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The value of demand forecast updates in managing component procurement for assembly systems

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ABSTRACT

This article examines the value of demand forecast updates in an assembly system where a single assembler must order components from independent suppliers with different lead times. By staggering each ordering time, the assembler can utilize the latest market information, as it is developed, to form a better forecast over time. The updated forecast can subsequently be used to decide the following procurement decision. The objective of this research is to understand the specific operating environment under which demand forecast updates are most beneficial. Using a uniform demand adjustment model, we are able to derive analytical results that allow us to quantify the impact of demand forecast updates. We show that forecast updates can drastically improve profitability by reducing the mismatch cost caused by demand uncertainty.

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Demand forecast updates; assembly systems; component procurement strategies; inventory management

1. Introduction

Market conditions can change rapidly, often without any warning. It is therefore critical for firms to continuously monitor these conditions and update their demand forecasts, so that production plans and inventory levels can be adjusted as needed. With advances in information technology, firms can now more readily than ever before collect the latest market information from sources such as point-of-sales terminals as it is developed and use this information to update their demand forecasts. These updated forecasts can subsequently be used to help manage operations more efficiently.

As firms continue to leverage better technologies over time, they have also become increasingly reliant on a global supplier network, chasing lower procurement costs. As supply chains lengthen, procurement lead times for particular components have also become longer. Consequently, it is not unusual for procurement lead times to vary drastically from component to component. To alleviate the problems associated with rapidly changing market conditions and high obsolescence costs, firms can take advantage of the differences in these procurement lead times by staggering their own ordering times. Doing so allows for an informational update between each ordering time, which can be used to further improve on demand forecasts. The objective of our research is to evaluate the value of demand forecast updates in this type of operating environment.

We use the following simplified version of a real-world problem to motivate our research. Consider an apparel assembler who sells a sweater consisting of two major components, namely, fabric and buttons. The fabric and buttons are sourced from two different suppliers with differing lead times. Due to the higher costs associated with producing fabric, an inexpensive overseas fabric supplier was selected. In contrast, a local company with a shorter lead-time was chosen as the button supplier. As is common with many apparel assemblers, the actual time to manufacture each sweater is negligible compared with the procurement lead times.

To begin, the assembler will first place an order with the fabric (long lead-time) supplier based on an initial estimate of customer demand. After some time has elapsed, the assembler places a second order with the button (short lead-time) supplier. In between the first and second ordering times, the assembler is able to collect additional demand information from trade shows, market surveys, point-of-sales terminals, and other sources of information. Using this additional demand information, the assembler is able to produce an updated demand forecast, before placing the second order.

One order each is allowed for the fabric supplier and the button supplier, as an emergency order to either will be highly disruptive to their respective production schedules. As such, once an order has been placed with the fabric supplier, an upper limit becomes set on the total number of sweaters that can be produced, as each sweater requires exactly one unit each from the fabric supplier and the button supplier. Consequently, the assembler needs to carefully evaluate the tradeoff between procuring too few units of fabric and thus limiting output and procuring too many units, which leads to excess wastage.

As market research can be expensive, our objective is to understand the value of demand forecast updates in the assembly system described above. Under what operating conditions would demand forecast updates be most beneficial? Our research aims to answer this question and also provide useful managerial insights regarding how the use of demand forecast updates can benefit the assembler, as well as the component suppliers, by reducing the adverse effects caused by market volatility.

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We first describe a general modeling framework and provide some basic analytical results on the optimal procurement policies for the assembler under demand forecast updates. Then by using a uniform demand adjustment model, we derive analytical expressions for the optimal procurement decisions and the associated expected optimal profits, both with and without demand forecast updates. After that, we develop a metric to measure the value of demand forecast updates and use the analytical results to quantify the impact of demand forecast updates in reducing the mismatch cost due to the underlying demand uncertainty for the assembler as well as the two component suppliers. We end by conducting an extensive set of numerical experiments under both uniform and normal demand adjustments to produce useful managerial insights to further understand the specific operating conditions under which demand forecast updates would provide the most benefits.

We make the following important contributions to the research literature. First, we derive closed-form analytical expressions for the optimal ordering quantities and the associated expected profit functions of the assembler for a general uniform demand adjustment model. These analytical expressions allow us to quantify the exact value of forecast updates, which was not previously possible. Thomas *et al.* (2009) only provide some limited analytical results for a very special case of our general model. Second, we have developed a performance metric, called mismatch cost reduction, which is useful for quantifying the value of demand forecast updates. Finally, we are able to combine our analytical results together with extensive numerical results to provide several useful managerial insights for understanding the specific operating environment under which demand forecast updates would be most valuable.

The rest of the article is organized as follows. Section 2 provides a literature review of relevant research. Section 3 describes the model formulation and provides some basic results under a general demand adjustment model. In Section 4, we focus on a model with uniform demand adjustments and derive analytical results on how demand forecast updates affect the order quantities and the associated expected profits of the assembler as well as the component suppliers. In Section 5, we conduct an extensive set of numerical experiments to extend our analysis and then discuss managerial insights. We conclude our article in Section 6. All mathematical proofs are provided in the Appendix.

2. Literature review

Our article is concerned with using demand forecast updates to coordinate component procurement decisions in assembly systems under stochastic demands. The importance of demand forecast updates in managing planning and inventory has been well-recognized and widely studied in the research literature. In this stream of research, it is assumed that firms can use sales or market information in an earlier stage to update their prior demand forecasts for improving their production or inventory decisions at later stages. Examples of this line of research include Bitran *et al.* (1986), Fisher and Raman (1996), Eppen and Iyer (1997), Gurnani and Tang (1999), Weng and Parlar (1999), Cattani and Hausman (2000), Choi *et al.* (2003, 2006), Sethi *et al.* (2003), Yan *et al.* (2003), Miyaoka and Hausman (2004), and Huang *et al.* (2005). Our article focuses specifically on component procurement decisions under an assembly structure.

There also exists a substantial body of research that studies the joint component procurement decisions in assembly systems under stochastic demands. Song and Zipkin (2003) provide a comprehensive review on earlier research that study this stream of research. More recent research that analyzes the structure of the optimal joint component procurement decisions in assembly systems under stochastic demands include Fu *et al.* (2006, 2009), Hsu *et al.* (2006, 2007), Fang *et al.* (2008), and Zhang *et al.* (2008). However, most research papers in this stream do not consider demand forecast updates in such decisions.

Two recent papers that investigate the impact of demand forecast updates in managing complementary component procurement in assembly systems are of particular relevance. Thomas *et al.* (2009) study the contractual relationship between an original equipment manufacturer and a contract assembler who assembles a product with two complementary components with different lead times. The original equipment manufacturer can choose to share demand forecast updates with the contract assembler. Their primary focus is to study whether it is beneficial for the original equipment manufacturer to share demand forecast updates and component overage risk with the contract assembler. Similarly, Yang *et al.* (2011) analyze a decentralized supply chain consisting of an assembler and two component suppliers with different lead times. The assembler uses the latest market information to update his demand forecast before ordering the short lead-time component. Their primary focus is to investigate what contractual mechanisms can coordinate a decentralized supply chain under demand forecast updates.

We adopt the same modeling framework as the centralized system in Thomas *et al.* (2009) and Yang *et al.* (2011), consisting of one assembler and two component suppliers with different procurement lead times. We are able to derive closed-form analytical expressions for the optimal order quantities and expected profit of the manufacturer for a uniform demand adjustment model, which extends the earlier result in Thomas *et al.* (2009), where both uniform demand adjustments are assumed to be identical. Our model is more general than theirs and allows for more managerial insights. In addition, we develop a performance metric for quantifying the exact value of demand forecast updates and illustrate the specific operating environment under which demand forecast updates would be most valuable.

Finally, Miyaoka and Hausman (2008) also use a similar modeling framework to analyze a supply chain consisting of a supplier and a manufacturer. The supplier first builds capacity based on original forecast information and the manufacturer later decides on the ordering quantity after observing updated demand information. These two decisions are thus equivalent to the procurement decisions of the two components in our model. However, their focus is on understanding the impact of updated demand information on supply chain coordination under different wholesale price purchasing arrangements in a decentralized setting.

3. Model formulation

We consider an assembler whose product consists of two complementary components with respective procurement lead times l_1 and l_2. We assume that $l_1 > l_2$ and refer to component 1 as the long lead-time component and component 2 the short lead-time component. Without loss of generality, we assume that the assembly time for the product is negligible when compared with the component procurement lead times.

We consider the following cost structure. Let p be the unit price for the product and c_i be the unit procurement cost for component i, $i = 1, 2$. To simplify our exposition, we assume that there is no salvage value for any excess component and there is no penalty cost for product shortage. However, our analysis can be easily extended to include a salvage value or shortage penalty. As is standard in newsvendor models, adding these two costs would simply have the effect of lowering the overage cost or increasing the underage cost, respectively, which would not change any of the basic insights of our article in any qualitative manner. In addition, we let $p > c_1 + c_2$ such that it would be profitable to assemble the product.

We assume that the assembler faces a one-time uncertain demand at some future time T and needs to decide the order quantity Q_i for component i at time $\tau_i = T - l_i$, $i = 1, 2$, with $T > l_1 > l_2$. At time τ_1, the assembler orders the long lead-time component 1 based on an initial best guess of customer demand. We denote this distribution by $F(x)$, which has a mean of μ and standard deviation of σ. Before ordering the short lead-time component 2 at time τ_2, the assembler observes the latest market signal a and updates his/her demand forecast. We denote the updated forecast distribution by $F(x|a)$.

To simplify potentially cumbersome notation, we develop an alternate demand model for our analysis by making some assumptions regarding $F(x|a)$. First, we assume that $F(x|a)$ has a mean of $\mu + a$ and standard deviation of $\hat{\sigma}$, such that the market signal a represents an adjustment to the initial mean μ of the original demand forecast. Also, we assume that a and $\hat{\sigma}$ are independent. This is reasonable since the signal a measures the change in market conditions during the time period $[\tau_1, \tau_2]$, and $\hat{\sigma}$ should only depend on the remaining amount of time from τ_2 until the start of the sales season T. Also, it is reasonable to assume that $\hat{\sigma} \leq \sigma$, as the forecast accuracy should improve the closer we get to the end of the sales season. Under these assumptions, we can express the final demand at time T as

$$D = \mu + A_1 + A_2, \tag{1}$$

where $G_i(.)$ represents the distribution functions of A_i, $i = 1, 2$, with $F(x - \mu - a|a) = G_2(x)$ and $F(x - \mu) = G_1 * G_2(x)$, where $G_1 * G_2(.)$ denotes the convolution of G_1 and G_2. Then, $G_i(x)$ has a mean of zero and standard deviation of σ_i, $i = 1, 2$, with $\sigma_1 = \sqrt{\sigma^2 - \hat{\sigma}^2}$ and $\sigma_2 = \hat{\sigma}$.

In our analysis we can use the demand model given in Equation (1) and work with distributions $G_1(.)$ and $G_2(.)$ instead of the original demand forecast distributions of $F(.)$ and $F(.|a)$. Using Equation (1), we can interpret μ as the initial mean demand forecast at time τ_1 and A_1 and A_2 as some (random) demand adjustments during time periods $[\tau_1, \tau_2]$ and $[\tau_2, T]$, respectively. We shall use $g_i(.)$ to denote the density functions of the random adjustment A_i, $i = 1, 2$.

The sequence of events is as follows: (i) the assembler first forms his/her initial demand forecast μ at time τ_1 and orders Q_1 units of the long lead-time component at time τ_1; (ii) the assembler observes A_1 at time τ_2 and orders Q_2 units of the short lead-time component 2; and (iii) final product demand D is realized at time T and the final product is assembled. The optimization problem for the assembler is to determine the optimal order quantity for components 1 and 2 at ordering epochs τ_1 and τ_2, respectively, to maximize his/her expected profit, taking into consideration the realization of demand adjustment A_1.

3.1. Optimal procurement strategies under general demand adjustments

We formulate the above assembler's optimization problem as a two-stage dynamic program and characterize the optimal order quantities Q_1 and Q_2. We follow the same approach as in Miyaoka and Hausman (2008) and Thomas *et al.* (2009) for the centralized system to derive the optimal order quantities. Specifically, let $\pi_1(Q_1)$ be the maximum expected profit of the assembler when ordering Q_1 units of component 1 at time τ_1 and $\pi_2(Q_2|Q_1; a_1)$ be the expected profit of the assembler when ordering Q_2 units of component 2 at time τ_2 given that the assembler has ordered Q_1 units of component 1 at time τ_1 and observes the realized first demand adjustment value of a_1. Then,

$$\pi_2(Q_2|Q_1; a_1) = \int_{-\infty}^{\infty} p \min(Q_1, Q_2, \mu + a_1 + a_2) g_2(a_2) da_2 - c_2 Q_2, \tag{2}$$

and the standard dynamic programming recursion can be written as

$$\pi_1(Q_1) = \int_{-\infty}^{\infty} \pi_2^*(Q_1; a_1) g_1(a_1) da_1 - c_1 Q_1, \tag{3}$$

with

$$\pi_2^*(Q_1; a_1) = \max_{Q_2} \pi_2(Q_2|Q_1; a_1).$$

We use standard backward induction to solve the above two-stage dynamic program analytically and derive the optimal order quantity Q_1 of component 1 at time τ_1.

