(p^{in}, q\right) \in \xi^F(p, q)$, $z\left(p^{in}, q\right)$ *is nonincreasing in* $p \in \mathbb{R}_+^\ell$.

Proof. For any price systems ${}^1p, {}^2p \in \mathbb{R}_+^\ell$ such that ${}^1p \geq {}^2p$, let ${}^1z \in \xi^F\left({}^1p, q\right)$ and ${}^2z \in \xi^F\left({}^2p, q\right)$ be two conditional factor demands. Without loss of generality, as reasoned for equation (3.3), assume that the set of commodities that appear in ${}^1z$ is the same as those in ${}^2z$. Then, we have

$$ {}^1p^{in} \cdot {}^1z \leq_F {}^1p^{in} \cdot {}^2z \quad \text{and} \quad {}^2p^{in} \cdot {}^2z \leq_F {}^2p^{in} \cdot {}^1z. $$

Hence, ${}^1p^{in} \cdot \left({}^1z - {}^2z\right) \leq_F 0 \leq_F {}^2p^{in} \cdot \left({}^1z - {}^2z\right)$. From this inequality, it follows that $\left({}^1p^{in} - {}^2p^{in}\right) \cdot \left({}^1z - {}^2z\right) \leq_F 0$. Therefore, when ${}^1p \geq {}^2p$, which implies ${}^1p^{in} - {}^2p^{in} \geq 0$, ${}^1z - {}^2z \leq_F 0$. That is, $z\left(p^{in}, q\right)$ is nonincreasing in $p \in \mathbb{R}_+^\ell$. □

Proposition 3.2. . *If the firm's order relation* $\leq_F$ *of real numbers is the same as the conventional one* $\leq$, *then the price of the firm's product* h_j^{out} *is equal to the marginal cost of producing this product. Symbolically,*

$$ p_{h_j^{out}}^{out} = \frac{\partial c(p, q)}{\partial q_{h_j^{out}}}, \quad \textit{for} \quad j = 1, 2, \ldots, s. \tag{3.5} $$

Proof. According to Forrest, Shao *et al.* [12], the firm has a clearly stated mission, which is formulated consistently with the firm's underlying system of values and beliefs; and its business goal in general is to optimally materialize, at least partially or remotely, the mission. So, in particular to the firm, its goal is to solve the following profit maximization problem for the purpose of materializing its stated mission. For any fixed price system $p \in \mathbb{R}_+^\ell$ and chosen output commodities h_1^{out}, h_2^{out},...,h_s^{out}, satisfying $h_1^{out} < h_2^{out} < \cdots < h_s^{out}$,

$$ \max\nolimits_{q \in \mathbb{R}_+^s}^F p^{out} \cdot q - c\left(p^{in}, q\right), \tag{3.6} $$

where p^{out} stands for the price system of the commodities in

$$ q = \left(q_{h_1^{out}}, q_{h_2^{out}}, \ldots, q_{h_s^{out}}\right), $$

and $c\left(p^{in}, q\right)$ the minimum cost given in equation (3.4). In this symbolic setup, assumed is that for every possible q-value, the firm has solved its cost minimization problem so that the cost function $c\left(p^{in}, q\right)$ is well defined and known.

Since the firm's order relation $\leq_F$ of real numbers is the same as the conventional one $\leq$, the first-order condition of the maximization problem in equation (3.1) holds true. That is, equation (3.5) holds true. □

Speaking differently, what Proposition 3.2 says is that when the firm's order relation of real numbers is the same as the conventional one, and the firm is able to maximize its profit conventionally, then the shadow prices of its products h_1^{out}, $h_2^{out}, \ldots, h_s^{out}$ are respectively the same as the market prices of these products.

4. Homogeneity of the optimal production correspondence

Let us define the optimal production correspondence of the firm [27] as the following partial, set-valued function $\eta^F\colon \mathbb{R}^\ell_+ \to Y$: For $p \in \mathbb{R}^\ell_+$, if there is $y \in Y$ satisfying that $p\cdot\, y =_F \max^F_{y^q \in Y} p \cdot y^q$, then

$$\eta^F(p) = \left\{ y \in Y : p \cdot y =_F \max_{y^q \in Y}^{F} p \cdot y^q \right\}. \tag{4.1}$$

Intuitively speaking, for each price system p, $\eta^F(p)$ is the subset of Y that contains all mission-maximizing productions, if this subset exists and is not empty.

