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CGEBox: A Flexible, Modular and Extendable Framework for CGE Analysis in GAMS

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We present CGEBox, an open-source and open-access framework for regional and global Computable General Equilibrium analysis implemented in the General Algebraic Modeling System (GAMS) software. It flexibly depicts different nestings in production and factor supply, supports different functional forms for demand and choices in modeling international trade sectors (Armington, Armington plus Constant Elasticity of Transformation to distribute supply, Melitz and Krugman model). Either a regional household approach or separate accounts for government and potentially multiple private households with related closures are available. Supply and factor markets can be dis-aggregated to sub-regions and an implementation for GTAP-AEZ is available. We compare the layout of different well-known global and single country CGE models and discuss to what extent our flexible framework can replicate these layouts. In a structural sensitivity analysis, we compare major results under multi-lateral trade liberalization and endowment changes in one country for different model configurations. These reflect important structural differences between the chosen examples as well as additional features such as the Melitz model or endogenous capital stocks driven by investments in a comparative-static setting. We find relative limited differences in global and regional welfare between models based on the Armington assumption, even if other features differ such as closures, nestings or functional form in demand. A discussion on further joint development of such a framework leads to our summary and conclusions.

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Keywords: Computable general equilibrium analysis; Modularity; Structural sensitivity analysis.

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1. Background and motivation

Computable General Equilibrium (CGE) models are perhaps the most widely used quantitative tool in economic policy impact analysis. For global CGE analysis, the Global Trade Analysis Project (Hertel, 1997) has provided for more than two decades the necessary data and parameters. Besides the GTAP Standard model (Hertel, 1997 and Corong et al., 2017) and its many variants, global CGE frameworks such as GLOBE (McDonald and Thierfelder, 2014), MIRAGE (Modelling International Relationships in Applied General Equilibrium, Decreux and Valin H., 2007) or ENVISAGE (Environmental Impact and Sustainability Applied General Equilibrium model, van der Mensbrugghe 2008) all draw on the Global Trade Analysis Project (GTAP) Data Base.[1] Even earlier, single country frameworks became popular such as the so-called IFPRI standard model (IPFRI-S, Lofgren et al., 2002) or STAGE (Standard Single Country CGE model, McDonald, 2015), which both have been extended in various directions such as multiple households and sub-national detail (cf. Dorosh and Thurlow, 2012), dealing with water issues (cf. Luckmann et al., 2014) or migration (Flaig et al., 2013). The GTAP Standard model is open source, which also holds for most of its variants such as GTAP-E (Energy-Environmental Version of the GTAP model, Burniaux and Truong, 2002) which offers detail for energy markets, GTAP-AGR (A Framework for Assessing the Implications of Multilateral Changes in Agricultural Policies, Keeney and Hertel 2005) with features relevant when analyzing agricultural issues or GTAP-AEZ (Agro-Ecological Zoned version of the GTAP model, Lee et al., 2005) related to land use. While the GTAP family mostly uses the specialized software package for General Equilibrium modeling GEMPACK (Codsi and Pearson, 1988), the other global models mentioned above are coded in GAMS (Brooke et al., 1998). Some of them, such as GLOBE, the IFPRI Standard model, STAGE or ENVISAGE are equally open source. Extended versions of these models developed for specific clients might however be copyright protected. Restricted access also holds for MIRAGE. Copyright protected models are not fully transparent as simulations cannot be replicated by outsiders. Furthermore, it is typically impossible to check if published documentation matches the actual code base.

Despite the availability of numerous CGE models of which we mentioned only a few, both the global and single country CGE models share a common basic structure such as representing the technology with nested Constant Elasticity of

[1] See Aguiar et al., 2016 for a description of the most recent public version of the GTAP Data Base.

Substitution (CES) structures.[2] Other building blocks such as modeling bilateral import demand based on the Armington assumption, using a Constant Elasticity of Transformation (CET) based approach to complement the Armington assumption on the supply side or CET nests to model sluggish factor supply are equally common. We will in the following sections show that this common core is quite large in the examples we analyze. But clearly, there are crucial structural differences in detail which can matter in policy impact analysis, besides differences in parameterization and the chosen regional and sectoral aggregation.

A modeler wishing to apply a combination of specific features is often confronted with the situation that no model with the desired layout can be readily accessed or even exists. That implies either extending one of the existing models to cover the desired features or compromising on the model layout. As familiarizing oneself with large-scale models is time-consuming, switching between models or model families for specific applications is unlikely. Furthermore, and perhaps more importantly, the community of CGE modelers as a whole does not share effectively costs related to the implementation of new features. Coding efforts are duplicated and potentially useful features not available in specific models. Additionally, the current situation makes it nearly impossible to pinpoint which differences in structure, parameterization, aggregation or shock design provoke deviating impacts found by different models in similar experiments.

We therefore propose in here a modeling framework which tries to address these shortcomings at least partly. Firstly, the code is not only open-source, but thought to be further developed based on open source joint development, i.e. a network of developers adds and shares extensions to the code, with the aim to better distribute development costs in the community. Secondly, we aim at "one code base, many models", i.e. a modular and flexible design which renders extensions or alternative implementations to a large extent mutually compatible. As a consequence, it should become easier both to flexibly adjust the model's structure for a specific study and to perform sensitivity analysis with structural features. That clearly also improves overall transparency, not at least as experiments are easier to replicate. Modularity is clearly also necessary to make it inviting to contribute, as it ensures that contributors remain rather free in deciding about the model design they later apply. To our knowledge, there are so far only a few attempts in the direction, mainly the GEMPACK based MAGNET (Modular Applied GeNeral Equilibrium Tool, Woltjer and Kuiper, 2013) model which however is not open source and probably less flexible compared to what is discussed in here, but provides other features with a focus on agriculture and food

[2] One notable exception is the IGEM model developed by Jorgenson and colleagues, which uses a translog flexible functional form (Jorgenson et al., 2013).

markets and the EU, such as depicting for instance milk and sugar production quotas.[3]

The paper is organized as follows. After a brief discussion of the concept of CGEBox, the second section reviews some of the more well-known CGE models with regard to specific features, such as the presentation of technology, firm behavior, trade, final demand, closures and how income distribution and accounts for private households and the government are modelled. We next show in section four, based on a partial multi-lateral trade liberalization experiment and a total factor productivity shock in one region, how differences in model layout—all realized with our modular system—impact major results, mimicking in these experiments major elements of the previously analyzed well-known models. Section five discusses institutional issues around shared development before we conclude and summarize.

2. Concept of CGEBox

GAMS based global CGE models which come close to the GTAP Standard model have been available for a while, such as the well-known GTAPinGAMS implementation (Rutherford and Arbor, 2005 and Lanz and Rutherford, 2016). However, the first faithful replication was only recently available, which provided the starting point for the work on CGEBox (Britz and van der Mensbrugghe, 2016) combined with a Graphical User Interface (GUI). A GUI was deemed useful to allow students to work in classes with the model without requiring a longer targeted course on the specific model implementation. At the same time, the GUI allows to efficiently configure the model and formulate shocks while it eases result analysis based on tables, graphs and maps. That might make use of the GUI inviting also for more seasoned policy analysts. However, the GAMS code can equally be used without the GUI. That work was presented in a pre-conference event of the GTAP conference 2016.

The original code is largely based on the GAMS code of ENVISAGE (van der Mensbrugghe, 2008) and therefore comprises many features found in ENVISAGE. That rendered it inviting to not only replicate version 7 of the GTAP Standard model, but to also allow for variants and extensions based on a modular concept (see Table 1). We define here as a module a building block of a CGE such as e.g. the equations, parameters and variables describing final demand or the production structure. Technically, a module consists of the software code related to such a building block, i.e. code which reads the necessary data and parameters for benchmarking, performs the benchmarking, generates the equations used for simulations and performs post-model processing. In the modular set-up of

[3] CGEBox discussed in here can be solved with an mixed complementarity (MCP) formulation, which allows introducing production or tariff rate quotas without further coding efforts.

CGEBox, building blocks can be exchanged or added to derive model variants. In some cases, variants can be realized by changing solely the parameterization of CGEBox. The documentation of CGEBox (Britz, 2018, provided in the supplementary materials available online with this paper) presents the overall concept, the core equations, its GUI and in detail the different modules mentioned below. Additionally, the documentation of the Standard GTAP model in GAMS, version 7 (van der Mensbrugghe, 2018) can be consulted with which CGEBox shares the core equations. The appendix documents the core equations of CGEBox and refers to equations in van der Mensbrugghe, 2018.

Different projects, mostly course work with students, led to modular implementations of a range of extensions (see Table 1), namely the concepts of GTAP-AEZ (Lee et al., 2005; see Britz, 2018[4] p151-157), which depicts land use at the level of Agro-Ecological Zones, GTAP-AGR (Keeney and Hertel, 2005; see Britz; 2018 p92-94) which introduces specific nestings in agricultural production and in factor supply to agricultural and non-agricultural sector, and GTAP-E (Burniaux and Truong, 2002; see Britz, 2018, p92) with its specific nestings for energy demand. The modules draw on the published parameters and data, and in the case of GTAP-AEZ, additionally on the GTAP-AEZ data base (Baldos, 2017). Recently, an implementation of the Melitz model (Britz and Jafari, 2018; see Britz, 2018, p127-150) was added, which can also be turned into a Krugman model, default parameters stem from literature. Equally, incorporation of multiple private households and other features relevant for work with household surveys were integrated (for the implementation see Britz, 2018, p58-72), based on features of myGTAP (Walmsley and Minor, 2013), ENVISAGE and GLOBE. The necessary data and parameters need to be provided by the modeler. Drawing on earlier work with regionalized single country CGE models, the model can depict the production side and factor markets at subnational level (see Britz, 2018, p73-78); currently, 280 SAMs at the NUTS 2 level for most European countries are available to use that feature. The regional SAMs were enriched with detail for agriculture from the CAPRI data base (Britz and Witzke, 2014) which also contributed land use data to integrate the NUTS2 resolution with GTAP-AEZ. Thanks to access to split factors used in the OECD's Metro model (OECD, 2016) the bilateral trade can be modelled differentiated by total intermediate, investment, government and final demand in a MRIO module (for the implementation see Britz, 2018, p79-82).

[4] Britz 2018 refers to documentation of CGEBox which is comprised in the supplementary material available online with the paper on the journal website, see "cgebox\doc\CGEBox_meth_tech_documentation.pdf".

Table 1:Modules and extensions in CGEBox

Module	Remarks
Data filter	Optionally removes small transactions from SAMS / trade matrices while maintaining closely important totals. Thought to support model applications with highly dis-aggregated data bases. Draws on code by T. Rutherford
GTAP-Standard V7	With extensions from ENVISAGE such as non-diagonal make matrix, CET on export side
Completely flexible nesting of production functions	Generate variants of the standard GTAP model currently available which differ in nesting of factors / intermediates / factor such as GTAP-E
Completely flexible nesting for factor supply	Generate variants of the standard GTAP model currently available which use nested CET structures to describe factor supply, such gas GTAP-AGR
Completely flexible sub-nests under final demand	Generate variants of the standard GTAP model currently available which use CES-subnests under the top-level final demand equation
CDE/LES/CD/AIDADS functions for final demand	LES Parameters derived from CDE parameterization, AIDADS parameters empirically estimated
GTAP-AEZ	Additionally: Land supply elasticities for natural land cover, additional nest, volume preserving CET for upper nests
GTAP-AGR	Applicable also to regional dis-aggregation different from original GEMPACK implementation, uses the flexible nesting approach, adjusts to sectoral detail.
GTAP-E	Based on flexible nesting approach, adjusts to sectoral detail
GTAP-Melitz	Includes a fixed cost nest based on the flexible nesting approach, sector coverage can be flexibly chosen. Can also be turned in a Krugman specification.
GTAP-MRIO	Differentiation of bilateral import demand by total intermediate demand and each final demand agent
GRDEM	Recursive-dynamic long-run version for baseline construction and counterfactual
myGTAP	Removes the regional household, supports multiple private households
Aggregate Armington aggregator for intermediate demand	Domestic and import shares for intermediate demand and related tax rates are not sector specific, removes a large share of equations
Aggregate Armington aggregator for all agents	Domestic and import shares for intermediate demand and related tax rates are not agent specific, removes a large share of equations
Third level nest for Armington / CET	Might avoid numerical problems with tiny shares, feature from GLOBE
Tariff lines	Allows a CET/CES dis-aggregation of selected bilateral trade links, explicit TRQ mechanism
Capital vintages	Draws on similar mechanism used in recursive-dynamic CGE models which differentiate vintage from new capital
NUTS2 break down for European countries	Breaks down production decisions and factor markets to sub-regional level, currently data available for NUTS2 administrative regions for Europe
Post-model reporting	Generates SAM like structure, calculates world totals, regional and sectoral totals based on additional GTAP agg file, welfare decomposition etc., feeds into GUI exploitation tools
Single region mode	Fixes import prices and let export demand react to lower Armington nest at export destinations.
Partial Equilibrium closure	Solves only one or some commodity markets and all factor markets, regional or household income exogenous
CO2 emissions	Can be combined with taxation or CO2 trading permits
Non-CO2 GHG emissions	Only post model reporting

Note: for details, refer to Britz, 2018

Based on the core code inherited from ENVISAGE, the model supports non-diagonal make matrices which can be combined with a CET nest to allocate production to multiple outputs and a CES nest on the demand side to differentiate between the same product being produced by several production activities (Britz, 2018 p22). The implementations of GTAP-AGR and GTAP-E draw on a flexible, multi-stage nesting approach which is realized via set-definitions in GAMS (Britz, 2018 p52-57). It allows adjusting the nesting in the production functions and in factor allocation across sectors without changing the equation structure of the model, even if multi-stage nests are used. A similar concept allows to aggregate commodities in final demand and use CES-nests inside these aggregates to allow for more flexibility in capturing cross-price effects in demand. Different final demand systems are supported (Britz, 2018, p29-30) as discussed below in section 3.5.1. As with most other CGE models, a larger set of typical closures for the different accounts are coded.

Furthermore, the code was set up such that the model can be used in comparative-static and recursive dynamic modes. That led to the development of G-RDEM (Britz and Roson, 2018; for the implementation see Britz, 2018, p94-126), a model for long-term baseline generation and analysis integrated in CGEBox. Furthermore, a single country CGE can be derived and the model can be turned into a partial equilibrium model by solving only one, some or all commodity markets while treating income as exogenous. The majority of these choices, such which modules to use and related options, can be specified through the GUI (see Britz, 2018, p183-202).

The data input tool in CGEBox combines features from ENVISAGE and GTAPinGAMS and allows to read directly, specifically, the latter contributed a filtering algorithm which allows elimination of tiny transactions from the global SAM (see Britz, 2018, p160-168). That algorithm was further improved and a pre-solve algorithm developed (see Britz, 2018, p202-205) with solves single country CGE models before the full global model, a combination which has proven to allow solving large model variants relatively fast (see Britz and van der Mensbrugghe, 2016).

Based on modular concept defined above, the different features and extensions can be switched on and off mostly independently from each other (see section 4). That flexibility raises almost naturally the question to what extent the framework can already replicate the structure of some well-known existing models. The next section therefore provides a stocktake by reviewing structural differences in selected CGE models and discusses to which extend they can be replicated in CGEBox, before we perform sensitivity analysis with some major differences.

Naturally, after decades of developments of CGE models for different research questions, it is impossible to define the universe of all CGE models, while it is already quite challenging to get an overview of those CGE models which are open-source. As such, many potentially interesting features developed for specific CGE models are missing in CGEBox which is hence not the CGE "superset" model, albeit additional features could be certainly added. We turn to that question again in section 5.

3. Structural differences between some selected CGE modelling frameworks

3.1 Data input

All global CGE models reviewed in here draw on the GTAP Data Base which is released in GEMPACK format. The models can hence either use GEMPACK based utilities distributed with the GTAP Data Base such as GTAPAgg (Horridge, 2006) to further transform the data before converting them to GAMS or convert the global SAM in its original format with full regional and sectoral detail and provide their own tools for further processing, for instance, for aggregation over sectors and regions. At least GLOBE and ENVISAGE are SAM based on the input side which implies that they integrate the different matrices which jointly represent the global GTAP Data Base into one single global SAM. The single country CGE models draw on country specific SAMs which can be flexible integrated, for instance as a spreadsheet. The SAM based character eases an overview on the model's structure as basically for any transaction in the (global) SAM, matching variables in prices and quantities must be defined. Next, these variables can be either fixed or an equation must be defined which renders them endogenous. The documentation of GLOBE (McDonald and Thierfelder, 2014) follows that principle rather stringently. Models such as GLOBE also re-balance the global SAM, for instance in order to resolve rounding errors.

As indicated above, the data driver of CGEBox is closely linked to ENVISAGE and embeds a filtering algorithm to remove small entries from the global SAM and re-balance it afterwards which also allows for manual corrections. In order to input data into CGEBox, the default solution is to employ the GTAPAgg utility to define a pre-aggregation of the GTAP Data Base. If only the GTAP Data Base without any auxiliary data is used, the output from GTAPAgg is used directly, otherwise, the GAMS code performs the aggregation to a desired level of sector and regional detail and only uses the aggregate definitions stored by GTAPAgg along with the database. A separate post-aggregation of the global SAM allows deriving a non-diagonal make structure. Equally, the code supports a SAM split based on user provided split factors (see Britz, 2018, p168-175) which in case for agri-food sectors can also be more or less automated derived from the around 130 sectors of the FABIO MRIO (see Britz, 2018, p222-234). As in the case of using the MRIO factors from METRO, data balancing is based on a Highest Posterior Density estimator (Heckelei et al., 2005) which ensures that the SAM entries introduced with the split and the dis-aggregated bilateral trade flows fit to the given global SAMs. The same concept is used to render sub-national data at NUTS2 level consistent to national ones at AEZ level.

3.2 Production technology

All CGE frameworks analyzed in here use nested CES functions to depict technology. While they all assume that the composition of Value Added from different primary factors is price dependent, they might allow or not for substitution in the input composition at others nodes of these nests and instead use a Leontief presentation. Furthermore, all global models depict intermediate demand for imports and domestic production based on the Armington assumption. However, demand shares might be differentiated by sector or not, or even be identical across all demand agents. In MIRAGE, the Armington model is extended for certain sectors to yield a Krugman model.

Perhaps the most basic layout is found in the Standard GTAP Model and GTAP in GAMS which assume Leontief relations between the value added nest and all intermediates. Each sector features its own Armington nest to source intermediate demand for each commodity from domestic origin and imports. The import composition from different origins is identical across all agents and hence not sector specific. In that regard, the standard layout of GLOBE goes even further by having identical shares for imports and domestic origin for each agent, i.e. including private, government consumption and investments. Variants of the GTAP model such as GTAP-AGR or GTAP-E introduce more complex nesting structures in the production function which might also involve combinations of primary factors and intermediates.

MIRAGE (Decreux and Valin, 2007, p. 12) assumes a Leontief relation between the Value Added and intermediate composite whereas the latter allows for substitution between individual intermediate commodities. The value added composite comprises a sub-nest which combines capital and skilled labor. As in the standard GTAP model, each sector splits up intermediate demand for each commodity between domestic origin and imports based on the Armington assumption, while the import shares are driven by a nest which is shared by all sectors and final demand. The production nesting in GLOBE (McDonald and Thierfelder, 2014, p. 23) provides a third approach as it assumes by default that the value added nest and the intermediate composite can be substituted while the intermediate composite remains a Leontief aggregate as in the standard GTAP model. Inside the value-added nest of GLOBE, labor is modeled as a CES nest of skilled and unskilled labor, an assumption also found for instance in the GTAP-E extension.

Table 2: Overview production function nesting in the different models

Model	Production function nesting		
	Value Added – Intermediate composite	Intermediate composite	Value Added
GTAP Standard	Leontief	Leontief	CES
GTAPinGAMS	Leontief	Leontief	CES
GLOBE	Leontief	CES	CES, with sub-nest for skilled/unskilled labor
MIRAGE	Leontief	CES	CES, with capital-skilled labor sub-nest
ENVISAGE	CES	CES with sub-nests for energy	CES, flexibility to have skilled bundled with unskilled or with capital

Source: Author summary

ENVISAGE allows for substitution between the value and intermediate composite, between intermediate composite and introduces a nesting to differentiate between energy commodities similar to GTAP-E. ENVISAGE supports in most cases a choice between a CES and CD-representation which requires two alternatives expressions for dual price aggregators.

Clearly, besides the specific nestings, substitution elasticities matter. They can in many cases be defined region and sector specific and might be even adjusted for specific applications. Table 2 above summarizes the major differences discussed above. Generally, the flexible nesting approach in CGEBox allows the tool to easily mimic the different nested CES structures employed in the models discussed in here. Equally, it inherits from ENVISAGE the possibility to use a Cobb-Douglas instead of a CES specification, a feature also found in GEMPACK and MPSGE based models. The flexible nesting approach can distribute the costs of an intermediate input, a primary factor or of nests to different nests, as currently used to depict fixed costs in the Melitz implementation (Britz and Jafari, 2018).

3.3 Factor supply and mobility

Most models consider the economy wide stock of primary factors as fixed. Allowing for price dependent factor supply e.g. based on a land supply or wage curve might render the model more realistic. Here ENVISAGE offers a rich choice as different functional forms can be chosen. However, endogenous factor stocks provide a challenge for welfare analysis: extended factor endowments allow for higher overall output and thus welfare gains while at the same time, a down-sloping factor supply curve implies some costs. As these costs are typically not linked to resource use in production, they must relate to utility losses, such as less leisure or increased negative externalities. Consequently, the downward sloping factor supply should be accounted for by a utility function in welfare analysis.

Furthermore, factors might be considered fully mobile, i.e. assuming homogeneity and the law of one price, partially (im)mobile based on a CET approach, typically termed sluggish factor supply, and fully immobile by rendering them sector specific. Most models allow for a flexible choice between these solutions. The GTAP Standard model renders natural resources such as minerals or fish stocks sector specific and thus immobile, land as sluggish and the other sectors as fully mobile. MIRAGE uses nested CET functions to split up factor supply for instance between agricultural and non-agricultural sectors. An extension is offered in variants of STAGE where physical units of factors and factor remuneration are distinguished in the data base which allows explicitly considering that moving factors between sectors affects average factor productivity (cf. Flaig et al., 2013). Somewhat similarly, ENVISAGE allows for segmented labor markets à la Harris-Todaro (Harris and Todaro, 1970), which regulates rural-to-urban migration based on wage differentials.

Many models allow fixing factor prices instead of factor stocks, an approach typically used for unskilled labor to endogenize the (un)employment rate. If the model is solved as an MCP, a reservation wage rate can be modelled as a price floor. In conjunction with a maximal stock for labor, a regime switch between unemployment at the fixed lower wage and flexible wages at full employment can be depicted. The model documentations suggest that only MIRAGE uses that mechanism as a default for unskilled labor in some developing countries, while it is supported by several other models. Our own tests suggest that solving large-scale models as an MCP can slow down the solution compared to solving a simple constrained system of equations. MCP solution time increases further if a shock requires a larger set of redefinitions where variables are at their bounds and the equations become slack, a case where a constrained system of equations is declared infeasible.

Generally, the possibility to solve the model as an MCP combined with the flexible nesting approach allows CGEBox to host the different variants for factor supply depicted above. The GTAP-AGR and GTAP-E extensions already employ flexible nesting structures and seem at least close to the solutions in ENVISAGE and MIRAGE. However, updating factor productivity when factors move between sectors is not yet supported.

3.4 Income distribution

The GTAP Standard Model and GTAPinGAMS use the concept of the regional household which collects factor and tax income and distributes it to private and government consumption and savings based on a modified Cobb-Douglas utility function where the share parameters depend on the utility of consumption expenditure with regard to income. MIRAGE simplifies that structure further by lumping final and government consumption together. The regional household approach with its single income collection and distribution node does not allow reflecting relations between earnings and expenditures of different agents.

