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Volume and Cost of Longboard using AirCrete

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Abstract

This project utilizes integral calculus to find the volume of a longboard-style surfboard. The cost for using AirCrete as the material for the longboard will then be calculated. Due to the longboard's shape, the longboard is divided into four sections, and the volume of each is found. The final volume for the longboard with the measurements provided is approximately 3821 cubic inches. The cost of making the longboard out of AirCrete was then calculated by using ratios. The final cost for the longboard is $4.70.

Keywords

longboard, volume, AirCrete, material cost

PROBLEM STATEMENT

Calculate the cost of a longboard style surfboard using AirCrete.

MOTIVATION

The uses of AirCrete in construction are widely researched, but its capability to float has not yet been explored in depth. Not only is AirCrete extremely durable and resistant to rot, but it is also environmentally friendly due to its composition of organic materials and use of fewer materials than regular concrete. It can also be recycled with ease. Fabricating a product such as a surfboard out of AirCrete will allow for small-scale testing and will not only help the surfboard industry become more sustainable, but also the project can be applied to other industries, for example, boats and shipping. It is important for this project to show how low-cost testing with subject material can be, while also giving guidelines for those who wish to make a longboard from this material.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

Because of its irregular shape, the longboard is divided into 4 sub-sections (Figure 1) in order to effectively calculate the total volume. Figure 1 can be dissected into a rectangular cube (α) , ¼ elliptical cylinders (β), the volume of revolution from an ellipse around the y-axis (c), and shorter ¼ elliptical cylinders (d). When these are added together with the equation below, they will give the total volume.

$$Volume_{total} = Volume_{\alpha} + 2Volume_{\beta} + Volume_{c} + 2Volume_{d}$$

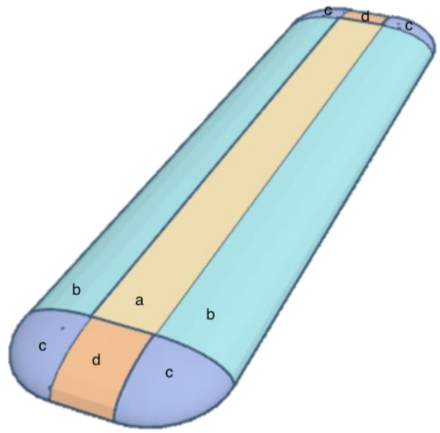

Figure 1

The volume for section “a” can be obtained utilizing the known formula (2) for the volume of a rectangular cuboid using the measurements from Figure 2

Figure 2

$$Volume_{\alpha} = l \cdot w \cdot h$$

$$Volume_{\alpha} = 65in \cdot 6in \cdot 3in = 1170in^3$$

The volume for section “β” can be calculated using integrals. Solve the equation of the ellipse for y. Then using that, the area underneath can be found through integrals. Use the measurements from Figure 3.

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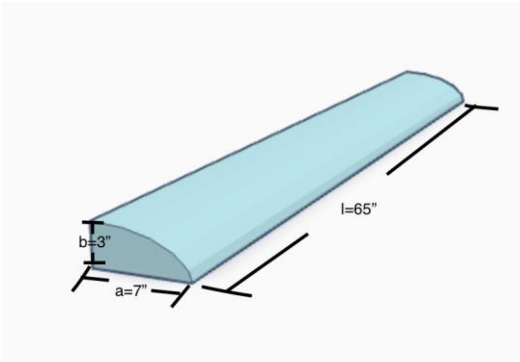

Figure 3

$$ellipse\ equation = \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$= \ \mathrm{y} = \mathrm{b}\sqrt{1 - \frac{x^2}{a^2}}$$

$$= y = 3\sqrt{1 - \frac{x^2}{7^2}}$$

$$\text{Area } of \frac{1}{4} ellipse = \int_0^a \mathrm{y}d\mathrm{x}$$

$$= \int_0^a 3\sqrt{1 - \frac{x^2}{7^2}}\,d\mathrm{x}$$

$$= \frac{3}{7}\int_0^a \sqrt{7^2 - x^2}\,dx$$

$$x = asint\ dx = acostdt$$
$$x = 7sint\ dx = 7costdt$$

$$= \frac{3}{7}\int_0^{\frac{\pi}{2}} \sqrt{7^2 - (7sint)^2} \cdot 7costdt$$

$$= 3\int_0^{\frac{\pi}{2}} \sqrt{7^2(1 - \sin^2 t)} \cdot costdt$$

Use trigonometric identities in order to make the integral easier to solve.