We first determine the optimal order quantity Q_2 of component 2 at time τ_2 given the ordering decision Q_1 at time τ_1 and the realized adjustment value a_1 at time τ_2 by solving the optimization problem $\max_{Q_2} \pi_2(Q_2|Q_1; a_1)$, where $\pi_2(Q_2|Q_1; a_1)$ is given in Equation (2). For unconstrained Q_1, this optimization problem reduces to the classic newsvendor model, and the optimal order quantity is given by the well-known fractile solution; that is,

$$q_2^*(a_1) = \mu + a_1 + G_2^{-1}\left(\frac{p - c_2}{p}\right). \tag{4}$$

Furthermore, it is clear from Equation (2) that the assembler should never order more than Q_1^* units of component 2, since exactly one unit of each component is required for each unit of the final product. It is well known from the classic newsvendor model that the expected profit function $\pi_2(Q_2|.)$ is concave in Q_2; thus it follows immediately that

$$Q_2^*(Q_1; a_1) = \min[Q_1, q_2^*(a_1)]. \tag{5}$$

In other words, the optimal order quantity of component 2 at time τ_2 depends critically on the realized adjustment value a_1. Specifically, if a_1 is below the threshold of

$$\zeta = Q_1 - \mu - G_2^{-1}\left(\frac{p - c_2}{p}\right), \tag{6}$$

then the optimal order quantity of component 2 is below Q_1. Otherwise, the optimal order quantity of component 2 is simply equal to Q_1.

We next use the result from the second-stage problem to derive the optimal order quantity of component 1 at time τ_1. Using recursion (3), this first-stage problem can be formulated as

$$\begin{aligned} \max_{Q_1} \pi_1(Q_1) &= \int_{-\infty}^{\infty} \pi_2^*(Q_1; a_1) g_1(a_1)da_1 - c_1 Q_1 \\ &= \int_{-\infty}^{\infty} \pi_2(Q_2^*(Q_1; a_1)|Q_1; a_1) g_1(a_1)da_1 - c_1 Q_1. \end{aligned} \tag{7}$$

where $Q_2^*(Q_1; a_1)$ is given in Equation (5). We can establish the following result.

Proposition 1. *$\pi_1(Q_1)$ in concave in Q_1, and the optimal order quantity Q_1^* must satisfy the following first-order condition:*

$$\begin{aligned} \frac{\partial \pi_1(Q_1^*)}{\partial Q_1} = \int_{Q_1^* - \mu - G_2^{-1}\left(\frac{p-c_2}{p}\right)}^{\infty} &\{(p - c_2) - pG_2(Q_1^* - \mu - a_1)\} \\ &\times g_1(a_1)da_1 - c_1 = 0. \end{aligned} \tag{8}$$

Proposition 1 provides the condition that can be used to characterize the optimal ordering quantity of the long lead-time component for any general demand adjustment distribution. For the remainder of this article, let π^* denote the corresponding optimal expected profit; i.e., $\pi^* = \pi_1(Q_1^*)$.

3.2. *Impact of demand forecast updates*

We can compare the optimal order quantities and the expected profit of the assembler under the two cases with and without demand forecast updates. Without demand forecast updates, the optimal order quantities of components 1 and 2 are the same and are given by the classic newsvendor model as

$$\tilde{Q}_1 = \tilde{Q}_2 = \mu + G_{1+2}^{-1}\left(\frac{p - c_1 - c_2}{p}\right), \tag{9}$$

where G_{1+2} denotes the distribution function of $(a_1 + a_2)$. Let $\tilde{\pi}$ denote the corresponding optimal expected profit with no forecast updates. We can establish the following results.

Proposition 2.

(i) $Q_1^* \geq \tilde{Q}_1$; *and*
(ii) $\pi^* \geq \tilde{\pi}$.

Proposition 2(i) shows that the assembler should order a higher quantity of the long lead-time component for the case with forecast updates compared with the case with no updates. We can explain this result as follows. As the order quantity of the long lead-time component imposes an upper limit on the maximum number of units that can be assembled, the assembler needs to provide an additional cushion in the event of a subsequent positive demand adjustment A_1 when he/she orders the long lead-time component. In addition, demand forecast updates can help protect the assembler from the risk associated with a negative adjustment A_1 by allowing him/her to reduce his/her order quantity for the short lead-time component. As such, the assembler's second order quantity is not "locked in" as is the case with no forecast updates, thus lowering the overall overstocking risk. These two factors allow the assembler to be more aggressive when ordering the long lead-time component. Proposition 2(ii) shows the intuitive result that the assembler cannot be worse off by using demand forecast updates.

4. Forecast updates under uniform demand adjustments

To quantify the value of demand forecast updates, we derive some analytical results for the case where both demand adjustments, A_1 and A_2, are assumed to be uniformly distributed. In particular, we assume that A_1 and A_2 are independent and uniformly distributed on supports $[-\overline{a}_1, \overline{a}_1]$ and $[-\overline{a}_2, \overline{a}_2]$, respectively. In addition, we set the initial forecast μ to be larger than $(\overline{a}_1 + \overline{a}_2)$ to avoid a possibly negative demand. To simplify notation, let $c = c_1 + c_2$, such that c represents the combined unit component costs.

Proposition 3. *Under demand forecast updates, we have the following results:*

(i) For $p \geq 2c_1 + c_2$, the optimal order quantity of the long lead-time component is given by

$$Q_1^* = \begin{cases} \mu + \overline{a}_2 \frac{(p-2c)}{p} & \text{if } \frac{\overline{a}_1}{\overline{a}_2} \leq \frac{2c_1}{p} \\ \mu + (\overline{a}_1 + \overline{a}_2) - \overline{a}_2 \frac{2c_2}{p} & \\ \quad - \sqrt{8\overline{a}_1\overline{a}_2 \frac{c_1}{p}} & \text{if } \frac{2c_1}{p} < \frac{\overline{a}_1}{\overline{a}_2} \leq \frac{(p-c_2)^2}{2pc_1} \\ \mu + \overline{a}_1\left(1 - \frac{2c_1}{p-c_2}\right) - \overline{a}_2 \frac{c_2}{p} & \text{if } \frac{(p-c_2)^2}{2pc_1} < \frac{\overline{a}_1}{\overline{a}_2} \end{cases} \tag{10}$$

the expected optimal order quantity of the short lead-time component is given by

$$E(Q_2^*) = \begin{cases} \mu + \overline{a}_2 \frac{(p-2c)}{p} & \text{if } \frac{\overline{a}_1}{\overline{a}_2} \leq \frac{2c_1}{p} \\ \mu + \overline{a}_2 \frac{(p-2c)}{p} & \text{if } \frac{2c_1}{p} < \frac{\overline{a}_1}{\overline{a}_2} \leq \frac{(p-c_2)^2}{2pc_1} \\ \mu - \overline{a}_1 \frac{c_1^2}{(p-c_2)^2} & \\ \quad - \frac{\overline{a}_2{}^2}{\overline{a}_1} \frac{(p-c_2)^2}{4p^2} & \\ \quad + \overline{a}_2 \frac{(p-c_1-2c_2)}{p} & \text{if } \frac{(p-c_2)^2}{2pc_1} < \frac{\overline{a}_1}{\overline{a}_2} \end{cases} \tag{11}$$

and the optimal expected profit is given by

$$\pi^* = \begin{cases} \mu(p-c) - \bar{a}_2 \frac{(p-c)c}{p} \\ \quad -\frac{\bar{a}_1{}^2}{\bar{a}_2}\frac{p}{12} & \text{if } \frac{\bar{a}_1}{\bar{a}_2} \le \frac{2c_1}{p} \\ \mu(p-c) - (\bar{a}_1+\bar{a}_2)c_1 \\ \quad -\bar{a}_2 \frac{(p-2c_1-c_2)c_2}{p} \\ \quad +\frac{4c_1}{3}\sqrt{2\bar{a}_1\bar{a}_2\frac{c_1}{p}} & \text{if } \frac{2c_1}{p} < \frac{\bar{a}_1}{\bar{a}_2} \le \frac{(p-c_2)^2}{2pc_1} \\ \mu(p-c) - \bar{a}_1\frac{(p-c)c_1}{p-c_2} \\ \quad -\frac{\bar{a}_2{}^2}{\bar{a}_1}\frac{(p-c_2)^3}{12p^2} - \bar{a}_2\frac{(p-c)c_2}{p} & \text{if } \frac{(p-c_2)^2}{2pc_1} < \frac{\bar{a}_1}{\bar{a}_2} \end{cases} \tag{12}$$

(ii) *For $p < 2c_1 + c_2$, the optimal order quantity of the long lead-time component is given by*

$$Q_1^* = \begin{cases} \mu + \bar{a}_2\frac{(p-2c)}{p} & \text{if } \frac{\bar{a}_1}{\bar{a}_2} \le \frac{2(p-c)}{p} \\ \mu - (\bar{a}_1+\bar{a}_2) \\ \quad +\sqrt{8\bar{a}_1\bar{a}_2\frac{(p-c)}{p}} & \text{if } \frac{2(p-c)}{p} < \frac{\bar{a}_1}{\bar{a}_2} \le \frac{(p-c_2)^2}{2(p-c)p} \\ \mu + \bar{a}_1\left(1-\frac{2c_1}{p-c_2}\right) \\ \quad -\bar{a}_2\frac{c_2}{p} & \text{if } \frac{(p-c_2)^2}{2(p-c)p} < \frac{\bar{a}_1}{\bar{a}_2} \end{cases} \tag{13}$$

the expected optimal order quantity of the short lead-time component is given by

$$E(Q_2^*) = \begin{cases} \mu + \bar{a}_2\frac{(p-2c)}{p} & \text{if } \frac{\bar{a}_1}{\bar{a}_2} \le \frac{2(p-c)}{p} \\ \mu - (\bar{a}_1+\bar{a}_2) \\ \quad +\sqrt{8\bar{a}_1\bar{a}_2\frac{(p-c)}{p}} & \text{if } \frac{2(p-c)}{p} < \frac{\bar{a}_1}{\bar{a}_2} \le \frac{(p-c_2)^2}{2(p-c)p} \\ \mu - \bar{a}_1\frac{c_1^2}{(p-c_2)^2} \\ \quad -\frac{\bar{a}_2{}^2}{\bar{a}_1}\frac{(p-c_2)^2}{4p^2} \\ \quad +\bar{a}_2\frac{(p-c_1-2c_2)}{p} & \text{if } \frac{(p-c_2)^2}{2(p-c)p} < \frac{\bar{a}_1}{\bar{a}_2} \end{cases} \tag{14}$$

and the optimal expected profit is given by

$$\pi^* = \begin{cases} \mu(p-c) \\ \quad -\bar{a}_2\frac{(p-c)c}{p} - \frac{\bar{a}_1{}^2}{\bar{a}_2}\frac{p}{12} & \text{if } \frac{\bar{a}_1}{\bar{a}_2} \le \frac{2(p-c)}{p} \\ \mu(p-c) \\ \quad -(\bar{a}_1+\bar{a}_2)(p-c) \\ \quad +\frac{4(p-c)}{3}\sqrt{2\bar{a}_1\bar{a}_2\frac{(p-c)}{p}} & \text{if } \frac{2(p-c)}{p} < \frac{\bar{a}_1}{\bar{a}_2} \\ & \quad \le \frac{(p-c_2)^2}{2(p-c)p} \\ \mu(p-c) \\ \quad -\bar{a}_1\frac{(p-c)c_1}{p-c_2} - \frac{\bar{a}_2{}^2}{\bar{a}_1}\frac{(p-c_2)^3}{12p^2} \\ \quad -\bar{a}_2\frac{(p-c)c_2}{p} & \text{if } \frac{(p-c_2)^2}{2(p-c)p} < \frac{\bar{a}_1}{\bar{a}_2} \end{cases} \tag{15}$$

We need to compare the results given in Proposition 3 with the corresponding results under no forecast updates in order to quantify the value of demand forecast updates. Under no demand forecast updates, the assembler will simply order the same amount of both components, and we use the notation $\tilde{Q}_i$ and $\tilde{\pi}$ to denote the optimal order quantities and the optimal expected profit in this case. The optimal results, $\tilde{Q}_i$ and $\tilde{\pi}$, can be easily derived from the classic newsvendor problem, and are given in Proposition 4.