GLOBE therefore refrains from a regional household approach. Rather, one or several representative private households receive a share of factor income net of factor taxes from which direct taxes are deducted based on ad valorem rates. The after-tax income of these households is distributed to savings and consumption with different closures for the saving rate available. The STAGE model adds intra-household transfers, transfers from enterprises and from government as additional income sources. That is rather similar to IFPRI-S which also considers enterprises as an intermediate layer between factor cost paid by sectors and factor income received by households (Lofgren et al., 2002, p. 19). ENVISAGE uses one representative private household[5] while also incorporating the bilateral remittances from GTAP's GMIG database and cross-border profit flows from GTAP's GDYN database.[6] All models depict tax income by a single government agent in quite some detail.

CGEBox uses either the regional household approach of the GTAP Standard Model or an approach drawing mostly on myGTAP (Walmsley and Minor, 2013), with elements added found in STAGE and ENVISAGE; i.e. it can consider remittances and international capital transfers as well as transfers between households in the same region. However, the enterprise approach from STAGE and IFPRI-S cannot be depicted directly by CGEBox. Furthermore, factor income shares for the different private households can be sector and factor specific. That allows defining for instance an agricultural household which owns the factors employed in agriculture plus some factor shares in other sectors.

3.5 Final demand and related account closures

3.5.1 Private consumption

The GTAP Standard model uses a Constant Difference in Elasticities (CDE) demand system for private consumption, which is also the default in CGEBox. GLOBE (McDonald and Thierfelder, 2014, p. 53), STAGE and IFPRI-S employ the LES demand system for final private demand. The LES demand system is less flexible compared to the CDE system which has three parameter vectors relating to commodities compared to two in the LES system. MIRAGE uses a CES system with commitment terms. ENVISAGE can translate produced outputs into

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[5] It is coded to allow for multiple households, though this feature has never been used.

[6] The latest version also includes government to government transfers, which captures amongst other things official development assistance (ODA), sourced from the myGTAP model.

commodities demanded by the household based on a transition matrix. Furthermore, savings can be added as a further argument in the LES demand function, which can also be extended to the AIDADS demand system. CGEBox can deploy a CDE, AIDADS, LES or CD system for private households and additionally introduce CES-Nests under aggregated product commodities which can depict the transition matrix from ENVISAGE. Such sub-nests are found in ENV-LINK and GTAP-E to model detail in energy demand.

3.5.2 Government demand

Government demand is lumped together with private demand in MIRAGE. GLOBE and STAGE use either fixed shares of government income in real terms or volumes to depict government demand. Government consumption is fixed in real terms in IFPRI-S. The GTAP Standard model uses a CD-utility function, i.e. fixed value shares. Based on the code of ENVISAGE, CGEBox can accommodate either a CD or CES demand system for the government, the latter hence also allows capturing the fixed in real terms representation found optionally in GLOBE and STAGE by setting the substitution elasticities to zero. Fixing government consumption for each commodity as in IFPRI-S is equally possible in CGEBox as a pre-compiled closure rule.

3.5.3 Savings and investment

All models implicitly consider in their standard layout government savings as residual. GLOBE and STAGE comprises a well-developed system to render different tax rate endogenous which allows fixing government saving. CGEBox allows the same type of closure; however, the choice of which tax rates are endogenously adjusted is more restricted. ENVISAGE drives government expenditures as a share of GDP, and fixes government savings. The government account is then closed by an endogenous shift of direct tax rates. With the exception of integrating savings into the LES demand system, CGEBox seems to be able to depict all variants to model final demand and the different closures for the final, government and savings account found in the discussed models.

MIRAGE as a dynamic model assumes in any one year that the existing capital stock is immobile. Allocation of regional and foreign savings to sector and regions in MIRAGE is driven by differences between capital returns, based on elasticities. It also lets capital revenue from foreign savings flow back to the source country. The FDI implementation shows thus some similarity to the global bank mechanism in the GTAP standard model, but tracks additionally the bilateral allocation of capital.

ENVISAGE uses a vintage concept where existing (depreciated) capital stock is immobile or sluggish, and new capital stock is fully mobile. Furthermore, each production sector is split into two activities: one that uses installed capital and the other *new* capital. That complex mechanism is not fully supported by the so-called capital vintage module of CGEBox, rather depreciated capital stock is considered immobile in CGEBox and investments define an endogenous stock of fully mobile new capital. That implies that the capital accumulation process depicted in a recursive-dynamic framework can be integrated in a comparative-static one in CGEBox by indicating over how many years the capital stock is depreciated in a comparative-static experiment.

3.6. Trade and imperfect competition

3.6.1 Armington specification and CET

All global models are based on the Armington assumption and their specific layouts can be seen as variants of the two-stage CES specification found in the GTAP standard model where the upper nest differentiates between domestic origin and aggregate imports and the lower nest between imports by origin. GLOBE adds a third nest on demand which splits up imports into two nests with bilateral trade flows which are large and small in shares. The small share nest is a Leontief aggregate; it is up to the user to set the related cut-off (McDonald and Thierfelder, 2014, p. 21). That nest with the small shares is aggregated with the more standard large-scale nest in Leontief fashion. That solution does hence not solve the "small shares stay small problem" often discussed as a dis-advantage of the Armington specification (cf. Himics and Britz, 2016), but rather helps to avoid numerical problems related to small trade shares. A mirroring implementation is used on the supply side based on three-stage CET nests.

GLOBE and MIRAGE deviate from the other global models as the different sectors and final demand share the top level Armington nest, i.e. all agents have equal shares of domestic and imported goods in their consumption. CGEBox allows on demand both the GLOBE solution where all Armington agents share both nests and an intermediate solution where all intermediate demand shares are equal. Equally, CGEBox features the third level based on small import and export shares found in GLOBE. MIRAGE features potentially also a third nest, however here, it is introduced in some sectors to distinguish between imports from developed and developing economies under the assumptions that qualities imported inside each of that group are more similar (Decreux and Valin, 2007, p. 10).

The single country CGE models do usually not differentiate between importers. STAGE (McDonald, 2015, p. 28) uses the Armington assumption to model domestic produce and imports as imperfect substitutes. The world market prices for imports and exports can be either fixed or in case of the export price can be based on downward sloping demand curves (Mc Donald, 2015, p. 54). A similar solution is used in IFPRI-S (Lofgren et al., 2002).

ENVISAGE can alternatively depict goods as homogenous to derive a net-trade specification with homogenous world market prices. That however requires reconstructing the SAM in order to level out e.g. differences in bilateral tax rates. The description also implies that the trade margins are absent for homogenous commodities. It is hence not clear if that extension is widely used. It is not supported by CGEBox. High substitutions elasticities in the Armington nests as used in the GTAP Standard model for instance for natural gas will probably yield similar results.

3.6.2 Imperfect competition

A recent extension of the GTAP model family is GTAP-HET (Akgul et al., 2016) which introduces heterogeneous firms based on Melitz 2003 into the GTAP structure, considering vertical differentiation inside sectors under monopolistic competition. A similar implementation is available for CGEBox (Britz and Jafari, 2018) and discussed in Dixon et al., 2016. MIRAGE considers imperfect competition for some sectors, however based on the more restricted model by Harrison et al., 1997 which draws on Krugman, 1979 where fixed costs occur only at sector level and are not differentiated by trade link. However, MIRAGE employs the imperfect competition framework in a multi-level CES framework where the lowest level differentiates the different varieties, while GTAP-HET and CGEBox use one nest only such that no differentiation of the substitution elasticities such as in MIRAGE is possible. An implementation of the Melitz/Krugman model maintaining the original two or three stage Armington structure is CGEBox is in prototype phase. CGEBox can simplify the Melitz model to yield the Harrison et al., 1997 implementation by setting the fixed costs on each trade link to zero which also implies that the number of varieties is not differentiated by trade link.

Another aspect of modeling international trade relates to international transport services. The Standard GTAP model introduces a global transport sector which allocates total global transport demand to the different regions based on a CD function, while the per unit demand for the transport margin on each trade link is a fixed Leontief coefficient. That structure is employed by GTAPinGAMS, MIRAGE and ENVISAGE as well. GLOBE uses a somewhat more complex system (McDonald and Thierfelder, 2014, p. 27) which requires allocating bilateral transport sector demands to the regions exporting the transport services. As a consequence, GLOBE has fully specified bilateral trade balances.

Summarizing, all models are based on the Armington assumption and, besides MIRAGE, assume perfect competition. Only GLOBE and ENVISAGE apply a CET on the export side.[7] CGEBox is currently only able in a prototype implementation to perfectly replicate MIRAGE as different substitution elasticities in import flows

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[7] In the case of ENVISAGE, it is an option. The default specification is perfect transformation as in the standard GTAP model.

for one commodity are not supported. Furthermore, CGEBox can so far not implement the more complex transport service sector implementation of GLOBE.

When used as a single region model, CGEBox can either fix international prices, use iso-elastic function to render them endogenous depending on export or import quantities or use the lower CES and CET nests at export and import destinations to render bilateral imports and exports and related prices endogenous. The CES/CET nests can also be replaced by the Melitz or Krugman implementation of trading partners in a single country setup.

3.7 Macro-Economic closures and numéraires

3.7.1 Numéraires

The behavioral functions of these neo-classical models are homogenous of degree zero in prices. Thus, all of the global CGE models have a single global numéraire that anchors the price system. For example, the standard GTAP model uses a global index of factor prices as the model numéraire and ENVISAGE uses an index of manufactured export prices from developed countries. Any single price or price index can be used as an anchor. GLOBE uses a somewhat different price mechanism, where, on top of a global numéraire, it also includes a regional price index, which is fixed. It introduces a 'nominal' exchange rate that converts domestic prices to 'international' prices in cross-border flows such as trade, transfers, etc. The choice of these numéraires does not affect the simulated quantity changes, but needs to be reflected when simulated price and value changes are analyzed, especially when comparing different regions. CGEBox either works with a fixed exchange rate, or, as e.g. in GLOBE, endogenizes the exchange rate and uses consumer, producer or factor price indices as regional numéraires.

3.7.2 Balance of payment and trade balance

The GTAP standard model features a so-called global bank which distributes foreign savings according to expected return to investments which are derived from returns to the given, fixed capital stock in each region. In GLOBE, STAGE and IFPRI-S, the capital account balance can be maintained by either fixing the foreign savings and solving for the exchange rate or fixing the exchange rates and solving for foreign savings. ENVISAGE uses fixed foreign savings in real terms. Similar to the GTAP model, it does not comprise exchange rates.

CGEBox can use the global bank mechanism, fix foreign savings in international currency, use fixed allocation shares of global foreign saving in international currency or derive the foreign savings based on the regional capital account balance, by fixing the factor price or consumer price index in addition to the exchange rate.

3.8 Software aspects

We restricted ourselves mostly to GAMS-based models while only mentioning the GEMPACK based realization of the GTAP Standard model and some widely used variants thereof. That implies that most models reviewed are realized in GAMS. GLOBE and STAGE use a very similar coding style and are linked to a Graphical User Interface (GUI). MIRAGE uses an EXCEL interface in combination with macros for specifying model options and result exploitation, the latter based on pivot tables which allow combined analysis of several experiments.

CGEBox uses clearly the most complex GAMS code implementation of the reviewed models in order to allow for a modular design. That implies GAMS pre-processor commands for conditional compilation to include certain blocks of equations and related code for data transformations and parameter calibration on demand. Equally, macros are used to substitute out variables and partly as well to support the modular design (see also the core model equations in the appendix). That certainly renders the code less self-explaining compared to the other models with a more straightforward and less flexible implementation. A specific feature of CGEBox is a quite extended post-model processing part which feeds into an exploration system which is shared with some other economic models (Britz et al., 2015). That is part of the GUI of CGEBox which also allows selecting shock files, closures and model features. The GUI is realized by a package which can generate GUIs for GAMS and R projects (Britz 2014) from a simple XML test file. That package also handles post-model analysis with tables, graphs and maps (Britz et al., 2015). For details, refer to the CGEBox documentation (Britz 2018) available as supplementary material.

4 To what extent does structural CGE layout matter?

4.1 Setting up structural experiments close to the different models

In this section, we provide a comparison of simulated impacts on the same data base and shock, using configurations of CGEBox which come as close as currently possible with its so-far implemented modular design to the layout of the global models discussed above. As we are clearly in most cases neither able to fully replicate the structure of these models nor aim at meeting their specific parameterization, we will name these structural sensitivity experiments after the models they are derived from, but put these names in quotes.[8]

The "GTAPinGAMS" experiment comes rather close to the actual specification of the GTAPinGAMS model. The Armington structure is identical to the GTAP

[8] The supplementary material comprises the document "Instructions_to_replicate.pdf" which details how the runs discussed in the following can be replicated. Replication requires a license for the GTAP data base.

Standard model. Private demand is depicted by a CES function[9], total government demand is fixed in real terms. Both government and investment demand are distributed to the different commodities based on a Leontief function.

In our "GLOBE" experiment, we aggregate all Armington agents, use a two-stage CET on the supply side to distribute output to domestic sales and exports in the upper and to trade flows in the second nest. Demand for the private household is depicted by a LES. Separate accounts for the representative private household and the government are introduced, and the regional household consequently removed. The production function does not allow for substitution between value added and intermediate composite, but inside both nests, non-zero substitution is used. The consumer price index is used as the regional numéraire, foreign savings in international currency are fixed and the capital account is closed by flexible exchange rates. While CGEBox can introduce the third level in the Armington and CET to depict small shares via Leontief, we refrain from that in here. What is clearly missing in our "GLOBE" experiments is the GLOBE approach to model international transport demand. Both features have probably a minor impact on results. We would also remind the reader again that we are not aiming at replicating the specific parameterization found in the models of which we try to capture the major structural differences to the GTAP Standard model.

The differences for the "MIRAGE" experiment against the MIRAGE model itself are more pronounced. As in the "GLOBE" experiment, we aggregate the Armington agent. We use a Krugman model to model monopolistic competition under vertical differentiation into varieties, but our implementation is much simpler compared with the one in MIRAGE. It lumps the Armington structure into one nest with common substitution elasticities, whereas MIRAGE features CES nests to differentiate between domestic and imports, within imports between developed and developing, in these two import bundles the different trade flows and finally varieties. MIRAGE also uses a different allocation model of foreign saving; we consider the global bank mechanism as most similar. MIRAGE uses a modified CES system with constant terms at the top level of a final demand system, which combines government and private household demand. We try to mimic that by fixing total government demand in real terms and using a Leontief relation to distribute that total, which means that in final demand net of investment, we now have constant terms as well. Furthermore, we introduce sluggish factor mobility between agricultural and non-agricultural sectors.

For "ENVISAGE", we introduce a CET on the export side, and, as in "MIRAGE", we model sluggish factor supply between agricultural and non-agricultural sectors. Additionally, we use the production nesting of GTAP-E while also considering a specific sub-nest under the LES demand function which

[9] The latest version of GTAPinGAMS (Lanz and Rutherford, 2016) can incorporate a CDE demand system.

substitutes between different types of energies. Government and investment demand use a CD utility function. Government savings and real government consumption are fixed; the government account is closed by updating direct taxes. Foreign savings are fixed as well.

Finally, we add two configurations which make use of modules already available in CGEBox. Firstly, we use the GTAP-AEZ module to dis-aggregate land-use, which also adds factor price dependent land supply to agriculture and forestry in each AEZ. Moreover, depreciated capital is sector specific, i.e. immobile, considering twenty years, while net investments define the final capital stock in use. That implies endogenous capital stocks in a comparative static setting. The configuration uses as well a CET nest to distribute output to domestic sales and exports. The CGEBox+ as the final configuration builds on the previous one. It replaces the CET for the manufacturing sector by the Melitz model with monopolistic competition, industry and trade-link specific fix costs and endogenous number of varieties on each trade link.

Solving each single configuration of the experiments discussed below including full model post-processing take less than half a minute; the predefined configurations can be chosen from the GUI and in conjunction with a batch facility, they can be quickly run on any type of shock. That underlines that structural sensitivity analysis with the flexible and modular approach in CGEBox is quite straightforward. The framework additionally supports sensitivity analysis for major parameters.

4.2 Results from a partial, multi-lateral trade liberalization

Our aim with the two following applications is clearly not a real-world policy experiment, but rather to show differences between configurations in exemplary types of shocks. In order to compare the model configuration, we report welfare changes using the equivalent variation approach and, additionally, relative changes in real GDP, as an indicator also often used in policy relevant application.

The first shock simulates a multi-lateral trade liberalization which reduces all bilateral import tariffs and export subsidies (not taxes) by 50% — even if current real world developments might hint in another direction. The global welfare changes under that shock are most pronounced under the Melitz model (CGEBox+), followed by "MIRAGE" based on the Krugman model. These outcomes are consistent with expectations from the so-called "new trade theory" based models compared with an Armington specification (cf. Jafari and Britz, 2018). The differences between the other, Armington based, configurations are more limited, and it is hard to derive a clear picture, such as between e.g. GLOBE and ENVISAGE which both use a CET transformation of supply to different destination and GTAP standard and GTAPinGAMS that do not. We will therefore look below at more fine grained changes in model structure.

Table 3: Equivalent variation globally and per region [const. US$ per capita], under the same multi-lateral trade experiment with different model configurations

	GTAP Standard	GTAP inGAMS	GLOBE	EN-VISAGE	CGEBOX	MIRAGE	CGEBOX plus
World	12	12	8	10	12	23	36
Australia & N. Zeal.	77	63	51	64	72	53	111
East Asia	35	34	17	25	33	43	83
Southeast Asia	8	7	4	4	7	15	32
South Asia	2	2	-2	0	1	1	5
North America	-4	8	16	9	4	46	42
Latin America	-1	-4	-7	-1	-1	-5	2
European Union 25	15	24	24	20	13	60	76
Mid. East & N. Africa	7	5	0	0	10	18	14
Sub-Saharan Africa	0	0	-1	0	2	-2	0
Rest of World	17	14	15	14	28	50	43

Source: Authors' calculations

Equally, differences in simulated welfare gains from the multi-lateral trade liberalization between the model variants appear relatively small when compared with the impact of changes to the regional aggregation used in the model (see Britz and van der Mensbrugghe, 2016). Note also the level of the welfare gains are at the lower limit of what was reported in Britz and van der Mensbrugghe, 2016, however based on the GTAP Version 8 Data Base, as the regional aggregation with ten world regions in our experiments is quite high.

We also tested differences under a 50% reduction of all consumer taxes and again found similar limited differences. Depending on the configuration, no region (CGEBox+) or one to two regions loses out from multi-lateral trade liberalization (see Table 3), in the latter case, there is no agreement among model variants that Latin America loses. However, consistently, the "Australia and New Zealand" aggregate is depicted as benefiting most on a per capita basis. Unlike the equivalent variation measures, real GDP increases in all variants with the exception of the "MIRAGE" configuration. It is interesting to note that while the money metric approach sees "Australia & New Zealand" consistently as benefitting most, the relative changes of real GDP for that region are in many cases even below the world average.

Table 4: Changes in real GDP [%], under the same multi-lateral trade experiment with different model configurations

	GTAP Standard	GTAP inGAMS	GLOBE	EN-VISAGE	CGEBOX	MIRAGE	CGEBOX plus
World	0.12%	0.12%	0.07%	0.08%	0.12%	0.27%	0.35%
Australia & N. Zeal.	0.08%	0.07%	0.05%	0.07%	0.08%	0.30%	0.16%
East Asia	0.24%	0.24%	0.15%	0.17%	0.26%	0.66%	0.81%
Southeast Asia	0.16%	0.17%	0.11%	0.12%	0.19%	-0.08%	0.91%
South Asia	0.37%	0.39%	0.20%	0.29%	0.37%	1.59%	0.74%
North America	0.02%	0.03%	0.02%	0.02%	0.02%	0.14%	0.09%
Latin America	0.07%	0.06%	0.03%	0.06%	0.08%	0.04%	0.15%
European Union 25	0.06%	0.07%	0.04%	0.04%	0.04%	0.23%	0.19%
Mid. East & N. Africa	0.36%	0.37%	0.13%	0.22%	0.40%	0.16%	0.59%
Sub-Saharan Africa	0.20%	0.20%	0.13%	0.18%	0.22%	0.55%	0.35%
Rest of World	0.12%	0.12%	0.07%	0.04%	0.13%	-0.36%	0.27%

Source: Authors' calculations

4.2 Results from a regional TFP shock

Assuming that differences between configurations might be greater under a much larger shock which additionally only roots in one region, such that, for instance, differences in how foreign savings are modeled could have a larger impact, we increase the total factor productivity (TFP) in all sectors in our "North American" region by 20%. Given its weight in the global economy, that generates a welfare impact per capita globally between around 540 and 640 USD, see table 5 below. The largest boost stems again from using a monopolistic competition model ("MIRAGE", "CGEBOX+"). Differences between individual countries are here more pronounced, depending on the configuration, citizens of the Australia-New Zealand region are simulated to lose -309 USD or win 115 USD from a TFP boost in North America. The differences in the money metric are to a larger part also reflected in the changes of real GDP, see table 6.

Table 5: Equivalent variation globally and per region [const. US$ per capita], under a 20% TFP boost in North America with different model configurations

	GTAP Standard	GTAP inGAMS	GLOBE	EN-VISAGE	CGEBOX	MIRAGE	CGEBOX plus
World	541	530	530	547	538	605	613
Australia & N. Zeal.	-309	-12	70	89	-94	115	-69
East Asia	-40	3	25	17	-5	25	-15
Southeast Asia	-9	3	12	9	-1	7	-5
South Asia	-9	2	3	5	-1	-22	-4
North America	8375	7766	7593	7879	8091	8725	9331
Latin America	-48	17	34	33	-9	0	-8
European Union 25	-116	8	69	72	-45	147	-102
Mid. East & N. Africa	19	23	23	22	10	-12	8
Sub-Saharan Africa	5	8	9	9	3	-4	3
Rest of World	66	67	68	51	24	126	16

Source: Authors' calculations

Table 6: Change in real GDP, under a 20% TFP boost in North America with different model configurations

	GTAP Standard	GTAP inGAMS	GLOBE	EN-VISAGE	CGEBOX	MIRAGE	CGEBOX plus
World	5.22%	5.19%	5.23%	5.39%	5.27%	6.16%	5.99%
Australia & N. Zeal.	-0.23%	-0.01%	0.02%	0.14%	-0.16%	0.38%	-0.15%
East Asia	-0.09%	0.03%	0.07%	0.08%	-0.09%	0.69%	-0.16%
Southeast Asia	-0.02%	0.02%	0.05%	0.09%	-0.11%	-2.78%	-0.17%
South Asia	-0.14%	0.00%	0.03%	0.02%	-0.10%	0.02%	-0.21%
North America	20.29%	20.02%	20.07%	20.56%	20.71%	23.04%	23.75%
Latin America	-0.18%	0.02%	0.04%	0.12%	-0.15%	0.11%	-0.18%
European Union 25	0.04%	0.00%	0.04%	0.07%	-0.14%	0.35%	-0.27%
Mid. East & N. Africa	-0.03%	0.04%	0.05%	0.12%	-0.10%	-0.32%	-0.11%
Sub-Saharan Africa	-0.05%	0.04%	0.06%	0.18%	-0.09%	-0.09%	-0.09%
Rest of World	0.00%	0.05%	0.05%	0.23%	-0.12%	0.42%	-0.16%

Source: Authors' calculations

To elucidate some of the key structural features, Table 7 shows the impacts of simulations which introduce stepwise the different structural changes and parameter updates to move from GTAP standard to the configuration labelled "ENVISAGE" as found in Table 5 above. The changes are expressed relative to the GTAP standard model results. All variants simulate in the "North America" region lower or very limited welfare gains, the lowest found in the "ENVISAGE" configuration. Introducing the CET or the vintage module which only renders non-depreciated capital mobile across the sectors considerably lowers the welfare gains simulated by the GTAP-Standard version. That might be an expected result due to less flexibility in adjusting supply to a shock by reducing factor mobility (vintage module), or in the case of the CET, of lower mobility of goods across destinations. However, these features shift welfare gains from the TFP increase in the "North America" region to the Rest-of-the-world, as global welfare increases, a perhaps unexpected effect. It is interesting to note that allowing for more flexibility in technology (substitution between ND and VA and inside ND, GTAP-AGR, GTAP-E) has very limited impacts on the simulated welfare gains.