This setting generalizes the conventional one where scholars automatically assume that each firm maximizes its profit, although this end is not true in real life [28]. By employing this general mission maximization problem, we will be expectedly able to resolve the difficulty that some well-established conclusions in microeconomics cannot be empirically applied [43] when the decision maker of concern is not an optimizer (e.g., neither a maximizer nor a minimizer) as in the conventional sense.

Evidently, there are three possibilities for the mission maximization problem in equation (3.5) or (3.6): the problem has multiple solutions, or a unique solution or no solution.

Proposition 4.1. *If the firm's order relation $\leq_F$ of real numbers satisfies the condition of positive multiplicativity, that is, for any scalar $\alpha > 0$ and $a, b \in \mathbb{R}$, $a \leq_F b \to \alpha a \leq_F \alpha b$, then the optimal production correspondence η^F is homogeneous of degree zero. Symbolically, for any scalar $\alpha > 0$, if $a \leq_F b \to \alpha a \leq_F \alpha b$, for any $a, b \in \mathbb{R}$, then $\eta^F(\alpha p) = \eta^F(p)$.*

Proof. Let α be a positive scalar. Then,

$$\begin{aligned}\eta^F(\alpha p) &= \left\{ y \in Y : \alpha p \cdot y =_F \max_{y^q \in Y}^{F} \alpha p \cdot y^q \right\} \\ &= \left\{ y \in Y : \alpha p \cdot y \geq_F \alpha p \cdot y^q, \forall y^q \in Y \right\}.\end{aligned}$$

So, the condition of positive multiplicativity of the order relation $\leqslant_F$ guarantees that

$$\left\{ y \in Y : \alpha p \cdot y \geq_F \alpha p \cdot y^q, \forall y^q \in Y \right\} = \left\{ y \in Y : p \cdot y \geq_F p \cdot y^q, \forall y^q \in Y \right\}.$$

Therefore, $\eta^F(\alpha p) = \eta^F(p)$. □

Example 4.2. Constructed here is a scenario where the set-valued function η^F is not homogeneous of degree zero. In particular, assume that a specific production of the firm, as shown in Figure 2, involves one unit of each of the commodity inputs A, A?, B, C?, C, where A? can be either A_1 or A_2, but not both, and similarly, C? can be either C_1 or C_2, but not both. That is, commodities A_1 and A_2 can substitute for each other and the same holds true for commodities C_1 and C_2. In Figure 2, the arrows stand for the sequence the corresponding commodities are fed into the production line one after another, while the weights the relevant dollar values created by the production sequence from one node to the next.

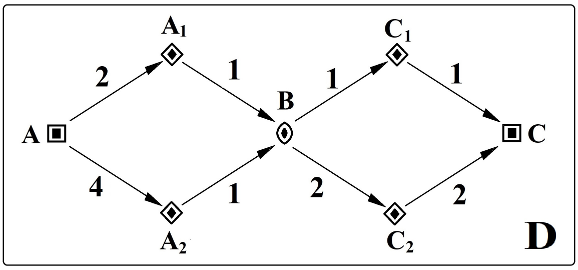

Figure 2. How product D can be produced

Without loss of generality, assume that each of these specific commodities costs \$1.00 a unit. The production produces only one output, named D, which can be sold at the market price that is equal to the sum of the path that leads to D.

The goal of the firm is to maximize the total profit of this production, while the firm orders real numbers by referring to the mod4 function. In particular, for any two real numbers x and y, $x < y$ if and only if $x\,(\text{mod}4) < y(\text{mod}4)$. In this case, there are four possible ways to produce D with their respective profits given as follows:

(a) A $\to A_1 \to$ B $\to C_1 \to$ C with profit $5 - 5 = 0$ (mod4) $= 0$;
(b) A $\to A_1 \to$ B $\to C_2 \to$ C with profit $7 - 5 = 2$ (mod4) $= 2$;
(c) A $\to A_2 \to$ B $\to C_1 \to$ C with profit $7 - 5 = 2$ (mod4) $= 2$; and
(d) A $\to A_2 \to$ B $\to C_2 \to$ C with profit $9 - 5 = 4$ (mod4) $= 0$;

Therefore, the maximum profit is equal to 2, as produced out of either the production A $\to A_1 \to$ B $\to C_2 \to$ C or A $\to A_2 \to$ B $\to C_1 \to$ C.