$$cos^2 t = 1 - \sin^2 t$$

$$= 21 \int_0^{\frac{\pi}{2}} \sqrt{(\cos^2 t)} \cdot costdt$$

$$= 21 \int_0^{\frac{\pi}{2}} cos^2 t\, dt$$

Use double angle formulas to help solve.

$$2cos^2 t = 1 + cos2t$$

$$= 21 \int_0^{\frac{\pi}{2}} \frac{1 + cos2t}{2} dt$$

$$= 21 \left(\frac{t}{2} + \frac{sin2t}{4}\right)_0^{\frac{\pi}{2}}$$

$$A = 21(\frac{\pi}{4} - 0) = 16.5\mathrm{in}^2$$

$$\mathrm{Volume} = \mathrm{A} \cdot \mathrm{h}$$

$$\mathrm{Volume}_\beta = 16.5\mathrm{in}^2 \cdot 65\mathrm{in} = 1072.5\mathrm{in}^3$$

The volume for section “c” can be calculated using the volume of rotation integrals. In this case, the equation of the ellipse is being rotated around the y-axis. Solve the equation of the ellipse for x. Then using that, rotate around the x-axis using integrals. Use the measurements from figure 4.

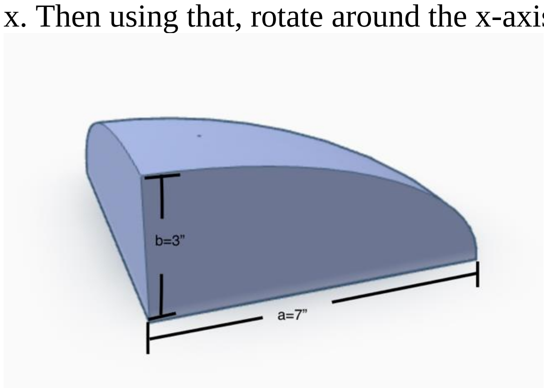

Figure 4

$$\text{Volume of revolution about y} - \text{axis} = \pi \int_c^d \mathrm{f(y)}^2 dy$$

$$ellipse\ equation\ = \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\mathrm{x} = \mathrm{a} \sqrt{1 - \frac{\mathrm{y}^2}{\mathrm{b}^2}} \mathrm{dy}$$

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$$x = 7\sqrt{1 - \frac{y^2}{3^2}} = f(y)$$

$$= \pi \int_0^3 \left(7\sqrt{1 - \frac{y^2}{3^2}}\right)^2 dy$$

$$= 49\pi \int_0^3 1 - \frac{y^2}{3^2} dy$$

$$= 49\pi(y - \frac{y^3}{27})_0^3$$

$$= 49\pi(2 - 0) = 307.87in^3 = Volume_c$$

The volume for section "d" can be calculated using the same method as for part "b". Using the same answer for the area underneath the ellipse curve and multiplying by the new length of part "d", the volume of this elliptic cylinder can be found. Use the measurements from Figure 5.

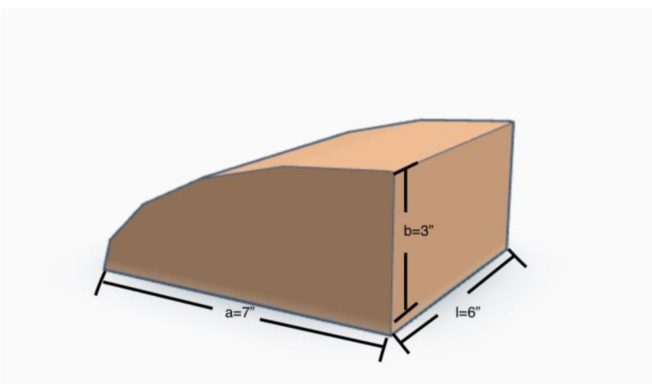

Figure 5

$$\text{Volume} = \text{A} \cdot \text{l}$$
$$\text{Volume}_\text{d} = 16.5\text{in}^2 \cdot 6\text{in} = 99\text{in}^3$$

Now all parts are added to find total volume

$$Volume_{total} = Volume_\alpha + 2Volume_\beta + 4Volume_c + 2Volume_d$$
$$\text{Volume total} = 1170\text{in}^3 + 2(1072.5\text{in}^3) + 307.87\text{in}^3 + 2(99\text{in}^3) = 3820.87\text{in}^3$$

Using information from DomeGaia [3], the original creators of AirCrete, convert their amount of water, foaming agent, and cement for 45 gal of AirCrete to the amount needed for the total volume calculated above for the longboard.

