

BATS Codes

Theory and Practice



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BATS Codes

Theory and Practice

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ABSTRACT

This book discusses an efficient random linear network coding scheme, called BATChed Sparse code, or BATS code, which is proposed for communication through multi-hop networks with packet loss. Multi-hop wireless networks have applications in the Internet of Things (IoT), space, and under-water network communications, where the packet loss rate per network link is high, and feedbacks have long delays and are unreliable. Traditional schemes like retransmission and fountain codes are not sufficient to resolve the packet loss so that the existing communication solutions for multi-hop wireless networks have either long delay or low throughput when the network length is longer than a few hops. These issues can be resolved by employing network coding in the network, but the high computational and storage costs of such schemes prohibit their implementation in many devices, in particular, IoT devices that typically have low computational power and very limited storage.

A BATS code consists of an outer code and an inner code. As a matrix generalization of a fountain code, the outer code generates a potentially unlimited number of batches, each of which consists of a certain number (called the batch size) of coded packets. The inner code comprises (random) linear network coding at the intermediate network nodes, which is applied on packets belonging to the same batch. When the batch size is 1, the outer code reduces to an LT code (or Raptor code if precode is applied), and network coding of the batches reduces to packet forwarding. BATS codes preserve the salient features of fountain codes, in particular, their rateless property and low encoding/decoding complexity. BATS codes also achieve the throughput gain of random linear network coding. This book focuses on the fundamental features and performance analysis of BATS codes, and includes some guidelines and examples on how to design a network protocol using BATS codes.

KEYWORDS

network coding, BATS code, multi-hop network, packet loss, degree distribution, finite-length analysis, BP decoding, inactivation decoding

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Preface

Driven by new applications, both the scale and the scope of network communications are going to expand significantly in the next several decades. The Internet of Things (IoT) is going to connect tens or hundreds of billions of devices together into networks. New network infrastructures like space communication networks formed by low-orbit satellites or unmanned aerial vehicles (UAVs) are being built and tested. The exploration of outer space and deep sea requires network communications in areas that have not been covered before.

One of the trends is that communication networks will employ more and more wireless links than today. Most existing communication networks, for example WiFi, cellular networks, and satellite networks, involve at most two wireless links, namely the first hop and the last hop. In contrast, multi-hop wireless networks will dominate many new applications.

Wireless links suffer from packet loss due to fading, shadowing, hand off, interference, and other effects. Different from packet loss due to congestion, packet loss due to the above effects in wireless links cannot be reduced by rate control. Extensive research on TCP for combating these wireless link effects was conducted around the year 2000, when WiFi and cellular data were becoming popular. For multi-hop wireless networks, however, modifying TCP cannot prevent the significant rate decrease as the number of hops increases. It is therefore necessary to design new network communication protocols based on a different philosophy for combating packet loss.

Network coding provides a general theory for designing network protocols and achieves the theoretical communication limit of wireless networks with packet loss. In this book, we discuss an efficient random linear network coding scheme called BATched Sparse code, or BATS code. Proposed for communication through networks with packet loss, a BATS code consists of an outer code and an inner code. As a matrix generalization of a fountain code, the outer code generates a potentially unlimited number of batches, each of which consists of a certain number (called the batch size) of coded packets. The inner code comprises (random) linear network coding at the intermediate network nodes, which is applied on the packets belonging to the same batch. When the batch size is 1, the outer code reduces to an LT code (or Raptor code if precode is applied), and network coding of the batches reduces to packet forwarding. BATS codes preserve the salient features of fountain codes, in particular, their rateless property and low encoding/decoding complexity. BATS codes also achieve the throughput gain of random linear network coding.

Applying network coding for communication networks is much more complicated than applying a new channel coding technique for wireless communications, for example, which involves only a modification of the physical layer of the network protocol and is transparent to all

the higher layers. When applying network coding, however, both the transport layer and the network layer must be completely redesigned, and the link/MAC layer and even the physical layer need to be properly tuned to optimize the performance. For a particular application, a network protocol based on BATS code must be designed with specific requirements and constraints. In this book, we provide guidelines and examples on how to design a network protocol using BATS codes.

ORGANIZATION

In Chapter 1, some background information is provided and various schemes for multi-hop networks are compared. We discuss in detail the fundamentals of the design and analysis of encoding, recoding, and decoding of BATS codes in Chapter 2–8. Chapter 2 presents the basic approaches to BATS code encoding and decoding. Chapter 3 introduces a simple BATS code network protocol. Chapter 4 discusses some advanced recoding techniques. The first four chapters are essential for readers who want to conduct BATS code-related research.

Chapters 5–9 comprise the major technical contents for the performance analysis and coding design for BATS codes. Chapter 5 analyzes the asymptotic performance of BP decoding. Chapter 6 discusses the achievable rates of BATS codes. Chapter 7 focuses on the finite-length analysis of BP decoding. Chapter 8 is devoted to inactivation decoding, including the finite-length analysis and practical design. Chapter 9 discusses how to apply BATS codes in a general network topology.

For readers with different interests, this book can be read in part as follows. To learn how to design a network protocol using BATS codes, Chapter 3 is the place to start with, and Chapters 4 and 9 include some further discussions. To understand how a degree distribution is designed, Section 5.1 should be read followed by Chapter 6 and Section 7.4. To study finite-length analysis, the first three sections of Chapter 7 and Section 8.2 should be read.

A major part of this book is rewriting and unifying the previous works of the authors [54, 95, 96, 97, 98, 101, 102]. There is also a significant part of the book consisting of new results that have not been published before. These include Chapter 4 on advanced recoding techniques, Sections 6.3 and 6.4 on BATS codes for multiple rank distributions, and Section 8.3 on practical design of inactivation decoding. Variations of BATS codes, e.g., BATS codes with variable batch sizes [98], quasi-universal BATS codes [91], and BATS codes with unequal error protection [90], are not discussed in this book.

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