Table 7: Differences Equivalent variation globally [const. US$ per capita], under a 20% TFP boost in World and North America for a stepwise introduction of the ENVISAGE configuration, compared to GTAP Standard

	World	North America
GTAP-Standard plus LES	0.05%	0.06%
GTAP-Standard plus substitution ND/VA and inside ND	-0.23%	-0.07%
GTAP-Standard plus CET	1.03%	-3.05%
GTAP-Standard plus GTAP-AGR	-0.02%	-0.07%
GTAP-Standard plus GTAP-E	-0.16%	0.17%
GTAP-Standard plus separate household and gov account	-0.10%	0.28%
GTAP-Standard plus separate household and gov account, gov demand fixed	1.59%	2.22%
GTAP-Standard plus capital vintage module	1.79%	-3.34%
GTAP-Standard plus LES and CET	1.09%	-3.01%
as before, plus substitution ND/VA and inside ND	0.76%	-3.15%
as before, plus GTAP-AGR	0.76%	-3.21%
as before, plus and GTAP-E	0.84%	-3.14%
as before, plus capital vintage module	1.55%	-4.12%
as before, plus separate household and gov account	1.60%	-4.26%
ENVISAGE configuration	1.07%	-5.92%

Source: Authors' calculations

Removing the regional household approach and fixing government consumption by adjusting the government's saving rate increases welfare both globally and in the "North America" region, while welfare decreases in "North America" if the feature is implemented jointly with some other features. The table also reveals some stronger interactions between the individual elements: the sum of all single changes, i.e. up to the line "capital vintage module" would suggest a combined change of around -4%, but the full ENVISAGE implementation leads to a drop compared to GTAP Standard. The main reason seems that fixing government consumption boosts welfare as a single feature, but leads to a drop when added to all other ones.

4.3 Discussion

Similar to us, there are examples in the literature which compare different CGE models or configurations. McKitrick, 1998 performs a comparison of a CGE model for Canada based on the CES functional form and a combination of CES and normalized quadratic, estimating the parameters for both variants econometrically, and finds sizeable difference. He draws the conclusion that the structural assumptions in CGE models are not sufficient to "dictate" simulated outcomes to the degree that the choice of functional forms or parameters do not matter. Compared to his exercise, differences found by us seem minor which might be linked to fact that our structural changes were firstly more modest. Secondly, in case of switching the functional form in final demand from the CDE to LES, we used the point income elasticity of the CDE to determine the marginal budget shares of the LES which might render the simulation behavior to some degree similar, at least as long as income effects dominate the outcome. Gibson and van Seventer, 2000 compare a configuration where savings drive investments with an alternative one with Keynesian inspired separate investment function and find very distinct differences between the two. As their neo-classical model is in line with all variants analyzed by us, their findings go beyond the comparisons performed by us. Tarr, 2013 compares outcomes of two configurations where one adds FDI in services and related NTMs while also accounting for endogenous productivity effects, finding larger differences in trade liberalization scenarios. Again, none of our configurations accounts for these effects. Other studies such as Brown and Stern, 2009 report published findings with different models on a policy change, but typically, neither the analyzed shock nor the underlying data base will be harmonized and renders it hard to pin-point differences to structural differences in models. For the GTAP-E model, Burniaux and Truong, 2002 compare the own and cross-price effects for the different sectors from a shock to the output tax. They find these becoming more negative for energy commodities due to increased substitution possibilities between them. Similarly, Keeney and Hertel, 2005 compare the GTAP-AGR model against the GTAP Standard model. Similar to our results, differences in overall welfare effects of trade liberalization between the two configurations are found as quite small.

5. Institutional issues around a jointly maintained modular and flexible CGE framework

We discuss here briefly the idea to maintain and further develop CGEBox as a joint effort where teams add not yet available features which they consider essential, sharing code and documentation. Doing so, each team could access a more powerful modeling system compared to a stand-alone development, at potentially lower development cost. Common courses could introduce new staff to the modeling platform.

Such a development process however poses a number of challenges. In a highly modular framework, new features must be tested under potentially all possible combinations of already existing ones, and with different data bases to account for sparsity and other more unusual data constellations. That requires a well-developed Quality Management System, for instance, comprising a larger test suite of shocks, a new challenge for current developers. Rigorous and well maintained documentation, including in-line comments, is essential to prevent a huge divide between developers and users due to a powerful, yet black box. Thus, common standards for documentation and coding must be agreed upon and followed, combined with technical solutions such as a shared Software Version System, provoking additional costs compared to independent development. The joint development of whole Operation Systems shows that these more technical impediments can be overcome.

However, developing own code from a given mathematical presentation instead of using an existing one can have teaching and training effects which might offset saved costs. Equally, open source development must be incentivized, by differentiated returns for contributors and users. Examples are grace periods during which solely the contributor can use a new module for publications or a peer-reviewed publication process for new developed modules similar to R-packages. The GTAP Center already goes in that direction with their technical paper series and the newly established journal.

Other critical points relates to trademarks and Intellectual Property Rights. All the models discussed above have been successfully marketed over more than a decade under their specific name, which leads to reputation and opportunities to raise funds, attract staff or eases peer reviewed publication of work based on the models. A strategy towards a common platform must allow the teams to keep their reputation which requires continued marketing of applications with "their model" under its established trademark such as "ENVISAGE, realized in CGEBox".

6. Summary and conclusions

We have shown that some well-known global and single country CGE frameworks (GTAP Standard model, GTAPinGAMS, GLOBE, ENVISAGE, MIRAGE, STAGE, IFPRI-S), all realized in GAMS, share a large common structural basis, but clearly differ in detail. Some interesting features found in some models are not available in others. We therefore developed a modular and extendable open-source and open-access CGE framework titled CGEBox which is able to replicate to a large extent the layout of these different models. CGEBox can be solved as a single country or global model, in comparative-static or dynamic mode. Even a partial equilibrium model for only some commodities found in the global SAM can be derived. Using a partial multi-lateral trade liberalization and TFP shock in a global, comparative-static setting as show cases, we highlight how major structural differences between the different models impact core results in our experiments.

More flexibility and/or complexity in the layout of a CGE model do not imply more robust or valid results. Each application of a CGE faces the challenge of an appropriate choice of how to model the shock, of the structural layout of the model and of its parameterization, issues we are not addressing in our paper. However, having the chance to compare results for different configurations can lead to more informed choices and for an assessment of uncertainties.

We conclude that it is by now technically relatively straightforward to set-up a modular and extendable code basis for CGE modeling in GAMS. That renders it inviting to jointly further develop such a platform where individuals and teams contribute new modules and share them with the others. Impediments to such a solution are clearly the required common coding and documentation standards, the more complex quality management to ensure that modules are interoperable and questions around IPR and trademarks as well as the benefits from an own-coded implementation of a feature.

We invite interested modelers wanting to contribute to contact the authors. Note that the code of CGEBox is subject to updates and extensions. While the code basis underlying the reported results is provided as supplementary material online with this paper, it is recommended to maintain an up-to-date copy based on a SVN client such as TortoiseSVN which is also the basis to contribute code in an organized set-up. The URL of the SVN repository is https://svn1.agp.uni-bonn.de/svn/cgebox/, user name cgebox and password cgebox.

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Appendix: Core model equations

The basic model equations are to a large extent identical to van der Mensbrugghe, D. 2018. "The Standard GTAP Model in GAMS", Version 7. *Journal of Global Economic Analysis*, *3*(1), 1-83. We refer below to the equations in that complete and excellent documentation (e.g. equation (x) in VDM 2018) and report differences where applicable. We refrain from repeating all equations in mathematical notation – the reader is invited to refer to VDM 2018. Instead, we insert screen shots of the actual code which we comment to ease understanding the technical implementation of the model. References to section below refer to the full documentation of CGEBox (Britz, 2018) available as supplementary material online.

Core sets

The equations of the basic model are comprised in the file "*model.gms*" and discussed in the following. The following general sets are used:

Table 8: Core sets used in model equations

Set name	Description
r, rp	Regions
rnat,nat1	nations (to differentiate from sub-regions in the case the NUTS2 level is active)
disr	Nations which are dis-aggregated to sub-regions
subr	sub-regions (only populated in the case the NUTS2 level is avtive)
aa	Armington agents (sectors, private household, government, savings, transport modes)
a	production activities
i, j, k	products
t	time
m	mode of transport
f	factors
fm	mobile factor (fully mobile or sluggish)
fnm	non-mobile factor, i.e. sector specific
h	households
gov	government (single item)
Inv	investment (single item)
fd	final demand groups, used in demand nests
dNest	demand nests
tNest	technology nests
fNest	factor supply nests

Note the lists of regions, activities, products and factors depend on the version of the GTAP data base used and the chosen aggregation. The list of demand, technology and factor supply nests is equally dynamic, depending which modules are active and/or on additional nests introduced by user provided files. The myGTAP extensions might introduce several private households in the set *h* which is otherwise a singleton. In the standard layout, i.e. without using the NUTS2 extensions, all regions are defined as nations and the list of dis-aggregated regions *disr* is empty.

As the model might run as a single region or as a partial equilibrium model or recursive dynamically, dynamic sets are used to indicate for which regions, product, activities and time points the equations in the current model instance should be generated:

rs	regions in current solve
ts	time point in current solve
aIn	activities in current model
iIn	products in current model

Furthermore, to support sparsity, i.e. to avoid that equations and variables are only generated for non-empty items, a larger set of parameters which serves as flags are used. The most important ones are listed here:

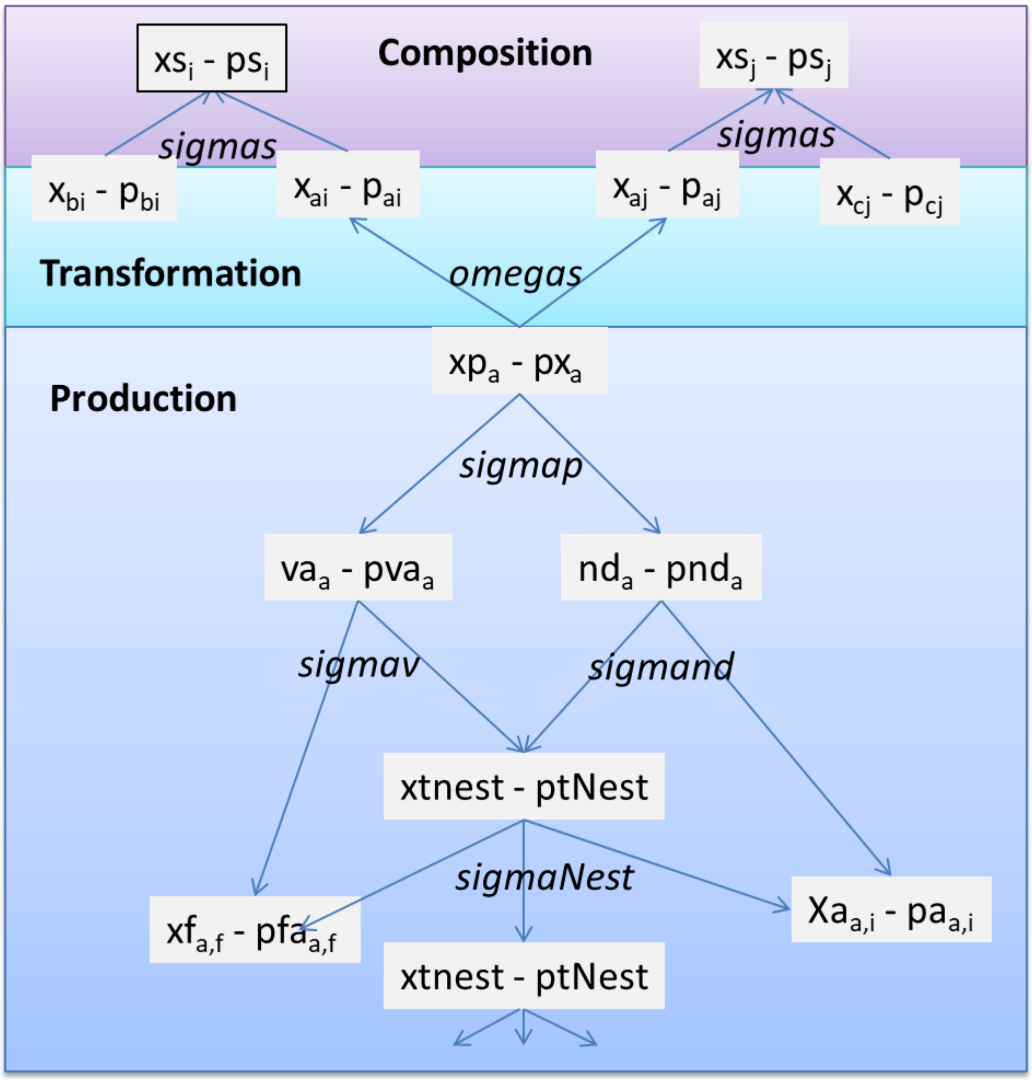
vaFlag(r,a)	value added for region *r* and activity *a* non-empty
ndFlag(r,a)	interm. composite for region *r* and activity *a* non-empty
xpFlag(r,a)	activity *a* for region *r* is non-empty
xfFlag(r,f,a)	primary factor *f* is used by activity *a* in region *r*
xaFlag(r,i,aa)	Armington agent *aa* demands product *i* in region *r*
xwFlag(r,i,rr)	Bilateral trade flag

Given these examples, the names of the other flags should be hopefully self-explanatory

Overview on the supply side

The following Figure 1 depicts the quantity and price variables as well as the substitution and transformation elasticities used on the supply side. The bottom part is defined for the production activities *a* with total output denoted with *xp* and related price *px*. It is composed of a value added composite *va* and an intermediate demand composite *nd*. The value added composite combines primary factor *f* and potentially technology sub-nests *tNest*. The intermediate demand composite *nd* combines intermediates defined as Armington demands *xa* of the activities and potentially technology sub-nests *tNest*. Technology sub-nests can combine other sub-nests, primary factors and intermediates in a nested fashion. Note firstly that primary factors or intermediates can be present in different shares in sub-nests and, secondly, that if the Melitz / Krugmann specification is used for a sector, fix costs are present in a separate sub-nest which does not contribute to *xp*.

Figure 1: Overview on production function nesting



The output of the activities *xp* can be transformed for the non-diagonal make case to different commodities *x* as shown in the middle box. If several activities produce the same commodity, the different *x* can be combined in supply *xs* based on a CES aggregator as shown in the top box.

For the upper two boxes, the code supports the case of finite and infinite transformation respectively substitution where the infinite case implies a linear aggregation and the case of one price. For the production nests, only finite transformation is supported including the CD case.

Production block

The model is set up to work with non-diagonal make matrices where one activity might produce several outputs and one output might be produced by several activities. The production block therefore is defined for activities *a* and not for the outputs *i*. Furthermore, in case regions are dis-aggregated to sub-regions, the production function is defined for these dis-aggregated regions. Accordingly, nations which are dis-aggregated to sub-regions *disr* are excluded from these equations. The production reflects the "Flexible nesting" approach which allows introducing CES-subnests under the value and the intermediate composite nests, or under other CES-subnests.

The nested production function for each activity *a* comprises a **top nest** which combines a value added *va* and intermediate demand *nd* composite with a substitution elasticity of *sigmap*. The production frontier can be shifted with the variable *axp*. The top nest is represented by its dual price aggregator in the equation *pxeq*. That equation considers three cases which are shown below: (1) *sigmap* is non-zero and different from unity with leads to the usual dual price aggregator for the CES case, (2) the CD case where *sigmap* is unity with a different dual price aggregator and (3) the Leontief case with *sigmap* equal to zero. The price for the intermediate composite is called *pnd* and that for the valued added one *pva*. The related technology shifters are the variables *lambdand* and *lambdava* while the share parameters are called *and* and *ava*.

Note that the unit cost price *px* might be substituted out from the model in the diagonal make case based on the macro *mm_px*. The equation is identical to VDM 2018, equation (3).

```
pxeq(rs(r),aIn(a),ts(t)) $ (xpFlag(r,a) $ (not diag(a)) $ (not disr(r)))..

   axp(r,a,t)*mm_px(r,a,t) =e=

       -- Non Leontief case with substitution between VA and ND

         (  (   and(r,a,t)*(pnd(r,a,t)/m_lambdaNd(r,a,t))**(1-sigmap(r,a))
              + ava(r,a,t)*(pva(r,a,t)/m_lambdava(r,a,t))**(1-sigmap(r,a))) ** (1/(1-sigmap(r,a)))
         ) $ (sigmap(r,a) $ (sigmaP(r,a) ne 1))

       --- CD case with substitution between VA and ND, sigmap=1

      +  (   axpCD(r,a,t)
              * ([(pnd(r,a,t)/(and(r,a,t)*m_lambdaNd(r,a,t)))**and(r,a,t)] $ and(r,a,t) + 1 $ (not and(r,a,t)))
              * ([(pva(r,a,t)/(ava(r,a,t)*m_lambdava(r,a,t)))**ava(r,a,t)] $ ava(r,a,t) + 1 $ (not ava(r,a,t)))
          )  $ (sigmap(r,a) eq 1)

       -- Leontief case (only introduced to ease work for solver), as in standard GTAP model

    +  (
          (   and(r,a,t)*(pnd(r,a,t)/m_lambdaNd(r,a,t))
            + ava(r,a,t)*(pva(r,a,t)/m_lambdava(r,a,t)) ))  $ (not sigmap(r,a))
    ;
```

The *mm_px* macro is in the usual case equal to the *m_xp* macro which is shown shown below. It directly uses the product specific supply price *ps* corrected for production taxes *prdtx* in case of a diagonal make relation for that activity as depicted by the flag *diag(a)*. If the activity produces several outputs, its unit cost price *px* is used instead. Note that the *xFlag* indicates which outputs *k* are produced by activity *a* in region *r*:

```
$$macro m_xp(r,a,t)  ( xp(r,a,t) $ (not diag(a)) + sum(k $ xFlag(r,a,k), xs(r,k,t)) $ diag(a) )
```

The demand for the **value added** composite *va* is defined in the equation *vaeq*, VDM 2018, Equation (1). It comprises the same symbols as shown above in the top level unit cost definition. Note that the equation treats the Leontief case where *sigmap* is zero differently by removing the prices from the equation which can speed up solution.

```
vaeq(rs(r),aIn(a),ts(t)) $ vaFlag(r,a) ..
   va(r,a,t)/va.scale(r,a,t) =e= ava(r,a,t) * m_xp(r,a,t)/va.scale(r,a,t)
                  * [   {(mm_px(r,a,t)/pva(r,a,t))**sigmap(r,a)} $ sigmap(r,a) + 1 $ (not sigmap(r,a)) ]
                  *   (axp(r,a,t)*m_lambdava(r,a,t))**(sigmap(r,a)-1) ;
```

Note that the *vaeq* equation is scaled with the scale field of the value added demand *va.scale*. Scaling factors are present basically in all equation relating to quantities or volumes to ease automated scaling by the solver and provide a more useful interpretation of the relative and absolute tolerances used by the solver.

The relevant activity output quantity driven the value added demand is defined in the macro *m_xp*. It uses directly the commodity supply *xs* in case of diagonal make matrix for that activity, i.e. diag(a) is not zero, otherwise, it introduces the activity output *xp* in the equation. Note the symmetry with the *m_px* macro shown above for the output price.


```
$$macro m_xp(r,a,t)  ( xp(r,a,t) $ (not diag(a)) + sum(k $ xFlag(r,a,k), xs(r,k,t)) $ diag(a) )
```


The equation *ndeq*, VDM 2018, Equation (2), identically structured as the *vaeq* equation above, drives the demand for the **intermediate demand** composite:

```
ndeq(rs(r),aIn(a),ts(t)) $ ndFlag(r,a) ..

   nd(r,a,t)/nd.scale(r,a,t) =e= and(r,a,t) * m_xp(r,a,t)/nd.scale(r,a,t)
                      * [   {(mm_px(r,a,t)/pnd(r,a,t))**sigmap(r,a)} $ sigmap(r,a) + 1 $ (not sigmap(r,a)) ]
                        *   (axp(r,a,t)*m_lambdaNd(r,a,t))**(sigmap(r,a)-1) ;
```

The demand for **primary factors** *xf* by each activity depends on a shifter variable *lambdaf*, the share parameter *af*, total value added demand *va* and the price relation between the price of the value added bundle *pva* and the sector specific factor price *pfa,* defined via the macro *m_pfa,* exponent the substitution elasticity *sigmav*:

```
xfeq(rs(r),f,aIn(a),ts(t)) $ (xfFlag(r,f,a) $ ( af(r,f,a,t)
                                               or sum(tNest_f_a(tNest,f,a), afNest(r,f,a,tNest,t)) ) ..

 xf(r,f,a,t)/xf.scale(r,f,a,t)
   =e=

    --- demand for factors in VA nest

       af(r,f,a,t)*va(r,a,t)/xf.scale(r,f,a,t)*(pva(r,a,t)/m_pfa(r,f,a,t))**sigmav(r,a)
               * m_lambdaf(r,f,a,t)**(sigmav(r,a)-1)

    --- demand for factor inside technology nest

    + sum(tNest_f_a(tNest,f,a) $ afNest(r,f,a,tNest,t),

   --- factor cost share times demand for technology nest

       afNest(r,f,a,tNest,t)*xtNest(r,tNest,a,t)/xf.scale(r,f,a,t)

   --- relative price impact (in case sigmndNest is not 0)

   * [ { (pTNest(r,tNest,a,t)/m_pfa(r,f,a,t))**sigmaNest(r,tNest,a)    } $ sigmaNest(r,tNest,a)
                                                                  +  1 $ (not sigmaNest(r,tNest,a)) ]
                * m_lambdaf(r,f,a,t)**(sigmaNest(r,tNest,a)-1));
```

That equation differs from VDM, equation (4) by the inclusion of the technology nests:

$$XF^{d}_{r,f,a} = \alpha^{f}_{r,f,a} VA_{r,a} \left(\frac{PVA_{ra}}{PF^{a}_{r,f,a}} \right)^{\sigma^{\upsilon}_{r,a}} \left(\lambda^{f}_{r,f,a} \right)^{\sigma^{\upsilon}_{r,a}-1}$$

$$+\alpha^{tNest}_{r,f,a} XTNEST_{r,tNest,a} \left(\frac{PNEST_{r,tNest,a}}{PF^{a}_{r,f,a}} \right)^{\sigma^{tNest}_{r,a}} \left(\lambda^{f}_{r,f,a} \right)^{\sigma^{tNest}_{r,a}-1} \quad (4^*)$$

The second part of the equation is not part of the standard model and only active if technology nests are used and is described in the section "Flexible nesting". It comprises the same elements: share parameters insides the nests *afNest*, the composite demand for the nest *xtNest*, the price relation which now uses the average price of the nest *pTNest* and the substitution elasticity *sigmaNest*. Note that the demand from technology nests is added, i.e. the model supports a layout where several technology nests and the value added nest can demand the same factor (or intermediate composite, see below) in different shares.

The dollar conditions might warrant some comments. The first one is the flag *xfFlag* indicating that the activity *a* is using that factor f, while the second ensures that also share parameters are present, either in the value added nests and/or some technology nests. That double security might secure against cases where due to numerical thresholds, share parameters are set to zero despite the fact that there some tiny quantity reported in the SAM.

