

Research Article

Exponential Growth of Solution for a Class of Reaction Diffusion Equation with Memory and Multiple Nonlinearities

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Abstract. In this paper, we consider the initial boundary value problem of a class of reaction diffusion equation with memory and multiple nonlinearities. We show the exponential growth of solution with L_p -norm using a differential inequality.

Keywords: reaction diffusion equation, exponential growth, memory term, multiple nonlinearities.

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1. Introduction

In this paper, we study the following initial boundary value problem for a class of reaction diffusion equation with memory and multiple nonlinearities

$$u_t - \Delta u + \int_0^t g(t-s)\Delta u(s)ds + |u|^{k-2}u_t = |u|^{p-2}u, \quad (1.1)$$

$$u(x, t) = 0, x \in \partial\Omega, \quad (1.2)$$

$$u(x, 0) = u_0(x), x \in \Omega, \quad (1.3)$$

where $k > 2, p > 2$ are real numbers and Ω is bounded domain in R^n with smooth boundary $\partial\Omega$ so that the divergence theorem can be applied. Here, g represents the kernel of memory term and Δ denotes the Laplace operator in Ω .

This type of problem not only is important from the theoretical point of view, but also arises in many physical applications and describes a great deal of models in applied science. It appears in the models of chemical reactions, heat transfer, population dynamics, and so on (see [1] and references therein). One of the most important field of such problems arises in the models of nonlinear viscoelasticity.

In absence of the nonlinear diffusion term $|u|^{k-2}u_t$, and when the function g vanishes identically (i.e. $g = 0$), then the equation (1.1) can be reduced to the following equation

$$u_t - \Delta u = |u|^{p-2}u, \quad (1.4)$$

A related problem to the equation (1.4) has attracted great attentions in the last two decades, and many results appeared on the existence, blow-up and asymptotic behavior of solutions. It



is well known that the nonlinear reaction term $|u|^{p-2}u$ drives the solution of (1.4) to blow up in finite time and the diffusion term is known to yield existence of global solution if the reaction term is removed from the equation [2]. The more general equation

$$u_t - \operatorname{div}(|\nabla u|^{m-2}\nabla u) = f(u), \tag{1.5}$$

has also attracted a great deal of people. The obtained results show that global existence and nonexistence depend roughly on m , the degree of nonlinearity in f , the dimension n , and the size of the initial data. In this regard, see the works of Levine [3], Kalantarov and Ladyzhenskaya [4], Levine et al. [5], Messaoudi [6], Liu et al. [7], Ayouch [8] and references therein. Pucci and Serrin [9] discussed the stability of the following equation

$$|u_t|^{l-2}u_t - \operatorname{div}(|\nabla u|^{m-2}\nabla u) = f(u), \tag{1.6}$$

Levine et al. [5] got the global existence and nonexistence of solution for (1.6). Pang et. al [10, 11] and Berrimi [12] gave the sufficient condition of blow-up result for certain solutions of (1.6) with positive or negative initial energy.

When $g = 0$, the class of equation (1.1) can also be as a special case of doubly nonlinear parabolic-type equations (or the porous medium equation) [5, 13]

$$\beta(u)_t - \Delta u = |u|^{p-2}u \tag{1.7}$$

if we take $\beta(u) = u + |u|^{m-2}u$. Such equation play an important role in physics and biology. It should be noted that questions of solvability, local and global in time, asymptotic behavior and blow-up of initial boundary value problems and initial value problems for equation of the type (1.7) were investigated by many authors. We only mention the work [13, 14] for this class equation.

