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Reliability Estimation Based on the Degradation Amount Distribution Using Composite Time Series Analysis and Grey Theory

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Abstract: *This paper puts forward a reliability estimation method by the Degradation Amount Distribution (DAD) of products, using a composite time series modeling procedure and grey theory based on a random failure threshold. Product DAD data are treated as a composite time series and described using a composite time series model to predict a long-term trend of degradation. The degradation test is processed for a certain electronic product and the degradation data is collected for reliability estimation. Comparison among the reliability evaluation by DAD composite time series analysis and grey theory, based on a constant and a random failure threshold, reliability evaluation by DAD regression analysis based on a random failure threshold, reliability evaluation by degradation path time series analysis, and real reliability of the electronic product is done. The results show that the reliability evaluation of the product using the method proposed is the most creditable of all.*

Keywords: *Reliability estimation, time series, grey theory, degradation amount distribution.*

1. Introduction

For long lifetime and high reliability products, it is difficult to obtain failure time data in a short time period. Hence, Degradation Testing (DT) is presented to deal with the cases when no fault data can be obtained, but degradation data of the primary parameter of the product is available.

At present there are mainly two ways to estimate the product reliability by DT: the first one based on the degradation path, that is, the product reliability estimation is obtained by prediction of each sample degradation path; another one based on Degradation Amount Distribution (DAD), that is, the product reliability estimation is obtained by prediction of all samples of DAD parameters.

Most previous works [5-11] use a deterministic model, such as the regression model to represent the degradation path or DAD parameters. However, the environmental variables have an important influence on the performance degradation process of many products. The reliability estimation must take into account the stochastic nature of the environmental variables.

A few investigations [1-4] study DT reliability estimation using the time series method because of its excellent stochastic and periodic information mining, in order to take into account the stochastic nature of the environmental variables. However, all reliability estimations using the time series method in present publications are based on the degradation path. It is important to study DT reliability estimation based on DAD using the time series method, since only this approach can be used in a random failure threshold situation, which is a common situation in practice.

In this paper two assumptions of DT are put forward:

(1) the sampling instants of all products are the same;

(2) the distribution pattern of the degradation amount at each sampling instant is unchanged, only the degradation parameters amount the distribution change.

2. DAD parameters estimation

In DT the degradation amounts of all product samples at the same time are usually subject to certain location-scale distribution, and the location and scale parameter at time t is denoted by μ_t and σ_t, which describes DAD at time t. DAD type is determined by a Goodness of Fit Test, such as Pearson chi-square test.

In this paper when the degradation amount distribution is location-scale distribution, the location and scale parameters at time t are denoted by μ_t and σ_t, which describes the degradation amount of the distributing situation at time t.

The estimation of the location and scale parameter of distribution of the degradation amount y_t at time t is obtained by MLE. This paper denotes the degradation amount of i-th product at time t as y_{ti}, when the total number of the products is n, the maximum likelihood function of the distribution of y_t is

(1) $$L(\beta_t)=\prod_{i=1}^{n} f(y_{ti},\beta_t).$$

The estimation of the location and scale parameter at time t is $\hat{\beta}_t=(\hat{\mu}_t,\hat{\sigma}_t)^{\mathrm{T}}$. By solving MLE equation of the location and scale parameter for each time, location and scale, the parameter series $\{\hat{\mu}_t\}$ and $\{\hat{\sigma}_t\}$, are obtained.

3. DAD stability analyses

The location and scale parameter series are arranged in a time order. Thus, $\{\hat{\mu}_t\}$ and $\{\hat{\sigma}_t\}$ are time series.

The location and scale parameters describe different characteristics of distribution of y_t. Hence, the time series model structure of $\{\hat{\mu}_t\}$ and $\{\hat{\sigma}_t\}$ is different. This paper models them respectively.

3.1. Location parameter time series modeling

In DT due to the monotonous degraded nature of the product performance, the periodic nature of the test equipment control and the stochastic nature of the environmental variables, the degradation amount y_t varies with time monotonously, periodically and randomly.

The location parameter describes the central tendency of the location of distribution of y_t. Hence, the location parameter varies with time monotonously, periodically and randomly, too.

