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RESEARCH ARTICLE

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Suction-ejection of a ping-pong ball in a falling water-filled cup

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Abstract. Dropping a water-filled cup with a ping-pong ball inside to the ground expels the ball much higher than its initial height. During free fall, the absence of gravity in the reference frame of the cup makes capillary forces dominant, causing the ball to be sucked into water. At impact, the high velocity ejection is due to the strong Archimedes' force caused by vertical acceleration. In this paper, we study the dynamics of the capillary sinking of the ball during free fall and the ejection speed at impact, using tracking and high-speed imaging. In particular, we show that at short-time, the sinking is governed by capillary and added mass forces.

Keywords: Capillary Suction / Surface Tension / Free Fall / Buoyancy.

1 Introduction

When a ping pong ball is dropped in a cup of water, it bounces back much higher than its initial height. This striking phenomenon is illustrated in Figure 1 (details of the experiment are given below). It looks similar at first glance to the classic cumulative canon problem, in which a light and a heavy balls are dropped together, resulting in the ejection of the light ball powered by the momentum transferred from the heavy ball. However, the efficient momentum transfer of the cumulative cannon problem seems highly unrealistic in the strongly dissipative ball-in-a-water-cup problem. Here, the physic is governed by the surface tension: the ball is first sucked into the water by capillary forces during the free fall of the water cup, and is then expelled from water by the huge vertical acceleration caused by the impact of the cup on the rigid floor [1].

This experiment is based on a viral video [2] and can expel the ball at speeds an order of three times larger than its velocity before impact. Understanding this counter-intuitive behaviour is what motivated our work and that of others [1]. The specificity of our approach is to focus on the short-time dynamics of the capillary suction, which is shown to be governed by the added mass effect (i.e. by the inertia of the displaced water), whereas Andreotti et al. [1] consider the long-time dynamics in which the ball reaches its free fall equilibrium.

The physics of the phenomenon is as follows. During the free fall of the cup, the gravity-dependent forces acting on the ball (weight and Archimedes' force) are zero in the reference frame of the cup, so the ball is subjected only to the capillary force and the inertia of the displaced water around the ball (added mass force [3]). Capillary forces cause the ball to sink deeper in the liquid as the system falls. When the cup hits the ground, it loses its downward velocity in a very short amount of time, resulting in a large acceleration, and hence a large apparent gravity in the frame of the cup. During the short duration of this impact, the ball experiences a strong Archimedes' force from the volume of water displaced by the capillary sinking, resulting in an upward velocity which can exceed by far the velocity before the collision.

By taking into account capillary and added mass forces, we present here a model for the dynamics of the capillary suction of the ball. Our model is in good agreement with our high-speed video tracking experiments, at least for short times, when the dynamics of the capillary suction is governed by the added mass effect. We however observe for longer times a strong decrease of sinking speed which may be coming from a return of gravity-dependent forces, as the cup's acceleration in laboratory frame is progressively reduced due to drag forces.

2 Methods

2.1 Experiments

Our experimental setup, sketched in Figure 2, consists in a beaker (a) filled with 200 mL of tap water in which a ping pong ball is placed, initially held at height H. The fall and collision with the ground is recorded by a high-speed camera (b) operating at 1080 fps with 1/3240 s exposure time. A heavy metal plate (c) is placed on the ground to

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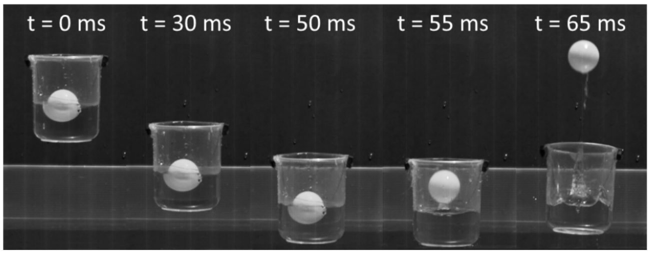

Fig. 1. Snapshots from a high-speed video of dynamics of a ping-pong ball placed in a water-filled beaker falling to the ground. During the free fall, the ball is sucked into the water by capillary forces. Upon collision, the ball is ejected at large velocity by the strongly enhanced Archimedes' force.