We should also point out that Polat [15] established a blow-up result for the solution with vanishing initial energy of the following initial boundary value problem

$$u_t - u_{xx} + |u|^{m-2}u_t = |u|^{p-2}u.$$

They also gave detailed results of the necessary and sufficient blow-up conditions together with blow-up rate estimates for the positive solution of the problem

$$(u^m)_t - \Delta u = f(u),$$

subjected to various boundary conditions. Korpusov [16, 17] have been obtained sufficient conditions for the blowup of a finite time and solvability for the following generalized Boussinesq equation

$$u_t - \Delta u - \Delta u_t + |u|^{m-2}u_t = u(u - \alpha)(u - \beta), \tag{1.8}$$

with initial boundary value (1.2) and (1.3) in R^3 for $\alpha, \beta > 0$ by concavity method [3, 4]. The result is extended by recent paper [18–21].

In absence of the nonlinear diffusion term $|u|^{k-2}u_t$, and with the presence of the viscoelastic term (i.e. $g \neq 0$), the equation (1.1) becomes

$$u_t - \Delta u + \int_0^t g(t-s)\Delta u(s)ds = |u|^{p-2}u, \tag{1.9}$$

it arises from the study of heat conduction in materials with memory. In this case, Messaoudi [22, 23] obtained a blow-up result for certain solutions with positive or negative initial energy and Giorgi [24] got the asymptotic behavior. Furthermore, for the quasilinear case

$$|u_t|^{l-2}u_t - \Delta u + \int_0^t g(t-s)\Delta u(s)ds = |u|^{p-2}u, \quad (1.10)$$

Messaoudi et al. [25, 26] established a general decay result from which the usual exponential and polynomial decay results are just only special cases for (1.10) without reaction term $|u|^{p-2}u$. Liu et al. [27] obtained a general decay of the energy function for the global solution and a blow-up result for the solution with both positive and negative initial energy for (1.10).

In this paper, we will investigate problem (1.1)–(1.3), which has few result of the problem to our knowledge. We will prove that L_p -norm of the solution grows as an exponential function. An essential tool to the proof is an idea used in [28, 29], which based on an auxiliary function (a small perturbation of the total energy), using a differential inequality and obtaining the result. This result extend the early paper. This article is organized as follows. Section 2 is concerned with some notations and statement of assumptions. In Section 3, we give the main result.

2. Preliminaries

In this section, we will give some notations and statement of assumptions for m, p, g . We denote $L^p(\Omega)$ by L^p and $H_0^1(\Omega)$ by H_0^1 , the usual Soblev space. The norm and inner of $L^p(\Omega)$ are denoted by $\|\cdot\|_p = \|\cdot\|_{L^p(\Omega)}$ and $(u, v) = \int_{\Omega} u(x)v(x)dx$ respectively. Especially, $\|\cdot\| = \|\cdot\|_{L^2(\Omega)}$ for $p = 2$.

For the relaxation function g and the number m and p , we assume that

(A1) $g : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is a differentiable function satisfying

$$g(0) > 0, 1 - \int_0^{+\infty} g(s)ds = l > 0;$$

(A2) there exists a nonincreasing function $\xi : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ such that

$$g'(s) \leq -\xi(s)g(s);$$

(A3) we also assume that

$$2 < k < p \leq \frac{2(n-1)}{n-2}, \quad \text{if } n \geq 3; \quad 2 < k < p < +\infty, \quad \text{if } n = 1, 2.$$

Similar to [15, 27], we call $u(x, t)$ a weak solution to problem (1.1)–(1.3) on $\Omega \times [0, T)$, if

$$u \in C(0, T; H_0^1) \cap C^1(0, T; L^2), \quad |u|^{k-2}u_t \in L^2(\Omega \times [0, T))$$

satisfying $u(x, 0) = u_0(x)$ and

$$\begin{aligned} \int_0^t \int_{\Omega} \left[\nabla u(s) \nabla v(s) - \int_0^s g(s-\tau) \nabla u(\tau) \nabla v(\tau) d\tau + u_t(s)v(s) \right. \\ \left. + |u|^{k-2}u_t v - |u|^{p-2}uv \right] dx ds = 0, \quad \forall v \in C(0, T; H_0^1), \quad \forall t \in [0, T). \end{aligned}$$

In this paper, we always assume that problem (1.1)–(1.3) exists a local solution (see [16, 17]).