Proposition 4. *With no demand forecast updates, we have the following results:*

(i) *For $p \ge 2(c_1+c_2)$, the optimal order quantities of both components are given by*

$$\tilde{Q}_1 = \tilde{Q}_2 = \begin{cases} \mu + \bar{a}_2\frac{(p-2c)}{p} & \text{if } \frac{\bar{a}_1}{\bar{a}_2} \le \frac{2c}{p} \\ \mu + (\bar{a}_1+\bar{a}_2) \\ \quad -\sqrt{8\bar{a}_1\bar{a}_2\frac{c}{p}} & \text{if } \frac{2c}{p} < \frac{\bar{a}_1}{\bar{a}_2} \le \frac{p}{2c} \\ \mu + \bar{a}_1\frac{(p-2c)}{p} & \text{if } \frac{p}{2c} < \frac{\bar{a}_1}{\bar{a}_2} \end{cases} \tag{16}$$

and the optimal expected profit is given by

$$\tilde{\pi} = \begin{cases} \mu(p-c) - \bar{a}_2\frac{(p-c)c}{p} - \frac{\bar{a}_1{}^2}{\bar{a}_2}\frac{p}{12} & \text{if } \frac{\bar{a}_1}{\bar{a}_2} \le \frac{2c}{p} \\ \mu(p-c) - (\bar{a}_1+\bar{a}_2)c \\ \quad +\frac{4c}{3}\sqrt{2\bar{a}_1\bar{a}_2\frac{c}{p}} & \text{if } \frac{2c}{p} < \frac{\bar{a}_1}{\bar{a}_2} \le \frac{p}{2c} \\ \mu(p-c) - \bar{a}_1\frac{(p-c)c}{p} - \frac{\bar{a}_2{}^2}{\bar{a}_1}\frac{p}{12} & \text{if } \frac{p}{2c} < \frac{\bar{a}_1}{\bar{a}_2} \end{cases} \tag{17}$$

(ii) *For $p < 2\,(c_1+c_2)$, the optimal order quantities of both components are given by*

$$\tilde{Q}_1 = \tilde{Q}_2 = \begin{cases} \mu + \bar{a}_2\frac{(p-2c)}{p} & \text{if } \frac{\bar{a}_1}{\bar{a}_2} \le \frac{2(p-c)}{p} \\ \mu - (\bar{a}_1+\bar{a}_2) \\ \quad +\sqrt{8\bar{a}_1\bar{a}_2\frac{(p-c)}{p}} & \text{if } \frac{2(p-c)}{p} < \frac{\bar{a}_1}{\bar{a}_2} \le \frac{p}{2(p-c)} \\ \mu + \bar{a}_1\frac{(p-2c)}{p} & \text{if } \frac{p}{2(p-c)} < \frac{\bar{a}_1}{\bar{a}_2} \end{cases} \tag{18}$$

and the optimal expected profit is given by

$$\tilde{\pi} = \begin{cases} \mu(p-c) - \bar{a}_2\frac{(p-c)c}{p} - \frac{\bar{a}_1{}^2}{\bar{a}_2}\frac{p}{12} & \text{if } \frac{\bar{a}_1}{\bar{a}_2} \le \frac{2(p-c)}{p} \\ \mu(p-c) - (\bar{a}_1+\bar{a}_2)(p-c) \\ \quad +\frac{4(p-c)}{3}\sqrt{2\bar{a}_1\bar{a}_2\frac{(p-c)}{p}} & \text{if } \frac{2(p-c)}{p} < \frac{\bar{a}_1}{\bar{a}_2} \\ & \quad \le \frac{p}{2(p-c)} \\ \mu(p-c) - \bar{a}_1\frac{(p-c)c}{p} - \frac{\bar{a}_2{}^2}{\bar{a}_1}\frac{p}{12} & \text{if } \frac{p}{2(p-c)} < \frac{\bar{a}_1}{\bar{a}_2} \end{cases} \tag{19}$$

We next derive a measure to quantify the value of forecast updates for the assembler. Instead of using the simple measure $(\pi^* - \tilde{\pi})$, which has the undesirable property of being sensitive to the forecast mean, we develop a more appropriate metric for measuring the benefit of forecast updates. First, note that the expected profit of the assembler is simply equal to $\mu(p-c)$ when there is no demand uncertainty, i.e., $A_1 = A_2 = 0$. Due to demand uncertainty, the assembler incurs an extra cost that result from a mismatch between the demand and supply of the two components. With no forecast updates, the expected profit of the assembler is equal to $\tilde{\pi}$ and thus the quantity $\tilde{m} = \mu(p - c) - \tilde{\pi}$ represents the *expected mismatch cost* due to demand uncertainty with no forecast updates. With forecast updates, the expected profit of the assembler is now equal to π^*, and the quantity $m^* = \mu(p-c) - \pi^*$ then represents the *expected mismatch cost* due to demand uncertainty with forecast updates. Therefore, $(\tilde{m} - m^*) = (\pi^* - \tilde{\pi})$ represents the reduction in expected mismatch cost for the assembler or, equivalently, the increase in expected profit due to forecast updates. Furthermore, the ratio $(\tilde{m} - m^*)/\tilde{m}$ represents the fraction of the total expected mismatch cost due to demand uncertainty that can be reduced using forecast updates.

We shall use the above ratio, called mismatch cost reduction (*MCR*), as our key metric in quantifying the value of forecast updates for the assembler. Note that $MCR = (\tilde{m} - m^*)/\tilde{m}$ is independent of the initial forecast μ and has a value of between zero and one. In particular, $MCR = 0$ implies that forecast updates do not reduce any expected mismatch cost and thus provides no value to the assembler. On the other hand, $MCR = 1$ implies that forecast updates can eliminate all expected mismatch cost due to demand uncertainty, which represents the maximum possible benefit for the assembler.

Using Propositions 3 and 4, we can derive the following results that show how the value of mismatch cost reduction changes with respect to different model parameters.

Proposition 5.

(i) *For $p \geq 2c_1 + c_2$, $MCR = 0$ if $\frac{\bar{a}_1}{\bar{a}_2} \leq \frac{2c_1}{p}$.*

(ii) *For $p < 2c_1 + c_2$, $MCR = 0$ if $\frac{\bar{a}_1}{\bar{a}_2} \leq \frac{(p-c_2)^2}{2p(p-c)}$. Furthermore, $\frac{(p-c_2)^2}{2p(p-c)}$ decreases to $\frac{2c_1}{p}$ as p increases to $2c_1 + c_2$.*

(iii) *As $\frac{\bar{a}_1}{\bar{a}_2} \to \infty$, $MCR = \frac{c_2}{c}\left(\frac{p-c}{p-c_2}\right)$.*

Proposition 5(i) and (ii) shows that the ratio $\frac{\bar{a}_1}{\bar{a}_2}$ must exceed a certain threshold for forecast updates to be valuable, and these thresholds are given in the high profit margin case (i) and the low profit margin case (ii). As the standard deviation of the uniform demand adjustment A_i is proportional to $\bar{a}_i$, Proposition 5 implies that the amount of demand uncertainty that can be resolved before ordering the long lead-time component relative to the remaining amount of uncertainty must exceed some threshold for demand forecast updates to be valuable. Furthermore, these thresholds are decreasing in p. In other words, as the product price (or profit margin) increases, the threshold for forecast updates to be valuable is reduced. This implies that forecast updates are more likely to be valuable for high-margin products. It is clear that the thresholds are decreasing in c_1, with $(c_1 + c_2)$ being held constant. This implies that forecast updates are more likely to be valuable when the unit cost of the long lead-time component comprises a smaller portion of the total component cost.

Proposition 5(iii) further provides the maximum possible value of mismatch cost reduction due to forecast updates when most of the demand uncertainty can be resolved before ordering the short lead-time component. This maximum possible value depends on the relative cost of the short lead-time component compared with the total component costs. For the trivial case $c_2 = 0$, we have $MCR = 0$, i.e., there is no value in forecast updating if the short lead-time component has zero cost, as the update is used to adjust the procurement quantity of this lead-time component based on the latest market signal. Suppose instead that $c_2 > 0$. As $(p-c)/(p-c_2)$ increases to 1 as $p \to \infty$ or $c_1 \to 0$, Proposition 5(iii) shows that forecast updates are especially valuable for high-margin products or when the unit cost of the long lead-time component comprises a smaller portion of the total component cost.

5. Managerial insights

In this section, we use both our analytical results and extensive numerical experiments to provide important managerial insights for understanding how demand forecast updates can impact the assembler and the two component suppliers. Applying the results in Propositions 3 and 4, we conduct a comprehensive set of numerical experiments under uniform demand adjustments to generate these insights. After that, we conduct another set of numerical experiments for the case where the demand adjustments are normally distributed to confirm the robustness of our findings.

5.1. Impact on the assembler

We conduct a comprehensive set of numerical experiments to understand how different model parameters may affect the mismatch cost reduction (*MCR*) with forecast updates. In our numerical experiments, we set $\mu = 100$, $c_1 + c_2 = 100$, and $\sigma_1^2 + \sigma_2^2 = 100$, where $\sigma_i = \frac{\bar{a}_i}{\sqrt{3}}$ is the standard deviation of the uniform demand adjustment A_i. Therefore, c_1 represents the proportion of unit cost of component 1 relative to the combined component cost $(c_1 + c_2)$ and σ_1^2 represents the proportion of combined variance $(\sigma_1^2 + \sigma_2^2)$ that can be resolved with forecast updates. We vary the values of c_1 from 0 to 100, σ_1^2 from 0 to 100, and p from 100 to 1000. To further support the qualitative insights observed under uniform demand adjustments, we also conduct the same set of numerical experiments using normal demand adjustments with the same values of μ, σ_1, and σ_2.

We first illustrate how each individual model parameter (p, σ_1^2, and c_1) affects the mismatch cost reduction. In particular, Figs. 1 to 3 illustrate the individual impact of the three model parameters on MCR under both uniform and normal demand adjustments. For these figures, we set $c_1 = c_2 = 50$, $\sigma_1^2 = \sigma_2^2 = 50$, and $p = 200$ as our base case, and then vary the value of each parameter from their base value.

Based on our numerical experiments, we can summarize our key observations as follows:

Observation 1.

(i) *MCR* increases as p increases, but the marginal increase in *MCR* decreases as p increases.

(ii) *MCR* becomes positive when σ_1^2 exceeds some threshold, and *MCR* increases as σ_1^2 increases.

(iii) *MCR* decreases to zero as the ratio $\frac{c_1}{c_1+c_2}$ increases to one.

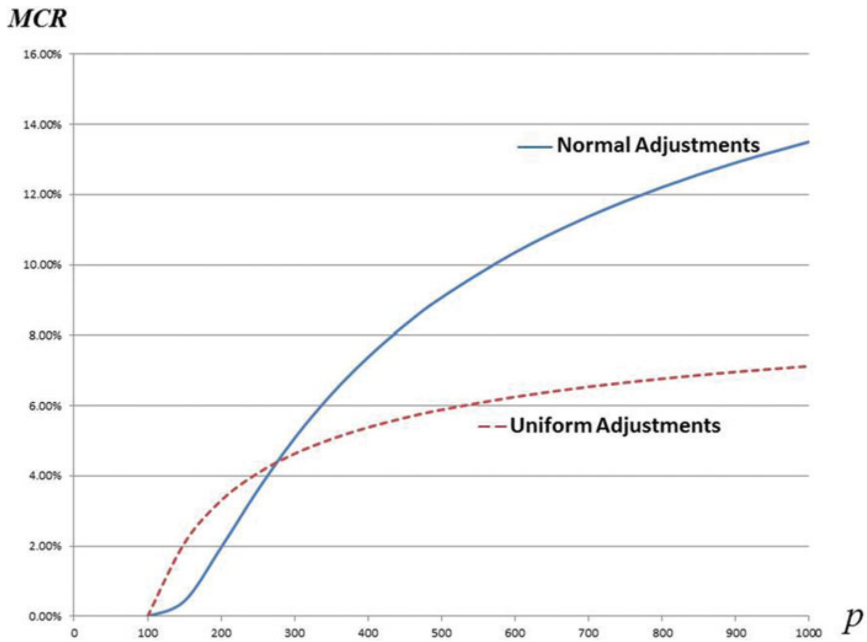

Figure Impact of p on MCR.

Observation 1(i) demonstrates the important result that the MCR is higher for products with a higher price (and thus higher profit margin). This result is rather interesting. First, it is intuitive that the total mismatched cost with no forecast updates, $\tilde{m}$, increases as p increases. Thus, Observation 1(i) implies that $(\tilde{m} - m^*)$ must also increase as p increases. This suggests that the increase in expected mismatch cost with forecast updates is less than that with no forecast updates as p increases, which might not be surprising. However, it is surprising that the ratio $MCR = \frac{(\tilde{m}-m^*)}{\tilde{m}}$ also increases as p increases, which suggests that forecast updates can reduce a larger portion of the total expected mismatch cost even though this total expected mismatched cost is higher as p increases. Therefore, Observation 1(i) provides the important insight that forecast updates are especially valuable for products with high profit margins.

Observation 1(ii) shows that forecast updates are especially valuable when most of the demand uncertainty can be resolved by forecast updates. We can explain this result as follows. A higher value of σ_1^2 helps to reduce the expected mismatch cost under forecast updates as the assembler can resolve a higher portion of the demand uncertainty before ordering the short lead-time component, whereas a higher value of σ_1^2 would have

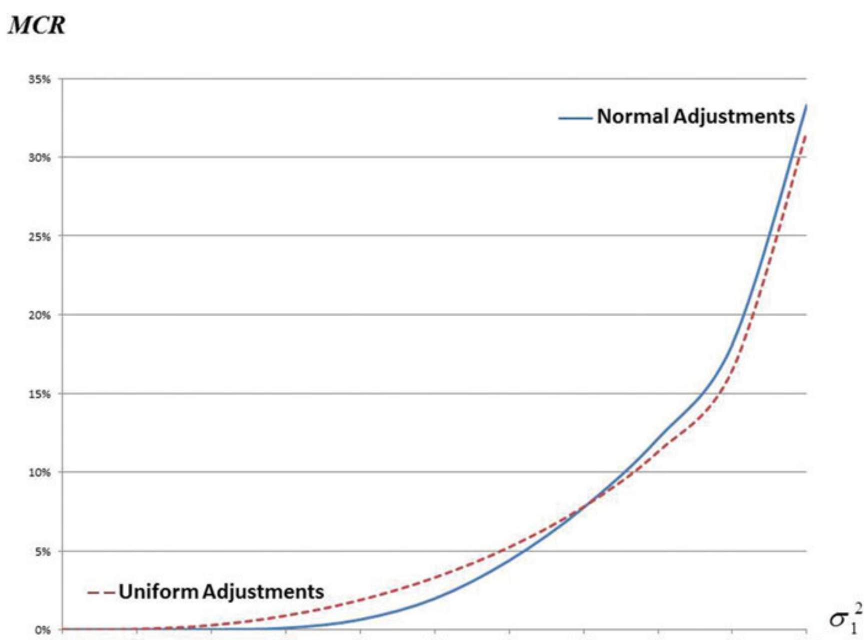

Figure Impact of σ_1^2 on MCR.