The **value added composite price** *pva* is defined in the *pvaeq* equation. It differentiates the cases where the substitution elasticity *sigmav* between primary factors is (1) not unity, i.e. CES or Leontief, (2) unity, i.e. the CD case:

```
pvaeq(rs(r),aIn(a),ts(t)) $ vaFlag(r,a) ..

 pva(r,a,t) =e=

   --- CES case (or Leontief)

   [ (

      --- contribution of factors to top VA nests

    sum(f $ af(r,f,a,t), af(r,f,a,t)*(m_pfa(r,f,a,t)/m_lambdaf(r,f,a,t))**(1-sigmav(r,a)))

      --- contribution of technology nests to top VA nests (not part of standard GTAP model)

   + sum(tNest_n_a("VA",tNest,a), aTNest(r,tNest,a,t)*pTNest(r,tNest,a,t)**(1-sigmav(r,a)))

    )** (1/(1-sigmav(r,a))) ] $ (sigmav(r,a) ne 1)

   --- CD case with sigmaV == 1

  + [  axVACD(r,a,t)

      --- contribution of factors to top VA nests

  *   prod(f $ af(r,f,a,t),(m_pfa(r,f,a,t)/(m_lambdaf(r,f,a,t)*af(r,f,a,t)))**af(r,f,a,t))

      --- contribution of technology nests to top VA nests (not part of standard GTAP model)

   * prod(tNest_n_a("VA",tNest,a) $ atNest(r,tNest,a,t), (pTNest(r,tNest,a,t)/aTNest(r,tNest,a,t))**aTNest(r,tNest,a,t))

   ] $ (sigmav(r,a) eq 1);
```

That equation differs from VDM 2018, equation (5) by the inclusion of the technology nests:

$$PVA_{r,a} = \left[\sum_f \alpha_{r,f,a}^f \left(\frac{PF_{r,f,a}^a}{\lambda_{r,f,a}^f} \right)^{1-\sigma_{r,a}^v} + \sum_{tNest \in VA} \alpha_{r,f,a}^{tNest} \left(\frac{PTNEST_{r,tNest,a}}{\lambda_{r,f,a}^{tNest}} \right)^{1-\sigma_{r,a}^v} \right]^{1/\left(1-\sigma_{r,a}^v\right)} \tag{5*}$$

Note that the flexible nesting approach allows to link nests into the value added composite such that both sums and products of the individual factors and over nests are introduced in the equation.

A similarly structured equation *pdneq* defines the **intermediate composite price** *pnd*. It is driven by the input coefficients *io* and their activity specific price *paint* defined via a macro and individual technology shifters *lambdaio* again captured by a macro. Note that the coefficients *io* describe shares inside the intermediate nest, and not relative to total output. As the standard GTAP model uses a Leontief representation for intermediate demand, the case where the substitution elasticity *sigmand* is zero is separated out here as well, such that we find three blocks (CES, CD and Leontief). Separating out the Leontief case reduces model complexity as the solver will define a linear instead of a non-linear price aggregator.

Note that here again we consider the cases where the intermediate demand is driven by the intermediate composite (standard model) and/or by technology nests.

```
pndEq(rs(r),aIn(a),ts(t)) $ ndFlag(r,a) ..

 pnd(r,a,t) =e=

         --- CES case

         [ (

          --- aggregate over all intermediate inputs not assigned to a technology tests

              sum(i $ io(r,i,a,t),
                         io(r,i,a,t)*(m_paint(r,i,a,t)/m_lambdaio(r,i,a,t))**(1-sigmand(r,a)))

          --- add demand for technology nests linked into the top ND undle

           + sum(tNest_n_a("ND",tNest,a) $ atNest(r,tNest,a,t),
                atNest(r,tNest,a,t) * ptNest(r,tNest,a,t)**(1-sigmand(r,a)))

             )**(1/(1-sigmand(r,a)))

         ] $ (sigmand(r,a) $ (sigmand(r,a) ne 1))


     --- CD case with sigmaND == 1

 +   [  axNdCD(r,a,t) *

      --- aggregate over all intermediate inputs not assigned to a technology tests

         {prod(i $ io(r,i,a,t),
                     (m_paint(r,i,a,t)/(io(r,i,a,t)*m_lambdaio(r,i,a,t)))**io(r,i,a,t))
           $ sum(i $ io(r,i,a,t), 1) + 1 $ (not sum(i $ io(r,i,a,t), 1)) }

      --- add demand for technology nests linked into the top ND undle

       *  {prod(tNest_n_a("ND",tNest,a), (ptNest(r,tNest,a,t)/atNest(r,tNest,a,t))**atNest(r,tNest,a,t))
             $ sum(tNest_n_a("ND",tNest,a), atNest(r,tNest,a,t))
            + 1 $ (not sum(tNest_n_a("ND",tNest,a), atNest(r,tNest,a,t))) }

     ] $ (sigmand(r,a) eq 1)


      --- Leontief case, as in standard GTAP model

   +  [

       --- aggregate over all intermediate inputs not assigned to a io-group

      sum(i $ io(r,i,a,t),
                  io(r,i,a,t)*(m_paint(r,i,a,t)/m_lambdaio(r,i,a,t)))

       --- add aggregate intermediate group demands (not part of standard GTAP Model)

        + sum(tNest_n_a("ND",tNest,a), atNest(r,tNest,a,t) * ptNest(r,tNest,a,t)) $ card(tNest)

      ] $ (not sigmand(r,a))
 ;
```

That equation thus differs again from VDM 2018 equation (7) due to the inclusion of the technology nests:

$$PND_{r,a} = \left[\sum_i \alpha^{io}_{r,i,a} \left(\frac{PA_{r,i,a}}{\lambda^{io}_{r,f,a}} \right)^{1-\sigma^{\upsilon}_{r,a}} + \sum_{tNest \in ND} \alpha^{tNest}_{r,i,a} \left(\frac{PTNEST_{r,tNest,a}}{\lambda^{tNest}_{r,i,a}} \right)^{1-\sigma^{nd}_{r,a}} \right]^{1/\left(1-\sigma^{nd}_{r,a}\right)} \quad (7^*)$$

The dual price aggregator equation *ptNestEq* for technology nests is depicted below. It considers the possible components: intermediates with the related share parameter *ioNest*, primary factors with their share parameter *afNest* and finally sub-nests with share parameters *atNest*. The price and shifters used for intermediates and primary factors are identical to those described above for the value added and intermediate composite nests. Due to the different dual price aggregator necessary for the CD case, the equation comprises two blocks.

```
ptNestEq(rs(r),tNest,aIn(a),ts(t)) $ (xpFlag(r,a) $ ( (atNest(r,tNest,a,t) gt 0)
                                   or sum(tNest_f_a(tNest,f,a), afNest(r,f,a,tNest,t))
                                   or sum(tNest_i_a(tNest,i,a), ioNest(r,i,a,tNest,t)) )) ..

   ptNest(r,tNest,a,t) =e=
   {[
*
*  --- aggregate over intermediate inputs assigned to technology nest
*
        sum(tNest_i_a(tNest,i,a) $ ioNest(r,i,a,tNest,t),
                                 ioNest(r,i,a,tNest,t)
                                    * (m_paint(r,i,a,t)/m_lambdaio(r,i,a,t))**(1-sigmaNest(r,tNest,a)))
*
*  --- aggregate over factors  assigned to technology nests
*
      + sum(tNest_f_a(tNest,f,a) $ afNest(r,f,a,tNest,t),
                                 afNest(r,f,a,tNest,t)
                                    * (m_pfa(r,f,a,t)/m_lambdaf(r,f,a,t))**(1-sigmaNest(r,tNest,a)))
*
*  --- aggregate over sub technology nests assigned to that technology nests
*
      + sum(tNest_n_a(tNest,tNest1,a) $ atNest(r,tNest1,a,t),
                                 atNest(r,tNest1,a,t)
                                    * ptNest(r,tNest1,a,t)**(1-sigmaNest(r,tNest,a)))
                                                                              ]**(1/(1-sigmaNest(r,tNest,a)))
  }
   $ (sigmaNest(r,tNest,a) ne 1)

*
*   ---- CES case with sigmaNest == 1 (shifter is missing ...)
*
  + [
*
*  --- aggregate over intermediate inputs assigned to technology nest
*
        prod(tNest_i_a(tNest,i,a) $ ioNest(r,i,a,tNest,t),
                                   (m_paint(r,i,a,t)/(ioNest(r,i,a,tNest,t)*m_lambdaio(r,i,a,t)))**ioNest(r,i,a,tNest,t))
*
*  --- aggregate over factors  assigned to technology nests
*
      * prod(tNest_f_a(tNest,f,a) $ afNest(r,f,a,tNest,t),
                                   (m_pfa(r,f,a,t)/(afNest(r,f,a,tNest,t)*m_lambdaf(r,f,a,t)))**afNest(r,f,a,tNest,t))
*
*  --- aggregate over sub technology nests assigned to that technology nests
*
      * prod(tNest_n_a(tNest,tNest1,a) $ atNest(r,tNest1,a,t),
                                   (ptNest(r,tNest1,a,t)/atNest(r,tNest1,a,t))**atNest(r,tNest1,a,t))/axTNest(r,tNest,a,t)
   ] $ (sigmaNest(r,tNest,a) eq 1);

  ;
```

Note that this equation is not part of the GTAP Standard as documented in VDM 2018, it reads in mathematical notation:

$$PNEST_{r,tNest,a} = \left[\begin{array}{l} \sum\limits_{i \in tNest} \alpha_{r,i,a}^{tNest} \left(\dfrac{PA_{r,i,a}}{\lambda_{r,i,a}^{tNest}} \right)^{1-\sigma_{r,a}^{tNest}} \\ + \sum\limits_{f} \alpha_{r,f,a}^{tNest} \left(\dfrac{PF_{r,f,a}^{a}}{\lambda_{r,f,a}^{tNest}} \right)^{1-\sigma_{r,a}^{tNest}} \\ + \sum\limits_{tNest1 \in tNest} \alpha_{r,tNest1,a}^{tNest} \left(PTNEST_{r,tNest1,a} \right)^{1-\sigma_{r,a}^{tNest}} \end{array} \right]^{1/\left(1-\sigma_{r,a}^{tNest}\right)}$$

The demand for technology nests *xtNest* is depicted by *xtNesteq* shown below. There are three identically structured cases: (1) the nest is linked into the intermediate composite ND, (2) into the value added composite VA or (3) into another technology nests. The three cases differ in the aggregate price used (*pnd*, *pva* or *ptNest*) and the substitution elasticity (*sigmaNd*, *sigmav* or *sigmaNest*). In all cases, the share parameter is denoted with *atNest* and the related price with *ptNest*. Equally, in all cases, in order to reduce complexity for the solver, the price relation is taken out when the substitution elasticity is zero, i.e. the Leontief case.

Note that the third case where the technology nest *tNest* is part of another nest requires the alias *tNest1* which depicts the nest which is higher up in the technology tree.

```
xtNestEq(rs(r),tNest,aIn(a),ts(t)) $ (xpFlag(r,a) $ atNest(r,tNest,a,t) $ (not  tNest_n_a("Top",tNest,a))) ..

   xtNest(r,tNest,a,t)/xtNest.scale(r,tNest,a,t) =e=
*
*       --- nest is part of top level intermediate demand
*
    +  ( atNest(r,tNest,a,t) * nd(r,a,t)/xtNest.scale(r,tNest,a,t)
                  * [ {(pnd(r,a,t)/ptNest(r,tNest,a,t))**sigmaNd(r,a)} $ sigmaNd(r,a) + 1 $ (not sigmaNd(r,a)) ] ) $ tNest_n_a("ND",tNest,a)
*
*       --- nest is part of top level VA demand
*
     + ( atNest(r,tNest,a,t) * va(r,a,t)/xtNest.scale(r,tNest,a,t)
                  * [ {(pva(r,a,t)/ptNest(r,tNest,a,t))**sigmav(r,a)} $ sigmav(r,a) + 1 $ (not sigmav(r,a)) ] ) $ tNest_n_a("VA",tNest,a)
*
*       --- nest is part of other nests
*
     + sum(tNest_n_a(tNest1,tNest,a), atNest(r,tNest,a,t) * xtNest(r,tNest1,a,t)/xtNest.scale(r,tNest,a,t)
                  * [ {(ptNest(r,tNest1,a,t)/ptNest(r,tNest,a,t))**sigmaNest(r,tNest1,a)} $ sigmaNest(r,tNest1,a) + 1 $ (not sigmaNest(r,tNest1,a)) ]);
```

Again, that equation is not part of the GTAP standard model version 7 as documented in VDM 2018. In mathematical notation it reads:

$$
\begin{aligned}
XTNEST_{r,tNest,a} = {} & \alpha^{va}_{r,tNest,a} VA_{r,a} \left(\frac{PVA_{r,a}}{PNEST_{r,tNest,a}} \right)^{\sigma^{v}_{r,a}} \\
& + \alpha^{nd}_{r,tNest,a} ND_{r,a} \left(\frac{ND_{r,a}}{PNEST_{r,tNest,a}} \right)^{\sigma^{nd}_{r,a}} \\
& + \sum_{tNest1} \alpha^{tNest1}_{r,tNest,a} XTNEST_{r,tNest1,a} \left(\frac{PNEST_{r,tNest1,a}}{PNEST_{r,tNest,a}} \right)^{\sigma^{tNest1}_{r,a}}
\end{aligned}
$$

More information on the nesting approach can be found above in the section "Flexible nesting".

The **case of multiple outputs from one activity** is depicted in the equation *xeq*. Equation (8) in VDM 2018. That case is shown when the flag *diag(a)* is not unity, i.e. a not diagonal activity. In that case, the *omegas* transformation elasticity distributes the total output *xp* to activity specific output *x* of each product *i* based on the share parameter *gx* and the activity specific prices for each product *i* termed *p* in relation to average per activity prices found in the macro *m_pp*. In case of infinite transformation, the prices have to be identical:

```
xeq(rsNat(rNat),aIn(a),i,ts(t)) $ (xFlag(rNat,a,i) $ (not diag(a))  )..

  0 =e= (    x(rNat,a,i,t)/xp.scale(rNat,a,t)
           - gx(rNat,a,i,t)*xp(rNat,a,t)/xp.scale(rNat,a,t) *(p(rNat,a,i,t)/m_pp(rNat,a,t))**omegas(rNat,a)
        ) $ (omegas(rNat,a) ne inf)

     +  (   p(rNat,a,i,t) $ (sigmas(rNat,i) ne inf)
          + ps(rNat,i,t) $ (sigmas(rNat,i) eq inf)
          -  (1 + prdtx(rNat,a,t))*m_px(rNat,a,t)
        )     $ (omegas(rNat,a) eq inf)
     ;
```

The related equation *xpeq*, equation (9) in VDM 2018, considers these two cases accordingly: with infinite transformation, total output *xp* is equal to the sum of the commodity outputs, either *xs* in the diagonal case or *x* otherwise. With finite transformation, the producer price as defined in the macro *m_pp* is derived from the dual CET price aggregator which uses the share parameters *gx*, the prices *p* or *ps* and the transformation elasticities *omegas*. The choice of *p* or *ps* depends on whether consumers differentiate between the same commodities being produced by different activities as depicted by the substitution elasticity *sigmas*.

The *marco m_pp* which fined the activity specific producer price charges the production tax *prdtx* on the unit cost *m_px*:

```
$$macro m_pp(r,a,t)   {(1 + prdtx(r,a,t))*m_px(r,a,t)}
```

The related equations *peq* and *pseq* to aggregate output of the same commodity from different activities are depicted next. Both are only active in the non-diagonal case (not *diag*(a) and not *diag*(i)). The first case depicts the relation between the price of the commodity *i* outputted by activity *a* termed *p* and the average supply price for the commodity *ps*. They are equal (second line) in case of infinite substitution, otherwise, the second equation defines the average supply price as non-linear weighted average. The first line in *peq* defines for the finite case the share of total supply *xs* demanded from activity *a* depicted by *x* based on the share parameter ax, the price relation and the substitution elasticity *sigmas*:

```
peq(rsNat(rNat),aIn(a),i,ts(t)) $ (xFlag(rNat,a,i) $ (not diag(a)) )..

 0 =e=      (
               x(rNat,a,i,t)/xp.scale(rNat,a,t)
             - ax(rNat,a,i,t)*xs(rNat,i,t)/xp.scale(rNat,a,t) *(ps(rNat,i,t)/p(rNat,a,i,t))**sigmas(rNat,i)
            ) $ (sigmas(rNat,i) ne inf)

        +   (
              p(rNat,a,i,t) - ps(rNat,i,t)
            ) $ (sigmas(rNat,i) eq inf)
    ;
```

The distribution of output from nations to different sub-regions is described in the section "Integration into the modeling framework" of the chapter "Sub-regional dis-aggregation of production and factor markets in CGEBox" in Britz 2016.

Factor markets

The **supply of fully mobile or sluggish factors** *xft* at national level *rsNat* is depicted by the equation *xfteq* in case where the factor supply is not fixed (.range eq 0), equation 69 in VDM 2018. It is driven by the factor price *pft* relative to the price of aggregate domestic absorption *pabs* and the factor supply elasticity *etaf*. If *etaf* is zero, the price dependent part becomes a constant of unity and *xft* is fixed to the constant *aft*. Note that endogenous factor supply is not part of the standard GTAP model. Demand for new capital as a new factor is part of the capital vintage module and depicted differently.

```
xfteq(rsNat(rNat),fm,ts(t) ) $ (xftEqFlag(rNat,fm) $ (xft.range(rNat,fm,t) ne 0) $ (Not sameas(fm,"newCap"))) ..

  xft(rNat,fm,t)/xft.scale(rNat,fm,t)
       =e= (aft(rNat,fm,t)*(pft(rNat,fm,t)/pwfact(t))**etaf(rNat,fm))/xft.scale(rNat,fm,t);
```

The related **economy wide average factor price** *pft* is defined by the equation *pfteq,* equation (71) in VDM 2018 expanded with factor supply nests, which distinguished the sluggish case with a dual price aggregator (first block) and the fully mobile case where the equation ensures market clearing (second block):

```
pfteq(rs(r),fm,ts(t)) $ (xftEqFlag(r,fm) and (pft.range(r,fm,t) ne 0) and (not disr(r))) ..

 0 =e=

      --- sluggish factor mobility, dual price aggregator

     [ pft(r,fm,t)

         --- factors directly linked into top CET nest

         -   sum(a $ (xfFlag(r,fm,a) $ (not sum(fNest_a_f(fNest,a,f),1))),
                    gf(r,fm,a,t)*pf(r,fm,a,t)**(1+omegaf(r,fm)))**(1/(1+omegaf(r,fm)))

         --- subnests linked into top CET nest

         -   sum(fNest_n_f("xft",fNest,fm) $ gfNest(r,fm,fNest,t),
                 gfNest(r,fm,fNest,t)*pfNest(r,fm,fNest,t)**(1+omegaf(r,fm)))**(1/(1+omegaf(r,fm)))

      ] $ (omegaf(r,fm) ne inf)

      --- fully mobile factors, uniform prices => phyiscal aggregation

    +  [ xft(r,fm,t)/xft.scale(r,fm,t)
          - sum(a $ (xfFlag(r,fm,a) $ (not sum(fNest_a_f(fNest,a,fm),1))), xf(r,fm,a,t))/xft.scale(r,fm,t)
          - sum(fNest_n_f("xft",fNest,fm) $ gfNest(r,fm,fNest,t),          xfNest(r,fm,fNest,t))/xft.scale(r,fm,t)

      ] $ (omegaf(r,fm) eq inf)
    ;
```

In case of sluggish factor supply, i.e. `$ (omegaf(r,fm) ne inf)`, the agent specific factor prices net of taxes *pf* are aggregated using the dual price aggregator in the first expression in square brackets, considering factor demand captured by the value added composite and by technology nests. In case of fully mobile factors (the second expression in square brackets), the price is not directly defined in the equation, but rather indirectly via market clearing.

Sector specific factor prices net of taxes *pf* are directly or indirectly defined in the equation *pfeq* shown below, equation (70) in VDM 2018. It considers five different cases. The first case considers sluggish factor supply where the factor is part of the value added nest. The usual CET distribution logic applies: the supply depends on the share parameter *gf* and the total supply *xft* as well as on the relation between the price paid in the sector *pf* relative to the average one *pft*, exponent the transformation elasticity *omegaf*. The second case where the factor is part of a factor supply nest under sluggish supply is identically is structured, however, the composite price now refers to a technology nest, i.e. *pfNest* with the related transformation elasticity *omegafNest*.

Next we have the two cases with fully mobile supply: either in case of economy wide full mobility or fully mobility inside in a nest. In both cases, the sector specific price is equal to the average one. The last case depicts immobile factors: here, the default case is that the immobile factor supply elasticity *etaff* is zero such that the factor demand *xf* must be equal to the given parameter *gf*.

```
pfeq(rs(r),f,aIn(A),ts(t)) $ (xfEqFlag(r,f,a) $ (pf.range(r,f,a,t) ne 0) $ (not disr(r)) )  ..
  0  =e=

    --- "sluggish" factor supply based on CET function, factor is part of top level nest

      [ xf(r,f,a,t)/xf.scale(r,f,a,t)
          -  gf(r,f,a,t)*xft(r,f,t)/xf.scale(r,f,a,t) * (pf(r,f,a,t)/pft(r,f,t))**omegaf(r,f)

      ]  $ (fm(f) $ (omegaf(r,f) ne inf) $ (not sum(fNest_a_f(fNest,a,f),1)) )


    --- "sluggish" factor supply based on CET function, factor is part of sub-nest
        (not part of standard GTAP)

    + [ xf(r,f,a,t)/xf.scale(r,f,a,t)
          - sum(fNest_a_f(fNest,a,f),
              gf(r,f,a,t)*xfNest(r,f,fNest,t)/xf.scale(r,f,a,t)
                     * (pf(r,f,a,t)/pfNest(r,f,fNest,t))**omegafNest(r,fNest,f))

      ]  $ (fm(f) $ sum(fNest_a_f(fNest,a,f) $ (omegafNest(r,fNest,f) ne inf),1))


  --- fully mobile factor: prices are equal across sectors

      ... factor is linked into top level nest
 +  [
        pf(r,f,a,t) - pft(r,f,t)

    ] $ ((fm(f) $ (omegaf(r,f) eq inf)) $ (not sum(fNest_a_f(fNest,a,f),1)))

      ... factor is linked into sub-nest
 +  [
        pf(r,f,a,t) - sum(fNest_a_f(fNest,a,f), pfNest(r,f,fNest,t))

    ] $ ((fm(f) $ sum(fNest_a_f(fNest,a,f) $ (omegafNest(r,fNest,f) eq inf),1)))

  --- immobile factors fnm, with factor supply elasticity

 +  [
       xf(r,f,a,t)/xf.scale(r,f,a,t) - gf(r,f,a,t)/xf.scale(r,f,a,t)
           *[ {(pf(r,f,a,t)/pwfact(t))**etaff(r,f,a)} $ etaff(r,f,a) + 1 $ (not etaff(r,f,a))]
    ] $(fnm(f)) ;
```

The **agent specific factor prices tax inclusive** are defined via the equation *pfaeq* and the macro m_pfa, equation (73) in VDM 2018:

```
pfaeq(rs(r),f,aIn(a),ts(t)) $ xfFlag(r,f,a) ..

  pfa(r,f,a,t) =E= m_pfa(r,f,a,t);
```

The macro *m_pfa* distinguishes the case of finite transformation of factor supply and the case of full factor mobility, i.e. infinite transformation and adds the national tax rates, i.e. subsidy rates *fctts*, tax rates *fcttx* and an economy factor tax shifter *fcctxShift*:

```
--- factor price of agents, equation (73) in VDM 2018

$$macro m_pfa(r,f,a,t) { \

   --- factors are immobile or sluggish: price before taxes differs by sector

     (pf(r,f,a,t) $ (fnm(f) or (omegaf(r,f) ne inf)) \

    --- fully mobile factor: all sector face a unifrom price before taxes

    + pft(r,f,t)  $ (fm(f) $ (omegaf(r,f) eq inf))) \

    --- add ad-valorem factor taxes

    * sum(r_r(r,rNat), ( 1 + fctts(rNat,f,a,t) + fcttx(rNat,f,a,t) + fcttxShift(rNat,t) $ fTaxShift(f))) }
```