Now, we introduce two functionals

$$E(t) = E(u) = \frac{1}{2} \|\nabla u\|^2 + \frac{1}{2} (g \otimes \nabla u)(t) - \frac{1}{2} \int_0^t g(s) ds \|\nabla u\|^2 - \frac{1}{p} \|u\|_p^p, \quad (2.1)$$

$$E(0) = \frac{1}{2} \|\nabla u_0\|^2 - \frac{1}{p} \|u_0\|_p^p, \quad (2.2)$$

where $u \in H_0^1$ and

$$(g \otimes \nabla v)(t) = \int_0^t g(t-s) \|\nabla v(s) - \nabla v(t)\|^2 ds.$$

Multiplying Equation (1.1) by u_t and integrating over Ω , by (A2) ($g'(t) \leq 0$), we have

$$E'(t) = -\|u_t\|^2 + \frac{1}{2} (g' \otimes \nabla u)(t) - \int_{\Omega} |u|^{k-2} u_t^2 dx - \frac{1}{2} g(t) \|\nabla u\|^2 \leq 0. \quad (2.3)$$

3. Exponential Growth of Solution

In this section, we prove the main result. Our technique is different from that in [5, 15, 27–29] because of the presence of nonlinear term $|u|^{k-2}u_t$ and the memory term.

Theorem 3.1. *Suppose that assumptions (A1), (A2) and (A3) hold, $u_0 \in H_0^1$ and u is a local solution of the system (1.1)–(1.3), and $E(0) < 0$. Furthermore, we assume that*

$$\int_0^{\infty} g(s) ds < \frac{p-2}{p-1}. \quad (3.1)$$

Then the solution of the system (1.1)–(1.3) grows exponentially.

Proof. We set

$$H(t) = -E(t). \quad (3.2)$$

By the definition of $H(t)$ and (2.3)

$$H'(t) = -E'(t) \geq 0. \quad (3.3)$$

Consequently, by $E(0) < 0$, we have

$$H(0) = -E(0) > 0. \quad (3.4)$$

Noting that

$$H(t) - \frac{1}{p} \|u\|_p^p = - \left[\frac{1}{2} \|\nabla u\|^2 + \frac{1}{2} (g \otimes \nabla u)(t) - \frac{1}{2} \int_0^t g(s) ds \|\nabla u\|^2 \right] < 0, \quad (3.5)$$

then (3.3), (3.4) and (3.5) imply

$$0 < H(0) \leq H(t) \leq \frac{1}{p} \|u\|_p^p. \quad (3.6)$$

□

Let us define the function

$$L(t) = H(t) + \frac{\epsilon}{2} \|u\|^2. \quad (3.7)$$

By taking the time derivative of (3.7) and by (1.1), we have

$$\begin{aligned} L'(t) &= H'(t) + \epsilon \int_{\Omega} uu_t dx \\ &= \|u_t\|^2 - \frac{1}{2}(g' \otimes \nabla u)(t) + \int_{\Omega} |u|^{k-2} u_t^2 dx + \frac{1}{2} g(t) \|\nabla u\|^2 \\ &\quad + \epsilon \|u\|_p^p - \epsilon \|\nabla u\|^2 + \epsilon \int_{\Omega} \int_0^t g(t-s) \nabla u(t) \nabla u(s) ds dx \\ &\quad - \epsilon \int_{\Omega} |u|^{k-2} uu_t dx \\ &\geq \|u_t\|^2 - \epsilon \|\nabla u\|^2 + \epsilon \|u\|_p^p + \epsilon \int_{\Omega} \int_0^t g(t-s) \nabla u(t) \nabla u(s) ds dx \\ &\quad + \int_{\Omega} |u|^{k-2} u_t^2 dx - \epsilon \int_{\Omega} |u|^{k-2} uu_t dx. \end{aligned} \quad (3.8)$$