Hence, this paper decomposes the location parameter time series $\{\hat{\mu}_t\}$ into three components, which are a trend, seasonal and random component, in order to take into account the monotonous degraded nature, the periodic nature and stochastic nature, and then describe them using a composite time series model.

The location parameter time series model is

$$\hat{\mu}_t = T_{\mu t} + S_{\mu t} + R_{\mu t},\ t = 1, 2, \cdots \tag{2}$$

Here, $T_{\mu t}$, $S_{\mu t}$ and $R_{\mu t}$ denote the trend, seasonal and random component at time t.

3.1.1. Trend component modeling

The monotonic degradation trend of $\hat{\mu}_t$ is represented by a linear or monotonic nonlinear regression model, such as a logarithmic function, exponential function, power function and so on. The model of the trend component $T_{\mu t}$ is

$$T_{\mu t} = b_\mu g_\mu(t) + T_{\mu 0}. \tag{3}$$

Here b_μ is the degradation rate of $\hat{\mu}_t$, $g_\mu(t)$ is a linear or monotonic nonlinear function, $T_{\mu 0}$ is the initial value of $T_{\mu t}$.

3.1.2. Season component modeling

Controlled by the test equipment, the test stress level usually fluctuates periodically around a set level. The product performance reflects the periodical fluctuation of the stress level. Removing the trend component from $\hat{\mu}_t$, this paper regards this fluctuation as a seasonal component $S_{\mu t}$ and represents it using the Hidden Periodicity (HP) regression model

$$S_{\mu t} = \sum_{j=1}^{q} a_j \cos(\omega_j t + \varphi_j). \tag{4}$$

Here q is the number of the angular frequency, α_j is j-th amplitude, ω_j is j-th angular frequency, φ_j is j-th phase.

3.1.3. Random component modeling

Removing $T_{\mu t}$ and $S_{\mu t}$ from $\hat{\mu}_t$, the random component $R_{\mu t}$ is represented by the Auto Regressive (AR) model

$$R_{\mu t} = \sum_{j=1}^{p_\mu} \eta_{\mu j} R_{\mu(t-j)} + \varepsilon_{\mu t}. \tag{5}$$

Here, p_μ is the order number of AR model, $\eta_{\mu j}$ is a coefficient of j-th order, $\varepsilon_{\mu t}$ is independent white noise.

Adding (3) and (4) to (5), a complex time series model of $\hat{\mu}_t$ is put forward,

$$\begin{aligned} \hat{\mu}_t &= T_{\mu t} + S_{\mu t} + R_{\mu t} = b_\mu g_\mu(t) + T_{\mu 0} + \\ &+ \sum_{j=1}^{q} a_j \cos(\omega_j t + \varphi_j) + \sum_{j=1}^{p_\mu} \eta_{\mu j} R_{\mu(t-j)} + \varepsilon_{\mu t}. \end{aligned} \tag{6}$$

3.2. Scale parameter time series modeling

The scale parameter describes the dispersion scale of distribution of y_t. In practice, the performance degraded nature of different product samples is different. Hence, there is a different degradation rate of different product samples. When the initial value of y_t of different product samples is the same, the dispersion scale of the distribution of y_t must increase with time. Hence, the scale parameter not only varies monotonically with time, but it also monotonically increases.

In DT the periodic nature of the test equipment control has the same effect on the life of different product samples. This means that the periodical variation trends of y_t of different product samples are the same. Hence, the dispersion scale of y_t does not vary periodically with time.

The influence of the stochastic nature of the environmental variables on different product samples is stochastic too. Hence, the random variation of y_t of different product samples is different, which is reflected in the scale parameter variation.

Hence, this paper decomposes the scale parameter time series $\{\hat{\sigma}_t\}$ in two components, which are a trend and a random component, in order to take into account the different degraded nature of the product performance and the different stochastic nature of the environmental variables, and then describe them using a composite time series model.

The scale parameter time series model is

$$\hat{\sigma}_t = T_{\sigma t} + R_{\sigma t},\ t = 1, 2, \cdots \tag{7}$$

Here $T_{\sigma t}$ and $R_{\sigma t}$ denote the trend and random component of $\hat{\sigma}_t$ at time t.