The “Flexible nesting” approach allows introducing factor supply nests which can also be linked into other factor supply nests. The equation *pfNestEq* defines the average factor price of such a nest. It distinguishes the cases of finite factor transformation in the first block and infinite one in the second. In the first block, the average price *pfNest* is defined via dual price aggregator, taking the share parameters (*gf* for factors and *gfNest* for sub-nest), the prices (*pf* for factors and *pfNest* for sub-nests) and the transformation elasticity *omegaFNest* into account. In case of infinite transformation handled by the second block, the price is indirectly defined from the adding up-condition of the factor quantities. That equation is not part of the GTAP standard model.

```
pfNestEq(rs(r),fm,fNest,ts(t)) $ gfNest(r,fm,fNest,t) ..

 0 =e=

     --- sluggish factor mobility, dual price aggregator for sub-nests

    (pfNest(r,fm,fNest,t)

         --- factors linked into sub nest

         - [     sum(fNest_a_f(fNest,a,fm) $ gf(r,fm,a,t),
                     gf(r,fm,a,t)*pf(r,fm,a,t)**(1+omegafNest(r,fNest,fm)))


         --- subnests linked into sub nest

             +    sum(fNest_n_f(fNest,fNest1,fm) $ gfNest(r,fm,fNest1,t),
                      gfNest(r,fm,fNest1,t)*pfNest(r,fm,fNest1,t)**(1+omegafNest(r,fNest,fm)))
           ]**(1/(1+omegafNest(r,fNest,fm)))

         ) $ (omegafNest(r,fNest,fm) ne inf)

     --- fully mobile factors, uniform prices => physical aggregation

  +  (xfNest(r,fm,fNest,t)/xft.scale(r,fm,t)
                 - sum(fNest_a_f(fNest,a,fm)      $ gf(r,fm,a,t),          xf(r,fm,a,t))/xft.scale(r,fm,t)
                 - sum(fNest_n_f(fNest,fNest1,f) $ gfNest(r,fm,fNest1,t), xfNest(r,fm,fNest1,t))/xft.scale(r,fm,t)

     ) $ (omegafNest(r,fNest,fm) eq inf)
  ;
```

The total factor supply to such a nest *xfNest* as defined in the *xfNestEq* again considers these two cases. In case of finite transformation in the first block, total supply either depends on the sector wide supply of that factor of *xft* and the price relations or on the amount supplied to the nest *fNest1* to which the sub-nest belongs. In case of infinite transformation, the sub-nest price is either equal to the sector-wide factor price *pft* or to the price of the upper nest *pfNest* indexed with *fNest1*.

```
xfNestEq(rs(r),fm,fNest,ts(t)) $ gfNest(r,fm,fNest,t) ..

  0 =e=

     --- "sluggish" factor supply from top level nest (xft,pft) to current nest

       ( xfNest(r,fm,fNest,t)/xft.scale(r,fm,t)
         - gfNest(r,fm,fNest,t) * xft(r,fm,t)/xft.scale(r,fm,t)
            * (pfNest(r,fm,fNest,t)/pft(r,fm,t))**omegaf(r,fm))
              $ ( (omegaf(r,fm) ne inf) $ fNest_n_f("xft",fNest,fm))

     --- "sluggish" factor supply from another nest fNest1 to the current nest

     + sum(fNest_n_f(fNest1,fNest,fm), xfNest(r,fm,fNest,t)/xft.scale(r,fm,t)
           - gfNest(r,fm,fNest,t) * xfNest(r,fm,fNest1,t)/xft.scale(r,fm,t)
                * (pfNest(r,fm,fNest,t)/pfNest(r,fm,fNest1,t))**omegafNest(r,fNest1,fm))
                  $ ( sum(fNest_n_f(fNest1,fNest,fm) $ (omegafNest(r,fNest1,fm) ne inf),1))

     --- non "sluggish" factor supply from top level nest (xft,pft) to current nest

     + ( pfNest(r,fm,fNest,t) - pft(r,fm,t) ) $ ( (omegaf(r,fm) eq inf) $ fNest_n_f("xft",fNest,fm))

     --- non "sluggish" factor supply from another nest to current nest

     + sum(fNest_n_f(fNest1,fNest,fm) $ (omegafNest(r,fNest1,fm) eq inf),
          pfNest(r,fm,fNest,t) - pfNest(r,fm,fNest1,t) )
 ;
```

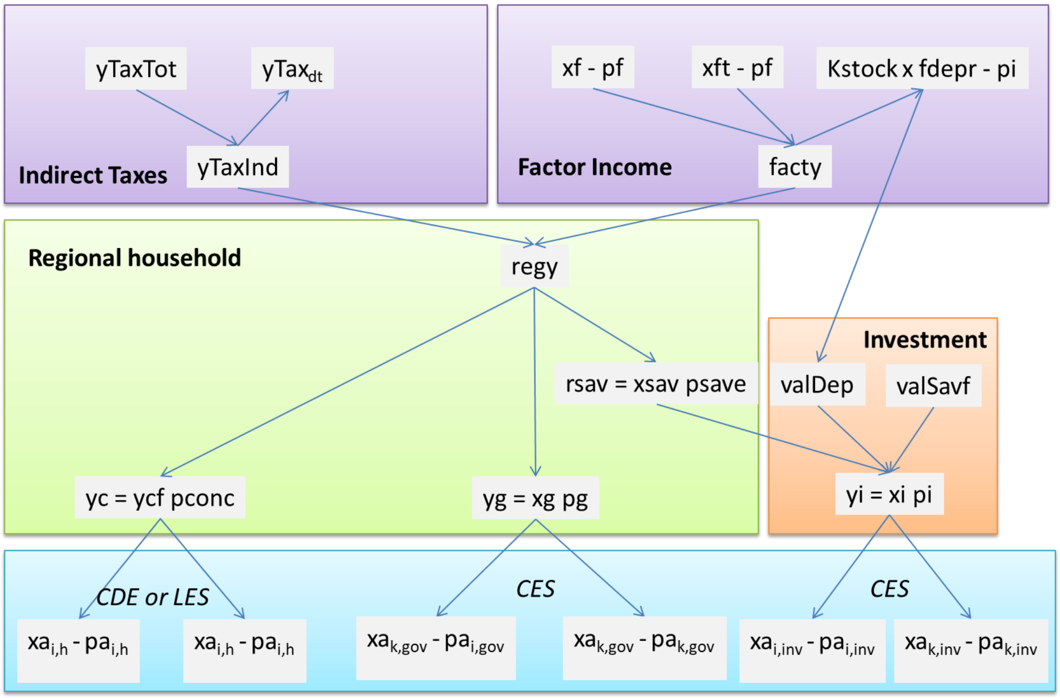
The factor supply from nation to sub-regions is described in the section "Integration into the modeling framework", of the chapter "Sub-regional dis-aggregation of production and factor markets in CGEBox". Note also that the "GTAP-AEZ" module will introduce land transformation at the level of Agro-Ecological Zones and replace some of the equations detailed above.

Income generation and distribution - overview

An overview on income generation and distribution under the regional household approach is depicted in Figure 2 below. Regional income is sourced (1) by factor income *facty* (factor remuneration including direct taxes) minus depreciation (*valDep*) and (2) by indirect taxes *yTaxInd*, i.e. all tax flows *yTaxTot* minus direct taxes $yTax_{dt}$ which are already comprised in the factor income.

Regional household income *regy* is distributed to final demand expenditures of private households *yc*, government *yg* and regional net savings *rsav*. Adding the value of depreciation *valDep* and of foreign savings *valSavf* to regional net savings *rsav* yields investment demand expenditures *yi*. The distribution of the final demand expenditures to the Amington demands for each product xa_i is based on CES demand systems for investments and the government which hence encompass the CD or Leontief case, whereas a CDE, LES or AIDADS demand system can be used to distribute private household expenditure *yc*.

Figure 2: Overview in income generation and distribution



Income generation

Regional income *regy*, i.e. economy wide income which can be spent on net savings and final consumption by government and private households, is generated from factor income including direct taxes *facty* and indirect taxes *yTaxInd* as defined in the *regYeq*, equation (26) in VDM 2018. Note that the *p_capTrans* parameter is part of the GRDEM module and otherwise zero:

```
*
* --- regional income, equation (26) in VDM 2018
*
  regYeq(rsNat(rNat),ts(t)) $ (regy.range(rNat,t) ne 0) ..

     regY(rNat,t)/regy.scale(rNat,t) =e= (factY(rNat,t) + yTaxInd(rNat,t)
                                         - p_capTrans(rNat,t)*pnum(t))/regy.scale(rNat,t);
```

Factor income including direct taxes *factY* is defined by the *factYeq*, equivalent to equation (25) in VDM 2018. It considers returns to primary factors, i.e. economy wide factor prices *pft* multiplied with economy wide factor use *xft* for mobile factors and sector specific factor use *xf* and related prices *pf* for immobile factors. Note that factor income comprises direct taxes. As returns to capital also cover depreciation, the value of depreciation is deducted, considering the depreciation rate *fdepr*, the average price of investments *pi* and the capital stock *kstock*:

```
factYeq(rsNat(rNat),ts(t))  ..

 factY(rNat,t)/facty.scale(rNat,t)
      =e= [    sum(fm $ xftFlag(rNat,fm), pft(rNat,fm,t)*xft(rNat,fm,t))
             + sum((fnm,a) $ xfFlag(rNat,fnm,a), pf(rNat,fnm,a,t)*xf(rNat,fnm,a,t))
             - fdepr(rNat,t)*pi(rNat,t)*kstock(rNat,t) ] / facty.scale(rNat,t);
```

Indirect tax income *yTaxInd* is calculated by the *ytaxIndeq* from total tax revenues *ytaxTot*, corrected for direct taxes (index "*dt*") comprised in factor income *factY* as defined above, see equation (24) in VDM 2018:

```
ytaxIndeq(rsNat(rNat),ts(t))  ..

  yTaxInd(rNat,t)/yTaxInd.scale(rNat,t) =e= (ytaxTot(rNat,t) - ytax(rNat,"dt",t))/yTaxInd.scale(rNat,t);
```

Total tax income *yTaxTot* considers all tax flow *gy* depicted in the model (see next equation) and is defined by the equation *ytaxTotEq*, equivalent to equation (23) in VDM 2018:

```
ytaxToteq(rsNat(rNat),ts(t))  ..

 ytaxTot(rNat,t)/yTaxTot.scale(rNat,t) =e= sum(gy, ytax(rNat,gy,t))/yTaxTot.scale(rNat,t);
```

Tax flows *yTax* for the different types of tax flow *gy* are defined by the equation *ytaxeq* , defined in equation (13)-(22) in VDM 2018. It considers the following blocks:

Production taxes *pt*, levied with the relative tax rate *prdtx* on sectoral revenues, i.e. output *m_px* times the related producer price *m_xp*. Note that fix cost might be present in the model if the Melitz/Krugmann extension is used, depicted by the technology nest "*Top*" on which also production taxes are charged.

```
--- Tax revenues from production tax, equation (13) in VDM 2018

   [  sum(a $ prdtx.l(rNat,a,t), prdtx(rNat,a,t)*(m_px(rNat,a,t)*m_xp(rNat,a,t)
     +sum(tNest $ tNest_n_a("Top",tNest,a),xtNest(rNat,tNest,a,t)*ptNest(rNat,tNest,a,t))))] $ sameas(gy,"pt")
```

Indirect taxes on private consumption *pc*, charged with rate *dintx* on domestic consumption of private household *m_xd* times the related domestic price *m_pd*, and with rate *mintx* on imports by private households *m_xm* times the average price of imports *pmt*:

```
Indirect tax revenues from private consumption, equation (15) in VDM 2018

[sum((i,h),    [  (dintx(rNat,i,h,t) + itxshft(rNat,i,t))*m_domPrice(rNat,i,h,t)*m_xd(rNat,i,h,t) ]  $ m_alphad(rNat,i,h,t)
             + [  (mintx(rNat,i,h,t) + itxshft(rNat,i,t))*pmt(rNat,i,t)*m_xm(rNat,i,h,t)          ]  $ m_alpham(rNat,i,h,t))] $ sameas(gy,"pc")
```

Note the product specific general tax shifter *itxShft* which is normally not active.

Two other blocks apply the very same logic to government and investment consumption, resulting in tax flows *gc* and *ic*:

```
Indirect tax revenues from government consumption, equation (16) in VDM 2018

[sum((i,gov),   [ (dintx(rNat,i,gov,t) + itxshft(rNat,i,t))*m_domPrice(rNat,i,gov,t)*m_xd(rNat,i,gov,t) ] $ m_alphad(rNat,i,gov,t)
             + [ (mintx(rNat,i,gov,t) + itxshft(rNat,i,t))*pmt(rNat,i,t)*m_xm(rNat,i,gov,t)             ] $ m_alpham(rNat,i,gov,t))] $sameas(gy,"gc")

Indirect tax revenues from investment consumption, equation (17) in VDM 2018

[sum((i,inv),    [ (dintx(rNat,i,inv,t) + itxshft(rNat,i,t))*m_domPrice(rNat,i,inv,t)*m_xd(rNat,i,inv,t) ] $ m_alphad(rNat,i,inv,t)
              + [ (mintx(rNat,i,inv,t) + itxshft(rNat,i,t))*pmt(rNat,i,t)*m_xm(rNat,i,inv,t)             ] $ m_alpham(rNat,i,inv,t))] $ sameas(gy,"ic")
```

Direct taxes *dt* are levied with factor specific rates *kappaF* on factor income, i.e. factor prices *pf* respectively *pft* times factor use *xf* respectively *xft*. An endogenous or exogenous direct tax shifter *kappaf* can be added:

```
Direct tax revenues, equation (22) in VDM 2018

[   sum(fm $ xftFlag(rNat,fm), (kappaf(rNat,fm,t)+kappaShft(rNat,t)) * pft(rNat,fm,t)*xft(rNat,fm,t))
  + sum((fnm,a) $ xfFlag(rNat,fnm,a), (kappaf(rNat,fnm,t)+kappaShft(rNat,t)) * pf(rNat,fnm,a,t)*xf(rNat,fnm,a,t)) ] $ sameas(gy,"dt")
```

Export tax revenues *et* are based on bilateral export tax rates *exptx* and potentially commodity specific export tax shifter *etax*. Note that depending on how exports are depicted (infinite transformation or not), different prices are used: (a) the bilateral export price *pe* if there is finite transformation between destination), (b) the average export price *pet* if there is infinite transformation between destination, but not between exports and domestic sales and (c) the supply price *ps* if all transformations are infinite. Finally, if the Melitz module is active, the firm price defined in the macro m_pFirm is used. The related quantity is defined in the macro *m_xws*.

```
--- Export tax revenues, equation (21) in VDM 2018

+    [sum((i,rNat1) $ ( (exptx.l(rNat,i,rNat1,t) + etax.l(rNat,i,t)) $ m_xwFlag(rNat,i,rNat1) ),
        (exptx(rNat,i,rNat1,t) + etax(rNat,i,t))
       * [ ( pe(rNat,i,rNat1,t)  $ (omegaw(rNat,i)  ne inf)
           + pet(rNat,i,t)       $ ((omegaw(rNat,i) eq inf) $ (omegax(rNat,i) ne inf))
           + ps(rNat,i,t)        $ (omegax(rNat,i)  eq inf))
           $$ifthen.iMel "%melitz%"=="on"
             $ (not iMel(i))
           + (m_pFirm(rNat,i,rNat1,t) $ iMel(i))
           $$endif.iMel
         ]
          *m_xws(rNat,i,rNat1,t))] $sameas(gy,"et")
```

Import taxes "*mt*" are defined from bilateral import taxes *imptx*, a commodity specific import tax shifter *mtax*, the bilateral c.i.f. prices defined via *%pmcif%* and the bilateral flows *xw*.

```
+    [sum((i,rNat1) $ m_xwFlag(rNat1,i,rNat), (imptx(rNat1,i,rNat,t) + mtax(rNat,i,t))
                                        *%pmcif%(rNat1,i,rNat,t)*xw(rNat1,i,rNat,t))] $sameas(gy,"mt")
```

Factor taxes *ft* paid by each activity are levied on the activity specific factor price *pf* and use *xf* with the rates *fcttx* and a factor tax shifter *fcttxShift*:

```
+    [sum((f,a) $ xfFlag(rNat,f,a), (fcttx(rNat,f,a,t) + fcttxShift(rNat,t) $ fTaxShift(f))
                                         * pf(rNat,f,a,t) * xf(rNat,f,a,t))] $sameas(gy,"ft")
```

The same logic (less the shifter) applies for factor subsidies *fs*:

```
+    [sum((f,a) $ xfFlag(rNat,f,a), fctts(rNat,f,a,t)      * pf(rNat,f,a,t) * xf(rNat,f,a,t))] $sameas(gy,"fs")
```

Finally, emission taxes *emis* can be introduced, levied on emissions *emis* with the potentially endogenous price *emisP*. Currently, these only relate to CO_2 emissions.

```
+    [emis(rNat,t) * emisP(rNat,t) $ permit(rNat,t) ]   $ sameas(gy,"emis")
```

Income distribution

If the regional household approach is used, savings, government and household demand are distributed based on a modified CD utility function where the private household demand share is driven by the utility of total private expenditure with regard to utility *phiP* and the original private demand share *betaP*, part of equation (28) of VDM 2018:

```
betaPhiEq(rsNat(rNat),ts(t))   ..
 betaPhi(rNat,t)  =E=  betaP(rNat,t)/phiP(rNat,t);
```

The updated share *betaP* termed *betaPhi* implies that the shares as defined in the benchmark do not add to unity any longer. Therefore, an intermediate variable *phiRegY* is defined which scales regional income to reflect the updated sum of the shares. Assume that *betaPhi* is increased compared to the benchmark. That implies that the second term on the LHS exceed unity. The total expenditure *phiRegy* is accordingly proportionally decreased to yield still total regional income *regY*, part of equation (28) in VDM 2018:

```
phiRegyEq(rsNat(rNat),ts(t))   ..
  phiRegy(rNat,t)*(betaPhi(rNat,t)  + betaG(rNat,t)  + betaS(rNat,t))/regy.scale(rNat,t)
       =E=  regY(rNat,t)/regy.scale(rNat,t);
```

That corrected income then dries the private, government and savings expenditures. The amount spent for private consumption *yc* is defined in the equation *yceq*, equation (29) in VDM 2018:

```
yceq(rsNat(rNat),h,ts(t))   ..

    yc(rNat,h,t)/yc.scale(rNat,h,t)  =e=  betaPHi(rNat,t)*phiregY(rNat,t)/yc.scale(rNat,h,t);
```

The amount spent for government consumption *yg* is defined in the equation *ygeq*, equation (30) in VDM 2018:

```
ygeq(rsNat(rNat),ts(t))   ..

   yg(rNat,t)/yg.scale(rNat,t)  =e=  betaG(rNat,t)*phiRegy(rNat,t)/yg.scale(rNat,t);
```

And finally, the amount of regional savings *rsav* is depicted in the equation *rsaveq*, equation (31) in VDM 2018:

```
rsaveq(rsNat(rNat),ts(t))   ..

 rsav(rNat,t)/rsav.scale(rNat,t)  =e=  betaS(rNat,t)*phiRegY(rNat,t)/rsav.scale(rNat,t);
```

Note that the regional household approach can be replaced by separate accounts, see the sub-section "Model equations" in the section "myGTAP module".

Household consumption

The **consumer price index** *pcons* is defined from the budget share *xcshr* and the Armington prices defined in the macro *m_pa*, equation (38) in VDM 2018:

```
*  Consumer expenditure deflator (approx.) -- PHHLDINDEX (1005)

pconseq(rs(rNat),h,ts(t)) $ (sum( i, xaFlag(rNat,i,h))  and ( (pcons.range(rNat,h,t) ne 0) or (not rres(rNat)) or (lcu.range(rNat,t) ne 0)) )  ..
    pcons(rNat,h,t) =e= sum( i $ xaFlag(rNat,i,h), xcshr(rNat,i,h,t)*m_pa(rNat,i,h,t)) ;
```

The Armington demands for household consumption can be defined either by a CDE demand system as used in the GTAP Standard model or a LES demand system as found in my other CGEs. Note that the LES system collapses to a CD system if the commitments are removed, such the model can host three different demand system for household consumption.

CDE case

In the standard GTAP model, a constant difference in elasticity (CDE) indirect demand system is used. The equations can be found "model\dem_cde.gms". The **final demand quantity** *xa* (the Armington demand) for each household *h* and product *i* are defined from the budget shares *xcshr* and the private consumption expenditures *yc*, see equation *xaceq*, equation (37) in VDM 2018:

```
xaceq(rs(r),iIn(i),h,ts(t)) $ xaFlag(r,i,h) ..
   xa(r,i,h,t) /xa.scale(r,i,h,t)*m_pa(r,i,h,t) =e=  xcshr(r,i,h,t)/xa.scale(r,i,h,t)*yc(r,h,t);
```

The **budget shares** *xcshr* as defined in the equation *xchsreq* are derived from unscaled shares *zcons*, scaled again by unity based on their sum *zConsSum*, equation (36) in VDM 2018:

```
xcshreq(rs(r),iIn(i),h,ts(t)) $ (xaFlag(r,i,h) $ (not sum(dNest_i_fd(dNest,i,h),1))) ..

 xcshr(r,i,h,t)/xcshr.scale(r,i,h,t) =e= zcons(r,i,h,t)/zConsSum(r,h,t) * 1/xcshr.scale(r,i,h,t);
```

The unscaled shares *zcons* as defined in the equation *zconseq* depend on utility *uh* (i.e. indirectly on expenditures) and the product prices defined in the macro *m_pa* relative to income *yc* per capita, defined from population *pop* for that household and three parameter vectors *alphaa*, *bh* and *eh*. That is an unnumbered equation in VDM 2018, noted before equation (36):

```
zconseq(rs(r),iIn(i),h,ts(t) ) $ (xaFlag(r,i,h) $ (not sum(dNest_i_fd(dNest,i,h),1))) ..

 zcons(r,i,h,t)/zcons.scale(r,i,h,t) =e= alphaa(r,i,h,t)*bh(r,i,t)

                  --- bh measures substitution effect, driven by price change
                      relative to income change

                  * (m_pa(r,i,h,t)**bh(r,i,t))

                  *  ((yc(r,h,t)/pop(r,h,t)*1000)**(-bh(r,i,t)))

                  --- expansion effect, dirven by utility (= income)

                  *  (uh(r,h,t)**(eh(r,i,t)*bh(r,i,t)))/zcons.scale(r,i,h,t);
```

The sum of these unscaled shares must also consider the case of sub-nests in demand, defined in the equation *zConsSumeq*, part of equation (36) in VDM 2018:

```
zConsSumeq(rs(r),h,ts(t)) $  sum(j,xaFlag(r,j,h)) ..

 zConsSum(r,h,t)/sum(j $xaFlag(r,j,h), zcons.scale(r,j,h,t))
      =e= [   sum(j $ (xaFlag(r,j,h) $ (not sum(dNest_i_fd(dNest,j,h),1))), zcons(r,j,h,t))
            + sum(dNest_n_fd("top",dNest,h) $ alphaDN(r,dNest,h,t), zconsDN(r,dNest,h,t))
          ]  /sum(j $xaFlag(r,j,h), zcons.scale(r,j,h,t)) ;
```

The **utility level** *u* is indirectly defined by the following equation in the equation *uheq*, equation (32) in VDM 2018:

```
uheq(rs(r),h,ts(t)) $ sum(i,xaFlag(r,i,h)) ..

 1 =e= sum(i $ (xaFlag(r,i,h) $ (not sum(dNest_i_fd(dNest,i,h),1))), zcons(r,i,h,t)/bh(r,i,t))

     --- add sub-nests under final demand function, not part of standard GTAP model

     + sum(dNest_n_fd("top",dNest,h) $ alphaDN(r,dNest,h,t), zconsDN(r,dNest,h,t)/bhDN(r,dNest,t));
```

Finally, the **elasticity of private expenditure versus private utility** *phip* is defined in the equation *phiPEq*. It updates the private consumption share in the regional household income distribution:

```
phiPeq(rs(r),ts(t)) $ ((phip.range(r,t) ne 0) $ rNat(r)) ..

 phiP(r,t) =e= sum((i,h) $ xaFlag(r,i,h), xcshr(r,i,h,t)*eh(r,i,t))/sum(h $ hFlag(r,h),1);
```

LES or CD case

The equations for the LES or CD case are found in the file "model\dem_les.gms". The LES case is not part of GTAP Standard model.