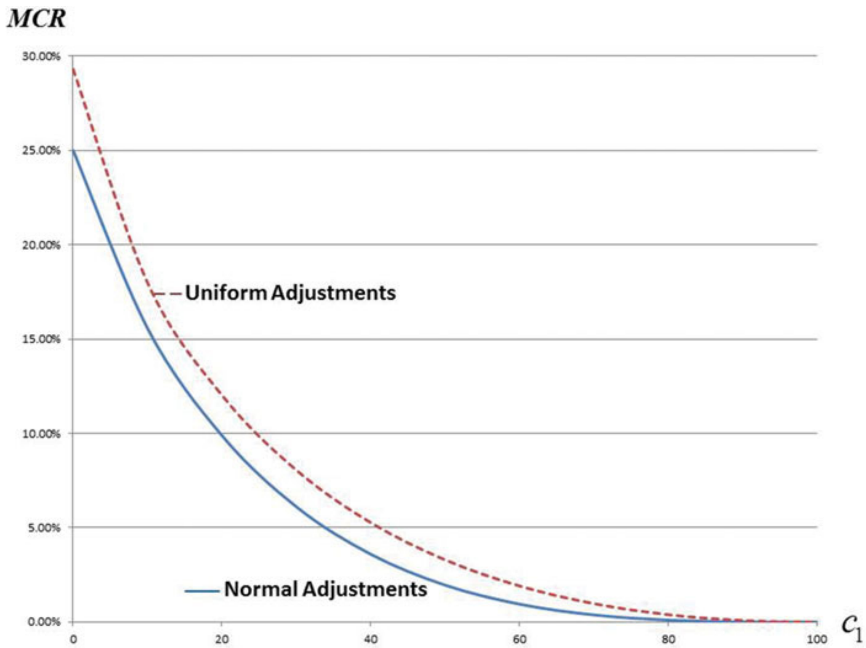

Figure Impact of c_1 on MCR.

little impact on the expected mismatch cost with no forecast updates when the total variance $\sigma_1^2 + \sigma_2^2$ remains constant. Consequently, *MCR* increases as σ_1^2 increases with a fixed value of $\sigma_1^2 + \sigma_2^2$.

Finally, note that the expected mismatch cost with no forecast updates remains the same as c_1 changes when the value of $c = c_1 + c_2$ is fixed. Therefore, Observation 1(iii) simply implies that as the ratio of $\frac{c_1}{c_1+c_2}$ increases, the expected mismatch cost under forecast updates increases, which is intuitive. Thus, Observation 1(iii) supports the intuitive result that as the unit cost of the long lead-time component is high relative to that of the short lead-time component, there is not much value in forecast updates, as the potential savings in adjusting the order quantity of the short lead-time component based on forecast updates become negligible.

We can conclude from our numerical results that forecast updates would be most valuable when (i) products have high profit margins; (ii) a high proportion of demand uncertainty can be resolved by forecast updates; and (iii) the unit cost of the short lead-time component is higher than that of the long lead-time component. Furthermore, Proposition 5 provides strong analytical support to this conclusion under uniform demand adjustments.

We next illustrate the joint impact of the three model parameters (c_1, σ_1, and p) on MCR in Figs. 4 to 6 for the uniform adjustments case. Figure 4 shows the joint impact of c_1 and σ_1^2 on *MCR* at $p = 200$ and $p = 400$. Notice that the value of *MCR* goes up sharply in both curves at small values of c_1 and high values of σ_1^2, which indicates that the joint impact of c_1 and σ_1 is significantly higher than the sum of the two individual impacts, especially at small values of c_1 and high values of σ_1^2 for which the value of forecast updates is high, as demonstrated in Observation 1(i) and (iii). This suggests a strong interaction effect between these two parameters.

Figure 5 illustrates the joint impact of p and σ_1^2 on *MCR* at $c_1 = 30$ and $c_1 = 70$. At different levels of p, the value of *MCR* appears to increase rather steadily as σ_2^2 increases. Figure 6 illustrates the joint impact of p and c_1 on *MCR* at $\sigma_1^2 = 30$ and $\sigma_1^2 = 70$. Similarly, the value of *MCR* appears to decrease rather steadily as c_1 increases at different levels of p. Thus, these two

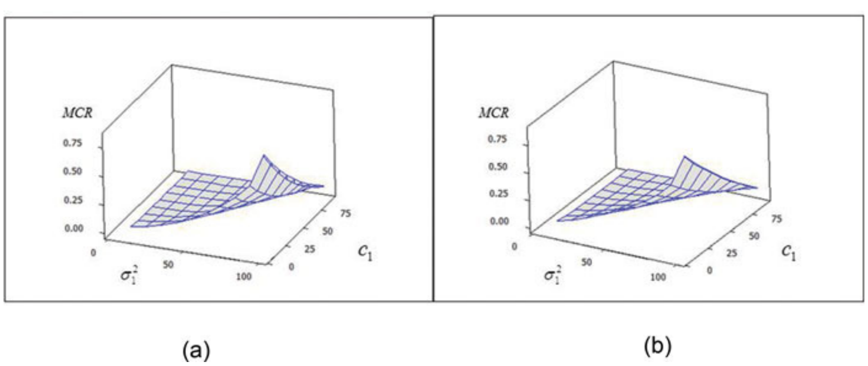

Figure Joint impact of c_1 and σ_1^2: (a) $p = 200$ and (b) $p = 400$.

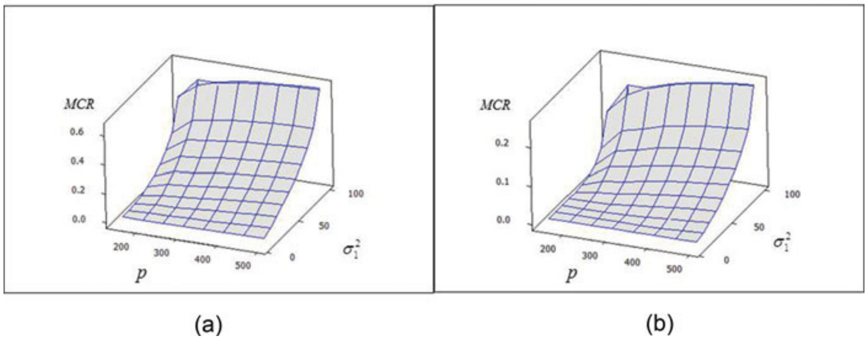

Figure Joint impact of p and σ_1^2: (a) $c_1 = 30$ and (b) $c_1 = 70$.

figures do not suggest a strong interaction effect between p and σ_1^2 or between p and c_1.

Overall, our numerical results show that there is a strong interaction effect between c_1 and σ_1, which suggests that the value of forecast updates is especially pronounced when the unit cost of the short lead-time component is high relative to that of the long lead-time component, and at the same time, a high degree of demand uncertainty can be resolved with forecast updates. We now summarize this key observation.

Observation 2. The joint impact of c_1 and σ_1 on *MCR* is significantly higher than the sum of the two individual effects, especially at small values of c_1 and high values of σ_1^2.

Furthermore, it is important to note that the value of forecast updates for the assembler can be substantial at low values of c_1 and high values of σ_1^2. The rationale behind the large MCR value is due to the fact that long lead-time components with a low cost allows the assembler to gamble on a higher order quantity of this component, which sets the upper ordering ceiling for the short lead-time component. In addition, the fact that most of the demand uncertainty can be resolved by forecast updates allows the assembler to accurately adjust the order quantity of the short lead-time component. These two factors combined allow the assembler to capture a potentially higher demand if demand adjustment a_1 is positive. On the other hand, if demand adjustment a_1 is negative, the assembler can accordingly reduce the order quantity of the short lead-time component, and the adverse effect is less serious as these short lead-time components are the expensive ones.

5.2. Impact on supplier of the long lead-time component

The quantity $(Q_1^* - \tilde{Q}_1)$ represents the change in order quantity for the long lead-time components due to forecast updates. Proposition 2 shows that the order quantity for the long lead-time components is always higher under forecast updates, which implies that this component supplier will always benefit due to forecast updates. Our numerical results further show that this extra order amount $(Q_1^* - \tilde{Q}_1)$ increases as the value of c_1 decreases, which implies that forecast updates are especially valuable to this supplier with a low unit cost. However, our numerical results show that this extra order amount is not necessarily monotone for other model parameters.

5.3. Impact on supplier of the short lead-time component

The quantity $[E(Q_2^*) - \tilde{Q}_2]$ represents the change in expected order quantity for the short lead-time components due to forecast updates, which can be used to measure the benefit to this supplier. Our numerical results show that the change in expected order quantity for the short lead-time components due to forecast updates can be either positive or negative, depending on the specific model parameters. For example, Fig. 7 provides a numerical illustration for the case where $[E(Q_2^*) - \tilde{Q}_2]$ is negative. In this figure, $c_1 = c_2 = 50$, $\sigma_1^2 + \sigma_2^2 = 100$ with the value of σ_1^2 varying from 0 to 100, and the two curves correspond to the two cases where $p = 250$ or $p = 500$. On the other hand, Fig. 8 provides a numerical illustration for the case where $[E(Q_2^*) - \tilde{Q}_2]$ is positive. In this figure, $p = 150$, $\sigma_1^2 + \sigma_2^2 = 100$, with a value of σ_1^2 from zero to 100, and the two curves correspond to the two cases where $(c_1, c_2) = (20, 80)$ or $(c_1, c_2) =$

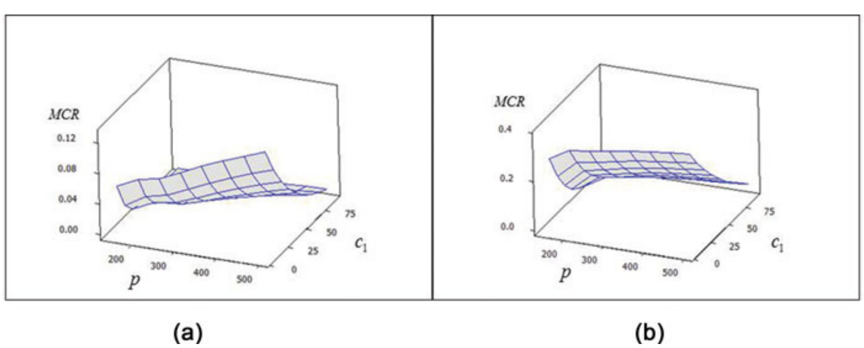

Figure Joint impact of p and c_1: (a) $\sigma_1^2 = 30$ and (b) $\sigma_1^2 = 70$.

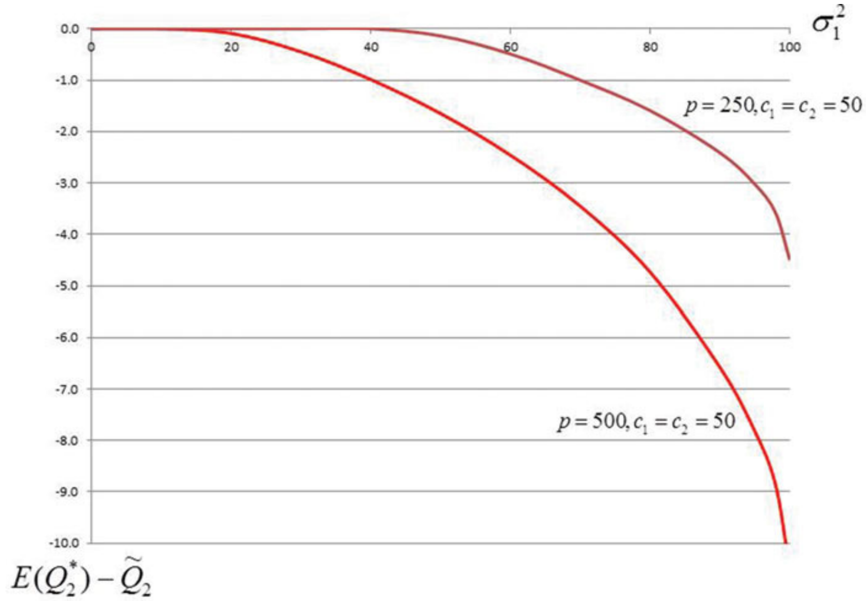

Figure A numerical case where $E(Q_2^*)$ is lower than $\tilde{Q}_2$.

(50, 50). Overall, we provide the following observations based on our numerical experiments.

Observation 3.

(i) $[E(Q_2^*) - \tilde{Q}_2] < 0$ when (i) a large amount of the demand uncertainty can be resolved using forecast updates; and (ii) profit margin is high.

(ii) $[E(Q_2^*) - \tilde{Q}_2] > 0$ when (i) a large amount of the demand uncertainty can be resolved using forecast updates, and (ii) the profit margin is low; and (iii) the unit component cost c_1 is low relative to c_2.

Observation 3 suggests that forecast updates can benefit or hurt the expected profit of the short lead-time supplier, depending on the specific model parameters. First, forecast updates would result in a lower expected order quantity of the short lead-time components when a large amount of the demand uncertainty can be resolved using forecast updates and the product margin is high. We can explain this result in Observation 3(i) as follows. With no forecast updates, the assembler will order the same amount of components 1 and 2, resulting in a high value of $\tilde{Q}_2$ due to a high profit margin using the newsvendor analysis. With forecast updates, the assembler can observe the first demand adjustment a_1 before ordering component 2, which would most likely result in a lower expected order quantity $E(Q_2^*)$ than $\tilde{Q}_2$.