If we choose scalar λ=3.2 to multiply each of the individual local values, then the corresponding productions are depicted in Figure 3; and the corresponding profits for the four productions are respectively equal to $0 \times 3.2\,(\text{mod}4) = 0$, $2 \times 3.2\,(\text{mod}4) = 1.6$, $2 \times 3.2\,(\text{mod}4) = 1.6$, and $4 \times 3.2\,(\text{mod}4) = 3.2$. That is, the maximum profit is equal to 3.2, as produced out of the production A $\to A_2$ $\to$ B $\to C_2 \to$ C.

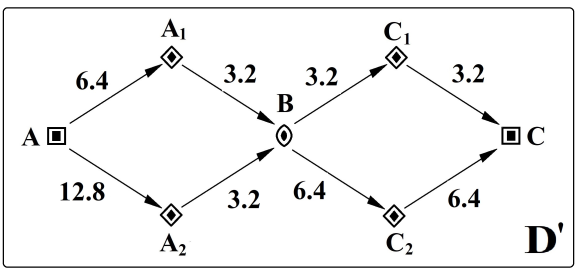

Figure 3. The price system is enlarged 3.2 times.

What has been shown here is that $\eta^F\,(3.2p)$ is the singleton A $\to A_2 \to$ B $\to$ $C_2 \to$ C, while $\eta^F\,(p)$ is equal to the set that contains the following two elements:

(i) A $\to A_1 \to$ B $\to C_2 \to$ C, and (ii) A $\to A_2 \to$ B $\to C_1 \to$ C. Hence, what is shown is that $\eta^F(3.2p) \neq_F \eta^F(p)$.

This example implies the following result.

Proposition 4.3. *The optimal production correspondence η^F in general is not homogeneous of degree zero on domain(η^F).*

5. Monotonicity of factor demands in prices of input commodities

For a price system $p \in \mathbb{R}^\ell_+$, if $\eta^F(p) \neq \emptyset$, then equation (4.1) implies that each $z = z(p) \in \eta^F(p)$, referred to as a factor demand at price p [27], solves

$$\max_{y\in Y}^F p \cdot y = \max_{y\in Y}^F \left(p^{out} \cdot f\left(y^{in}\right) + p^{in} \cdot y^{in}\right).$$

Proposition 5.1. *Assume that the firm's order relation $\leq_F$ of real numbers satisfies the condition of positive multiplicativity, that is, for any scalar $\alpha > 0$ and $a, b \in \mathbb{R}$, $a \leq_F b \to \alpha a \leq_F \alpha b$. Let $p \in \mathbb{R}^\ell_+$ be a price system, satisfying $\eta^F(p) \neq \emptyset$. If $z = z(p) \in \eta^F(p)$, then for each input commodity h_j^{in}, $z_{h_j^{in}}(p)$ is non-decreasing in $p_{h_j^{in}}$.*

Proof. Let us pick two price systems p, $p' \in \mathbb{R}^\ell_+$ and two factor demands $z \in \eta^F(p)$, $z' \in \eta^F(p')$. Then the definition of the optimal production correspondence implies that

$$p \cdot z =_F \max\nolimits_{y\in Y}^F p \cdot y \quad \text{and} \quad p' \cdot z' =_F \max\nolimits_{y\in Y}^F p' \cdot y.$$

Hence, we have $p \cdot z \geq_F p \cdot z'$ and $p' \cdot z' \geq_F p' \cdot z$. So, $p \cdot (z - z') \geq_F 0 \geq_F p' \cdot (z - z')$ follows. By combining the two ends of this inequality, we produce $(p' - p) \cdot (z' - z) \geq_F 0$. This inequality of vectors implies that for any input commodity h_j^{in},

$$\left(p'_{h_j^{in}} - p_{h_j^{in}}\right) \cdot \left(z'_{h_j^{in}} - z_{h_j^{in}}\right) \geq_F 0.$$