3.2.1. Trend component modelling

The monotone increasing trend of $\hat{\sigma}_t$ is represented by a linear or monotonic nonlinear regression model. The model of the trend component $T_{\sigma t}$ is

(8) $$T_{\sigma t} = b_\sigma g_\sigma(t) + T_{\sigma 0}.$$

Here b_σ is the degradation rate of $\hat{\sigma}_t$, $g_\sigma(t)$ is a linear or monotonic nonlinear increasing function, $T_{\sigma 0}$ is the initial value of $T_{\sigma t}$.

3.2.2. Random component modeling

Removing $T_{\sigma t}$ from $\hat{\sigma}_t$, the random component $R_{\sigma t}$ is represented by AR model

(9) $$R_{\sigma t} = \sum_{j=1}^{p_\sigma} \eta_{\sigma j} R_{\sigma(t-j)} + \varepsilon_{\sigma t},$$

Here, p_σ is the order number of AR model, $\eta_{\sigma j}$ is a coefficient of j-th order, $\varepsilon_{\sigma t}$ is independent white noise.

Adding (2) to (6), a complex time series model of $\hat{\sigma}_t$ is put forward,

(10) $$\begin{aligned}\hat{\sigma}_t &= T_{\sigma t} + R_{\sigma t} = \\ &= b_\sigma g_\sigma(t) + T_{\sigma 0} + \sum_{j=1}^{p_\sigma} \eta_{\sigma j} R_{\sigma(t-j)} + \varepsilon_{\sigma t}.\end{aligned}$$

4. Reliability estimation

In DT a fault occurs as the product performance level achieves a specified failure threshold. The product reliability is the probability of achieving it. In this paper the reliability estimation is obtained by prediction of DAD parameters.

4.1. Parameter prediction

The l-step prediction of $\hat{\mu}_t$ is obtained using the best linear unbiased prediction of (6). The prediction formula is

(11) $$\begin{aligned}\hat{\mu}_{\tau+l} &= T_{\mu(\tau+l)} + S_{\mu(\tau+l)} + R_{\mu(\tau+l)} = \\ &= b_\mu g_\mu(\tau+l) + T_{\mu 0} + \\ &+ \sum_{j=1}^{q} a_j \cos\left(\omega_j(\tau+l) + \varphi_j\right) + \sum_{j=1}^{p_\mu} \eta_{\mu j} R_{\mu(\tau+l-j)}.\end{aligned}$$

Here τ is the time scale before prediction.

The l-step prediction of $\hat{\sigma}_t$ is obtained using the best linear unbiased prediction of (10). The prediction formula is

(12) $$\hat{\sigma}_{(\tau+l)} = T_{\sigma(\tau+l)} + R_{\sigma(\tau+l)} = b_\sigma g_\sigma(\tau+l) + T_{\sigma 0} + \sum_{j=1}^{p_\sigma} \eta_{\sigma j} R_{\sigma(\tau+l-j)}.$$

4.2. Random failure threshold reliability estimation

Traditional researches often suppose that the product failure threshold is a constant, and the reliability evaluation based on a constant failure threshold is obtained by traditional reliability formulas.

However, in practice the failure threshold is most often a random variable due to the stochastic nature of failure occurrence. This paper denotes the failure threshold random variable as *D*. The reliability evaluation based on a random failure threshold is obtained by formulas as follows:

If y_t increases with time, the failure threshold *D* is not less than the initial value of y_t, which is denoted by D_0, then the product reliability is

(13)
$$R_t = 1 - P\{y_t \geq D\} = \iint\limits_{y_t \geq D} f_D(D) f_{y_t}(y_t)\, dD dy_t = \\ = \int_{D_0}^{+\infty}\int_{D}^{+\infty} f_D(D) f_{y_t}(y_t)\, dD dy_t.$$

Here $f_D(D)$ is the probability distribution function of the failure threshold *D*, $f_{y_t}(y_t)$ is the probability distribution function of y_t, which is determined by $\hat{\mu}_t$ and $\hat{\sigma}_t$.

If y_t decreases with time, the failure threshold *D* is not greater than D_0, then the product reliability is

(14)
$$R_t = 1 - P\{y_t \leq D\} = \iint\limits_{y_t \leq D} f_D(D) f_{y_t}(y_t)\, dD dy_t = \\ = \int_{-\infty}^{D_0}\int_{-\infty}^{D} f_D(D) f_{y_t}(y_t)\, dD dy_t.$$

5. Grey prediction

The prediction precision of a time series model depends on the sample size. Hence, accurate prediction of a time series model needs plenty of samples. It is difficult to apply it into practice. Grey theory is a prediction method based on data of a small sample size and indeterminacy. This paper puts forward a grey failure time prediction method.