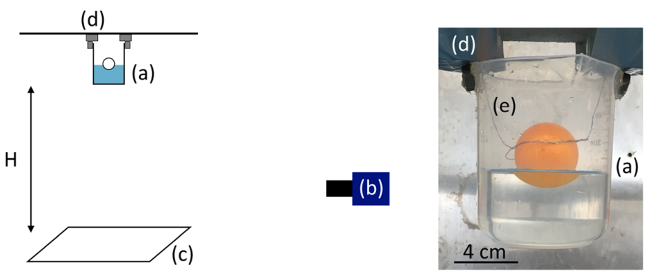

Fig. 2. Left: Schematic of the experimental setup. Right: close-up picture of the water-filled beaker with the ping-pong ball. (a) transparent beaker, (b) high-speed camera, (c) metal plate, (d) electromagnets, (e) metal wire.

ensure a stiff impact. The setup is lighted by a 1000 W spotlight, and a black background was used to improve the image quality for automated tracking of the ball and beaker.

The beaker is held up using two identical electromagnets, 20 N each (d). The magnets maintain the beaker upright in the vertical plane, allowing for a negligible pitch angle. They are switched off simultaneously, to let the beaker fall with a nearly zero tilt angle. Thanks to this system, the angle between the beaker and the ground upon impact was less than 5°; the residual angle was due to imperfect synchronization of the electromagnets causing a slight initial angular deviation, and still represents a limiting factor of our setup. A small loop of metal wire (e) was used to maintain the ball at the center of the beaker.

Three different ping pong balls with distinct wetting properties were used. The wetting property was varied by sanding regular balls with sandpaper of two different grains. Smaller grains lead to better wetting. The wetting property of a ball is quantified by the contact angle θ, given by the Young-Dupré law

$$\cos\theta = \frac{\gamma_{SG} - \gamma_{SL}}{\gamma_{LG}}, \quad (1)$$

where γ_{SG}, γ_{SL} and γ_{LG} are the surface tension of solid-gas surface, solid-liquid surface and liquid-gas surface, respectively. Contact angles were measured by placing a small water drop on the different balls (see Fig. 3). We obtain $\theta = 48.1^{\circ}$ (most hydrophobic), 65.5° (intermediate)

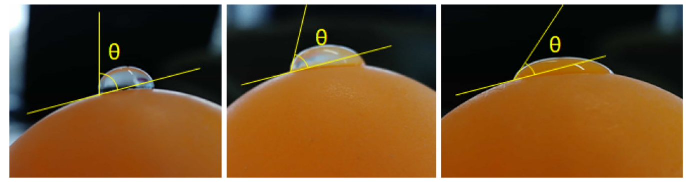

Fig. 3. Measurement of the contact angle using a water drop on ping-pong balls with different wetting properties. From left to right, $\theta = 79.5^{\circ}$, 65.5° and 48.1°.

and 79.5° (most hydrophylic). These values are the average of 6 measurements made at different location on the ball, because imperfections in sanding could cause spatial fluctuations.

The beaker's dropping height is varied between 21 to 66 cm; below 21 cm, the ball hardly jumps from the water, and beyond 66 cm the plastic beaker is likely to break. Seven dropping heights and 3 ping-pong balls with different contact angles were considered. For each case, 5 videos were acquired, amounting to a total of about 100 videos.

2.2 Model

We consider in the following two phases in the problem, illustrated in the video snapshots in Figure 1. In the "Free fall phase", from the release of the cup to right before it hits the ground, the apparent gravity is zero because of the downwards acceleration. In the "Impact phase", the cup is in contact with the ground and looses its velocity, resulting in a very large acceleration and a rapid ejection of the ball.

