Errata: Convergence Analysis of Evolutionary Algorithms That Are Based on the Paradigm of Information Geometry

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Four lines below Equation (7), page 682, it must correctly read:

$$\mathbf{Q} = -\frac{1}{2}\mathbf{H}_f(\mathbf{x}) = -\frac{1}{2}\nabla\nabla^{\mathrm{T}}f(\mathbf{x}).$$

In Equation (27), page 685, the logarithms are missing, it must correctly read:

$$I_{ij}(\boldsymbol{\theta}) = \int \frac{\partial \ln p(\boldsymbol{x}|\boldsymbol{\theta})}{\partial \theta_i} \frac{\partial \ln p(\boldsymbol{x}|\boldsymbol{\theta})}{\partial \theta_j} p(\boldsymbol{x}|\boldsymbol{\theta}) \, \mathrm{d}^N \boldsymbol{x} = -\int \frac{\partial^2 \ln p(\boldsymbol{x}|\boldsymbol{\theta})}{\partial \theta_i \partial \theta_j} p(\boldsymbol{x}|\boldsymbol{\theta}) \, \mathrm{d}^N \boldsymbol{x}$$
(27)

On page 687, below Equation (36), the derivations leading to Equation (40) must be corrected¹ (the final result in Equation (41) does not change by this correction):

Treating the **C**-related part in (35) using $\frac{\partial C_{ab}}{\partial C_{cd}} = \frac{1}{2} \left(\delta_{ac} \delta_{bd} + \delta_{ad} \delta_{bc} \right)$ (here, symmetry of **C** must be taken into account) yields

$$I_{(\alpha_{1}\alpha_{2}),(\beta_{1}\beta_{2})} = \frac{1}{2} \sum_{k,l,m,n} C_{kl}^{-1} \frac{\partial C_{lm}}{\partial C_{\alpha_{1}\alpha_{2}}} C_{mn}^{-1} \frac{\partial C_{nk}}{\partial C_{\beta_{1}\beta_{2}}}$$
$$= \frac{1}{8} \sum_{k,l,m,n} C_{kl}^{-1} (\delta_{l\alpha_{1}}\delta_{m\alpha_{2}} + \delta_{l\alpha_{2}}\delta_{m\alpha_{1}}) C_{mn}^{-1} (\delta_{n\beta_{1}}\delta_{k\beta_{2}} + \delta_{n\beta_{2}}\delta_{k\beta_{1}}).$$
(37)

Thus, one gets for the **C**-related part of θ (taking the symmetry of **C**⁻¹ into account)

C:
$$I_{(\alpha_1\alpha_2),(\beta_1\beta_2)} = \frac{1}{4} \left(C_{\alpha_1\beta_1}^{-1} C_{\alpha_2\beta_2}^{-1} + C_{\alpha_1\beta_2}^{-1} C_{\alpha_2\beta_1}^{-1} \right).$$
 (38)

The non-numbered equation below Equation (40), page 687, must be adopted accordingly:

The correctness of

$$\mathbf{C}: \quad I_{(\alpha_1\alpha_2),(\beta_1\beta_2)}^{-1} = 2C_{\alpha_1\beta_2}C_{\beta_1\alpha_2}$$
(40)

¹The author is grateful to Zhenhua Li for pointing out this mistake to be corrected here.

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is proven directly by checking

$$\sum_{\beta_1,\beta_2} I_{(\alpha_1\alpha_2),(\beta_1\beta_2)}^{-1} I_{(\beta_1\beta_2),(\gamma_1\gamma_2)} = \frac{1}{2} \left(\delta_{\alpha_1\gamma_2} \delta_{\alpha_2\gamma_1} + \delta_{\alpha_1\gamma_1} \delta_{\alpha_2\gamma_2} \right).$$

On page 694, the derivation of Equation (83) must be corrected (again without consequences for the result in Equation (84)):

$$\frac{\partial \sigma_{f}}{\partial C_{mn}} = \frac{1}{2} \frac{1}{\sigma_{f}} \frac{\partial}{\partial C_{mn}} \left(\sum_{i,j,k,l} (a_{i} - 2\overline{x}_{k}Q_{ki})C_{ij}(a_{j} - 2\overline{x}_{l}Q_{lj}) + 2Q_{ij}C_{jk}Q_{kl}C_{li} \right) \\
= \frac{1}{2} \frac{1}{\sigma_{f}} \sum_{i,j,k,l} \left((a_{i} - 2\overline{x}_{k}Q_{ki}) \frac{1}{2} (\delta_{im}\delta_{jn} + \delta_{in}\delta_{jm})(a_{j} - 2\overline{x}_{l}Q_{lj}) \\
+ Q_{ij} (\delta_{jm}\delta_{kn} + \delta_{jn}\delta_{km})Q_{kl}C_{li} + Q_{ij}C_{jk}Q_{kl}(\delta_{lm}\delta_{in} + \delta_{ln}\delta_{im}) \right) \\
= \frac{1}{2} \frac{1}{\sigma_{f}} \sum_{k,l} \left((a_{m} - 2\overline{x}_{k}Q_{km})(a_{n} - 2\overline{x}_{l}Q_{ln}) + 4Q_{mk}C_{kl}Q_{ln} \right) \tag{83}$$