The system grey prediction nesting method of the grey theory nests GM(1, 1) in GM(1, *N*) to obtain prediction values based on a prediction model. According to it, the first data is removed from the series and a one-step prediction data is added to this series. Modeling this new series can update the model parameters. This procedure is repeated until a defined step. The prediction precision of this nesting prediction is higher than the one of direct prediction.

This paper predicts a location and scale parameter time series model using the system grey prediction nesting method. The prediction procedure of the degradation data is as follows. Fig. 1 shows the flow chart of this prediction procedure.

Step 1. h is set as the total number of prediction times; u denotes the prediction times, $u = 1, 2, \cdots, h$.

Step 2. $\{x_t\}$, $t = 1, 2, \cdots, N$, denotes a not-predicted time series of a location or a scale parameter.

Step 3. The parameters of (6) or (10) are updated at u-th prediction.

Step 4. An one-step prediction data x_{N+1} is obtained by the optimal unbiased prediction model from (11) and (12).

Step 5. x_1 is removed from $\{x_t\}$, $t = 1, 2, \cdots, N$, and x_{N+1} is added to it, then a new series $\{x_t, x_{N+1}\}$, $t = 2, 3, \cdots, N$, is obtained.

Step 6. $\{x_t, x_{N+1}\}$, $t = 2, 3, \cdots, N$, is regarded as a not-predicted series, Steps 2-4 are repeated.

Step 7. Step 5 and 6 are repeated $h-1$ times, x_{N+u}, $u = 1, 2, \cdots, h$, is obtained one by one.

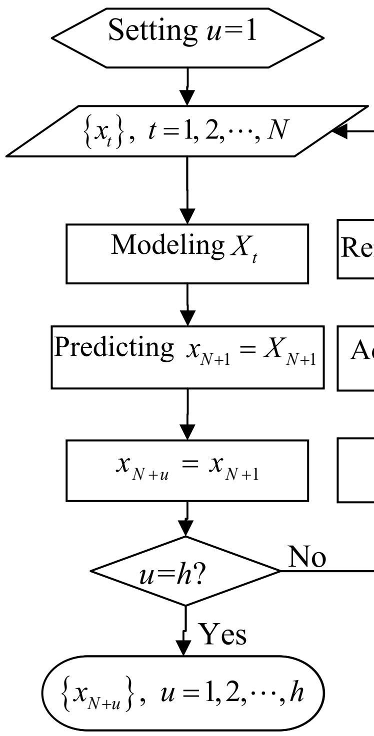

Fig. 1. Flow chart of the prediction procedure

6. Example verification

A DT of 13 certain products is conducted as an example to verify the method proposed. The sampling interval is 1 min. The construction of DT system is shown in Fig. 2.

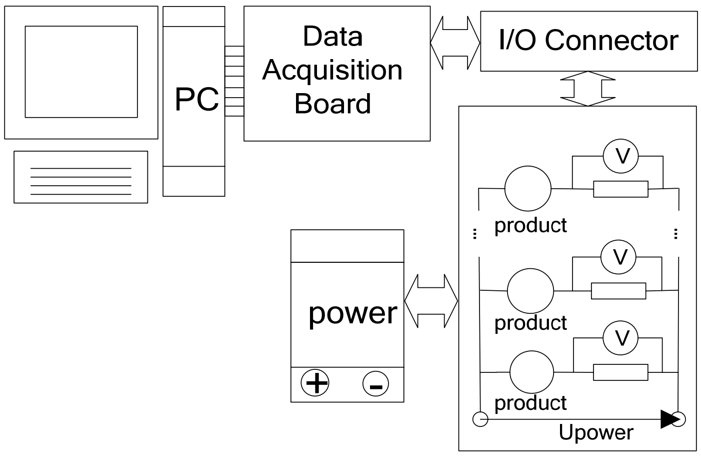

Fig. 2. The construction of the degradation testing system

In this paper DT has been conducted for 17 000 min. DT data of each product is preprocessed for eliminating the influence of its initial value difference. Fig. 3 shows the preprocessed DT data until all products failure.