We mainly focus on the dynamics of the capillary suction during the free fall phase. The capillary force is of order of γR, with γ the surface tension and R the ball radius. This force is of order of 1 mN, which in normal conditions is negligible compared to the ball weight and Archimede's force, which are of order of 0.1 N. But during the free fall phase, inertia forces counter the weight and the Archimedes' force in the reference frame of the cup. The capillary force is therefore the only force at play: the meniscus formed at the circular contact line pulls the ball deeper into the water, at a depth of order of R. This sinking process lasts in principle until the surface of the fluid becomes perfectly horizontal, but it is usually interrupted by the impact of the cup on the ground. The depth of the ball at the impact is therefore governed by the dynamics of the sinking.

We propose here a simple model for the dynamics of the capillary sinking in the free-fall phase, based on a Newtonian description of the system, retaining only dominant forces. We consider the ball in the reference frame of the beaker, in which inertia acts as a correction to the gravitational acceleration g.

We note $g^* = g - a$ the apparent gravity, with a the acceleration of the reference frame of the beaker. During the free fall phase, since the inertia force counters the weight and the Archimedes' force, we have $g^* \simeq 0$, whereas in the impact phase we have $g^* \gg g$. In the reference frame of the cup, the weight of the ball corrected by the

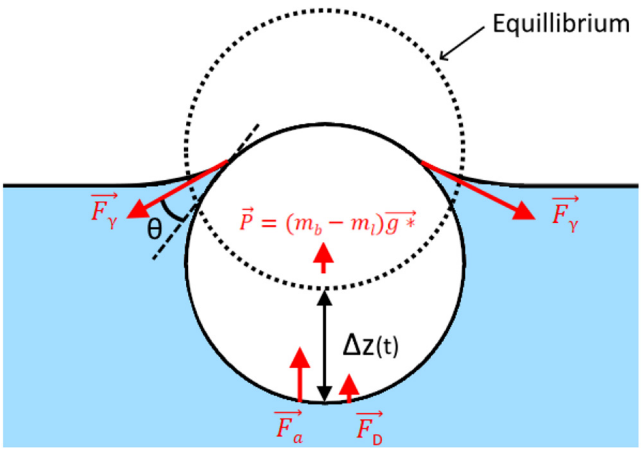

Fig. 4. Forces acting on the ball in the reference frame of the cup. θ is the contact angle, and the red arrows show the capillary force F_γ, the apparent weight modified by the Archimedes' P (with g^* the apparent gravity), the added mass force F_a and the drag force F_D. Δz represents the difference between the position of the ball at equilibrium (dotted circle) and during the free fall of the cup (black circle).

Archimedes' force is $P \simeq R^3(\rho_b - \rho_l)g^*$, where ρ_b is the density of the ball and ρ_l the density of water.

As the ball gradually sinks, it moves the water around it. The ball therefore experiences a resistance, which originates from the inertia of the displaced water. This resistance, known as the added mass force, depends on the ball acceleration, and should not be confused with the drag force which depends on the ball velocity [3]. It is usually relevant when a light object moves through a fluid of larger density, as in the classical problem of air bubbles rising in sparkling water [4].

We note z the height of the ball in the reference frame of the cup (see Fig. 4). Applying Newton's second law for the ball in this reference frame yields

$$m_b \ddot{z} = F_a + F_\gamma + F_D + P, \tag{2}$$

where m_b is the mass of the ball, F_a is the added mass force, F_γ the capillary force, and F_D the drag force.

The added mass force takes the form $F_a \sim \rho_l R^3 \ddot{z}$. The ball is much lighter than water: the ratio between the acceleration of the ball and the added mass force is $m_b \ddot{z}/F_a \sim \rho_b/\rho_l \sim 0.08 \ll 1$. Therefore we can neglect the inertia of the ball in the following.

The drag force is essentially due to inertia in our problem. The Reynolds number Re, which compares the inertial forces to viscous forces, is large, $Re = R\dot{z}/\eta \sim 1000$, with η dynamic viscosity of water, so the drag force writes $F_D \sim \rho_l R^2 \dot{z}^2$.