On the other hand, the assembler would choose a low quantity of Q_1 and Q_2 for a product with a low profit margin with no forecast updates. With forecast updates, the assembler would

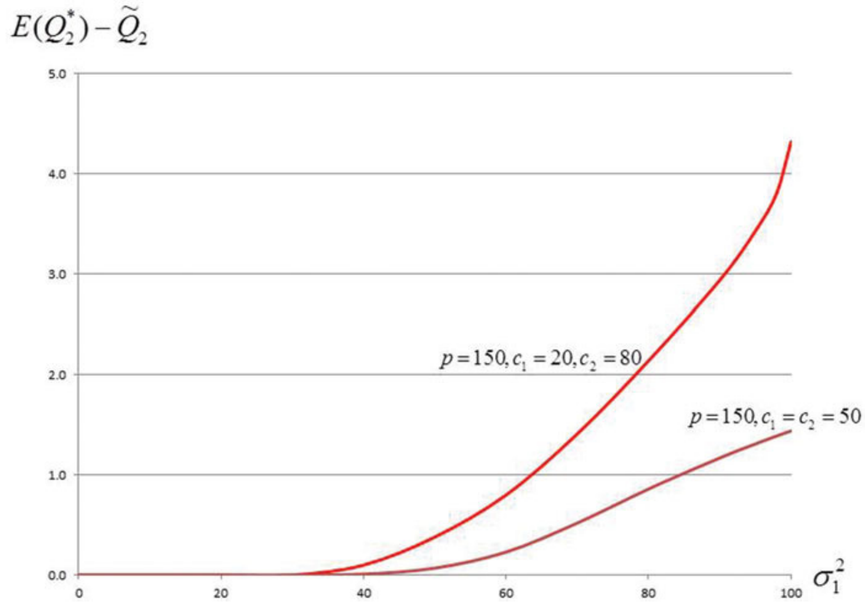

Figure A numerical case where $E(Q_2^*)$ is higher than $\tilde{Q}_2$.

observe the first demand adjustment before ordering component 2, which would most likely result in a higher expected order quantity $E(Q_2^*)$ than $\tilde{Q}_2$. However, since Q_1^* sets an upper bound for Q_2^*, $E(Q_2^*) > \tilde{Q}_2$ only if Q_1^* is large, which requires that the unit component cost c_1 is low relative to c_2. This explains Observation 3(ii).

We can use the results in Propositions 3 and 4 to provide analytical support for the insights provided in Observation 2. First, observe that when $\bar{a}_1/\bar{a}_2$ is small, it follows from Propositions 3 and 4 that

$$\tilde{Q}_2 = E(Q_2^*) = \mu + \bar{a}_2 \frac{(p-2c)}{p}.$$

This implies that when a large amount of demand uncertainty remains unsolved with forecast updates, it would not affect the expected order quantity of the short lead-time components. In other words, forecast updates have no significant impact on this supplier.

When $\bar{a}_1/\bar{a}_2$ is sufficiently large, it follows from Equations (11) and (14) that with forecast updates

$$E(Q_2^*) = \mu - \frac{\bar{a}_1 c_1^2}{\left(p-c_2\right)^2} - \frac{\bar{a}_2{}^2\left(p-c_2\right)^2}{4\bar{a}_1 p^2} + \frac{\bar{a}_2\left(p-c_1-2c_2\right)}{p},$$

which implies that

$$E(Q_2^*) = \mu - \bar{a}_1 \frac{c_1^2}{\left(p-c_2\right)^2} \tag{20}$$

as $\bar{a}_2 \to 0$. Also, when $\bar{a}_1/\bar{a}_2$ is sufficiently large, it follows from Equations (16) and (18) that with no forecast updates,

$$\tilde{Q}_2 = \mu + \bar{a}_1 \frac{(p-2c)}{p}. \tag{21}$$

Suppose that $p \geq 2c$. We have $\tilde{Q}_2 \geq \mu > E(Q_2^*)$ by comparing Equation (20) and Equation (21). This shows that when the profit margin is high, the expected order amount of the short lead-time components with forecast updates is always lower than that with no forecast updates. In other words, forecast updates would hurt this supplier when the product has a high profit margin.

Suppose that $p < 2c$. In this case, it follows from Equations (20) and (21) that $\tilde{Q}_2 < E(Q_2^*)$ only when

$$\frac{(2c-p)}{p} > \frac{c_1^2}{\left(p-c_2\right)^2}$$

or equivalently,

$$f(c_2) = \frac{(2c-p)}{p} - \left(\frac{c-c_2}{p-c_2}\right)^2 > 0.$$

For fixed c, it is straightforward to show that $f(0) = -(p-c)^2/p < 0$, $f(c) = (2c-p)/p > 0$, and $f(c_2)$ is increasing in c_2. This implies that $f(c_2) > 0$ when the value of c_2 is high relative to c_1. This shows that $\tilde{Q}_2 < E(Q_2^*)$ only when $p < 2c$ and c_2 is high relative to c_1. In other words, forecast updates would only be beneficial to this supplier when the product margin is low and the unit cost of the short lead-time component is high relative to that of the long lead-time component.

6. Conclusions

We analyze the value of demand forecast updates for reducing the expected mismatch cost due to demand uncertainty in an assembly system that requires two complementary components with different procurement lead times. Using a uniform demand adjustment model, we derive closed-form analytical expressions for the optimal order quantities of the two components and the expected profit of the assembler. We use these analytical results together with comprehensive numerical experiments to generate managerial insights on how forecast updates can provide substantial value to the assembler and the component suppliers. These results help to further our understanding of the specific operating conditions under which the value of forecast updates would be most beneficial to the assembler and the component suppliers.

Specifically, we show that demand forecast updates would be most valuable to the assembler when (i) the unit cost of the short lead-time component is higher than that of the long lead-time component; (ii) a large proportion of demand uncertainty can be resolved by forecast updates; and (iii) the product has a high profit margin. Furthermore, our results suggest that the joint impact of the relative component costs and the relative amount of demand uncertainty that can be resolved by forecast updates can be significantly higher than the sum of these two individual effects. Therefore, demand forecast updates are especially valuable to the assembler when the unit cost of the short lead-time component is high relative to that of the long lead-time component and the amount of demand uncertainty that can be resolved by forecast updates is large relative to the remaining demand uncertainty. Under this type of operating environment, forecast updates can greatly reduce the mismatch cost due to demand uncertainty for the assembler. As market research is generally expensive, our insights can assist managers in determining when the benefits of demand forecast updating outweigh the high costs of conducting market research. Finally, our results also illustrate how forecast updates can affect the two component suppliers. Although the long lead-time component supplier will always benefit from a higher order quantity from the assembler due to forecast updates, we have found the surprising result that forecast updates can either benefit or hurt the short lead-time supplier, depending on the model parameters.

An important future research topic is to generalize our results to multiple component suppliers with different lead times. Although it is straightforward to extend our modeling framework to more than two component suppliers, the problem becomes analytically intractable and computationally challenging. Nevertheless, additional research results in this direction could prove to be illuminating.

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Appendix

Proof of Proposition 1. From Equations (2) and (5), we can split the integral in $\pi_1(Q_1)$ given in Equation (7) based on the critical threshold ζ defined in Equation (6) as

$$\begin{aligned}
\pi_1(Q_1) &= \int_{\zeta}^{\infty}\left\{\int_{-\infty}^{\infty} p\min(Q_1,\mu+a_1+a_2)g_2(a_2)da_2 - c_2Q_1\right\}\\
&\quad\times g_1(a_1)da_1\\
&\quad+\int_{-\infty}^{\zeta}\left\{\int_{-\infty}^{\infty} p\min(q_2^*(a_1),\mu+a_1+a_2)g_2(a_2)da_2\right.\\
&\quad\left.-\,c_2q_2^*(a_1)\right\}g_1(a_1)da_1 - c_1Q_1\\
&= \int_{\zeta}^{\infty}\left\{(p-c_2)Q_1 - p\int_{-\infty}^{Q_1-\mu-a_1} G_2(a_2)da_2\right\}g_1(a_1)da_1\\
&\quad+\int_{-\infty}^{\zeta}\left\{(p-c_2)\left[\mu+a_1+G_2^{-1}\left(\frac{p-c_2}{p}\right)\right]\right.\\
&\quad\left.-p\int_{-\infty}^{G_2^{-1}(\frac{p-c_2}{p})} G_2(a_2)da_2\right\}g_1(a_1)da_1 - c_1Q_1.
\end{aligned}$$

Applying the Leibnitz integration rule, we can differentiate $\pi_1(Q_1)$ given above to obtain:

$$\begin{aligned}
\frac{\partial\pi_1(Q_1)}{\partial Q_1} &= \int_{\zeta}^{\infty}\{(p-c_2)-pG_2(Q_1-\mu-a_1)\}g_1(a_1)da_1\\
&\quad-\frac{\partial\zeta}{\partial Q_1}\left\{(p-c_2)Q_1 - p\int_{-\infty}^{Q_1-\mu-\zeta} G_2(a_2)da_2)\right\}g_1(\zeta)\\
&\quad+\frac{\partial\zeta}{\partial Q_1}\left\{(p-c_2)\left[\mu+\zeta+G_2^{-1}\left(\frac{p-c_2}{p}\right)\right]\right.\\
&\quad\left.-p\int_{-\infty}^{G_2^{-1}(\frac{p-c_2}{p})} G_2(a_2)da_2\right\}g_1(\zeta) - c_1\\
&= \int_{\zeta}^{\infty}\{(p-c_2)-pG_2(Q_1-\mu-a_1)\}g_1(a_1)da_1 - c_1,
\end{aligned} \tag{A1}$$

where the last equality follows from the definition that $\zeta = Q_1 - \mu - G_2^{-1}((p-c_2)/p)$. Applying the Leibnitz integration rule again to Equation (A1), we can obtain:

$$\begin{aligned}
\frac{\delta^2\pi_1(Q_1)}{\delta Q_1^2} &= \int_{\zeta}^{\infty}\{-pg_2(Q_1-\mu-a_1)\}g_1(a_1)da_1\\
&\quad-\{(p-c_2)-pG_2(Q_1-\mu-\zeta)\}g_1(\zeta)\\
&= -\int_{\zeta}^{\infty} pg_2(Q_1-\mu-a_1)g_1(a_1)da_1 < 0.
\end{aligned}$$

Therefore, we prove that $\pi_1(Q_1)$ is concave in Q_1, which implies that the optimal order quantity Q_1^* must satisfy the first-order condition $\partial\pi_1(Q_1)/\partial Q_1 = 0$. ∎

Proof of Proposition 2.

(i) For $Q_1 = 0$, we have $G_2(Q_1 - \mu - a_1) = 0$ and $\zeta = -\mu - G_2^{-1}((p - c_2)/p)$. It follows from Equation (A1) that

$$\frac{\partial\pi_1(Q_1)}{\partial Q_1} = \int_{-\mu-G_2^{-1}(\frac{p-c_2}{p})}^{\infty} (p - c_2)g_1(a_1)da_1 - c_1$$
$$= \int_{-\bar{a}_1}^{\bar{a}_1} (p - c_2)g_1(a_1)da_1 - c_1 = p - c_2 - c_1 > 0,$$

where the second equality follows from the assumption that $\mu > \bar{a}_1 + \bar{a}_2$ to avoid negative demand. For $Q_1 \to \infty$, we have $G_2(Q_1 - \mu - a_1) = 1$ and $\zeta \to \infty$, which implies that

$$\frac{\partial\pi_1(Q_1)}{\partial Q_1} = \int_{\zeta}^{\infty} -c_2 g_1(a_1)da_1 - c_1 \to -c_1 < 0.$$

Also, we show that $\partial\pi_1(Q_1)/\partial Q_1$ is decreasing in Q_1 in Proposition 2. Since $\partial\pi_1(Q_1^*)/\partial Q_1 = 0$, it suffices to show that $\partial\pi_1(\tilde{Q}_1)/\partial Q_1 \geq 0$, that is,

$$\frac{\partial\pi_1(\tilde{Q}_1)}{\partial Q_1} = p\int_{\tilde{Q}_1-\mu-G_2^{-1}\left(\frac{p-c_2}{p}\right)}^{\infty} \left\{\int_{\tilde{Q}_1-\mu-a_1}^{\infty} g_2(a_2)da_2\right\} \times g_1(a_1)da_1 - c_1 - c_2 \times \bar{G}_1\left(\tilde{Q}_1 - \mu - G_2^{-1}\left(\frac{p-c_2}{p}\right)\right). \quad (A2)$$

Using Equation (9), we have

$$p\int_{-\infty}^{\infty}\left\{\int_{\tilde{Q}_1-\mu-a_1}^{\infty} g_2(a_2)da_2\right\} g_1(a_1)da_1 = c_1 + c_2. \quad (A3)$$

Combining Equations (A2) and (A3), we obtain:

$$\frac{\partial\pi_1(\tilde{Q}_1)}{\partial Q_1} = c_2G_1\left(\tilde{Q}_1 - \mu - G_2^{-1}\left(\frac{p-c_2}{p}\right)\right) - p\int_{-\infty}^{\tilde{Q}_1-\mu-G_2^{-1}\left(\frac{p-c_2}{p}\right)}\left\{\int_{\tilde{Q}_1-\mu-a_1}^{\infty} g_2(a_2)da_2\right\} \times g_1(a_1)da_1$$
$$\geq c_2G_1\left(\tilde{Q}_1 - \mu - G_2^{-1}\left(\frac{p-c_2}{p}\right)\right) - p\int_{-\infty}^{\tilde{Q}_1-\mu-G_2^{-1}\left(\frac{p-c_2}{p}\right)}\left\{\int_{G_2^{-1}\left(\frac{p-c_2}{p}\right)}^{\infty} g_2(a_2)da_2\right\} \times g_1(a_1)da_1$$
$$= c_2G_1\left(\tilde{Q}_1 - \mu - G_2^{-1}\left(\frac{p-c_2}{p}\right)\right) - p\int_{-\infty}^{\tilde{Q}_1-\mu-G_2^{-1}\left(\frac{p-c_2}{p}\right)}\left\{1 - G_2\left(G_2^{-1}(\frac{p-c_2}{p})\right)\right\} \times g_1(a_1)da_1$$
$$= c_2G_1\left(\tilde{Q}_1 - \mu - G_2^{-1}\left(\frac{p-c_2}{p}\right)\right) - c_2G_1\left(\tilde{Q}_1 - \mu - G_2^{-1}\left(\frac{p-c_2}{p}\right)\right) = 0.$$

This proves that $Q_1^* \geq \tilde{Q}_1$.