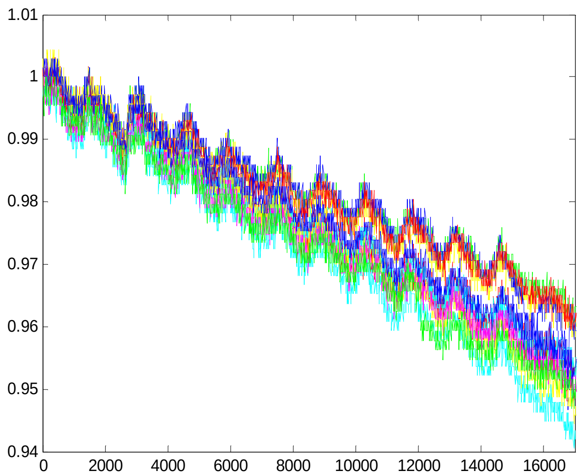

Fig. 3. Preprocessed DT data

Firstly, DAD pattern of DT data is determined by Pearson chi-square Goodness of Fit Test. Table 1 shows the test. The judging threshold is 7.815.

Table 1. Pearson chi-square test of DAD

DAD pattern	Chi-square test
Normal	4.7426
Lognormal	4.6992
Weibull	8.0535

According to Table 1, lognormal distribution is the most appropriate distribution. Thus, the logarithm of the degradation amount is with normal distribution. It is regarded as degradation data in this example.

Fig. 4 shows the trend component of the degradation data.

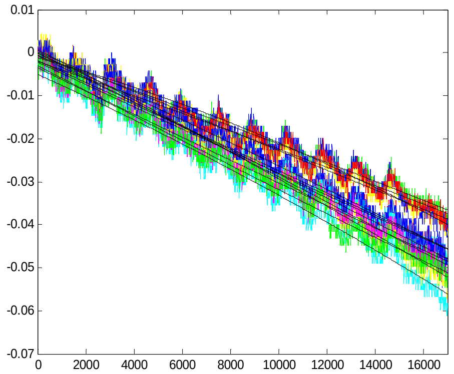

Fig. 4. Trend component of the degradation data

The random component of the degradation data is obtained by removing the trend component; Fig. 5 shows them.

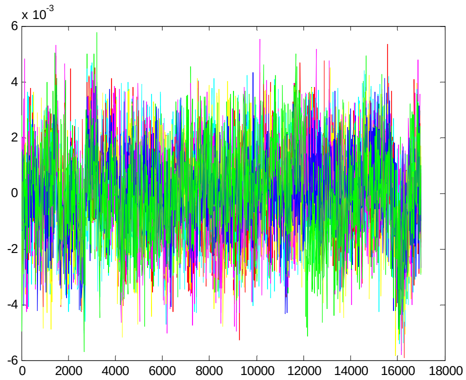

Fig. 5. Random component of the degradation data

A stationary test result for the random component of the degradation data shows that it is stationary.

Secondly, the location and scale parameter of DAD is modeled by a composite time series model. Both the location and scale parameter trend components are power functions.

Thirdly, the predictions of DAD parameters are obtained. Figs 6 and 7 show the location scale parameter prediction.