Equation (2) becomes

$$\rho_l R^3 \ddot{z} \approx \rho_l R^2 \dot{z}^2 + \gamma R \cos\theta, \tag{3}$$

yielding

$$\ddot{z} \approx \frac{\dot{z}^2}{R} + \frac{\gamma \cos\theta}{\rho_l R^2} \tag{4}$$

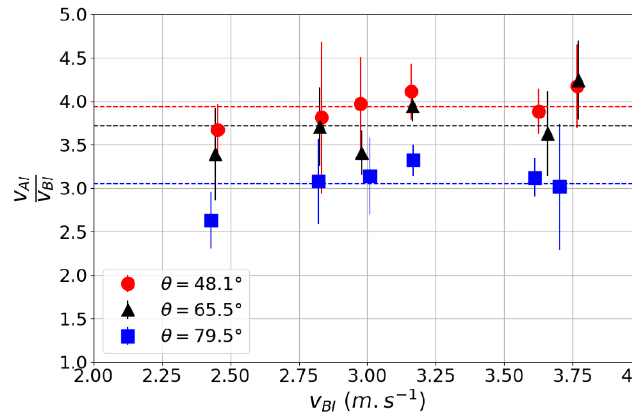

Fig. 5. Velocity ratio as a function of ball velocity before impact. We find highest speed transmission for the ball with highest wetting property.

3 Results

For a typical dropping height $H \simeq 0.5$ m, the duration of the free fall, $T = \sqrt{2H/g}$, is of order of 0.3 s, and its velocity before impact, $V_{BI} = \sqrt{2Hg}$, is of order of 3 m.s^{-1}. The duration of the impact phase could be estimated from inspection of high-speed video of the impact performed at 40 000 fps. We find that $\tau_{impact} \simeq 1$ ms, yielding a typical acceleration $V_{BI}/\tau_{impact} \simeq 3000$ m.s^{-2}, i.e. 300 times the gravitational acceleration.

We measured the speed of the system right before impact (V_{BI}) and the speed of the ball right after impact (V_{AI}). We systematically observed the ball being projected out of the beaker with higher speed than it had before impact, implying a transfer of kinetic energy to the ball during the collision phase. To quantify this energy transfer, we plot in Figure 5 the velocity ratio V_{AI}/V_{BI} as a function of the pre-impact velocity V_{BI}, for each ping pong ball. This ratio does not show significant dependence with V_{BI}, but a clear dependency with the contact angle. The velocity ratio averaged over V_{BI} is plotted in Figure 6 as a function of the contact angle θ. We observe a clear decrease with θ, implying that a better wetting enhances the energy transmission. This clearly demonstrates that surface tension is the linchpin of the ball's behaviour in the free-fall phase.

As shown in Figure 1, we observe that by the end of the free fall phase, the ball has sunk into the water. We measured this sinking (relative to the ball's initial equilibrium position) right before impact for different dropping heights and contact angles. The results are summarized in Figure 7, with each data point corresponding to 35 experiments. It is clear that a better wetting causes the ball to sink deeper in the same amount of time.

In order to better understand the mechanics behind it, we tracked the real-time sinking of a ball (contact angle $\theta \approx 63°$) for a dropping height $H = 66$ cm with the tracking software "Tracker" [5]. A parallax correction was applied to the data. Figure 8 presents the results compared to a fit with a quadratic law. It shows a very good

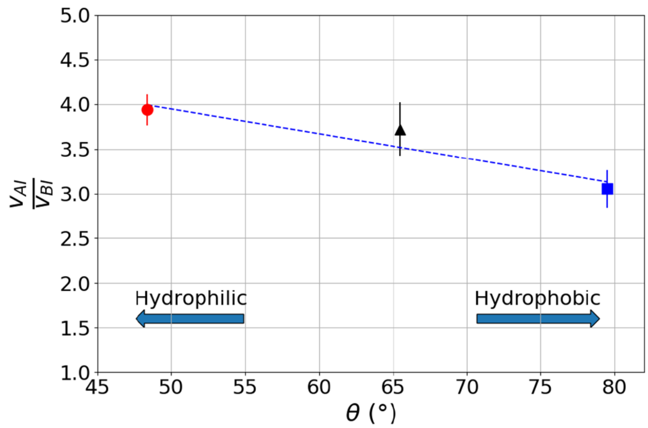

Fig. 6. Speed ratio as a function the contact angle θ. Transmission of kinetic energy decreases for more hydrophobic balls.