To estimate the last term in the right-hand side of (3.8), by using the following Young's inequality

$$ab \leq \delta^{-1} a^2 + \delta b^2,$$

we have

$$\begin{aligned} \int_{\Omega} |u|^{k-2} uu_t dx &= \int_{\Omega} |u|^{\frac{k-2}{2}} u_t |u|^{\frac{k-2}{2}} u dx \\ &\leq \delta^{-1} \int_{\Omega} |u|^{k-2} u_t^2 dx + \delta \int_{\Omega} |u|^k dx. \end{aligned}$$

Therefore, combining with

$$\begin{aligned} &\int_{\Omega} \int_0^t g(t-s) \nabla u(t) \nabla u(s) ds dx \\ &= \int_0^t g(s) ds \|\nabla u(t)\|^2 + \int_0^t g(t-s) \int_{\Omega} \nabla u(t) (\nabla u(s) - \nabla u(t)) dx ds \\ &\geq \frac{1}{2} \int_0^t g(s) ds \|\nabla u(t)\|^2 - \frac{1}{2} (g \otimes \nabla u)(t), \end{aligned}$$

we have

$$\begin{aligned} L'(t) &\geq \|u_t\|^2 + \epsilon \left(\frac{1}{2} \int_0^t g(s) ds - 1 \right) \|\nabla u\|^2 + \epsilon \|u\|_p^p \\ &\quad - \epsilon \delta \|u\|_k^k + (1 - \epsilon \delta^{-1}) \int_{\Omega} |u|^{k-2} u_t^2 dx - \frac{\epsilon}{2} (g \otimes \nabla u). \end{aligned} \quad (3.9)$$

By using

$$\|u\|_p^p = p H(t) + \frac{p}{2} \left(1 - \int_0^t g(s) ds \right) \|\nabla u\|^2 + \frac{p}{2} (g \otimes \nabla u),$$

(3.9) becomes

$$\begin{aligned}
 L'(t) &\geq \|u_t\|^2 + \epsilon \left(\frac{1}{2} \int_0^t g(s) ds - 1 \right) \|\nabla u\|^2 \\
 &\quad + \epsilon \left[p H(t) + \frac{p}{2} (g \otimes \nabla u) + \frac{p}{2} \left(1 - \int_0^t g(s) ds \right) \|\nabla u\|^2 \right] \\
 &\quad - \epsilon \delta \|u\|_k^k + (1 - \epsilon \delta^{-1}) \int_{\Omega} |u|^{k-2} u_t^2 dx - \frac{\epsilon}{2} (g \otimes \nabla u) \\
 &\geq \|u_t\|^2 + (1 - \epsilon \delta^{-1}) \int_{\Omega} |u|^{k-2} u_t^2 dx \\
 &\quad + \epsilon a_1 \|\nabla u\|^2 + \epsilon a_2 (g \otimes \nabla u) + \epsilon p H(t) - \epsilon \delta \|u\|_k^k,
 \end{aligned} \tag{3.10}$$

where $a_1 = \frac{1-p}{2} \int_0^\infty g(s) ds + \frac{p-2}{2} > 0$ by (3.1), $a_2 = \frac{p}{2} - 1 > 0$. Noting that $p > k > 2$ and embedding theorem, we have

$$\|u\|_k^k \leq C \|u\|_p^k \leq C (\|u\|_p^p)^{\frac{k}{p}}, \tag{3.11}$$

where $C > 0$ is a positive embedding constant. Since $0 < \frac{k}{p} < 1$, now applying the inequality $x^l \leq (x+1) \leq (1+\frac{1}{z})(x+z)$, which holds for all $x \geq 0, 0 \leq l \leq 1, z > 0$, in particular, taking $x = \|u\|_p^p, l = \frac{k}{p}, z = H(0)$, we obtain