(ii) This result is clear as the optimal value could only decrease when we simply add a constraint on the decision variables to the same optimization problem. ∎

Proof of Proposition 3. As $\bar{a}_i$ is uniformly distributed on support $[-\bar{a}_i, \bar{a}_i]$ such that $g_i(x) = 1/2\bar{a}_i$ and $G_i(x) = (\bar{a}_i + x)/2\bar{a}_i$ for $x \in [-\bar{a}_i, \bar{a}_i]$, the optimal order quantity for the short lead-time component for the problem with unconstrained Q_1 given by Equation (4) is equal to

$$q_2^*(a_1) = \mu + a_1 + \left[-\bar{a}_2 + 2\bar{a}_2\left(\frac{p-c_2}{p}\right)\right] = \mu + a_1 + \bar{a}_2\frac{(p-2c_2)}{p}. \quad (A4)$$

Also, the maximum expected profit function $\pi_1(Q_1)$ given by Equation (3) can be expressed as

$$\pi_1(Q_1) = \frac{1}{4\bar{a}_1\bar{a}_2}\int_{-\bar{a}_2}^{\bar{a}_2}\int_{-\bar{a}_1}^{\bar{a}_1}\Big[p\min(\mu + a_1 + a_2, Q_1, q_2^*(a_1)) - c_2\min(Q_1, q_2^*(a_1))\Big]da_1da_2 - c_1Q_1. \quad (A5)$$

Proposition 1 shows that $\pi_1(Q_1)$ is concave in Q_1. Therefore, any feasible value of Q_1 satisfying the first-order condition $\pi_1'(Q_1) = 0$ is an optimal solution that maximizes the expected profit. We next analyze the function $\pi_1(Q_1)$ for different ranges of Q_1 based on the underlying values of the model parameters and derive a number of sufficient conditions for any feasible Q_1 that satisfies the first-order condition.

For $Q_1 \geq \mu + \bar{a}_1 + \bar{a}_2((p - 2c_2)/p)$, it follows from Equation (A4) that $Q_1 \geq q_2^*(a_1)$ for all $a_1 \in [-\bar{a}_1, \bar{a}_1]$, and so

$$\pi_1(Q_1) = \frac{1}{4\bar{a}_1\bar{a}_2}\int_{-\bar{a}_2}^{\bar{a}_2}\int_{-\bar{a}_1}^{\bar{a}_1}\Big[p\min(\mu + a_1 + a_2, q_2^*(a_1)) - c_2q_2^*(a_1)\Big]da_1da_2 - c_1Q_1.$$

Then, $\pi_1'(Q_1) = -c_1$, which implies that $\pi_1(Q_1)$ is strictly decreasing in Q_1 for $Q_1 \geq \mu + \bar{a}_1 + \bar{a}_2\frac{(p-2c_2)}{p}$. Thus, $Q_1^* \leq \mu + \bar{a}_1 + \bar{a}_2\frac{(p-2c_2)}{p}$. As the final demand D is given by Equation (1), we have $D \in [\mu - \bar{a}_1 - \bar{a}_2, \mu + \bar{a}_1 + \bar{a}_2]$. Thus, it suffices to analyze the profit function $\pi_1(Q_1)$ for $\mu - \bar{a}_1 - \bar{a}_2 \leq Q_1 \leq \mu + \bar{a}_1 + \bar{a}_2\frac{(p-2c_2)}{p}$. We divide our analysis into two possible cases:

(a) $-\bar{a}_1 + \bar{a}_2\frac{(p-2c_2)}{p} \leq \bar{a}_1 - \bar{a}_2$;

(b) $-\bar{a}_1 + \bar{a}_2\frac{(p-2c_2)}{p} > \bar{a}_1 - \bar{a}_2$.

These two cases correspond to the conditions: $\frac{\bar{a}_1}{\bar{a}_2} \geq \frac{p-c_2}{p}$ or $\frac{\bar{a}_1}{\bar{a}_2} < \frac{p-c_2}{p}$, respectively.

Case (a): $-\bar{a}_1 + \bar{a}_2\frac{(p-2c_2)}{p} \leq \bar{a}_1 - \bar{a}_2$.

We perform our analysis for three possible ranges for Q_1:

(a1) $\mu - \bar{a}_1 - \bar{a}_2 \leq Q_1 \leq \mu - \bar{a}_1 + \bar{a}_2\frac{(p-2c_2)}{p}$.

In this case, $Q_1 \le q_2^*(a_1)$ for all $a_1 \in [-\bar{a}_1, \bar{a}_1]$. Therefore, $Q_2^* = Q_1^*$, the profit function (A5) can be expressed as

$$\pi_1(Q_1) = \frac{1}{2\bar{a}_1}\int_{Q_1-\mu+\bar{a}_2}^{\bar{a}_1}\left\{p(\mu+a_1) - c_2Q_1 - \frac{p}{2\bar{a}_2}\times\int_{-\bar{a}_2}^{\bar{a}_2}\left[a_2-(Q_1-\mu-a_1)\right]da_2\right\}da_1 + \frac{1}{2\bar{a}_1}\int_{-\bar{a}_1}^{Q_1-\mu+\bar{a}_2}\left\{p(\mu+a_1)-c_2Q_1-\frac{p}{2\bar{a}_2}\times\int_{Q_1-\mu-a_1}^{\bar{a}_2}\left[a_2-(Q_1-\mu-a_1)\right]da_2\right\}da_1 - c_1Q_1,$$

and

$$\pi_1'(Q_1) = -\frac{1}{8\bar{a}_1\bar{a}_2}\left\{8\bar{a}_1\bar{a}_2c_1 + 8\bar{a}_1\bar{a}_2c_2 + p\left[\bar{a}_1{}^2 - 2\bar{a}_1(3\bar{a}_2 + \mu - Q_1) + (\bar{a}_2-\mu+Q_1)^2\right]\right\}.$$

It is straightforward to show that:

$$z_1 = \mu - (\bar{a}_1+\bar{a}_2) + \sqrt{8\bar{a}_1\bar{a}_2\frac{(p-c)}{p}} \tag{A6}$$

is the only possible feasible root in the range $[\mu-\bar{a}_1-\bar{a}_2, \mu-\bar{a}_1+\bar{a}_2((p-2c_2)/p)]$ that can satisfy the first-order condition $\pi_1'(Q_1)=0$. Clearly, $\mu-\bar{a}_1-\bar{a}_2 \le z_1$. Therefore, z_1 is feasible if

$$z_1 = \mu - (\bar{a}_1+\bar{a}_2) + \sqrt{8\bar{a}_1\bar{a}_2\frac{(p-c)}{p}} \le \mu - \bar{a}_1 + \bar{a}_2\frac{(p-2c_2)}{p}$$

or equivalently,

$$\frac{\bar{a}_1}{\bar{a}_2} \le \frac{(p-c_2)^2}{2p(p-c)}.$$

We thus have the following sufficient optimality condition:

Condition 1: $Q_1^* = Q_2^* = z_1$ if

$$\frac{p-c_2}{p} \le \frac{\bar{a}_1}{\bar{a}_2} \le \frac{(p-c_2)^2}{2p(p-c)}.$$

(a2) $\mu-\bar{a}_1+\bar{a}_2\frac{(p-2c_2)}{p} \le Q_1 \le \mu+\bar{a}_1-\bar{a}_2$.

In this case, it follows from Equation (A4) that $Q_1 \ge q_2^*(a_1)$ if and only if $\zeta \equiv Q_1-\mu-\bar{a}_2((p-2c_2)/p) \ge a_1$. Therefore, the profit function (A5) can be expressed as

$$\pi_1(Q_1) = \frac{1}{2\bar{a}_1}\int_{\zeta}^{\bar{a}_1}\left\{p(\mu+a_1)-c_2Q_1 - \frac{p}{2\bar{a}_2}\int_{-\bar{a}_2}^{\bar{a}_2}\left[a_2-(Q_1-\mu-a_1)\right]^+da_2\right\}da_1 + \frac{1}{2\bar{a}_1}\int_{-\bar{a}_1}^{\zeta}\left\{p(\mu+a_1)-c_2\left[\bar{a}_2\frac{(p-2c_2)}{p}+\mu+a_1\right] - \frac{p}{2\bar{a}_2}\int_{\bar{a}_2\frac{(p-2c_2)}{p}}^{\bar{a}_2}\left[a_2-\bar{a}_2\frac{(p-2c_2)}{p}\right]da_2\right\}da_1 - c_1Q_1,$$

and

$$\pi_1'(Q_1) = \frac{1}{2\bar{a}_1p}\left\{-2\bar{a}_1c_1p + (c_2-p)\left[\bar{a}_2c_2 - p(\mu+\bar{a}_1-Q_1)\right]\right\}.$$

Then,

$$z_2 = \mu + \bar{a}_1\left(1-\frac{2c_1}{p-c_2}\right) - \bar{a}_2\frac{c_2}{p}, \tag{A7}$$

satisfies the first-order condition, and z_2 is feasible if

$$\mu-\bar{a}_1+\bar{a}_2\frac{(p-2c_2)}{p} \le z_2 \le \mu+\bar{a}_1-\bar{a}_2.$$

We can easily show that $\mu-\bar{a}_1+\bar{a}_2((p-2c_2)/p) \le z_2$ is equivalent to

$$\frac{(p-c_2)^2}{2p(p-c)} \le \frac{\bar{a}_1}{\bar{a}_2},$$

and $z_2 \le \mu+\bar{a}_1-\bar{a}_2$ is equivalent to

$$\frac{(p-c_2)^2}{2pc_1} \le \frac{\bar{a}_1}{\bar{a}_2}.$$

Also, in this case,

$$E[Q_2^*] = \frac{1}{2\bar{a}_1}\int_{-\bar{a}_1}^{Q_1^*-\mu-\bar{a}_2\frac{(p-2c_2)}{p}} q_2^*(a_1)da_1 + \frac{1}{2\bar{a}_1}\int_{Q_1^*-\mu-\bar{a}_2\frac{(p-2c_2)}{p}}^{\bar{a}_1} Q_1^*da_1$$
$$= \frac{1}{2\bar{a}_1}\int_{-\bar{a}_1}^{z_2-\mu-\bar{a}_2\frac{(p-2c_2)}{p}}[\mu+a_1+\bar{a}_2\frac{(p-2c_2)}{p}]da_1 + \frac{1}{2\bar{a}_1}\int_{z_2-\mu-\bar{a}_2\frac{(p-2c_2)}{p}}^{\bar{a}_1} z_2da_1$$
$$= \mu - \bar{a}_1\frac{c_1^2}{(p-c_2)^2} - \frac{\bar{a}_2{}^2}{\bar{a}_1}\frac{(p-c_2)^2}{4p^2} + \bar{a}_2\frac{(p-c_1-2c_2)}{p}.$$

We have the following sufficient optimality condition:

Condition 2: $Q_1^* = z_2$ and

$$E(Q_2^*) = \mu - \bar{a}_1\frac{c_1^2}{(p-c_2)^2} - \frac{\bar{a}_2{}^2}{\bar{a}_1}\frac{(p-c_2)^2}{4p^2} + \bar{a}_2\frac{(p-c_1-2c_2)}{p}$$

if

$$\frac{p-c_2}{p} \le \frac{\bar{a}_1}{\bar{a}_2},\ \frac{(p-c_2)^2}{2p(p-c)} \le \frac{\bar{a}_1}{\bar{a}_2} \text{ and } \frac{(p-c_2)^2}{2pc_1} \le \frac{\bar{a}_1}{\bar{a}_2}.$$

(a3) $\mu+\bar{a}_1-\bar{a}_2 \le Q_1 \le \mu+\bar{a}_1+\bar{a}_2\frac{(p-2c_2)}{p}$.

Similar to case (a2), we have $Q_1 \ge q_2^*(a_1)$ if and only if $\zeta \equiv Q_1-\mu-\bar{a}_2((p-2c_2)/p) \ge a_1$. Therefore,

$$\pi_1(Q_1) = \frac{1}{2\bar{a}_1}\int_{\zeta}^{\bar{a}_1}\left\{p(\mu+a_1)-c_2Q_1 - \frac{p}{2\bar{a}_2}\int_{Q_1-\mu-a_1}^{\bar{a}_2}\left[a_2-(Q_1-\mu-a_1)\right]da_2\right\}da_1 + \frac{1}{2\bar{a}_1}\int_{-\bar{a}_1}^{\zeta}\left\{p(\mu+a_1)-c_2\left[\bar{a}_2\frac{(p-2c_2)}{p}+\mu+a_1\right]\right.$$

$$-\frac{p}{2\bar{a}_2}\int_{\bar{a}_2\frac{(p-2c_2)}{p}}^{\bar{a}_2}\left[a_2-\bar{a}_2\frac{(p-2c_2)}{p}\right]da_2\Big\}da_1$$
$$-c_1Q_1,$$

and

$$\pi_1'(Q_1)=\frac{1}{8\bar{a}_1\bar{a}_2p}\Big\{4\bar{a}_2{}^2c_2^2+p[-8\bar{a}_1\bar{a}_2c_1+p(\mu+\bar{a}_1+\bar{a}_2-Q_1)^2]$$
$$-4\bar{a}_2c_2p(\mu+\bar{a}_1+\bar{a}_2-Q_1)\Big\}.$$

It is straightforward to show that

$$z_3=\mu+(\bar{a}_1+\bar{a}_2)-\bar{a}_2\frac{2c_2}{p}-\sqrt{8\bar{a}_1\bar{a}_2\frac{c_1}{p}} \quad \text{(A8)}$$

is the only possible feasible root in the range $[\mu+\bar{a}_1-\bar{a}_2, \mu+\bar{a}_1+\bar{a}_2((p-2c_2)/p)]$ that can satisfy the first-order condition. Clearly, $z_3\le\mu+\bar{a}_1+\bar{a}_2((p-2c_2)/p)$. Therefore, z_3 is feasible if $\mu+\bar{a}_1-\bar{a}_2\le z_3$, or equivalently,

$$\frac{\bar{a}_1}{\bar{a}_2}\le\frac{(p-c_2)^2}{2pc_1}.$$

Also,

$$E[Q_2^*]=\frac{1}{2\bar{a}_1}\int_{-\bar{a}_1}^{Q_1^*-\mu-\bar{a}_2\frac{(p-2c_2)}{p}}q_2^*(a_1)da_1$$
$$+\frac{1}{2\bar{a}_1}\int_{Q_1^*-\mu-\bar{a}_2\frac{(p-2c_2)}{p}}^{\bar{a}_1}Q_1^*da_1$$
$$=\frac{1}{2\bar{a}_1}\int_{-\bar{a}_1}^{z_3-\mu-\bar{a}_2\frac{(p-2c_2)}{p}}\left[\mu+a_1+\bar{a}_2\frac{(p-2c_2)}{p}\right]da_1$$
$$+\frac{1}{2\bar{a}_1}\int_{z_3-\mu-\bar{a}_2\frac{(p-2c_2)}{p}}^{\bar{a}_1}z_3da_1$$
$$=\mu+\bar{a}_2\frac{(p-2c)}{p}.$$

Thus, we have the following sufficient optimality condition:

Condition 3: $Q_1^*=z_3$ and $E(Q_2^*)=\mu+\bar{a}_2(\frac{p-2c}{p})$ if

$$\frac{p-c_2}{p}\le\frac{\bar{a}_1}{\bar{a}_2}\le\frac{(p-c_2)^2}{2pc_1}.$$

Case (b): $-\bar{a}_1+\bar{a}_2\frac{(p-2c_2)}{p}>\bar{a}_1-\bar{a}_2$.