So, the assumed condition of positive multiplicativity implies that if $p'_{h_j^{in}} - p_{h_j^{out}} >_F 0$, then $z'_{h_j^{in}} - z_{h_j^{in}} \geq_F 0$. $\square$

Example 5.2. Constructed here is a scenario where for a particular order relation $\leq_F$ the firm has for real numbers, $z_{h_j^{in}}(p)$ is not non-decreasing in $p_{h_j^{in}}$. To this end, assume that $\leq_F = \leq_{\text{mod}(4)}$, where for real numbers x and $y \in \mathbb{R}$,

$$x <_{\text{mod}(4)} y \quad \text{if and only if} \quad x \bmod(4) < y \bmod(4),$$

where the order $<$ is the conventional one defined on $\mathbb{R}$, $x \bmod(4)$ is the remainder of $x \div 4$ and $y \bmod(4)$ the remainder of $y \div 4$, such that $0 \leq x \bmod(4) < 4$ and $0 \leq y \bmod(4) < 4$.

Let $p'_{h_j^{in}} - p_{h_j^{out}} =_{\text{mod}(4)} 2 >_{\text{mod}(4)} 0$ and $z'_{h_j^{in}} - z_{h_j^{in}} =_{\text{mod}(4)} -2$. Then, we have

$$\left(p'_{h_j^{in}} - p_{h_j^{in}}\right) \cdot \left(z'_{h_j^{in}} - z_{h_j^{in}}\right) \geq_{\text{mod}(4)} 0.$$

However, $z'_{h_j^{in}} - z_{h_j^{in}} <_{\text{mod}(4)} 0$. In other words, for Proposition 5.1 to hold true, the assumed positive multiplicativity is also a sufficient condition.

6. Aggregated supply/demand and the maximization of total productions

In this section, instead of looking at the firm as an individual, independent business entity, we examine the economy as a whole that consists of a collection of many interacting agents, such as producers and consumers. In particular, Subsection 6.1 focuses on the aggregated supply and aggregated demand in an economy; and Subsection 6.2 addresses the problem of maximization of total productions in an economy.

6.1. The firm and its representation in a Euclidean space

Each agent in the economy, be it an individual person, or a firm or an organization, chooses a plan $A = (a_1, a_2, \ldots, a_\ell) \in \mathbb{R}^\ell$ of action for the purpose of first staying alive and then thriving. Interactions exist in the fashion of input and output connections, forming various kinds of supply-chain ecosystems [3, 18, 38]. In particular, an agent's inputs are the outputs of some other agents and vice versa unless the agent is an ultimate consumer in the consumer product market.

Assume that the economy of our concern has n producers. Producer $j(= 1,$ 2, ..., $n)$ makes and carries out a production plan, which specifies the quantities of all input and output commodities. That is, the production plan (or simply production) of producer j is an element $y_j = (y_{j1}, y_{j2}, \ldots, y_{j\ell}) \in \mathbb{R}^\ell$ with outputs written as positive numbers and inputs negative numbers, as assumed before. If producer j chooses its production plan $y_j \in \mathbb{R}^\ell$, $j = 1, 2, \ldots, n$, then

$$y = y_1 + y_2 + \cdots + y_n = \sum_{j=1}^{n} y_j = \left(\sum_{j=1}^{n} y_{j1}, \sum_{j=1}^{n} y_{j2}, \ldots, \sum_{j=1}^{n} y_{j\ell} \right)$$

stands for the total production or total supply to the consumer market, where supplies to producers are counted twice, once as outputs and once as inputs so that they cancel each other in this summation.

Let Y_j be the set of all feasible production plans of producer j, meaning that each $y_j \in Y_j$ is technically materializable for producer j within its boundary conditions and meets the moral codes of its system of values and beliefs. Then, the set

$$Y = Y_1 + Y_2 + \cdots + Y_n = \sum_{j=1}^{n} Y_j = \left\{ \sum_{j=1}^{n} y_j : y_j \in Y_j, j = 1, 2, \ldots, n \right\}$$

represents the set of total productions of the producers or the production possibilities of the entire economy.