The **Armington demands** *xa* in the LES case reflect the constant term *gammaLES*, often termed commitment, and a share *alphaLES* on non-committed income *yCNonCom* divided by the Armington price defined in the macro *m_pa*:

```
xacLesEq(rsNat(r),i,h,ts(t)) $ (xaFlag(r,i,h) $ (not sum(dNest_i_fd(dNest,i,h),1))) ..
 xa(r,i,h,t) /xa.scale(r,i,h,t)
   =e= (    gammaLES(r,i,h,t)*pop(r,h,t)
          + (alphaLES(r,i,h,t)/m_pa(r,i,h,t) * yCNonCom(r,h,t)) $ alphaLes.l(r,i,h,t))/xa.scale(r,i,h,t);
```

That equation replace equation (37) in VDM 2018, CDE case:

$$XA_{r,i,h} = \gamma^{LES}_{r,i,h} pop_{r,h} + \alpha^{LES}_{r,i,h} yCNonCom_{r,h} \tag{37*}$$

The same functional relation is also used to define sub-nests demands as defined in the equation *xdNestLesEq*:

```
xdNestLesEq(rsNat(r),dNest,h,ts(t)) $ sum(dNest_n_fd("top",dNest,h), alphaLES.l(r,dNest,h,t)) ..
 xdNest(r,dNest,h,t) /xdNEst.scale(r,dnest,h,t)
   =e= (gammaLES(r,dNest,h,t)*pop(r,h,t) + alphaLES(r,dnest,h,t)/pdNest(r,dNest,h,t) * yCNonCom(r,h,t))/xdNest.scale(r,dNest,h,t);
```

Non-committed income *yCNonCom* as defined in the equation *yCNonComEq* reflects total private consumption expenditure *yc* minus the value of the commitments, i.e. the gamma parameters multiplied with the Armington prices defined in the macro *m_pa*:

```
yCNonComEq(rsNat(r),h,ts(t)) $ sum(i,xaFlag(r,i,h))  ..
  ycNonCom(r,h,t)/yc.scale(r,h,t)
   =E= (yc(r,h,t) - sum(i $ (xaFlag(r,i,h) $ alphaLES.l(r,i,h,t)), gammaLES(r,i,h,t)*pop(r,h,t) * m_pa(r,i,h,t))
                  - sum(dNest_n_fd("top",dNest,h), gammaLES(r,dNest,h,t)*pop(r,h,t) * pdNest(r,dNest,h,t)))/yc.scale(r,h,t);
```

In mathematical notation:

$$ycNonCom_{r,h} = \left(yc_{r,h} - \sum_i \gamma^{LES}_{r,i,h} pop_{r,h} PA_{r,i,h} - \sum_{dNest \in top} \gamma^{LES}_{r,dNest,h} pop_{r,h} PDNest_{r,i,h} \right)$$

The **budget shares** *xcshr* are defined from the Armington demands, prices and expenditures in the equation *xcshrLESeq*:

```
xcshrLESeq(rsNat(r),i,h,ts(t)) $ xaFlag(r,i,h) ..
  xcshr(r,i,h,t)*yc(r,h,t)/xa.scale(r,i,h,t) =e= xa(r,i,h,t)*m_pa(r,i,h,t)/xa.scale(r,i,h,t);
```

These budget share equations replace equation (36) in VDM 2018 for the CDE case:

$$s^{p}_{r,i,h} yc_{r,h} = XA_{r,i,h} PA_{r,i,h} \tag{36*}$$

Finally, the utility for the private households *uh* is defined in the equation *uhLESeq*:

```
uhLESeq(rsNat(r),h,ts(t)) $ (sum(i,xaFlag(r,i,h)) $ (p_demSystem eq LES)) ..

   uh(rsNat,h,t)
          =E= (yc(rsNat,h,t)
              - sum(i $ (xaFlag(rsNat,i,h) $ alphaLES.l(rsNat,i,h,t)),
                        gammaLES(rsNat,i,h,t)*pop(rsNat,h,t) * m_pa(rsNat,i,h,t))
              - sum(dNest_n_fd("top",dNest,h),
                        gammaLES(rsNat,dNest,h,t)*pop(rsNat,h,t) * pdNest(rsNat,dNest,h,t))
             )*(  prod(i $ (xaFlag(rsNat,i,h) $ alphaLES.l(rsNat,i,h,t)),
                             (alphaLES.l(rsNat,i,h,t)/m_pa(rsNat,i,h,t))**alphaLES.l(rsNat,i,h,t))
               * prod(dNest_n_fd("top",dNest,h) $ alphaLES.l(rsNat,dNest,h,t),
                             (alphaLES(rsNat,dNest,h,t)/pdNest(rsNat,dNest,h,t))**alphaLES(rsNat,dNest,h,t)))

             * 1/uhLES(rsNat,h,t);
```

Note that the CD case is comprised in the equations above if the commitment terms are set to zero. The equation above replaces equation (32) from VDM 2018 for the CDE case:

$$U_r^h = \frac{1}{UBas_r^h} \prod_i \left(XA_{r,i,h} - \gamma_{r,i,h}^{LES} pop_{r,h} \right)^{\alpha_{r,i,h}^{LES}} \prod_{tNest \in top} \left(XdNest_{r,tNest,h} - \gamma_{r,tNest,h}^{LES} pop_{r,h} \right)^{\alpha_{r,tNest,h}^{LES}}$$

(32*)

The econometrically estimated AIDADS system as part of G-RDEM modules is detailed in section "An AIDADS demand system with detail for food consumption".

Government consumption

First the reader is reminded that under the regional household approach, there is not separate household account and hence no direct link between tax revenues and government expenditures. That can be changed by using the "myGTAP module".

Government consumption *yg* under the regional household approach is a share *betag* of regional income *regy*, corrected for the endogenous share of private spent captured by *phiRegy* (see above for savings), equation (39) in VDM 2018:

```
ygeq(rsNat(rNat),ts(t))  ..

   yg(rNat,t)/yg.scale(rNat,t)  =e=  betaG(rNat,t)*phiRegy(rNat,t)/yg.scale(rNat,t);
```

The physical demand aggregate *xg* is derived from the price index *pg* defined by the equation *pgeq*, equation (40) in VDM 2018, expanded to account for demand nests. The price index *pg* is defined under the assumption of CES / CD / Leontief demand for government using the typical dual price aggregator based on the government specific Armington prices defined by the macro *m_pa*, a preference shifter variable *lambag* and the share parameters *alphaa* for government. Which case is used is defined by the related substitution elasticity *sigmag*. Note that the equation also considers the case that government demand uses CES sub-nests which aggregates products to product groups:

```
pgeq(rs(rNat),ts(t)) $ axg(rNat,t) ..

  pg(rNat,t) =E=

            --- CD case with sigmag==1

           ( [ prod((i,gov) $ ((not sum(dNest_i_fd(dnest,i,gov),1)) $ alphaa.l(rNat,i,gov,t)),
                    (m_pa(rNat,i,gov,t)/alphaa(rNat,i,gov,t))**alphaa(rNat,i,gov,t))

               -- CES-subnests, not part of Standard GTAP model

               * prod(dNest_n_fd("top",dnest,gov) $ alphaDN(rNat,dNest,gov,t),
                     (pdNest(rNat,dNest,gov,t)/alphaDN(rNat,dNest,gov,t))**alphaDN(rNat,dNest,gov,t))
                ]/axg(rNat,t) ) $ (sigmag(rNat) eq 1)

            --- CES or Leontief case with sigmag<>1

    +          ([ sum((i,gov) $ ((not sum(dNest_i_fd(dnest,i,gov),1)) $ alphaa.l(rNat,i,gov,t)),
                  alphaa(rNat,i,gov,t)*m_pa(rNat,i,gov,t)**(1-sigmag(rNat)))

               -- CES-subnests, not part of Standard GTAP model

               + sum(dNest_n_fd("top",dnest,gov) $ alphaDN(rNat,dNest,gov,t),
                  alphaDN(rNat,dNest,gov,t)*pdNest(rNat,dNest,gov,t)**(1-sigmag(rNat)))

               ]**(1/(1-sigmag(rNat)))/axg(rNat,t)
                   ) $ (sigmag(rNat) ne 1);
```

The macro *m_pa* is usually defined as follows. If both a share parameter for imports *alpham* and for domestic sales *alphad* is given, it uses the definition of the Armington price *m_padef* given below. Otherwise, it uses directly either the macro for the domestic price *m_pdp* or for the import prices *m_pmp*. Finally, if neither of the two share parameters is given, the Armington price is used, a case relevant when the Melitz module is active.

```
$$macro m_pa(r,i,aa,t)   { \

                         --- Armington price and matching
                             equation is only present in model if both domestic and import demand are non-zero

                         [   m_padef(r,i,aa,t)              $ (alpham(r,i,aa,t) $ alphad(r,i,aa,t) ) \

                         --- if only domestic demand, use domestic price

                          + (m_pdp(r,i,aa,t)*alphad(r,i,aa,t)) $ (not alpham(r,i,aa,t))\

                         --- if only import demand, use import price

                          + (m_pmp(r,i,aa,t)*alpham(r,i,aa,t)) $ (not alphad(r,i,aa,t)) \

 $$ifi "%modulesGTAP_MELITZ%"=="on"  + pa(r,i,aa,t) $ ( (not alpham(r,i,aa,t)) $ (not alphad(r,i,aa,t)) )\
```

The macro *m_padef* can either introduce the Armington price *pa* or can replace it with the dual price aggregator, equation (47) in VDM 2018:

```
$$macro m_padef(r,i,aa,t) {  (     (alphad(r,i,aa,t)*m_pdp(r,i,aa,t)**(1-sigmam(r,i)))    \
                                 + (alpham(r,i,aa,t)*m_pmp(r,i,aa,t)**(1-sigmam(r,i))) )**(1/(1-sigmam(r,i))) }
```

The physical demand *xg* is distributed in the equation *xageq*, equation (39) in VDM 2018, to demand for individual products based on given share parameters *alphaa*, Armington prices captured by the m_pa macro and the average price *pg* based on the substitution elasticity *sigmag*:

```
xageq(rs(rNat),iIn(i),gov,ts(t)) $ (xaFlag(rNat,i,gov) $ (not sum(dNest_i_fd(dnest,i,gov),1)) $ alphaa.l(rNat,i,gov,t)) ..

 xa(rNat,i,gov,t)/xa.scale(rNat,i,gov,t)
    =e= alphaa(rNat,i,gov,t)*xg(rNat,t)/xa.scale(rNat,i,gov,t)*(pg(rNat,t)/m_pa(rNat,i,gov,t))**sigmag(rNat) ;
```

Note that that in the case of demand nests for government consumption, not part of the GTAP standard model, additional equations are used as described in the section Demand sub-nests. The demand nest equation for the government is defined as:

$$XdNest_{r,dNest,gov} = \alpha^{top}_{r,dNest,gov} XG_r \left(\frac{PG_r}{PdNest_{r,dNest,gov}} \right)^{\sigma^g} + \alpha^{dNest1}_{r,dNest,gov} XdNest_{r,gov} \left(\frac{PdNest_{r,dNest1,gov}}{PdNest_{r,dNest,gov}} \right)^{\sigma^g}$$

Investments and savings

Gross investment expenditures *yi* are composed of the value of depreciation *valDep* and of regional *rsav* and foreign savings *valSavf* by the equation *yieq*, equation (85) in VDM 2018:

```
yieq(rsNat(rNat),ts(t)) $ ( (not rres(rNat)) or singleCountry or (card(rNat) eq 1) or (pnum.range(t) gt 0))  ) ..

  yi(rNat,t)/yi.scale(rNat,t)  =e= (
                                  ---- value of depreciation
                                  valDep(rNat,t)
                                  ---- savings of the regional household
                                + rsav(rNat,t)
                                  ---- Value of foreign savings in local currency
                                + valsavf(rNat,t))/yi.scale(rNat,t);
```

The price index of investments *pi* is defined by the equation *pieq*, equation (43) in VDM 2018 expanded for demand nests. For a detailed explanation of the equation, refer to the explanation for government expenditures above:

```
pieq(rsNat(rNat),ts(t))  ..

 pi(rNat,t)  =e=

            --- CD case with sigmai==1

             ( [ prod((i,inv) $ ((not sum(dNest_i_fd(dnest,i,inv),1)) $ alphaa.l(rNat,i,inv,t)),
                    (m_pa(rNat,i,inv,t)/(lambdai(rNat,i,t)*alphaa(rNat,i,inv,t)))**alphaa(rNat,i,inv,t))

              -- CES-subnests, not part of Standard GTAP model

              * prod(dNest_n_fd("top",dnest,inv) $ alphaDN(rNat,dNest,inv,t),
                     (pdNest(rNat,dNest,inv,t)/alphaDN(rNat,dNest,inv,t))**alphaDN(rNat,dNest,inv,t))
               ]/axi(rNat,t) ) $ (sigmai(rNat) eq 1)

            --- CES or Leontief case with sigmai<>1

   +          ([ sum((i,inv) $ ((not sum(dNest_i_fd(dnest,i,inv),1))  $ alphaa.l(rNat,i,inv,t)),
                 alphaa(rNat,i,inv,t)*(m_pa(rNat,i,inv,t)/lambdai(rNat,i,t))**(1-sigmai(rNat)))

              -- CES-subnests, not part of Standard GTAP model

              + sum(dNest_n_fd("top",dnest,inv) $ alphaDN(rNat,dNest,inv,t),
                 alphaDN(rNat,dNest,inv,t)*pdNest(rNat,dNest,inv,t)**(1-sigmai(rNat)))

              ]**(1/(1-sigmai(rNat)))/axi(rNat,t)
                  ) $ (sigmai(rNat) ne 1)
   ;
```

The **total physical investment demand** *xi* is derived from the price index *pi* defined above and total investment expenditure *yi,* as defined in equation *xieq*, equation (44) in VDM 2018:

```
xieq(rsNat(rNat),ts(t))  ..
  yi(rNat,t)/xi.scale(rNat,t)  =E= pi(rNat,t)*xi(rNat,t)/xi.scale(rNat,t);
```

Product specific investment demand *xa* depicted in the equation *xaieq*, equation (42) in VDM 2018, reflects the share parameters *alphaa*, the substitution elasticity *sigmai* and the shifter variable *lamdbai*:

```
xaieq(rs(rNat),iIn(i),inv,ts(t)) $ (xaFlag(rNat,i,inv) $ (not sum(dNest_i_fd(dnest,i,inv),1)) $ alphaa.l(rNat,i,inv,t)) ..

 xa(rNat,i,inv,t)/xa.scale(rNat,i,inv,t) =e=
   alphaa(rNat,i,inv,t)*xi(rNat,t)/xa.scale(rNat,i,inv,t)
     * [ ((lambdai(rNat,i,t)*pi(rNat,t)/m_pa(rNat,i,inv,t))**sigmai(rNat)/lambdai(rNat,i,t) ) $ (sigmai(rNat) ne 0)
        + lambdai(rNat,i,t)  $ (sigmai(rNat) eq 0)];
```

The **value of depreciation** *valdep* is a given share *depr* on the variably capital stock *kstock* which together define the physical depreciation, multiplied with the average price of savings *pi*, equation (85) in VDM 2018:

```
valDepEq(rsNat(rNat),ts(t))  ..

  valDep(rNat,t)/valDep.scale(rNat,t) =E=  pi(rNat,t)*depr(rNat,t)*kstock(rNat,t)/valDep.scale(rNat,t);
```

Regional savings *rsav* are a given share *betas* of regional income, corrected for expansion effects, equation (31) in VDM 2018:

```
rsaveq(rsNat(rNat),ts(t))  ..

 rsav(rNat,t)/rsav.scale(rNat,t)  =e= betaS(rNat,t)*phiRegY(rNat,t)/rsav.scale(rNat,t);
```

The correction implied by phiRegY ensures that the shares of savings *betaS*, government *betaG* and private consumption *betaPhi* add up to unity, part of equation (28) in VDM 2018:

```
phiRegyEq(rsNat(rNat),ts(t))  ..
  phiRegy(rNat,t)*(betaPhi(rNat,t) + betaG(rNat,t) + betaS(rNat,t))/regy.scale(rNat,t)
      =E= regY(rNat,t)/regy.scale(rNat,t);
```

The **physical amount of regional savings** *xsav* is based on the savings expenditures *rsav* and the average price of savings *psave*, equation (34) in VDM 2018:

```
xsaveq(rsNat(rNat),ts(t))  ..

  xsav(rNat,t)*psave(rNat,t)/xsav.scale(rNat,t) =e= rsav(rNat,t)/xsav.scale(rNat,t);
```

The **regional value of foreign savings** *valSavf* is defined from the value in foreign currency *savf* and the exchange rate *lcu*, part of equation (85) in VDM 2018:

```
valSavfEq(rsNat(rNat),ts(t))  ..

 valSavf(rNat,t)/savf.scale(rNat,t) =E= savf(rNat,t)*lcu(rNat,t)/savf.scale(rNat,t);
```

The **foreign savings** in foreign currency *savf* can be driven by different mechanisms. We start by discussing the so-called global bank mechanism which uses expected returns to foreign savings to distribute global net investment.

Global bank

The global bank mechanism distributes foreign savings across regions such that expected returns to net investments are equal across regions. The different steps in the allocation procedure are described by the following variables and equations.

Regional physical net investment *netInv* is the difference between gross investment demand *xi* and physical depreciation derived from the capital stock *kStock* and the depreciation rate *depr*, part of equation (86) in VDM 2018:

```
netInvEq(rsNat(rNat),ts(t)) ..

  netInv(rNat,t)/netinv.scale(rNat,t) =E= [xi(rNat,t) - depr(rNat,t)*kstock(rNat,t)]/netinv.scale(rNat,t);
```

The **beginning of period capital stock** *kStock* is defined in the equation *kStockEq*, equation (75) in VDM 2018. It converts with the factor *krat* capital use *xft* (for mobile or sluggish capital) or non-mobile capital use *xf* into the aggregate capital stock, where capital (types) are define by the set *cap*:

```
kstockeq(rsNat(rNat),ts(t)) $ ( (kstock.range(rNat,t) ne 0) ) ..

 kstock(rNat,t)/kapend.scale(rNat,t) =e= sum(cap, xft(rNat,cap,t))/(krat(rNat,t)*kapend.scale(rNat,t));
```

The **end of period capital stock** *kapEnd* is derived by deducting the depreciation rate *depr* times the number of depreciation years *nDeprYears* and adding gross investment times the number of depreciation years, equation (76) in VDM 2018:

```
kapEndeq(rsNat(rNat),ts(t))   ..

 kapEnd(rNat,t)/kapend.scale(rNat,t)
     =e= [
              (1 - depr(rNat,t)*nDeprYears(t))*kstock(rNat,t)  + xi(rNat,t)*nDeprYears(t)

        ]/kapend.scale(rNat,t);
```

The **average returns of capital after taxes** *arent* is defined in the equation *arenteq*, equation (77) in VDM 2018, from mobile capital prices *pft*, considering direct taxes *kappaf* and their potential shifter *kappaShft* as well as the factor which converts yearly capital use into stock values *krat*:

```
arenteq(rsNat(rNat),ts(t)) $ (rorFlag ne fixedForeignSavings) ..

  arent(rNat,t)/arent.scale(rNat,t)
     =e= [sum((cap) $ xftFlag(rNat,cap),
                 (1-kappaf(rNat,cap,t))*pft(rNat,cap,t)*xft(rNat,cap,t))
          /  kstock(rNat,t)] /arent.scale(rNat,t);
```

The **net rate of return to capital** *rorc* as defined in the equation *rorceq*, equation (77) in VDM 2018, corrects these gross returns factors *arent* for the depreciation rate *fdepr*:

```
rorceq(rsNat(rNat),ts(t)) $ (rorFlag ne fixedForeignSavings) ..

  rorc(rNat,t)/rorc.scale(rNat,t) =e= (arent(rNat,t)/pi(rNat,t) - fdepr(rNat,t))/rorc.scale(rNat,t);
```

The **expected net returns to capital** *rore*, defined in the equation *roreeq*, corrects the net rate of return to capital *rorc* by the expression *rorePart1* which is smaller the larger the relative increase of the capital stock. *rorePart1* takes the change in end of period capital stock *kapEnd* relative to beginning of period stock *kStock* exponent the elasticity *RorFlex*. The two equations *rorePart1Eq* and *roreEq* are jointly equivalent to equation (79) in VDM 2018.

```
rorePart1Eq(rsNat(rNat),ts(t)) $ (RorFlag ne fixedForeignSavings)..

 rorePart1(rNat,t)/rorePart1.scale(rNat,t)
     =E= [(kapEnd(rNat,t)/kstock(rNat,t))**(-RoRFlex(rNat,t))]/rorePart1.scale(rNat,t) ;

--- Expected rate of return, equation (79) in VDM 2018

roreeq(rsNat(rNat),ts(t)) $ (RorFlag ne fixedForeignSavings)..

  rore(rNat,t)/rore.scale(rNat,t)  =e=  rorc(rNat,t)/rore.scale(rNat,t)  * rorePart1(rNat,t);
```

As shown above, two equations are used to define that relation to avoid numerical problems in the solver.

Specifically, the global bank mechanism aims at equalizing the expected net returns *rore* across regions by changing the distribution of foreign savings *fsav* to the different regions. In the benchmark, a risk parameter *risk* ensures that the average global return *rorg* are lined up with the expected returns in each region *rore*. Assume now that the price of mobile capital in a region after direct taxes increases, e.g. by tax reform. That will increase the expected returns and thus attract foreign saving. Increasing the foreign savings in a region will in turn increase total savings and thus investments *xi*. That will change the end of period capital *kapEnd* which will affect the relation between end and beginning stock and thus decrease expected rate *rore*.

The **value of global net investment** *gblValNetInv* as defined in the equation *gvlValNetInvEq* is derived from the regional net investments *netInv*, their regional prices *pi* and the exchange rate *lcu*, part of equation (82) in VDM 2018:

```
gblValNetInvEq(ts(t)) ..

    gblValNetInv(t)/sum(rNat,netInv.scale(rNat,t))

       =E= sum(rNat,  (pi(rNat,t)/lcu(rNat,t)     *netInv(rNat,t))    $ rsNat(rNat)
                    +(pi.l(rNat,t)/lcu.l(rNat,t)*netInv.l(rNat,t)) $ (not rsNat(rNat)))
              /sum(rNat,netInv.scale(rNat,t));
```

The RHS sums up over all regions, however, regions not in the current solve, i.e. not rsNat(rNat), enter with the exogenous given variable values .l. That reflects the case where the model is run in single country mode or in pre-solve mode.