Again, we consider three possible ranges for Q_1:

(b1) $\mu-\bar{a}_1-\bar{a}_2\le Q_1\le\mu+\bar{a}_1-\bar{a}_2$.

In this case, the profit function (A5) is the same as that in case (a1). Therefore, z_1 as defined in Equation (A6) also satisfies the first-order condition in this case and is feasible if $\mu-\bar{a}_1-\bar{a}_2\le z_1\le\mu+\bar{a}_1-\bar{a}_2$. Clearly, $\mu-\bar{a}_1-\bar{a}_2\le z_1$. Thus, z_1 is feasible if $z_1\le\mu+\bar{a}_1-\bar{a}_2$, or equivalently, $\frac{2(p-c)}{p}\le\frac{\bar{a}_1}{\bar{a}_2}$. We have the following sufficient optimality condition:

Condition 4: $Q_1^*=Q_2^*=z_1$ if

$$\frac{2(p-c)}{p}\le\frac{\bar{a}_1}{\bar{a}_2}\le\frac{(p-c_2)}{p}.$$

(b2) $\mu+\bar{a}_1-\bar{a}_2\le Q_1\le\mu-\bar{a}_1+\bar{a}_2((p-2c_2)/p)$.

In this case, $Q_1\le q_2^*(a_1)$ for all $a_1\in[-\bar{a}_1,\bar{a}_1]$. Therefore, $Q_2^*=Q_1^*$. Also,

$$\pi_1(Q_1)=\frac{1}{2\bar{a}_1}\int_{-\bar{a}_1}^{\bar{a}_1}\Big\{p(\mu+a_1)-c_2Q_1$$
$$-\frac{p}{2\bar{a}_2}\int_{Q_1-\mu-a_1}^{\bar{a}_2}[a_2-(Q_1-\mu-a_1)]da_2\Big\}da_1-c_1Q_1,$$

and

$$\pi_1'(Q_1)=\frac{1}{2\bar{a}_2}\Big\{-2\bar{a}_2c_1-2\bar{a}_2c_2+p(\mu+\bar{a}_2-Q_1)\Big\}.$$

Then,

$$z_4=\mu+\bar{a}_2\frac{(p-2c)}{p} \quad \text{(A9)}$$

satisfies the above first-order condition, and z_4 is feasible if $\mu+\bar{a}_1-\bar{a}_2\le z_4\le\mu-\bar{a}_1+\bar{a}_2\frac{(p-2c_2)}{p}$. We can easily show that $\mu+\bar{a}_1-\bar{a}_2\le z_4$ is equivalent to

$$\frac{\bar{a}_1}{\bar{a}_2}\le\frac{2(p-c)}{p},$$

and $z_4\le\mu-\bar{a}_1+\bar{a}_2\frac{(p-2c_2)}{p}$ is equivalent to

$$\frac{\bar{a}_1}{\bar{a}_2}\le\frac{2c_1}{p}.$$

We have the following sufficient optimality condition:

Condition 5: $Q_1^*=Q_2^*=z_4$ if

$$\frac{\bar{a}_1}{\bar{a}_2}\le\frac{p-c_2}{p},\ \frac{\bar{a}_1}{\bar{a}_2}\le\frac{2(p-c)}{p}\text{ and }\frac{\bar{a}_1}{\bar{a}_2}\le\frac{2c_1}{p}.$$

(b3) $\mu-\bar{a}_1+\bar{a}_2\frac{(p-2c_2)}{p}\le Q_1\le\mu+\bar{a}_1+\bar{a}_2\frac{(p-2c_2)}{p}$.

In this case, the profit function (A5) is the same as that in case (a3). Therefore, z_3 as defined in (A8) also satisfies the first-order condition in this case, and is feasible if

$$\mu-\bar{a}_1+\bar{a}_2\frac{(p-2c_2)}{p}\le z_3\le\mu+\bar{a}_1+\bar{a}_2\frac{(p-2c_2)}{p}.$$

Clearly,

$$z_3\le\mu+\bar{a}_1+\bar{a}_2\frac{(p-2c_2)}{p}.$$

Thus, z_3 is feasible if

$$\mu-\bar{a}_1+\bar{a}_2\frac{(p-2c_2)}{p}\le z_3$$

or equivalently,

$$\frac{2c_1}{p}\le\frac{\bar{a}_1}{\bar{a}_2}.$$

We have the following sufficient optimality condition:

Condition 6: $Q_1^*=z_3$ and $E(Q_2^*)=\mu+\bar{a}_2(\frac{p-2c}{p})$ if $\frac{2c_1}{p}\le\frac{\bar{a}_1}{\bar{a}_2}\le\frac{p-c_2}{p}$.

To prove part (i), assume that $p\ge 2c_1+c_2$. Suppose that

$$\frac{\bar{a}_1}{\bar{a}_2}\le\frac{2c_1}{p}.$$

The assumption $p \geq 2c_1 + c_2$ implies that

$$\frac{2c_1}{p} \leq \frac{2(p-c)}{p} \text{ and } \frac{2c_1}{p} \leq \frac{p-c_2}{p}.$$

Therefore, Condition 5 holds, and $Q_1^* = z_4$. Suppose that

$$\frac{2c_1}{p} < \frac{\bar{a}_1}{\bar{a}_2} < \frac{(p-c_2)^2}{2pc_1}.$$

If

$$\frac{\bar{a}_1}{\bar{a}_2} \leq \frac{p-c_2}{p},$$

then Condition 6 holds, and $Q_1^* = z_3$. On the other hand, if

$$\frac{\bar{a}_1}{\bar{a}_2} > \frac{p-c_2}{p},$$

then Condition 3 holds, and $Q_1^* = z_3$. In either case, we have $Q_1^* = z_3$. Finally, suppose that

$$\frac{\bar{a}_1}{\bar{a}_2} \geq \frac{(p-c_2)^2}{2pc_1}.$$

The assumption $p \geq 2c_1 + c_2$ implies that

$$\frac{(p-c_2)^2}{2pc_1} \geq \frac{(p-c_2)^2}{2p(p-c)} \text{ and } \frac{(p-c_2)^2}{2pc_1} \geq \frac{(p-c_2)}{p}.$$

Therefore, Condition 2 holds, and $Q_1^* = z_2$.

We can substitute the optimal order quantity into the profit function in each case to obtain the corresponding optimal expected profits. The derivation is straightforward, and so we omit the details here. This proves part (i).

To prove part (ii), now assume that $p < 2c_1 + c_2$. Suppose that

$$\frac{\bar{a}_1}{\bar{a}_2} \leq \frac{2(p-c)}{p}.$$

The assumption $p < 2c_1 + c_2$ implies that

$$\frac{2(p-c)}{p} < \frac{2c_1}{p} \text{ and } \frac{2(p-c)}{p} < \frac{p-c_2}{p}.$$

Therefore, Condition 5 holds, and $Q_1^* = z_4$. Suppose that

$$\frac{2(p-c)}{p} < \frac{\bar{a}_1}{\bar{a}_2} < \frac{(p-c_2)^2}{2p(p-c)}.$$

If

$$\frac{\bar{a}_1}{\bar{a}_2} \leq \frac{p-c_2}{p},$$

then Condition 4 holds, and $Q_1^* = z_1$. On the other hand, if

$$\frac{\bar{a}_1}{\bar{a}_2} > \frac{p-c_2}{p},$$

then Condition 1 holds, and $Q_1^* = z_1$. In either case, we have $Q_1^* = z_1$. Finally, suppose that

$$\frac{\bar{a}_1}{\bar{a}_2} \geq \frac{(p-c_2)^2}{2p(p-c)}.$$

The assumption $p < 2c_1 + c_2$ implies that

$$\frac{(p-c_2)^2}{2p(p-c)} > \frac{p-c_2}{p} \text{ and } \frac{(p-c_2)^2}{2p(p-c)} > \frac{(p-c_2)^2}{2pc_1}.$$

Therefore, Condition 2 holds, and $Q_1^* = z_2$.

Again, we can substitute the optimal order quantity into the profit function in each case to obtain the corresponding optimal expected profits. This completes the proof. ∎

Proof of Proposition 4. With no demand forecast updates, the optimal order quantities of both components are the same and given by (9), corresponding to the classic newsvendor problem. The maximum expected profit function $\pi_1(Q_1)$ can be expressed as

$$\pi_1(Q_1) = \int_{-\bar{a}_1-\bar{a}_2}^{\bar{a}_1+\bar{a}_2} p \min(\mu + x, Q_1) g_{1+2}(x) dx - cQ_1, \tag{A10}$$

where $g_{1+2}(x)$ represents the density function of $x = a_1 + a_2$. It is well known from the classic newsvendor problem that the profit function given in Equation (A10) is concave in Q_1. Therefore, any feasible value of Q_1 satisfying the first-order condition $\pi_1'(Q_1) = 0$ is an optimal solution that maximizes the expected profit. We follow the same approach as in the proof of Proposition 3 to derive a number of sufficient conditions for any feasible Q_1 that satisfies the first-order condition based on the underlying values of the model parameters. We divide our analysis into two possible cases: (a) $\bar{a}_1/\bar{a}_2 \geq 1$ or (b) $\bar{a}_1/\bar{a}_2 \leq 1$.

Case (a): $\bar{a}_1/\bar{a}_2 \geq 1$. In this case, it can be easily shown that the density function $g_{1+2}(x)$ is given by

$$g_{1+2}(x) = \begin{cases} \dfrac{\bar{a}_1 + \bar{a}_2 + x}{4\bar{a}_1\bar{a}_2} & \text{if } x \in [-\bar{a}_1 - \bar{a}_2, -\bar{a}_1 + \bar{a}_2] \\ \dfrac{1}{2\bar{a}_1} & \text{if } x \in [-\bar{a}_1 + \bar{a}_2, \bar{a}_1 - \bar{a}_2] \\ \dfrac{\bar{a}_1 + \bar{a}_2 - x}{4\bar{a}_1\bar{a}_2} & \text{if } x \in [\bar{a}_1 - \bar{a}_2, \bar{a}_1 + \bar{a}_2] \end{cases}.$$

We analyze $\pi_1(Q_1)$ for three possible ranges for Q_1.

(a1) $\mu - \bar{a}_1 - \bar{a}_2 \leq Q_1 \leq \mu - \bar{a}_1 + \bar{a}_2$.

In this range for Q_1, the first-order condition $\pi_1'(Q_1) = 0$ can be expressed as

$$\int_{\mu-\bar{a}_1-\bar{a}_2}^{Q_1} \frac{\bar{a}_1 + \bar{a}_2 + x}{4\bar{a}_1\bar{a}_2} dx = \frac{p-c}{p}.$$

It is then straightforward to show that

$$\tilde{z}_1 = \mu - (\bar{a}_1 + \bar{a}_2) + \sqrt{8\bar{a}_1\bar{a}_2 \frac{(p-c)}{p}} \tag{A11}$$

satisfies the first-order condition $\pi'(\tilde{z}_1) = 0$. Also, $\tilde{z}_1 \leq \mu - \bar{a}_1 + \bar{a}_2$ is equivalent to

$$\frac{\bar{a}_1}{\bar{a}_2} \leq \frac{p}{2(p-c)}.$$

We thus have the following sufficient optimality condition:

Condition 1: $\tilde{Q}_1 = \tilde{z}_1$ if

$$1 \le \frac{\bar{a}_1}{\bar{a}_2} \le \frac{p}{2(p-c)}.$$

(a2) $\mu - \bar{a}_1 + \bar{a}_2 \le Q_1 \le \mu + \bar{a}_1 - \bar{a}_2$.