$$(\|u\|_p^p)^{\frac{k}{p}} \leq \left(1 + \frac{1}{H(0)} \right) (\|u\|_p^p + H(0)),$$

then from (3.6) and (3.11)

$$\|u\|_k^k \leq C \|u\|_p^k \leq C_1 \|u\|_p^p,$$

we have

$$\begin{aligned}
 L'(t) &\geq \|u_t\|^2 + (1 - \epsilon \delta^{-1}) \int_{\Omega} |u|^{k-2} u_t^2 dx \\
 &\quad + \epsilon a_1 \|\nabla u\|^2 + \epsilon a_2 (g \otimes \nabla u) + \epsilon p H(t) - \epsilon \delta C_1 \|u\|_p^p,
 \end{aligned} \tag{3.12}$$

Taking $0 < a_3 < \min\{a_1, a_2\}$, and by $2H(t) \geq -\|\nabla u\|^2 - (g \otimes \nabla u) + \frac{2}{p} \|u\|_p^p$, we have

$$\begin{aligned}
 L'(t) &\geq \|u_t\|^2 + (1 - \epsilon \delta^{-1}) \int_{\Omega} |u|^{k-2} u_t^2 dx + \epsilon (a_1 - a_3) \|\nabla u\|^2 + \epsilon p H(t) \\
 &\quad + \epsilon (a_2 - a_3) (g \otimes \nabla u) + \epsilon \left(\frac{2}{p} a_2 - \delta C_1 \right) \|u\|_p^p \\
 &\quad + \epsilon a_3 \left[\|\nabla u\|^2 + (g \otimes \nabla u) - \frac{2}{p} \|u\|_p^p \right] \\
 &= \|u_t\|^2 + (1 - \epsilon \delta^{-1}) \int_{\Omega} |u|^{k-2} u_t^2 dx + \epsilon (a_1 - a_3) \|\nabla u\|^2 \\
 &\quad + \epsilon (a_2 - a_3) (g \otimes \nabla u) + \epsilon \left(\frac{2}{p} a_3 - \delta C_1 \right) \|u\|_p^p + \epsilon (p - 2a_3) H(t).
 \end{aligned} \tag{3.13}$$

Taking δ small enough such that $\frac{2}{p}a_3 - \delta C_1 > 0$, and then taking ϵ small enough such that $1 - \epsilon\delta^{-1} > 0$, and noting that $p - 2a_3 > 0$, then

$$L'(t) \geq C_2 (H(t) + \|u_t\|^2 + \|\nabla u\|^2 + (g \otimes \nabla u) + \|u\|_p^p). \quad (3.14)$$

On the other hand, by the definition of $H(t)$, we get

$$\begin{aligned} L'(t) &\leq H(t) + \|u\|^2 \leq C_3 (H(t) + \|\nabla u\|^2) \\ &\leq C_3 (H(t) + \|u_t\|^2 + \|\nabla u\|^2 + (g \otimes \nabla u) + \|u\|_p^p). \end{aligned} \quad (3.15)$$

From (3.14) and (3.15), we obtain the differential inequality

$$L'(t) \geq rL(t). \quad (3.16)$$

Integration of (3.16) between 0 and t gives us

$$L(t) \geq L(0)\exp(rt). \quad (3.17)$$

From (3.8) and with ϵ small enough, we have

$$L(t) \leq H(t) \leq \frac{1}{p}\|u\|_p^p. \quad (3.18)$$

By (3.17) and (3.18), we deduce

$$\|u\|_p^p \geq C\exp(rt).$$

Therefore, we conclude that the solution in the L_p -norm grows exponentially.

Remark. By the same method (similar to [29]), we can also get the similar result to problem (1.1)–(1.3) with positive initial energy.

Concluding

In this paper, the initial boundary value problem of a class of reaction diffusion equation with memory and multiple nonlinearities is considered. Using a differential inequality, exponential growth of solution with L_p -norm is proved for negative and positive initial energy.

Competing Interests

The authors declare no competing interests.

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