Proposition 6.1. *For each aggregated production $y = y_1 + y_2 + \cdots + y_n \in Y = Y_1 + Y_2 + \cdots + Y_n$ such that $y_j \in Y_j$, for $j = 1, 2, \ldots, n$, and each physical commodity h, the aggregated supply of h is greater than or equal to the aggregated demand of h. Symbolically,*

$$y_h \geq 0 \quad or \quad \sum_{j=1}^{n} y_{jh} \geq 0.$$

Proof. If there is a physical commodity h such that for some $y_j, y_{jh} < 0$, then this commodity h has to be produced by at least one other producer. So, in general, there are two sets $\{y_{j_p}\}_{j_p \in I_p}$ and $\{y_{j_q}\}_{j_q \in I_q}$ of producers, for some index sets I_p and $I_q \subseteq \{1,\ 2,\ \ldots,\ n\}$, such that the former ones input commodity h in their productions, while the latter ones produce h as outputs of their productions. Note that in real life, these two index sets I_p and I_q do not have to be disjoint. That is, $y_{j_p h} < 0$ and $y_{j_q h} > 0$ for each j_p and j_q such that

$$\sum_{j_q} y_{j_q h} + \sum_{j_p} y_{j_p h} \geq 0.$$

That is, the total input of commodity h by all producers is sufficiently covered by the total output of commodity h from all the producers. $\square$

The importance of Proposition 6.1 is demonstrated by the following conclusion: the assumption $Y \supset \left(-\mathbb{R}^{\ell}_{-}\right)$, as introduced by Debreu [7, p. 42], cannot hold true in general. In particular, Gerard Debreu intended to mean by introducing this assumption that it is possible for a total production to notify all of its outputs or for all producers to dispose of all commodities. However, Proposition 6.1 says that any commodity input of a producer has to come from at least one producer who produces the very commodity.

6.2. The maximization of total productions in an economy

In this subsection, assume that each producer is a price taker, that it maximizes the realization of its mission by choosing a production y_j, and that it uses its unique system of values and beliefs to define the meaning of maxima and to construct the method of maximization. Because in this paper every commodity and its price are time and location specific, each producer is required to choose a production so that its inputs and outputs are optimally distributed over both time and space. Such a desired production is known as one of producer *j's* equilibrium productions [7] with respect to the price system p.

Let $p \in \mathbb{R}^{\ell}_{+}$ be a price system and $y_j \in Y_j$ a production. The profit π_j of producer j is $p \cdot y_j$ and the total profit π of all the producers is

$$\pi = \sum_{j=1}^{n} p \cdot y_j = p \cdot \sum_{j=1}^{n} y_j = p \cdot y.$$

Assume that producer *j's* order of real numbers is $\leq_j$, when the firm solves for optimal decisions, while the conventional order between real numbers is $\leq$. For the entire economy, let us define the collective order $\leq_E$ of real numbers as follows: For any u and $v \in \mathbb{R}$,

$$u \leq_E v \quad \text{if and only if} \quad u \leq_j v, \quad \text{for each} \quad j = 1, 2, \ldots, n, \tag{6.1}$$

where the society is assumed to be democratic.

For the following proposition, we assume that each producer *j's* order of real numbers is the same as the conventional one.

Proposition 6.2. *For a given price system $p \in \mathbb{R}_+^\ell$, and a total production $y = \sum_{i=1}^n y_i \in \sum_{i=1}^n Y_i$, satisfying $y_j \in Y_j$, for each $j = 1, 2, \ldots, n$, the following statements are equivalent:*

(a) $p \cdot y =_E \max_{y^q \in Y}^E p \cdot y^q$*;*

(b) $p \cdot y_j =_j \max_{y_j^q \in Y_j}^j p \cdot y_j^q$*, for* $j = 1, 2, \ldots, n$.

Proof. (a) $\Rightarrow$ (b) By contradiction, let us assume that for $y_j \in Y_j$, for $j = 1, 2, \ldots, n$,

$$p \cdot y = p \cdot y_1 + p \cdot y_2 + \cdots + p \cdot y_n =_E \max_{y^q \in Y}^E p \cdot y^q;$$

however, there is a producer j, for some $1 \le j \le n$, such that

$$p \cdot y_j <_j \max_{y_j^q \in Y_j}^j p \cdot y_j^q.$$

Hence, we have $\max_{y^q \in Y}^E p \cdot y^q =_E p \cdot y_1 + p \cdot y_2 + \cdots + p \cdot y_n <_E p \cdot y_1 + \cdots + p \cdot y_{j-1} + \max_{y_j^q \in Y_j}^j p \cdot y_j^q + p \cdot y_{j+1} + \cdots + p \cdot y_n$, a contradiction. That means $p \cdot y =_E \max_{y^q \in Y}^E p \cdot y^q$.