In this range for Q_1, the first-order condition can be expressed as

$$\int_{\mu-\bar{a}_1-\bar{a}_2}^{\mu-\bar{a}_1+\bar{a}_2} \frac{\bar{a}_1+\bar{a}_2+x}{4\bar{a}_1\bar{a}_2}dx + \int_{\mu-\bar{a}_1+\bar{a}_2}^{Q_1} \frac{1}{2\bar{a}_1}dx = \frac{p-c}{p},$$

and

$$\tilde{z}_2 = \mu + \bar{a}_1\frac{(p-2c)}{p} \tag{A12}$$

satisfies the first-order condition. Also, $\mu - \bar{a}_1 + \bar{a}_2 \le \tilde{z}_2$ is equivalent to

$$\frac{\bar{a}_1}{\bar{a}_2} \ge \frac{p}{2(p-c)},$$

and $\tilde{z}_2 \le \mu + \bar{a}_1 - \bar{a}_2$ is equivalent to

$$\frac{\bar{a}_1}{\bar{a}_2} \ge \frac{p}{2c}.$$

We have the following sufficient optimality condition:

Condition 2: $\tilde{Q}_1 = \tilde{z}_2$ if

$$\frac{\bar{a}_1}{\bar{a}_2} \ge 1,\ \frac{\bar{a}_1}{\bar{a}_2} \ge \frac{p}{2(p-c)},\ \text{and}\ \frac{\bar{a}_1}{\bar{a}_2} \ge \frac{p}{2c}.$$

(a3) $\mu + \bar{a}_1 - \bar{a}_2 \le Q_1 \le \mu + \bar{a}_1 + \bar{a}_2$.

In this range for Q_1, the first-order condition can be expressed as

$$\int_{\mu-\bar{a}_1-\bar{a}_2}^{\mu-\bar{a}_1+\bar{a}_2} \frac{\bar{a}_1+\bar{a}_2+x}{4\bar{a}_1\bar{a}_2}dx + \int_{\mu-\bar{a}_1+\bar{a}_2}^{\mu+\bar{a}_1-\bar{a}_2} \frac{1}{2\bar{a}_1}dx + \int_{\mu+\bar{a}_1-\bar{a}_2}^{Q_1} \frac{\bar{a}_1+\bar{a}_2-x}{4\bar{a}_1\bar{a}_2}dx = \frac{p-c}{p}.$$

It is straightforward to show that:

$$\tilde{z}_3 = \mu + (\bar{a}_1+\bar{a}_2) - \sqrt{8\bar{a}_1\bar{a}_2\frac{c}{p}}, \tag{A13}$$

satisfies the first-order condition. Also, $\mu + \bar{a}_1 - \bar{a}_2 \le \tilde{z}_3$ is equivalent to

$$\frac{\bar{a}_1}{\bar{a}_2} \le \frac{p}{2c}.$$

We have the following sufficient optimality condition:

Condition 3: $\tilde{Q}_1 = \tilde{z}_3$ if

$$1 \le \frac{\bar{a}_1}{\bar{a}_2} \le \frac{p}{2c}.$$

Case (b): $\bar{a}_1/\bar{a}_2 \le 1$. In this case, the density function $g_{1+2}(x)$ is given by

$$g_{1+2}(x) = \begin{cases} \dfrac{\bar{a}_1+\bar{a}_2+x}{4\bar{a}_1\bar{a}_2} & \text{if } x \in [-\bar{a}_1-\bar{a}_2, \bar{a}_1-\bar{a}_2] \\ \dfrac{1}{2\bar{a}_2} & \text{if } x \in [\bar{a}_1-\bar{a}_2, -\bar{a}_1+\bar{a}_2] \\ \dfrac{\bar{a}_1+\bar{a}_2-x}{4\bar{a}_1\bar{a}_2} & \text{if } x \in [-\bar{a}_1+\bar{a}_2, \bar{a}_1+\bar{a}_2] \end{cases}.$$

Again, we analyze $\pi_1(Q_1)$ for three possible ranges for Q_1.

(b1) $\mu - \bar{a}_1 - \bar{a}_2 \le Q_1 \le \mu + \bar{a}_1 - \bar{a}_2$.

In this range for Q_1, the first-order condition is the same as that in case (a1). Therefore, $\tilde{z}_1$ as defined in Equation (A11) satisfies the first-order condition. Also, $\tilde{z}_1 \le \mu + \bar{a}_1 - \bar{a}_2$ is equivalent to

$$\frac{2(p-c)}{p} \le \frac{\bar{a}_1}{\bar{a}_2}.$$

We have the following sufficient optimality condition:

Condition 4: $\tilde{Q}_1 = \tilde{z}_1$ if

$$\frac{2(p-c)}{p} \le \frac{\bar{a}_1}{\bar{a}_2} \le 1.$$

(b2) $\mu + \bar{a}_1 - \bar{a}_2 \le Q_1 \le \mu - \bar{a}_1 + \bar{a}_2$.

In this range for Q_1, the first-order condition can be expressed as

$$\int_{\mu-\bar{a}_1-\bar{a}_2}^{\mu+\bar{a}_1-\bar{a}_2} \frac{\bar{a}_1+\bar{a}_2+x}{4\bar{a}_1\bar{a}_2}dx + \int_{\mu+\bar{a}_1-\bar{a}_2}^{Q_1} \frac{1}{2\bar{a}_1}dx = \frac{p-c}{p},$$

and

$$\tilde{z}_4 = \mu + \bar{a}_2\frac{(p-2c)}{p} \tag{A14}$$

satisfies the first-order condition. Also, $\mu + \bar{a}_1 - \bar{a}_2 \le \tilde{z}_4$ is equivalent to

$$\frac{\bar{a}_1}{\bar{a}_2} \le \frac{2(p-c)}{p},$$

and $q_{b2} \le \mu - \bar{a}_1 + \bar{a}_2$ is equivalent to

$$\frac{\bar{a}_1}{\bar{a}_2} \le \frac{2c}{p}.$$

We have the following sufficient optimality condition:

Condition 5: $\tilde{Q}_1 = \tilde{z}_4$ if

$$\frac{\bar{a}_1}{\bar{a}_2} \le 1,\ \frac{\bar{a}_1}{\bar{a}_2} \le \frac{2(p-c)}{p},\ \text{and}\ \frac{\bar{a}_1}{\bar{a}_2} \le \frac{2c}{p}.$$

(b3) $\mu - \bar{a}_1 + \bar{a}_2 \le Q_1 \le \mu + \bar{a}_1 + \bar{a}_2$.

In this range for Q_1, the first-order condition can be expressed as

$$\int_{\mu-\bar{a}_1-\bar{a}_2}^{\mu+\bar{a}_1-\bar{a}_2} \frac{\bar{a}_1+\bar{a}_2+x}{4\bar{a}_1\bar{a}_2}dx + \int_{\mu+\bar{a}_1-\bar{a}_2}^{\mu-\bar{a}_1+\bar{a}_2} \frac{1}{2\bar{a}_2}dx + \int_{\mu-\bar{a}_1+\bar{a}_2}^{Q_1} \frac{\bar{a}_1+\bar{a}_2-x}{4\bar{a}_1\bar{a}_2}dx = \frac{p-c}{p}.$$

It can be shown that $\tilde{z}_3$ as defined in Equation (A13) satisfies the above first-order condition. Also, $\mu - \bar{a}_1 + \bar{a}_2 \le \tilde{z}_3$ is equivalent to

$$\frac{2c}{p} \le \frac{\bar{a}_1}{\bar{a}_2}.$$

We have the following sufficient optimality condition:

Condition 6: $\tilde{Q}_1 = \tilde{z}_3$ if

$$\frac{2c}{p} \le \frac{\bar{a}_1}{\bar{a}_2} \le 1.$$

To prove part (i), assume that $p \ge 2(c_1 + c_2) = 2c$. Suppose that

$$\frac{\bar{a}_1}{\bar{a}_2} \le \frac{2c}{p}.$$

The assumption $p \ge 2c$ implies that

$$\frac{2c}{p} \le 1 \text{ and } \frac{2c}{p} \le \frac{p-c}{p}.$$

Therefore, Condition 5 holds, and $\tilde{Q}_1 = \tilde{z}_4$. Suppose that

$$\frac{2c}{p} < \frac{\bar{a}_1}{\bar{a}_2} < \frac{p}{2c}.$$

If $\bar{a}_1/\bar{a}_2 \le 1$, then Condition 6 holds, and $\tilde{Q}_1 = \tilde{z}_3$. On the other hand, if $\bar{a}_1/\bar{a}_2 > 1$, then Condition 3 holds, and $\tilde{Q}_1 = \tilde{z}_3$. In either case, we have $\tilde{Q}_1 = \tilde{z}_3$. Finally, suppose that

$$\frac{\bar{a}_1}{\bar{a}_2} \ge \frac{p}{2c}.$$

The assumption $p \ge 2c$ implies that:

$$\frac{p}{2c} \ge 1 \text{ and } \frac{p}{2c} \ge \frac{p}{2(p-c)}.$$

Therefore, Condition 2 holds, and $\tilde{Q}_1 = \tilde{z}_2$.

To prove part (ii), assume that $p < 2c$. Suppose that

$$\frac{\bar{a}_1}{\bar{a}_2} \le \frac{2(p-c)}{p}.$$

The assumption $p < 2c$ implies that:

$$\frac{2(p-c)}{p} < 1 \text{ and } \frac{2(p-c)}{p} < \frac{2c}{p}.$$

Therefore, Condition 5 holds, and $\tilde{Q}_1 = \tilde{z}_4$. Suppose that

$$\frac{2(p-c)}{p} < \frac{\bar{a}_1}{\bar{a}_2} < \frac{p}{2(p-c)}.$$

If $\bar{a}_1/\bar{a}_2 \le 1$, then Condition 4 holds, and $\tilde{Q}_1 = \tilde{z}_1$. On the other hand, if $\bar{a}_1/\bar{a}_2 > 1$, then Condition 1 holds, and $\tilde{Q}_1 = \tilde{z}_1$. In either case, we have $\tilde{Q}_1 = \tilde{z}_1$. Finally, suppose that

$$\frac{\bar{a}_1}{\bar{a}_2} \ge \frac{p}{2(p-c)}.$$

The assumption $p < 2c$ implies that:

$$\frac{p}{2(p-c)} > 1 \text{ and } \frac{p}{2(p-c)} > \frac{p}{2c}.$$

Therefore, Condition 2 holds, and $\tilde{Q}_1 = \tilde{z}_2$.

We can then substitute the optimal order quantities into the profit functions to obtain the corresponding optimal expected profits. The derivation is straightforward, and so we omit the details here. ∎

Proof of Proposition 5.

(i) Suppose that $p \ge 2c_1 + c_2$ and

$$\frac{\bar{a}_1}{\bar{a}_2} \le \frac{2c_1}{p}.$$

Proposition 3(i) shows that:

$$\pi^* = \mu(p-c) - \bar{a}_2\frac{(p-c)c}{p} - \frac{\bar{a}_1{}^2}{\bar{a}_2}\frac{p}{12}.$$

Note that $p \ge 2c_1 + c_2$ is equivalent to $c_1 \le p - c$ and so

$$\frac{\bar{a}_1}{\bar{a}_2} \le \frac{2c_1}{p} \le \frac{2(p-c)}{p}.$$

It then follows from Proposition 4 that $\tilde{\pi} = \pi^*$, and thus $MCR = 0$.

(ii) Suppose that $p < 2c_1 + c_2$. This implies that $p < 2(c_1 + c_2)$, and so π^* is given by Proposition 3(ii) and $\tilde{\pi}$ is given by Proposition 4(ii). For

$$\frac{\bar{a}_1}{\bar{a}_2} \le \frac{2(p-c)}{p},$$

we have

$$\pi^* = \tilde{\pi} = \mu(p-c) - \bar{a}_2\frac{(p-c)c}{p} - \frac{\bar{a}_1{}^2}{\bar{a}_2}\frac{p}{12}.$$

Note that $(p-c_2)^2 \le p^2$, and so

$$\frac{(p-c_2)^2}{2(p-c)p} \le \frac{p}{2(p-c)}.$$

Thus, for

$$\frac{2(p-c)}{p} < \frac{\bar{a}_1}{\bar{a}_2} \le \frac{(p-c_2)^2}{2(p-c)p},$$

we have

$$\pi^* = \tilde{\pi} = \mu(p-c) - (\bar{a}_1 + \bar{a}_2)(p-c) + \frac{4(p-c)}{3}\sqrt{2\bar{a}_1\bar{a}_2\frac{(p-c)}{p}},$$

and thus $MCR = 0$.

Assume that $p \le 2c_1 + c_2$, and let

$$f(p) = \frac{(p-c_2)^2}{2p(p-c)}.$$

Then,

$$\begin{aligned} f'(p) &= \frac{2p(p-c)2(p-c_2) - (p-c_2)^2 2(2p-c)}{4p^2(p-c)^2} \\ &= \frac{2(p-c_2)[2p(p-c) - (p-c_2)(2p-c)]}{4p^2(p-c)^2} \end{aligned}$$

$$= \frac{2(p-c_2)[p(c_2-c_1)-c_2(c_1+c_2)]}{4p^2(p-c)^2}$$
$$\le \frac{2(p-c_2)[(2c_1+c_2)(c_2-c_1)-c_2(c_1+c_2)]}{4p^2(p-c)^2}$$
$$= \frac{2(p-c_2)[-2c_1^2]}{4p^2(p-c)^2} < 0.$$

Thus, $f(p)$ is decreasing in p for $p \le 2c_1 + c_2$. It is clear that

$$f(2c_1+c_2) = \frac{2c_1}{p}.$$

(iii) As $\overline{a}_1/\overline{a}_2 \to \infty$, we have $\overline{a}_2 \to 0$, and it follows from Propositions 3 and 4 that

$$m^* \to \overline{a}_1 \frac{(p-c)c_1}{p-c_2} \text{ and } \tilde{m} \to \frac{(p-c)c}{p}.$$

Then,

$$MCR = \frac{\tilde{m}-m^*}{\tilde{m}} \to \frac{\frac{c}{p}-\frac{c_1}{p-c_2}}{\frac{c}{p}} = 1 - \frac{c_1 p}{(p-c_2)c}$$
$$= \frac{c_2}{c}\left(\frac{p-c}{p-c_2}\right).$$

This completes the proof. ∎