Regional net investments *netInv* as defined in the *netInvEq*, part of equation (86) in VDM 2018, are equal to aggregated gross investment demand *xi* minus depreciation, i.e. the depreciation rates *fdepr* times the beginning of year capital stocks *kStock*:

```
netInvEq(rsNat(rNat),ts(t)) ..

  netInv(rNat,t)/netinv.scale(rNat,t) =E= [xi(rNat,t) - depr(rNat,t)*kstock(rNat,t)]/netinv.scale(rNat,t);
```

The **total global net investments** *xigbl* are defined in the equation *xigbleq* as the summing up of the regional net investments *netInv*, equation (86) in VDM 2018:

```
xigbleq(ts(t)) ..

   xigbl(t)/sum(rNat, netInv.scale(rNat,t))
        =e= sum(rNat,
                        netInv(rNat,t)    $ rsNat(rNat)
                    ---add values of countries not in current solve,but in the global framework
                     + netInv.l(rNat,t) $ (not rsNat(rNat))
             )/sum(rNat,netInv.scale(rNat,t)) ;
```

The **average global expected returns to capital** *rorg* is the value – net investment *netinv* times saving prices *pi* – weighted average of the regional expected returns *rore* as defined in the two equation *rorgeg* and *gblValNetInv1Eq*:

```
gblValNetInv1Eq(ts(t)) $ (RoRFlag eq fixedAllocationOfInv) ..

  gblValNetInv1(t)/sum(rNat,netInv.scale(rnat,t))
      =E= sum(rNat,   (rore(rNat,t)    * pi(rNat,t)/lcu(rNat,t)      * netInv(rNat,t)) $ rsNat(rNat)
                    + (rore.l(rNat,t)  * pi.l(rNat,t)/lcu.l(rNat,t)  * netInv.l(rNat,t)) $ (not rs(rNat)))
                  /sum(rNat,netInv.scale(rNat,t)) ;


rorgeq(ts(t)) $ (RoRFlag eq fixedAllocationOfInv) ..
   rorg(t)/rorg.scale(t) =e= gblValNetInv1(t) / gblValNetInv(t) * 1/rorg.scale(t);
```

The **distribution of the net investments in case of the global bank mechanism** (*RoRFlag eq equalReturnToInv*) is steered by the first part of the following *savfeq*, equation (80) in VDM 2018, which requires for each region that risk adjusted expected returns are equal to the global average:

```
savfeq(rsNat(rNat),ts(t)) $( (      (RoRFlag eq equalReturnToInv)
                                 or ((RoRFlag eq fixedAllocationOfInv) $ (not rres(rNat))))
                           $   (savf.range(rNat,t) ne 0))..
 0    =e=

 --- equal relative changes in expected returns to investments
     across regions, rorg in the global currency

       ([rore(rNat,t)*risk(rNat,t) - rorg(t)]/rore.scale(rNat,t) )       $ (RoRFlag eq equalReturnToInv)

  --- fixed allocation of global investment based on parameter chiInv

    +  [(netInv(rNat,t) - chiInv(rNat,t)*xigbl(t))/netInv.scale(rNat,t)]  $ (RoRFlag eq fixedAllocationOfInv);
```

Fixed allocation of foreign savings

That case is depicted in the second block of the equation *savfeq* if the *RoRFlag* is set equal to “fixedAllocationOfInv” and is based on given parameters *chiInv* which reflect the benchmark distribution. Note that the mechanism refers to “capShrFix” in VDM 2018:

```
savfeq(rsNat(rNat),ts(t)) $( (      (RoRFlag eq equalReturnToInv)
                              or ((RoRFlag eq fixedAllocationOfInv) $ (not rres(rNat))))
                          $    (savf.range(rNat,t) ne 0))..
  0   =e=

  --- equal relative changes in expected returns to investments
      across regions, rorg in the global currency

        ([rore(rNat,t)*risk(rNat,t) - rorg(t)]/rore.scale(rNat,t) )        $ (RoRFlag eq equalReturnToInv)

   --- fixed allocation of global investment based on parameter chiInv

     +  [(netInv(rNat,t) - chiInv(rNat,t)*xigbl(t))/netInv.scale(rNat,t)]  $ (RoRFlag eq fixedAllocationOfInv);
```

Note that the residual region is excluded from the mechanism and defined via the capital account balance.

Capital account and balance of payments

The capital account balance *capAcctEq*, equation (84) in VDM 2018, ensures that the sum of the foreign savings *savf* is zero:

```
capAccteq(ts(t)) $ ( (    (sum(rsNat $ (savf.range(rsNat,t) ne 0),1) gt 1)
                     or (      (RoRFlag eq fixedForeignSavings) and sum(rres(rsNat),1)
                          and ( (pnum.range(t) gt 0) or (not singleCountry) ))) ) ..

   0 =e= sum(rNat, savf(rNat,t) $ rsNat(rNat) + savf.l(rNat,t) $ (not rsNat(rNat)) - p_capTrans(rnat,t)*pnum(t))
        /sum(rres,savf.scale(rres,t));
```

The equation is only active if (1) there are at least two countries in the current solve where foreign savings are not fixed or (2) the residual region is in the current solve and the foreign savings for the other regions are fixed and the global model is used (not singleCountry) or the numéraire is not fixed.

The balance of payments equation *bopEq* is only a check for the correct setup of the model, i.e. the *bopSlack* should be equal to zero given the accuracies of the solver and original tiny numerical imbalances in the SAM:

```
 bopEq(rsNat(rNat),ts(t))  ..

        (valsavf(rNat,t) + bopSlack(rNat,t) - p_capTrans(rNat,t)*pnum(t))
        /savf.scale(rNat,t)
         =E= [ - sum( (i,rNat1)  $ m_xwFlag(rNat,i,rNat1), m_xws(rNat,i,rNat1,t)*%pefob%(rNat,i,rNat1,t))
               - sum( (m,tmg) $ xaFlag(rNat,m,tmg), xa(rNat,m,tmg,t)*m_pa(rNat,m,tmg,t)*lcu(rNat,t))
               + sum( (rNat1,i)  $ m_xwFlag(rNat1,i,rNat), xw(rNat1,i,rNat,t)*%pmcif%(rNat1,i,rNat,t))]
       /savf.scale(rNat,t)
 ;
```

Demand sub-nests

Demand sub-nests aggregate individual Armington demands in final demand by private household, government or savings to aggregates based on CES utility function. The resulting nests can be either part of the top-level demand function of the function or linked into other demand nests. That mechanism allows increasing the flexibility of depicting substitution relations between individual products.

The **average price for a demand nest** *pdNest* for the nest *dNest* and the demand agent *fdn* is defined by dual price aggregators which distinguish the CD from the CES/Leontief case depending on the substitution elasticity *sigmaFDNest*. In both cases, the price index reflects the contribution of individual products *i* based on their share parameter *alphaa* and the contribution of sub-nests based on their share parameter *alphaDN*.

```
pdNestEq(rsNat(rNat),dnest,fdn,ts(t)) $ (alphaDN(rNat,dNest,fdn,t)) ..

  pdNest(rNat,dNest,fdn,t)
      =E=

          --- CD case with sigmaFDNest==1

         [

            ---- contribution of individual commodity demands to price aggregator

            prod(dNest_i_fd(dNest,i,fdn) $ alphaa.l(rNat,i,fdn,t),
                        (m_pa(rNat,i,fdn,t)/(alphaa(rNat,i,fdn,t)*(1+(lambdai(rNat,i,t)-1)$ sameas(fdn,"inv"))))**alphaa(rNat,i,fdn,t))

            ---- contribution of sub-nests under the current nest to price aggregator

          *  prod(dNest_n_fd(dNest,dNest1,fdn) $ alphaDN(rNat,dNest1,fdn,t),
                        (pdNest(rNat,dNest1,fdn,t)/alphaDN(rNat,dNest1,fdn,t))**alphaDN(rNat,dNest1,fdn,t))

          * axdNestCD(rNat,dNest,fdn,t)

         ] $ (sigmaFDNest(rNat,dNest,fdn) eq 1)

  + [
     { sum(dNest_i_fd(dNest,i,fdn) $ alphaa.l(rNat,i,fdn,t),
             alphaa(rNat,i,fdn,t) * (m_pa(rNat,i,fdn,t)/(1+(lambdai(rNat,i,t)-1)$ sameas(fdn,"inv")))**(1-sigmaFDNest(rNat,dNest,fdn))
         )

     + sum(dNest_n_fd(dNest,dNest1,fdn) $ alphaDN(rNat,dNest1,fdn,t),
             alphaDN(rNat,dNest1,fdn,t) * pdNest(rNat,dNest1,fdn,t)**(1-sigmaFDNest(rNat,dNest,fdn))
         )

     } **(1/(1-sigmaFDNest(rNat,dNest,fdn)))

    ]  $ (sigmaFDNest(rNat,dNest,fdn) ne 1);
```

In mathematical notation:

$$PdNest_{r,dNest,fdn} = \left[\sum_{i \in dNest} \alpha^{dNest}_{r,i,fdn} \left(\frac{PA_{r,i,fdn}}{\lambda_{r,i,fdn}} \right)^{\sigma^{dNest}_{fdn}} + \sum_{dNest1 \in dNest} \alpha^{denst}_{r,dNest1,fdn} \left(PdNest_{r,dNest1,fdn} \right)^{\sigma^{dNest}_{fdn}} \right]^{1/\left(1-\sigma^{dNest}_{fdn}\right)}$$

With the list of final demands $fdn = \{gov, inv, h\}$

The notation already underlines that such nests can comprise other nests.

The **demand for a sub-nest** *xdNest* depends on the total demand (inv or gov) or is driven by a sub-nest:

```
xdNest(rNat,dNest,fdn,t)/sum(i, xa.scale(rNat,i,fdn,t))
     =E=

        [

         --- driven by top nest demand in case of gov/inv (CES or CD)
             Final household demand has separate equation to reflec the more complex CDE case

         + sum(dNest_n_fd("top",dnest,"gov") $ alphaDN(rNat,dNest,"gov",t),
                 alphaDN(rNat,dNest,"gov",t) * xg(rNat,t)  * (pg(rNat,t)/pdNest(rNat,dNest,"gov",t))**sigmaG(rNat)) $ sameas(fdn,"gov")

         + sum(dNest_n_fd("top",dnest,"inv") $ alphaDN(rNat,dNest,"inv",t),
                 alphaDN(rNat,dNest,"inv",t) * xi(rNat,t)  * (pi(rNat,t)/pdNest(rNat,dNest,"inv",t))**sigmai(rNat)) $ sameas(fdn,"inv")

         -- driven by another nest

         + sum(dNest_n_fd(dNest1,dNest,fdn) $ alphaDN(rNat,dNest,fdn,t),
                 alphaDN(rNat,dNest,fdn,t) * xdNest(rNat,dNest1,fdn,t) * (pdNest(rNat,dNest1,fdn,t)/pdNest(rNat,dNest,fdn,t))**sigmaFDNest(rNat,d

        ]/sum(i, xa.scale(rNat,i,fdn,t));
```

For the final household case with a CDE demand function (see dem_cde.gms), the following equation is used:

```
zconsDnEq(rs(r),dNest,h,ts(t) ) $ sum(dNest_n_fd("top",dNest,h), alphaDN(r,dNest,h,t)) ..

 zconsDN(r,dNest,h,t)/zconsDN.scale(r,dNest,h,t) =e= alphaDN(r,dNest,h,t)*bhDN(r,dnest,t)
                   * (pdNest(r,dNest,h,t)**bhDN(r,dNest,t))
                   *  ((yc(r,h,t)/pop(r,h,t)*1000)**(-bhDN(r,dNest,t)))
                   *  (uh(r,h,t)**(ehDN(r,dNest,t)*bhDN(r,dNest,t)))/zconsDN.scale(r,dNest,h,t);
```

The **Armington demands** *xa* **driven by a sub-nest** are defined in the equation *xdDNesteq* and use the usual CES-structure, i.e. the share parameter *alphaa*, the sub-nest total demand *xdNest* and the price relation exponent the substitution elasticity sigmaFDNest as well a preference shifter *lambdai*:

```
xaDNestEq(rsNat(rNat),i,fdn,ts(t)) $ (xaFlag(rNat,i,fdn) $ sum(dNest_i_fd(dNest,i,fdn),1) $ iIn(i)) ..

 xa(rNat,i,fdn,t)/xa.scale(rNat,i,fdn,t) =e=
   sum(dNest_i_fd(dNest,i,fdn),
         alphaa(rNat,i,fdn,t)*xdNest(rNat,dnest,fdn,t)/xa.scale(rNat,i,fdn,t)

           --- not the preference shifter lambdai for investment demand

           * [

                --- CES case with price effect

                {
                  ((1+(lambdai(rNat,i,t)-1)$ sameas(fdn,"inv"))*pdNest(rNat,dNest,fdn,t)/m_pa(rNat,i,fdn,t))
                         **sigmaFDNest(rNat,dNest,fdn)
                          /((1+(lambdai(rNat,i,t)-1)$ sameas(fdn,"inv")))
                 }   $ (sigmaFDNest(rNat,dNest,fdn) ne 0)

                --- Leontief case with fixed composition, no price effect

               + {
                    (1+(lambdai(rNat,i,t)-1) $ sameas(fdn,"inv"))
                 }  $ (sigmaFDNest(rNat,dNest,fdn) eq 0)
               ]);
```

International trade and domestic sales, and related prices

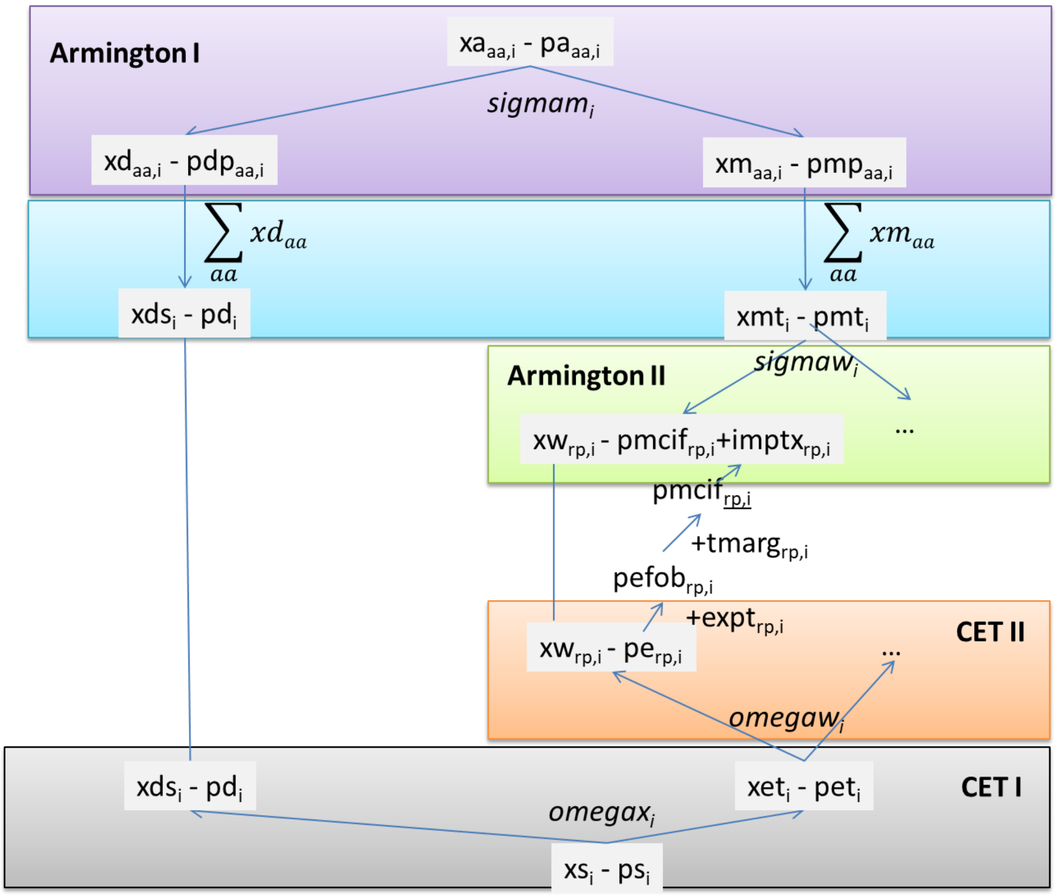
Overview

The model uses in its standard layout a two-stage Armington system where the shares of the lower nests representing bilateral imports are identical across the different Armington agents. The Armington approach can be complemented by a CET to distribute supply in each region based on finite transformation. Alternatives to the standard layout in demand are: a further aggregation in the Armington system where also the upper nested is shared across sectors or across all agents, the "MRIO extension" and the heterogenous firm extension, see the sub-section "Melitz model" in Britz 2016.

The graphic below depicts these main relations in international trade and the distribution of supply. The top level Armington nest in the uppermost box distributes the Armington demand for each agent to demand from domestic origin and imports. Next, these demands are aggregated over the agents. The total import demand is then split up into demands of the different exporters *xw*, driven by the cif price plus import taxes. The difference between the cif and fob price are the endogenous transport margins. Taking export taxation or subsidization into account, the export prices *pe* in each exporter region are derived.

Distribution of supply *xs* to total exports *xet* and domestic sales *xds* is driven by a transformation nest depicted in the lowest box. The transformation can also be infinite as the default case. Distribution of the total exports *xet* to different destination *xw* is handled by a second transformation nest, again, with infinite transformation as the default case.

Figure 3: Overview on distribution of supply and sourcing of demand



Note that the set of Armington agents depends on the chosen structure of the model. In the default layout, it comprises the list of sectors, one aggregate private household, government and investment demand. Alternatively, that differentiation can be completely removed, i.e. the shares in the upper nests are identical for each agent, or can be defined identical across sectors. The "MRIO extension" introduces an own set of equations which allows to dis-aggregate also the lower Armington nests, i.e. the bilateral demands, by agent. Furthermore, note that the Melitz and Krugman extensions use only one nest considering love-of-variety.

Individual equations

The **agent specific prices for imports** *pmp* as defined in the equation *pmpeq*, equation (46) in VDM 2018, reflects the average import price *pmt* and agent specific import taxes *mintx* plus tax shifts *itxshft* and emission taxes:

```
pmpeq(rsNat(rNat),iIn(i),aa,ts(t)) $ (alpham(rNat,i,aa,t) $ (not iMrio(i)) $ (not a(aa) or sum(sameas(aa,aIn),1))) ..

  pmp(rNat,i,aa,t) =e= pmt(rNat,i,t)*(1 + mintx(rNat,i,aa,t) + itxshft(rNat,i,t))

    --- add per unit emission tax

    $$ifi %ModulesCO2_Emissions%==on  + emisp(rNat,t)* (emii0(rNat,i,aa)/xma0(rNat,i,aa)) $ xma0(rNat,i,aa)
;
```

Note that the equation is not active if the "MRIO extension" is switched on for that product (*not iMrio(i)*) as in that case, the basis to derive the agent specific is specific to group of agents and not equal to *pmt*.

Equally, the equation requires that the share parameters for imports *alpham* is not zero. The Melitz extension sets the share parameter to zero for the production handled in the Melitz model and thus also removes the equation for these products. Finally, the model will for the default case substitute out the *pmp* prices for intermediate demand (not a(aa)).

Similar, the **agent specific prices for domestic origin** *pdp* reflect price in the domestic markets, defined in the equation *pdeq*, equation (45) in VDM 2018. These are equal to sectoral prices *ps* under infinite transformation or equal to domestic sales prices *pd* in case of non-infinite transformation. Taxes are added as in the case imports above:

```
pdpeq(rsNat(rNat),iIn(i),aa,ts(t)) $ (alphad(rNat,i,aa,t) $ (not a(aa) or sum(sameas(aa,aIn),1))) ..

 pdp(rNat,i,aa,t) =e= (    pd(rNat,i,t) $ (omegax(rNat,i) ne inf)
                         + ps(rNat,i,t) $ (omegax(rNat,i) eq inf)) *(1 + dintx(rNat,i,aa,t) + itxshft(rNat,i,t))

    --- add per unit emission tax (not part of GTAP Standard)

    $$ifi %ModulesCO2_Emissions%==on  + emisp(rNat,t) * (emid0(rNat,i,aa)/xda0(rNat,i,aa)) $ xda0(rNat,i,aa)
;
```

Note again that the Melitz extension will delete the *alphad* parameters for the products handled by imperfect competition to replace the equation *pdpeq* as it uses a different pricing system.

The **Armington price** of the different agents *pa* is defined in the *paeq*, equation (47) in VDM 2018. That dual price aggregator as usually reflects the given shares for domestic *alphad* and imported *alpham* origin and the related prices as defined above as well as the substitution elasticity between imports and domestic origin *sigmam* which is not agent specific:

```
paeq(rsNat(rNat),iIn(i),aa,ts(t)) $ (  [                  ((alphad(rNat,i,aa,t) $ alpham(rNat,i,aa,t)))
                                         or (gtapStd and (alphad(rNat,i,aa,t) or alpham(rNat,i,aa,t)))  ]
                                       $ (not a(aa) or sum(sameas(aa,aIn),1))

         WB: If additional prices are substitued out (switched on in interface)
             the Armington price aggregator for firm demands (agents aa)
             is calculated in the macro m_pa and the equation is not active

        $$iftheni.subsPrices "%subsArmPrices%"=="on"
             $ (1 eq 2)
        $$endif.subsPrices
    )..

 pa(rNat,i,aa,t) =e=

    (   [ (alphad(rNat,i,aa,t)*m_pdp(rNat,i,aa,t)**(1-sigmam(rNat,i))) ]  $ alphad(rNat,i,aa,t)
       +[ (alpham(rNat,i,aa,t)*m_pmp(rNat,i,aa,t)**(1-sigmam(rNat,i))) ]  $ alpham(rNat,i,aa,t))**(1/(1-sigmam(rNat,i)));
```

Note here that the equation is normally substituted out if only domestic or import demand is present. That is not the case if the standard GTAP layout is used or the substitution is explicitly switched off on the interface.

Domestic demand *xd* by the different agents is driven by the share parameter *alpham* and times the total Armington demand defined in the macro *m_xa*, times the price relation exponent the substitution elasticity, as defined in the equation *xdeq*, equation (48) in VDM 2018:

```
xdeq(rsNat(rNat),iIn(i),aa,ts(t)) $ (   (alpham(rNat,i,aa,t) or gtapStd)
                                      $ alphad(rNat,i,aa,t) $ (not a(aa) or sum(sameas(aa,aIn),1))) ..

 xd(rNat,i,aa,t)/xd.scale(rNat,i,aa,t)  =e=   alphad(rNat,i,aa,t)*mm_xa(rNat,i,aa,t)/xd.scale(rNat,i,aa,t)
                                          * (m_pa(rNat,i,aa,t)/m_pdp(rNat,i,aa,t))**sigmam(rNat,i);
```

It is linearly aggregated over agents to **total domestic sales** *xds* in the equation *pdeq*, equation (67) in VDM 2018:

```
pdeq(rsNat(rNat),iIn(i),ts(t)) $ xdFlag(rNat,i) ..

   xds(rNat,i,t)/xds.scale(rNat,i,t)
           =e= sum(aa $ m_alphad(rNat,i,aa,t), m_xd(rNat,i,aa,t))/xds.scale(rNat,i,t);
```

Imported demand by each agent *xm* is defined accordingly in the equation *xmeq*, equation (49) in VDM 2018:

```
xmeq(rsNat(rNat),iIn(i),aa,ts(t))  $ (  (alphad(rNat,i,aa,t) or gtapStd)
                                      $ alpham(rNat,i,aa,t) $ (not a(aa) or sum(sameas(aa,aIn),1)) )  ..

 xm(rNat,i,aa,t)/xm.scale(rNat,i,aa,t)  =e=    alpham(rNat,i,aa,t)*mm_xa(rNat,i,aa,t)/xm.scale(rNat,i,aa,t)
                                         * (m_pa(rNat,i,aa,t)/m_pmp(rNat,i,aa,t))**sigmam(rNat,i);
```

Total import demand *xmt* is in the equation *xmteq* is defined as an adding up over the demand of the individual Armington agents, equation (50) in VDM 2018:

```
xmteq(rsNat(rNat),iIn(i),ts(t)) $ xmtFlag(rNat,i) ..

 xmt(rNat,i,t)/xmt.scale(rNat,i,t) =e= sum(aa $ alpham(rNat,i,aa,t), m_xm(rNat,i,aa,t))/xmt.scale(rNat,i,t);
```

The bilateral **cost, insurance and freight prices** *pmcif* are defined by the equation *pmcifeq* and the macro *m_pmcif*. As the default, these prices are substituted out from the model. The *rrComb* set is used in case of only one region being solved to depict the bilateral trade links to include.

```
pmcifeq(rNat1,iIn(i),rNat,ts(t)) $ (m_xwFlag(rNat1,i,rNat) $ rrComb(rNat1,rNat)) ..

  pmcif(rNat1,i,rNat,t) =E= m_pmcif(rNat1,i,rNat,t);
```

The macro *m_pmcif* is defined as follows, equation (65) in VDM 2018:

```
$$macro m_pmcif(r,i,rp,t) { [ m_pefob(r,i,rp,t)/lcu(r,t) \
                              + (sum(m $ amgm(m,r,i,rp), amgm(m,r,i,rp)*ptmg(m,t)\
                                     /m_lambdamg(m,r,i,rp,t))*tmarg(r,i,rp,t)) $ tmarg.l(r,i,rp,t)\
                              ]*lcu(rp,t) }
```

It converts the bilateral fob (free on board) price defined in the macro *m_pefob* to international currency (division by *lcu*) and adds the per unit transport margin cost in international currency. These costs are defined as the transport mode *m* (see, air …) specific shares *amgm* on the given transport margin *tmarg*, updated with the mode specific average global price for that mode *ptmg*. The mode specific costs can be shifted by *m_lambdamg*. The resulting costs - fob plus transport margin – in international currency and finally converted in local currency again by the multiplication by *lcu*.