(b) $\Leftarrow$ (a) Once again, we prove this conclusion by contradiction. To this end, we assume

$$p \cdot y_j =_j \max_{y_j^q \in Y_j}^j p \cdot y_j^q, \ \textit{for } j = 1, 2, \ldots, n,$$

while

$$p \cdot y =_E p \cdot y_1 + p \cdot y_2 + \cdots + p \cdot y_n <_E \max_{y^q \in Y}^E p \cdot y^q.$$

Hence, there are $y_j^* \in Y_j$, for$j = 1, 2, \ldots, n$, such that

$$p \cdot y_1 + p \cdot y_2 + \cdots + p \cdot y_n <_E p \cdot y_1^* + p \cdot y_2^* + \cdots + p \cdot y_n^*.$$

So, for some j, $1 \le j \le n$, we have

$$p \cdot y_j =_j \max_{y_j^q \in Y_j}^j p \cdot y_j^q <_j p \cdot y_j^*,$$

a contradiction. Therefore, the assumption $p \cdot y <_E \max_{y^q \in Y}^E p \cdot y^q$ does not hold true. $\square$

Speaking differently, the equivalent statements in Proposition 6.2 imply

$$\max_{y^q \in Y}^E p \cdot y^q =_E \max_{y_1^q \in Y_1}^1 p \cdot y_1^q + \max_{y_2^q \in Y_2}^2 p \cdot y_2^q + \cdots + \max_{y_n^q \in Y_n}^n p \cdot y_n^q. \tag{6.2}$$

However, the following example shows that this equation is not generally true when the producers are allowed to individually order real numbers differently from the conventional one and from each other. To confirm this end, we will construct the following Example 6.3. And, to make this construction self-contained, let us employ the generalized modular function defined for all real numbers in $\mathbb{R}$ [10]. In particular, for a fixed $a \in \mathbb{R}$ such that $a > 0$, the linear order relation $\le_{\mathrm{mod}(a)}$ of real numbers is defined as follows: For x and $y \in \mathbb{R}$,

$$x <_{\mathrm{mod}(a)} y \quad \text{if and only if} \quad x \bmod (a) < y \bmod (a),$$

where the order relation $<$ is the conventional one defined on $\mathbb{R}$, $x \bmod (a)$ is the remainder of $x \div a$ and $y \bmod (a)$ the remainder of $y \div a$ such that $0 \le x \bmod(a) < a$ and $0 \le y \bmod(a) < a$. In general, if $b(>0)$ is the remainder of $x \div a$, then b stands for the point on a circle of radius a, known as modulus, that is of a circular distance b in the counterclockwise direction from point 0; and when $b = x \bmod (a) < 0$, b represents the point on the circle that is of a circular distance b in the clockwise direction from point 0. When all the numbers used in discussion, such as a, x, and y above, are limited to the set $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, +1, +2, +3, \ldots\}$ of integers, the afore-defined order relation $\le_{\mathrm{mod}(a)}$ degenerates into the one widely studied in number theory [6].

Twelve-hour clocks, 7-day weeks, months of various numbers of days represent some of the familiar uses of modular operations in real life.

Example 6.3. To simplify our discussion, let us consider an economy that consists only of two producers, named 1 and 2, and that these producers order real numbers by using mod4 function, that is, $\le_{\mathrm{mod}(4)}$. As in the previous examples, we assume that one unit of each commodity is imported into the production line and is produced out of the line, where the production lines of producer 1 and 2 are respectively given in Figures 4 and 5. In particular, the arrows stand for the sequence for the commodities to be fed into the production line; and, the weights of the edges represent the relevant profits generated by the production sequence.