To find the ratio between their total volume of 45 gal and the volume for the longboard, inches cubed must be converted into gallons. Then dividing that by the 45 gallons will give a ratio that can be multiped by the amount of each material to give the amount needed for the longboard.

$$\frac{3820.87in^3}{1} \cdot \frac{1gal}{231in^3} = 16.5gal$$

$$\frac{16.5gal}{45gal} = 0.368$$

11 gallons of water were used to make the 45gal, so it can be multiplied by the ratio to give the amount that is needed for the longboard. This is the same process for the other materials.

$$11gal \cdot\ 0.368 = 4gal \text{ of water}$$

$$2cups \cdot\ 0.368 = 0.736cups \text{ of foaming agent}$$

$$94lbs\ \cdot\ 0.368 = 34.6lbs \text{ of cement}$$

The American Water Works Association stated that tap water cost about $.004 a gallon [4], so using that value, the cost of the water can be found.

$$\frac{\$0.004}{1\text{gal}} \cdot 4\text{gal} = \$0.02$$

At Target, Seventh generation natural dish soap, as found to be one of the best foaming agents by DomeGaia [1] cost $2.89, so using that value the cost of the foaming agent can be found.

$$\frac{\$2.89}{2.375\text{cups}} \cdot 0.736\text{cups} = \$0.9$$

At Lowes, the price of Portland cement, recommended by DomeGaia was $10.28, so using that value the cost of the cement can be found.

$$\frac{\$10.28}{94\text{lbs}} \cdot 34.6\text{lbs} = \$3.78$$

Adding all the costs together gives the total cost for the materials needed to make the longboard out of AirCrete.

$$\text{Cost}_{\text{total}} = \$0.02\ +\ \$0.9\ + \$3.78 = \$4.7$$

Finally, the cost per inch cubed was calculated by diving the total price by the total volume of the longboard.

$$\frac{\$4.7}{3820.87\text{in}^3} = \$0.001 \text{ per cubic inch}$$

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Discussion

The project successfully calculates the theoretical volume and gives an estimated cost of materials for the construction of a longboard-style surfboard fabricated with AirCrete. The volume was found to be 3820.87in^3 . At this volume the cost of the materials to make AirCrete was low at only $4.70. This shows that the material is inexpensive to research and should be explored more. However, this is without the costs of the machinery needed including: plastic sheeting, a mold, a concrete mixer, an air compressor, and a drill with a mixing paddle. Calculations for the price are made based on the ratios outlined by DomeGaia the original creator of AirCrete and local Tampa Florida store prices. The price per cubic inch was found to give a guideline for future projects. AirCrete was the material chosen for this project based on its ability to float and its sustainability while being a relatively unresearched material.

Conclusion and Recommendations

The volume for the longboard was calculated utilizing integral calculus. Based on the volume calculated, ratios from DomeGaia were used to find the total cost of materials for producing the AirCrete longboard. The price per cubic inch was calculated. After all the calculations are performed, it can be concluded that the longboard in the proposed problem has a volume of 3821in^3 and has a very low material cost of $4.7.

Based on the lack of information about the use of AirCrete for floatation, the low cost of materials, and its sustainability, it seems that AirCrete should be explored more. Usually, surfboards are made from materials that are not sustainable and break easily without a way to be recycled. Using AirCrete to build a surfboard is an idea that could address these issues with current surfboards.

Future project suggestions include finding the surface area of the longboard in order to find the cost of materials for covering the board in Entropy resin (an epoxy-like substance made from tree sap[2]) and hemp fabric. This would act as a fiberglass agent that would increase the durability, strength, and water resistance of the board. Establishing the buoyancy of the material would also be a future project that should be undergone.

Nomenclature

V	Volume inches cubed
A	Area inches squared
Lbs	Pounds
Gal	Gallons

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