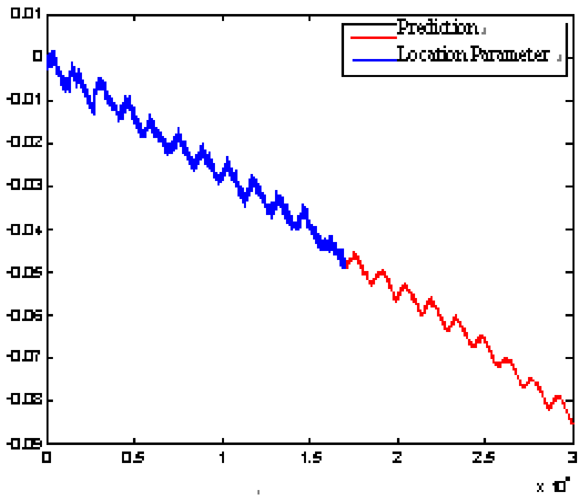

Fig. 6. Location parameter prediction

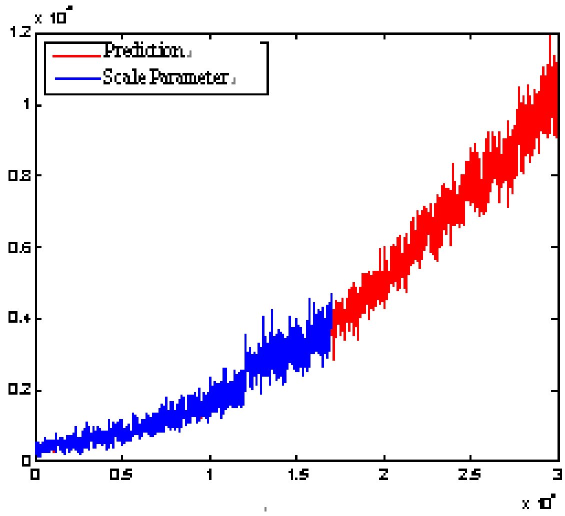

Fig.7. Scale parameter prediction

According to practical experience, the constant failure threshold of the product is 93% and the random failure threshold distribution function of the product is lognormal distribution.

Fig. 8 shows the reliability evaluation by DAD time series model and grey theory, based on a constant and random failure threshold (the blue curve and the red curve), the reliability evaluation by DAD regression model based on a random failure threshold (the black solid curve), the reliability evaluation by a degradation path time series model based on a constant failure threshold (the black dotted curve), and the real reliability (the green curve) for comparison. Here, the real reliability is obtained by the real degradation data and the real failure time data of the 13 products considered.

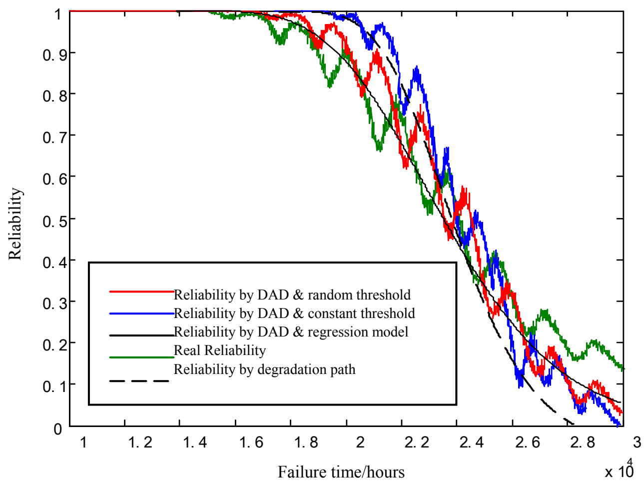

Fig. 8. Reliability evaluation

According to Fig. 8, it is obvious that the reliability evaluation curve by DAD based on a random failure threshold is closer to the real reliability curve than the reliability evaluation curve by DAD based on a constant failure threshold.

Since the reliability evaluation by the degradation path can only be used in a constant failure threshold, its curve is close to the reliability evaluation curve by DAD based on the constant failure threshold, but not close to the real reliability curve.

Compared to the reliability evaluation curve by DAD regression model, the reliability evaluation curve by DAD time series model and grey prediction is random and fluctuating, which can reflect the implementation of the stochastic and periodic nature of the test equipment and the environmental variables on the product reliability.

Hence, the reliability evaluation curve by DAD time series model and grey prediction, based on a random failure threshold is closer to the real reliability curve of all.

7. Conclusion

This paper proposes a DT reliability estimation method by DAD using a composite time series modeling procedure and grey theory based on a random failure threshold.

According to the example verification, there are two advantages of the method proposed in this paper:

One is that compared with the reliability evaluation by the degradation path, the reliability evaluation by the proposed method can be used in a random failure threshold, which is more practical;

The other is that compared with the reliability evaluation by a traditional regression model, the reliability evaluation by the proposed method can reflect the implementation of the stochastic and periodic nature of the test equipment and environmental variables on the product reliability, which evaluates more accurately the reliability.

Hence, the reliability evaluation based on the method proposed is more creditable than others known in literature.

There are also a few disadvantages of the method proposed in this paper, for example that it will not be working if the sampling instants of all the products are not the same. This problem can be solved by using unequal interval time series modeling procedure, which can be brought into future research plan.

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