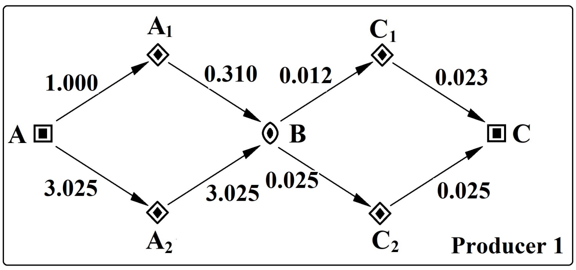

Figure 4. Flow chart of producer 1's productions

For producer 1, its potential commodity inputs are A, $A_?$, B, $C_?$, C, where $A_?$ can be either A_1 or A_2, but not both, and similarly, $C_?$ can be either C_1 or C_2, but not both. That is, commodities A_1 and A_2 (respectively, C_1 and C_2) are substitutes of each other. Hence, the set Y_1 of all production possibilities of producer 1 contains the following elements (or paths) and the corresponding profits are 1.345, 1.36, 6.085 mod(4) = 2.085, and 6.1 mod(4) = 2.1, respectively:

$$I_{11} : A \to A_1 \to B \to C_1 \to C \ ;$$
$$I_{12} : A \to A_1 \to B \to C_2 \to C \ ;$$
$$I_{21} : A \to A_2 \to B \to C_1 \to C \ ;$$
$$I_{22} : A \to A_2 \to B \to C_2 \to C \ .$$

Therefore, we have

$$\max\nolimits^1_{y \in Y_1} p \cdot y =_1 2.1 \tag{6.3}$$

Similarly for producer 2, its commodity inputs are U, $U_?$, V, $W_?$, W, where $U_?$ (respectively, $W_?$) can be either U_1 or U_2 (respectively, either W_1 or W_2), but not both.

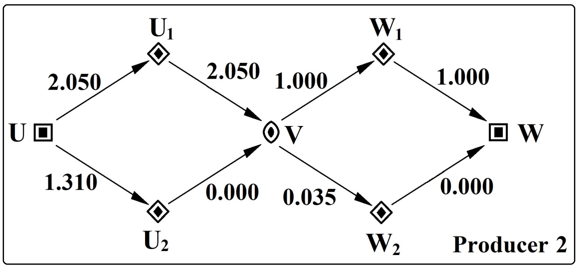

Figure 5. Flow chart of producer 2's productions

The set Y_1 of production possibilities of producer 2 contains the following elements (or paths) and the corresponding profits are 6.1 mod(4) = 2.1, 4.135 mod(4) = 0.135, 3.31, and 1.315, respectively.

$$J_{11} : U \to U_1 \to V \to W_1 \to W \ ;$$
$$J_{12} : U \to U_1 \to V \to W_2 \to W \ ;$$
$$J_{21} : U \to U_2 \to V \to W_1 \to W \ ;$$
$$J_{22} : U \to U_2 \to V \to W_2 \to W \ .$$

That is, we have

$$\max{}^2_{y \in Y_2} p \cdot y =_2 3.31 \tag{6.4}$$

Therefore, from equations (6.3) and (6.4), we have

$$\max{}^1_{y \in Y_1} p \cdot y + \max{}^2_{y \in Y_2} p \cdot y =_{\mathrm{mod}(4)} 2.1 + 3.31 \ \mathrm{mod}(4) =_{\mathrm{mod}(4)} 1.41 \tag{6.5}$$

To compute $\max^E_{y \in Y} p \cdot y$, we first find the set $Y = \{y_1 + y_2 : y_1 \in Y_1, y_2 \in Y_2\}$ of total productions of the economy. The economy's order of real numbers is equal to $\leqslant_E = \leqslant_{\mathrm{mod}(4)}$. The computational results of $p \cdot y = p \cdot y_1 + p \cdot y_2$ mod(4) are shown in Table 1.

Table 1. Computation of $p \cdot y_1 + p \cdot y_2$ mod(4)

p2 / p1	2.100	0.135	3.310	1.345
1.345	3.445	1.480	0.655	2.690
1.360	3.460	1.495	0.670	2.705
2.085	0.185	2.220	1.395	3.430
2.100	0.200	2.235	1.410	3.445

Note: p1 = producer 1; p2 = producer 2

Therefore, we obtain $\max^E_{y\in Y} p\cdot y = 3.46$. So, by referencing back to equation (6.5), we have

$$\max^E_{y\in Y} p\cdot y >_E \overset{1}{\max_{y_1^q\in Y_1}} p\cdot y_1^q + \overset{2}{\max_{y_2^q\in Y_2}} p\cdot y_2^q + \cdots + \overset{n}{\max_{y_n^q\in Y_n}} p\cdot y_n^q.$$

That end implies that equation (6.2) does not generally hold in terms of systems of values and beliefs.