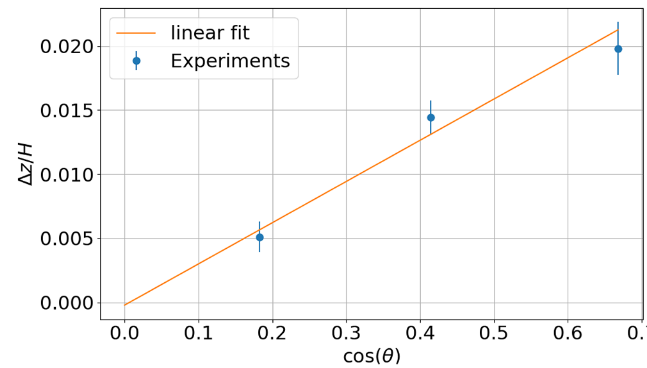

Fig. 7. Relative sinking of the ball as a function of the contact angle for a given initial drop height, $H = 66$ cm.

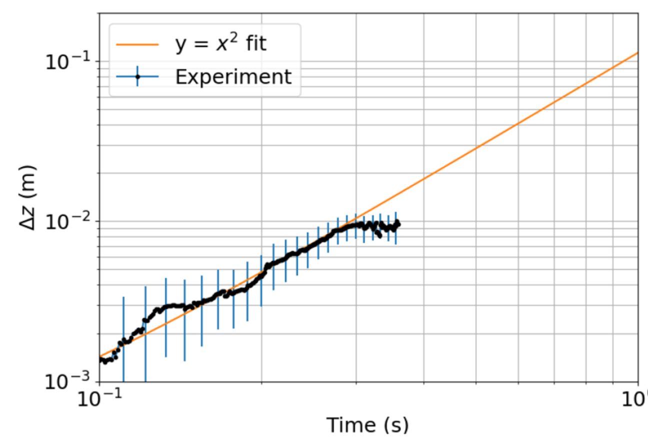

Fig. 8. Relative sinking Δz as a function of time for a beaker dropped from the height of 66 cm. The contact angle is $\theta = 63^o$.

agreement with the fitted law at short time, having a correlation coefficient of $\rho \simeq 0.98$ for data taken for $t < 0.3s$. Beyond $t = 0.3$ s, the sinking appears to saturate.

4 Discussion

We focus on the short-time dynamics of the sinking process. For small times ($t \ll \tau \sim \sqrt{\rho_l R^3/\gamma \cos\theta}$), the drag contribution can be neglected, so equation (4) reduces to

$$\ddot{z} \approx \frac{\gamma \cos\theta}{\rho_l R^2}, \tag{5}$$

which predicts that the sinking depth of the ball (relative to its initial equilibrium position) is

$$\Delta z \approx \frac{\gamma \cos\theta}{\rho_l R^2} t^2. \tag{6}$$

This quadratic law is expected to hold only for short time, before the ball reaches its new equilibrium position with a flat surface. Assuming that the free fall phase duration $T = \sqrt{2H/g}$ is shorter than this equilibrium time, the sinking depth just before impact is

$$\Delta z \approx \frac{2H\gamma}{\rho_l g R^2} \cos\theta. \tag{7}$$

We can compare those results with that of our sinking measurements, summarized in Figure 7. The good collapse of the sinking depth when normalized by the dropping height H, and the linear variation of $\Delta z/H$ with $\cos\theta$, are in excellent agreement with our prediction (7). This confirms that more hydrophilic balls sink deeper during the free fall phase.