Note that with a dense bilateral trade matrix, the number of variables relating to bilateral relations increases quadratic in the number of regions and linear in the number of sectors. Under full density, using 50 regions and 50 sectors implies hence 50x50x50 = 125,000 non-zero elements for each variable defined bilaterally. That explains why substitutions of the e.g. the f.o.b. and c.i.f. prices and the bilateral trade margins can dramatically reduce model size.

Bilateral **free on board prices** *pefob* are defined by the macro *mm_pefob*, equation (64) in VDM 2018. They reflect: bilateral export taxes *exptx* and a product specific export tax shifter *etax* levied on the relevant price which depends if and how a transformation of output is used as seen below. If no CET approach is used, the supply price *ps* is the basis for fob calculation. In case there is only a CET between total exports and domestic sales, but none between bilateral export flows, the average price of exports *pet* is used, otherwise, bilateral export prices *pe* define the basis for fob prices.

```
$$macro mm_pefob(r,i,rp,t) { \

       --- export taxes

       (1 + exptx(r,i,rp,t) + etax(r,i,t)) \

   --- the following might substitutes out pe depending on the CETs
       between domestic sales and export resp. between exports

           * (  \
               --- use bilateral import prices if a CET between export destination is active
               pe(r,i,rp,t) $ psFlag(r,i,"pe") \
               --- use the export prices if there is only a CET between total exports and domestic use
             + pet(r,i,t)    $ psFlag(r,i,"pet") \
               --- otherwise, use supply price
             + ps(r,i,t)     $ psFlag(r,i,"ps")        )}
```

Similar to the case of cif prices, the fob prices *pefob* are defined from that macro in the equation *pefobeq*. Again, the default case is that these equations are substituted out from the model.

```
pefobeq(rNat1,iIn(i),rNat,ts(t)) $ (m_xwFlag(rNat1,i,rNat) $ rrComb(rNat1,rNat)) ..

  pefob(rNat1,i,rNat,t) =E= m_pefob(rNat1,i,rNat,t);
```

The user can define on the interface if these prices are substituted from the model.

The **average price of imports** *pmt* is defined in equation *pmteq*, equation (52) in VDM 2018, and considers three cases. The first case applies for substitution elasticity different from unity and for shares not considered small. It uses the standard dual price aggregator using the bilateral demand share parameters *amw*, the cif price defined by *%pmcif%* plus bilateral import taxes *imptx* plus a product specific import tax shifter *mtax* and reflects a preference shifter defined in the macro *m_lambdam*. The second case uses the dual price aggregator for the CD case where the substitution elasticity is unity. The third case reflects small shares which are treated à la Leontief.

```
pmteq(rNat,iIn(i),ts(t)) $ ( (pmt.range(rNat,i,t) ne 0) $ (not pmtElas(rNat,i)) $ xmtFlag(rNat,i) ) ..

 pmt(rNat,i,t) =e=

      --- CES case with sigmaw <> 1

      [sum(rNat1 $ (xwFlag(rNat1,i,rNat)
           $$ifi "%hasSmallShare%"=="on"   $ (not isSmallImpShare(rNat1,i,rNat))
                   ),
             amw(rNat1,i,rNat,t)
                   *([(1 + imptx(rNat1,i,rNat,t) + mtax(rNat,i,t)) * %pmcif%(rNat1,i,rNat,t)/chipm(rNat1,i,rNat) ]
                                                                   /m_lambdam(rNat1,i,rNat,t))**(1-sigmaw(rNat,i))
           )**(1/(1-sigmaw(rNat,i))) * axPmtCD(rNat,i,t)
       ] $ ((sigmaw(rNat,i) ne 1) $ axPmtCD(rNat,i,t))

      --- CD case with sigmaw == 1

    + [prod(rNat1 $ (xwFlag(rNat1,i,rNat)
           $$ifi "%hasSmallShare%"=="on"   $ (not isSmallImpShare(rNat1,i,rNat))
                    ),
                   ([(1 + imptx(rNat1,i,rNat,t) + mtax(rNat,i,t)) * %pmcif%(rNat1,i,rNat,t)/chipm(rNat1,i,rNat) ]
                    /(m_lambdam(rNat1,i,rNat,t)*amw(rNat1,i,rNat,t)))**amw(rNat1,i,rNat,t)
            ) * axPmtCD(rNat,i,t)
      ] $ ((sigmaw(rNat,i) eq 1) $ axPmtCD(rNat,i,t))

  $$iftheni.smallShare "%hasSmallShare%"=="on"
       + sum(rNat1 $ isSmallImpShare(rNat1,i,rNat), amw(rNat1,i,rNat,t)
                 *([(1 + imptx(rNat1,i,rNat,t) + mtax(rNat,i,t))
                                      * %pmcif%(rNat1,i,rNat,t)/chipm(rNat1,i,rNat) ]/m_lambdam(rNat1,i,rNat,t)))
  $$endif.smallShare
  ;
```

Note that the single country case can either use the lower level Armington / CET equations of the trading partner of the country solved, or use import and export elasticities. To host that case, the *pmteq* equation will not be introduced in the model if import prices are elasticity driven.

The allocation of total imports *xmt* to the **bilateral imports** xw is defined in the equation *xweq*, equation (52) in VDM and is based on the share parameters *amw*, the substitution between origins *sigmaw* and the relevant price relation, i.e. the average import price *pmt* divided by the cif price *%pmcif%* plus bilateral import taxes *imptx* plus a potential import tax shifter *mtax*. Preference shifters as defined in the macro *m_lambdam* can be used as well.

```
xweq(rNat1,iIn(i),rNat,ts(t)) $ (xwFlag(rNat1,i,rNat) $ (not iMrio(i)) $ rrComb(rNat1,rNat)) ..

 xw(rNat1,i,rNat,t)/xw.scale(rNat1,i,rNat,t) =e= amw(rNat1,i,rNat,t)*xmt(rNat,i,t)/xw.scale(rNat1,i,rNat,t)

                          * [ {(pmt(rNat,i,t)/[(1 + imptx(rNat1,i,rNat,t) + mtax(rNat,i,t))

                             --- Note WB: cif price is defined via macro %pmcif%, normally not a variable in model

                           * %pmcif%(rNat1,i,rNat,t)/chipm(rNat1,i,rNat) ])**sigmaw(rNat,i)
                             *  m_lambdam(rNat1,i,rNat,t)**(sigmaw(rNat,i) - 1)}

                            $$iftheni.smallShare "%hasSmallShare%"=="on"
                                $ (not isSmallImpShare(rNat1,i,rNat))
                                   + m_lambdam(rNat1,i,rNat,t) $ isSmallImpShare(rNat1,i,rNat)
                            $$endif.smallShare
                           ];
```

Note that in opposite to VDM 2018, the import price defined in equation (66) is always substituted out.

That macro *m_lambdam* is introduced to avoid that for each bilateral trade link in the model, a variable must be fixed to unity if no shifter is present. The shifter variable is used if it either fixed, i.e. the range is zero, or its starting value is not zero. Otherwise, the constant 1 is used as shown in the first line of the macro.

```
$$macro m_lambdam(r,i,rp,t) {[ 1                  $ ( (lambdam.range(r,i,rp,t) ne 0) and (lambdam.l(r,i,rp,t) eq 0 )) \
                             + lambdam(r,i,rp,t) $ ( (lambdam.range(r,i,rp,t) eq 0) or  (lambdam.l(r,i,rp,t) ne 0 )) ]  $ card(lambdam)\
                               + 1 $ (card(lambdam) eq 0) }
```

Similar macros are used for other shifter variables as well.

As the model allows non-infinite transformation of outputs, the following equation *xdseq* (implicitly), equation (53) in VDM 2018, defines **domestic sales** *xds*. The first case is that of infinite transformation as found in the GTAP standard model where by definition the price of domestic sales *pd* is equal to the average supply price *ps*. The second case distributes total supply *xs* of a product to domestic sales based on the share parameter *gd* and the relation between the domestic sales price and average supply times exponent the transformation elasticity *omegax:*

```
xdseq(rsNat(rNat),iIn(i),ts(t)) $ (xdFlag(rNat,i) $ (omegax(rNat,i) ne inf) ) ..

 0 =e=

    --- infinite CET <=> prices are equal, GTAP standard

       (pd(rNat,i,t) - ps(rNat,i,t)) $ (omegax(rNat,i) eq inf)

    --- CET mechanism otherwise

    +  (xds(rNat,i,t)/xds.scale(rNat,i,t)
         - gd(rNat,i,t)*xs(rNat,i,t)/xds.scale(rNat,i,t)
            *(pd(rNat,i,t)/ps(rNat,i,t))**omegax(rNat,i))   $ (omegax(rNat,i) ne inf);
```

A similar equation defines **total exports** *xet* in the equation *xeteq*, equation (54) in VDM 2018, The relevant share parameter is *ge* while the average price of exports is called *pet*:

```
xeteq(rsNat(rNat),iIn(i),ts(t)) $ (xetFlag(rNat,i) $ (xet.range(rNat,i,t) ne 0) $ (pet.range(rNat,i,t) ne 0)  )..

 xet(rNat,i,t)/xet.scale(rNat,i,t)
     =e=

         --- case with CET

         (ge(rNat,i,t)*xs(rNat,i,t)/xet.scale(rNat,i,t)
             *(pet(rNat,i,t)/ps(rNat,i,t))**omegax(rNat,i))  $ (omegax(rNat,i) ne inf)

          --- infinite CET: physical aggregation and elasticity driven (single country option)

       +  (  sum( rp $ m_xwFlag(rNat,i,rp), m_xws(rNat,i,rp,t)/xet.scale(rNat,i,t)) $ (not petElas(rNat,i))
             + (axPet(rNat,i,t) * pet(rNat,i,t)**petElas(rNat,i))                    $      petElas(rNat,i)
              )  $ (omegaw(rNat,i) eq inf);
```

Note the special case for the single country model where exports are driven by an export elasticity.

The average **supply price** *ps* is defined in the equation *xseq*, equation (55) in VDM 2018. In case of infinite transformation and thus a linear aggregator – the first block – the sum of domestic sales *xds* and exports *xet* must be equal to physical output *xs*. In case of not-infinite transformation, a dual price aggregator is used based on the share parameters *gd* and *ge*, related prices *pd* and *pet* and the transformation elasticity *omegax*:

```
xseq(rsNat(rNat),iIn(i),ts(t)) $ (xsFlag(rNat,i) $ (ps.range(rNat,i,t) ne 0))..
 0 =e=

    --- infinite CET <=> physical balancing

    +  (xs(rNat,i,t)/xs.scale(rNat,i,t) - (xds(rNat,i,t)/xs.scale(rNat,i,t)) $ xdFlag(rNat,i)
                                        - (xet(rNat,i,t)/xs.scale(rNat,i,t)) $ xetFlag(rNat,i) ) $ (omegax(rNat,i) eq inf)

    --- dual price aggregator

    +  (ps(rNat,i,t) - (  gd(rNat,i,t)*pd(rNat,i,t)**(1+omegax(rNat,i))
                        + ge(rNat,i,t)*pet(rNat,i,t)**(1+omegax(rNat,i)))**(1/(1+omegax(rNat,i))) ) $ (omegax(rNat,i) ne inf)
    ;
```

Bilateral export supply is by definition equal to bilateral export demand *xw*, that equality is used to define indirectly the bilateral export price *pe* in case of non-infinite transformation in the first block, as defined in the equation *peeq*, equation (56) in VDM 2018. Otherwise, bilateral export prices *pe* and the average export prices *pet* are by definition equal:

```
peeq(rNat,iIn(i),rNat1,ts(t)) $ (m_xwFlag(rNat,i,rNat1) $ psFlag(rNat,i,"pe") $ rrComb(rNat,rNat1)) ..

 0 =e= ( m_xws(rNat,i,rNat1,t)/xw.scale(rNat,i,rNat1,t)
           - gw(rNat,i,rNat1,t)*xet(rNat,i,t)/xw.scale(rNat,i,rNat1,t)
            * [  1                                                    $ isSmallExpShare(rNat,i,rNat1)
                + { (pe(rNat,i,rNat1,t)/pet(rNat,i,t))**omegaw(rNat,i) } $ (not isSmallExpShare(rNat,i,rNat1)) ]
      ) $ (omegaw(rNat,i) ne inf)

      --- case with law of one price (infinite transformation)

    +  ( pe(rNat,i,rNat1,t) - pet(rNat,i,t) $ (omegax(rNat,i) ne inf)
                            - ps(rNat,i,t)  $ (omegax(rNat,i) eq inf) )
       $ (omegaw(rNat,i) eq inf) ;
```

The **aggregate price of export** *pet* defined in the equation *peteq*, equation (57) in VDM 2018, is either defined from a dual price aggregator in case of non-infinite transformation or equal to the supply price *ps* in case of infinite transformation. Note the inclusion of the special case of small export shares handled via Leontief:

```
peteq(rNat,iIn(i),ts(t)) $ (xetFlag(rNat,i) $ sum(rsNat,rrComb(rNat,rsNat)) $ (pet.range(rNat,i,t) ne 0)) ..

 pet(rNat,i,t) =e= [  axPet(rNat,i,t)
                    * sum(rNat1 $ (m_xwFlag(rNat,i,rNat1) $ (not isSmallExpShare(rNat,i,rNat1))),
                           gw(rNat,i,rNat1,t) * pe(rNat,i,rNat1,t)**(1+omegaw(rNat,i))
                       )**(1/(1+omegaw(rNat,i)))

                  --- Leontief treatment of small export shares

                  + sum(rNat1 $ (m_xwFlag(rNat,i,rNat1) $ isSmallExpShare(rNat,i,rNat1)),
                          gw(rNat,i,rNat1,t)*pe(rNat,i,rNat1,t)) $ card(isSmallExpShare)

              ]      $ (omegaw(rNat,i) ne inf)

              +  ps(rNat,i,t) $ (omegaw(rNat,i) eq inf);
```

The **global demand for transport services** *xtmg* of mode *m* is based on a Leontief approach and defined in the equation *xtmgeq,* summarizing equations (58), (59) and (61) in VDM 2018. The given bilateral transport margin demand *tmarg* are distributed to the different transport modes *m* based on the share parameter *amgm* and multiplied with the bilateral transport flows defined in the macro m_xws, reflecting a potential demand shifter *m_lambdamg*. Note that substitution is between regions providing shares on international transport by transport mode, and not between different modes:

```
xtmgeq(m,ts(t)) $ (xtmg.range(m,t) ne 0) ..

 xtmg(m,t)/xtmg.scale(m,t)
   =e= sum((rNat,i,rNat1) $ amgm(m,rNat,i,rNat1),

                            ---- equation (58) - bilateral import flows
                                 times per unit demand for transport services
                            amgm(m,rNat,i,rNat1)
                             * (  m_xws(rNat,i,rNat1,t)    $ rrComb(rNat,rNat1)
                                  $$onexpand
                                + m_xwsl(rNat,i,rNat1,t) $ (not rrComb(rNat,rNat1))
                                  $$offexpand
                               )

                           --- equation (59) multiplied with trade margins

                           * tmarg(rNat,i,rNat1,t)/m_lambdamg(m,rNat,i,rNat1,t)
           )/xtmg.scale(m,t);
```

The **region specific demand for each transport mode** *xa,* defined in equation *xatmgeq*, equation (62) in VDM 2018, is based on a CES demand system which reflects the average global price for each transport mode *ptmg* and the regional specific price defined in the macro *m_pa* and the substitution elasticity *sigmamg*:

```
xatmgeq(rsNat(rNat),m,tmg,ts(t)) $ (xaFlag(rNat,m,tmg) $ iIn(m)) ..

 xa(rNat,m,tmg,t)/xa.scale(rNat,m,tmg,t)
       =e= alphaa(rNat,m,tmg,t)*xtmg(m,t)/xa.scale(rNat,m,tmg,t)
                          *( ptmg(m,t)/(m_pa(rNat,m,tmg,t)/lcu(rNat,t)))**sigmamg(m)  ;
```

The global average price for each transport mode *ptmg* is defined in the equation *ptmgeq*, equation (63) in VDM 2018, via a dual price aggregator which distinguishes the CD and CES/Leontief case:

```
ptmgeq(m,ts(t)) $ (ptmg.range(m,t) ne 0) ..

 ptmg(m,t) =e=

     --- case with elasticy <> 1 (CES)

     (  (sum( (rNat,tmg) $ alphaa.l(rNat,m,tmg,t),
             alphaa(rNat,m,tmg,t)
               *( (m_pa(rNat,m,tmg,t)/lcu(rNat,t))     $ rsNat(rNat)
                + (pa.l(rNat,m,tmg,t)/lcu.l(rNat,t))   $ (not rsNat(rNat)))**(1-sigmamg(m)) )**(1/(1-sigmamg(m))))/axmg(m,t)
     ) $ (sigmamg(m) ne 1)

     --- Cobb-double case with elasticity == 1

   +  ( sum(tmg, prod(rNat $ alphaa.l(rNat,m,tmg,t),
                 ((   (m_pa(rNat,m,tmg,t)/lcu(rNat,t))    $ rsNat(rNat)
                    + (pa.l(rNat,m,tmg,t)/lcu.l(rNat,t))  $ (not rsNat(rNat)))
               /alphaa(rNat,m,tmg,t))**(alphaa(rNat,m,tmg,t))))/axmg(m,t)
      ) $(sigmamg(m) eq 1)
   ;
```

Price indices

The model defines different price indices which can be used as regional (or global) numéraires and/or for reporting purposes.

Average factor prices *pft* and total stock *xft* for non-non-mobile factors, are defined in the *pftFnmEq*:

```
 pftFnmEq(rs(r),fnm,ts(t)) $ (xftEqFlag(r,fnm) or disr(r)) ..

  pft(r,fnm,t)*xft(r,fnm,t)/sum(a $ xfFlag(r,fnm,a), xf.scale(r,fnm,a,t))
     =E= sum(a $ xfFlag(r,fnm,a), xf(r,fnm,a,t)*pf(r,fnm,a,t))/sum(a $ xfFlag(r,fnm,a), xf.scale(r,fnm,a,t));

 xftFnmEq(rs(r),fnm,ts(t)) $ ((xftEqFlag(r,fnm) or disr(r)) and (not sameas(fnm,"newCap"))) ..

  xft(r,fnm,t)/sum(a $ xfFlag(r,fnm,a), xf.scale(r,fnm,a,t))
     =E= sum(a $ xfFlag(r,fnm,a), xf(r,fnm,a,t))/sum(a $ xfFlag(r,fnm,a), xf.scale(r,fnm,a,t));
```

Regional factor price indices *pfact* are defined in the equation *pfacteq* based on factor prices *pft* and weights *phif* and are used to define them in the benchmark, whereas in shock or follow up years, equations (93) and (94) in VDM 2018:

```
*
* --- Regional index of factor prices
*
  $$macro m_qfactr(tp,tq)  sum (f, (   (pft(rNat,f,tp)/lcu(rNat,tp))     $ (ts(tp) and rsNat(rNat))        \
                                     + (pft.l(rNat,f,tp)/lcu.l(rNat,tp)) $ ( (not ts(tp)) or (not rsNat(rNat))))  \
                                *  (    xft(rNat,f,tq)       $ (ts(tq) and rsNat(rNat))  \
                                     +  xft.l(rnat,f,tq)     $ ( (not ts(tq)) or (not rsNat(rNat)))) )

  $$macro m_qfactw(tp,tq)  sum ( rNat, m_qfactr(tp,tq))

  pfacteq(rsNat(rNat),ts(tRun)) $ ( ((pfact.range(rNat,tRun) ne 0) or (not rres(rNat)) or (lcu.range(rNat,tRun) ne 0))  ) ..

   pfact(rNat,tRun) =e= sum(f, phif(rNat,f,tRun)*pft(rNat,f,tRun)) $ sameas(tRun,"%t0%")

                      + [ pfact.l(rNat,tRun-1) * sqrt(  m_qfactr(tRun,tRun-1)/m_qfactr(tRun-1,tRun-1)
                                                      * m_qfactr(tRun,tRun)   /m_qfactr(tRun-1,tRun)) ] $ (not sameas(tRun,"%t0%"));
*
```

The **average world price of factors** *pwfact* as defined in the equation *pwfacteq* uses weights *phifw* and reflects the exchange rates *lcu* to aggregate the regional factor prices *pft*. It is the global numéraire price in the model, equations (95) and (96) in VDM 2018.

```
*
* --- average world price of factors, equation (95) and (96) in VDM 2018
*     fixed to numeraire with the following equation
*
  $$macro m_qfactw(tp,tq)  sum ( rNat, m_qfactr(tp,tq))
  pwfacteq(ts(tRun)) ..

   pwfact(tRun) =e= sum((rNat,f), phifw(rNat,f,tRun)*(  pft(rNat,f,tRun)    $ rsNat(rNat)
                                                      + pft.l(rNat,f,tRun) $ (not rsNat(rNat)) )/lcu(rNat,tRun)) $ sameas(tRun,"%t0%")
*
                 + [ pwfact.l(tRun-1) * sqrt(  m_qfactw(tRun,tRun-1)/m_qfactw(tRun-1,tRun-1)
                                             * m_qfactw(tRun,tRun)   /m_qfactw(tRun-1,tRun)) ] $ (not sameas(tRun,"%t0%"));
*
```

Regional producer price indices *pprod* as defined in the equation *pprodeq* are based on weights *phii*:

```
pprodeq(rsNat(rNat),ts(t)) $ ( ((pprod.range(rNat,t) ne 0) or (not rres(rNat)) or (lcu.range(rNat,t) ne 0)) ) ..
   pprod(rNat,t) =e= sum(i, phii(rNat,i,t)*ps(rNat,i,t)) ;
```

Average domestic consumption prices *pabs* are an average of the Armington prices for the different types of final demand *fd* (final demand prices for households, government, investment and domestic supply of trade margins) and weights *phia*. That is an approximate version of equation (90) in VDM 2018. That price index is not used elsewhere in the model.

```
pabseq(rsNat(rNat),ts(t)) ..
  pabs(rNat,t) =e= sum((i,fd), phia(rNat,i,fd,t)*m_pa(rNat,i,fd,t)) ;
```

Table 9: Prices in the model

Variable	Content	Indices
px	Unit costs of production	*r,a,t*
pp	Producer price	*r,a,t*
pva	Price of value added composite	*r,a,t*
pnd	Price of intermediate bundle	*r,a,t*
pf	Activity specific factor price, tax exclusive	*r,f,a,t*
pfa	Activity specific factor price, tax inclusive	*r,f,a,t*
ptnest	Price of technology nest	*r,tNest,a,t*
pft	Aggregate price of factors	*r,f,t*
p	Price of output	*r,a,i,t*
ps	Price of domestic supply	*r,i,t*
pe	Prices for bilateral export supply	*r,i,rp,t*
pet	Average price of export supply	*r,i,t*
pefob	Border price of exports (free on board)	*r,i,rp,t*
pmcif	Border price of imports (cost, insurance, freight)	*r,i,rp,t*
pm	Bilateral price of imports, tax inclusive	*r,i,rp,t*
pmt	Average price of imports	*r,i,t*
pmtMrio	Price of aggregate imports, by mrio agent	*r,i,mrioA,t*
pd	Price of domestically produced good	*r,i,t*
pdp	Purchaser price of domestic good	*r,i,aa,t*
pdNest	Price aggregator for sub-nest below final demand equations	*r,dNest,fd,t*
pa	Armington prices	*r,i,aa,t*
pm	Bilateral price of imports, tax inclusive	*r,i,rp,t*
pmp	Public expenditure price deflator	*r,i,aa,t*
pcons	Consumer price deflator	*r,h,t*
pi	Investment expenditure price deflator	*r,t*
pg	Public expenditure price deflator	*r,t*
pfact	Public expenditure price deflator	*r,t*
pprod	World factor price index	*r,t*
pwfact	World factor price index	*t*
ptmg	Global price index of transport services by mode	*m,t*