In terms of real life, economies in general do not have such a linear order $\leq_E$ of real numbers that is consistent with that of each individual producer. That is, equation (6.1) generally does not hold true. For instance, if in Example 6.3, producer 1's order of real numbers is $\leq_{\mathrm{mod}(3)}$, and producer 2's is $\leq_{\mathrm{mod}(4)}$, then real numbers 1 and 3.2 cannot be ordered in the economy, because these producers have inconsistent order relations:

$$3.2 \leq_{\mathrm{mod}(3)} 1 \text{ and } 1 \leq_{\mathrm{mod}(4)} 3.2.$$

That is, in this case, the economy's order $\leq_E$ of real numbers is not linear.

Closely relevant to this end is Adam Smith's "invisible hand" as initially introduced in 1759 in Part IV and Chapter 1 of his work The Theory of Moral Sentiments. In particular, the imagined "invisible hand" pronounces that although individuals are selfish, their self-interest centered actions collectively produce unintended greater social benefits and public goods [41]. Here, what does the word "greater" mean? According to the discussion above, the greater community of selfish individuals most likely does not have any order of real numbers that is consistent with that of every producer, as long as the producers order real numbers differently. Speaking differently, in general, there is not an unanimously acknowledged method to decipher the meaning of "greater social benefits and public good." It is because in any economy of more than two economic agents there are different systems of values and beliefs [9], and these differences define an inconsistent economywide order $\leq_E$ of real numbers. In terms of the literature, based on Greenwald and Stiglitz [16], Joseph E. Stiglitz [4] believes that the invisible hand is often not there. In comparison, what we achieved here definitively confirms analytically that Stiglitz's belief is correct.

7. Conclusion

This paper establishes a series of 7 propositions by employing the methodology of Euclidean spaces on the bases of the four natural endowments of firms. These conclusions extend some of the well-known results from the prevalent producer theory [27, 32] to the general case of no matter what system of values and beliefs a firm may possibly embrace. At the same time, we construct 4 counterexamples to confirm the fact that some of the fundamental results in the prevalent producer theory only hold true under very specific conditions.

By highlighting the real-life fact that firms generally employ different decision criteria of priority and employ their specific means to optimize the realization of their missions [17, 44, 46], this paper is able to partially actualize the goal of research outlined in the introduction section earlier. Because this paper starts its analytical reasoning on the concept of firms' natural endowments, it opens up a large area of research, where most, if not all, of the established results in economics need to be checked to see whether or not they still hold true when the order of real numbers is not the same as the conventional one.

Specific to this paper, due to its novel methodological approach, which was initially adopted by Debreu [7], and its emphasis on firms' natural endowments [10, 12], this work establishes, among others, the following main results:

- Each conditional factor demand is a nonincreasing function of prices (Proposition 3.1);
- A firm's optimal production correspondence in general is not homogeneous of degree zero (Proposition 4.3), which is different from what is known before [27];
- In a functional economy, the aggregated supply of a commodity is more than or equal to the aggregated demand of the commodity (Proposition 6.1), when the time factor is ignored;
- Micro players' actions on their self-interests do not generally lead to unintended greater macro-level social benefits and public good (Example 6.3), as commonly believed and known as the "invisible hand" [41].

To summarize, it is indeed true that this work is 100% based on the set theory of Euclidean spaces. However, because this paper considers how a firm prioritizes its decision alternatives on the basis of its system of values and beliefs, which has been totally ignored in the literature, results established herein are expected to be more practically relevant than the corresponding results derived previously.

As for potential future research along the lines drawn in the previous sections, there are many topics one can look at closely. More specifically, among all potentials for future research, one can formalize additional ways on how decision-making managers and entrepreneurs prioritize their available alternatives in real life. Only by doing so, we can hopefully answer the loud calls for the desperate need to reconstruct the existent economic theories [5, 17, 25, 36, 43, 44, 46].

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