In addition, Figure 8 reveals that the short-time sinking of the ball does follow a quadratic law in time. This is also in good accordance with our model, with a correlation coefficient $\rho \simeq 0.98$ close to 1. This demonstrates that no forces other that capillarity and inertia play a role. As we have seen that ball inertia is negligible compared to that of water, we can conclude that the sinking dynamics are only governed by added mass and capillary forces. Other experiments with different dropping heights show similar results.

The transition time at $t \simeq 0.3$ s (which corresponds to a fall of 0.45 m) could suggest that at large time the ball has approached its new equilibrium position, which corresponds here to $\Delta z \simeq 1$ cm. However, this interpretation is in contradiction with the non-flat water surface profile observed in Figure 1 even right before the impact, which shows that the pure capillary equilibrium is not reached in our experiments. A possible interpretation for this non-flat surface at large time is that the gravitational force in the reference frame of the beaker is not strictly zero. Indeed, friction with air is expected to progressively reduce the acceleration of the beaker, resulting in a residual non-zero apparent gravity in the reference frame of the beaker at large time. From the drag coefficient of a cylindrical beaker [6], we can estimate that after a fall of 0.3 s, friction results in a residual acceleration of $g^* \sim 0.1g$ in the beaker. As this residual acceleration grow, a turning point should be reached by the ball, at which the modified gravity balances capillary forces.

5 Dead ends

The beaker was initially held manually using a simple twine attached to the cup. However, it was difficult to ensure reproducible conditions with this system: small angular variations in the drop conditions led to significant deviations during the free fall and uncontrolled ejection of the ball. To study this phenomenon, it is crucial to have a setup allowing reproducible drop conditions as well as minimizing angular fluctuations. Otherwise, the statistical dispersion of the data is too large to deal with.

Additionally, meniscus attraction causes the ball to rapidly drift radially and stick to the side of the beaker (an effect sometimes called Cheerios effect [7]). Because only a portion of the meniscus then remains, and the ball is subjected to friction forces against the beaker side. This induces additional variability as well as overall lower projection speed. We advise attempts at reproducing this phenomenon include some way of maintaining the ball in the center. We used a small loop made of metal wire to prevent the ball from straying without affecting its behaviour too much.

6 Conclusion

In this paper, we studied the origin of the ball ejection in this hydrodynamic version of the cumulative cannon problem. Our experiments confirm the key role played by the surface tension in this problem: The ball sinks in the water during the free fall phase because of capillary forces, when the weight and the Archimedes' force are nearly balanced by the inertia of the free fall. The ball ejection is due to the sudden vertical acceleration of the beaker during the collision, of the order of hundreds of G. The water displaced by capillary sinking exerts on the ball a very large Archimedes' force which expels it out of the beaker. The key parameter that controls the energy ratio transferred to the ball is the wetting property of the ball: a more hydrophilic ball is pulled faster (and therefore deeper) into the water, resulting in a stronger Archimedes' force.

Our analysis shows that the dynamics of the sinking of the ball in this problem is governed by the capillary force and the added mass force, i.e. by the inertia of the displaced water around the ball. These forces lead to a sinking following a quadratic law in time, in excellent agreement with our high-speed camera tracking experiments at short time. For larger time, the sinking of the ball saturates, which we attribute to a slowing down of the free fall due to friction of the beaker with air.

We note that this problem shares a number of interesting similarities with the formation of a jet in the falling water-filled tube problem [8]. In this problem, the contact angle with a hydrophilic tube produces a pronounced cavity during the free fall, which reverses and produces a concentrated jet at the impact. Similarly, in our experiment, a "water cavity" is created as the ball sinks into the water. Drawing a parallel with this work could be interesting to have a better understanding of pressure distribution building up under the ball, at the origin of the jet forming under it at impact. Though our experiment is of a more complicated nature than the free jet in Ref. [8], it might provide a stronger model to understand momentum transfer from water to the ball at impact.

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