

# Horizontal fiscal imbalance in Germany

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Regional inequalities are currently a challenge for the majority of the countries, in particular the large ones. The problem of public income redistribution emerges due to possible differentiation of the economic development level of territorial units. The most often considered problem is the vertical distribution. The horizontal division of income is far less frequently considered. Horizontal fiscal imbalance or regional tax inequalities seem to be graver than the vertical imbalance, particularly in developing countries. The public finance system, in particular in federations, is often very complex. Public finance of federations and federated states are not often based on the same assumptions. This leads to differences among regions, both vertical and horizontal. The use of the presented measures helps identify those differences and permits developing mechanisms equalising those inequalities. It should be remembered that those measures may have certain drawbacks, and they mainly focus on certain specific values of income redistribution. Thereby several measures should be applied in measurements and the obtained results should be compared. There are no up-to-date measurements and comparisons of horizontal fiscal imbalance among countries. The aim of this paper is to measure horizontal fiscal imbalance in Germany, especially after reunification, which represents one of two models of federalism. At the beginning it shows the static and dynamic measurements presented in the literature that can be used to measure the horizontal fiscal imbalance. And then it is followed by the results of calculations for Germany in the period 1970-2013. As expected, horizontal imbalance was much lower before than after the reunification of Germany. After the reunification there were large disparities between "old" and "new" länder. This imbalance is gradually reduced. In comparison with the results obtained for the USA (Kowalik, 2014) it can be said that in Germany the disparities are much smaller between länder than in the United States between the individual states.

**JEL Classifications:** H77, H73, O11, O18, O47, R12

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## Introduction

The decentralisation of public finances connected with the state federal structure, apart from benefits also causes specific problems and subsequently requires solution. The main problem is the decentralisation of tax authority and financial equalisation. Due to possible differentiation of the economic development level of territorial units of a federation, the problem of public income redistribution emerges - both horizontal and vertical. The most often considered one is the vertical distribution of income among individual levels of public authority. At the same time units richer than others can occur at each lower government level. The difference arising from the resources possessed at the same level may be defined as horizontal fiscal imbalance or horizontal fiscal gap<sup>1</sup>. It occurs when it is

<sup>1</sup> The two notions are used interchangeably.

not possible to achieve income matching the needs of the authorities at the same level. A certain incongruity between income and expenses at different levels is unavoidable in the case of all federations (see more in Kowalik (2012; 2014)).

Analysis of the fiscal imbalance gives possibilities of comparative studies of states, especially those with a federal form. According to Bitner and Cichocki (2012), comparative research on local government subsector finance is particularly rare in public finance literature. There are no up-to-date measurements and comparisons of horizontal fiscal imbalance among countries. Results obtained by the end of the 20th century can be found in English-language literature. R. Bird and A.V. Tarasov as well as R. Shankar and A. Shah measured horizontal fiscal imbalance. The calculations were conducted basing on data before 2000 in the case of Bird and Tarasov (2002, p.52) and before 1998 in the case of Shankar and Shah (2003, pp.1426, 1428, 1432-1433, 1438). Apparently, the European Union is an exception since it calculates the regional GDP for its regions within the NUTS classification. The comparisons are made on this base.

The aim of this paper is to measure horizontal fiscal imbalance in Germany in the period of 1970-2013, especially after reunification.

This article is a part of author's research regarding vertical and horizontal fiscal imbalances in federal and unitary countries. In the publication (Kowalik, 2014) the author has provided the estimates of horizontal fiscal imbalances in the United States of America which represent one of the two models of federalism - cooperative federalism, where state and local governments compete for different laws and powers. And the second model is the cooperative federalism, where national, state and local government work jointly and govern together. This model is represented by Germany. As the author has been interested also in this kind of model disparities the research presented below will be complimentary to the case of United States.

## Static and dynamic measures of horizontal fiscal imbalance

Various measures may be used for measuring horizontal differences<sup>1</sup>. The static measures of horizontal fiscal imbalance are the minimum and maximum indicators, range, maximum to minimum, simple and weighted coefficient of variation, Gini and Theil indexes or Hoover and Coulter coefficients.

Coefficient of minimum (maximum) as percent of national average define formulas (Bird and Tarasov, 2002, p.12):

$$\frac{y_{\min}}{\bar{y}} \cdot 100\% \text{ and } \frac{y_{\max}}{\bar{y}} \cdot 100\%, \quad (1)$$

where:  $y_{\max}$  - region with maximum parameter<sup>2</sup> per capita (e.g. GDP),  $y_{\min}$  - region with minimum parameter per capita,  $\bar{y}$  - national average of given parameter.

Range is a measure characterising the empirical area of variation of the examined feature; however, it does not provide information on the diversification of individual values. It is presented e.g. by Cowell (2011, p.155):

<sup>1</sup> They may be static - they show inequalities in the given moment, or dynamic - they reflect historical trends. Dynamic measures are based on the hypothesis of convergence or divergence.

<sup>2</sup> This parameter may be the regional GDP, regional income, regional expenses, regional own income, regional total income, inter-government transfers, gross added value, personal available income.

$$R = y_{\max} - y_{\min}, \quad (2)$$

Maximum-to-minimum ratio can specify a formula (Bird and Tarasov, 2002, p.12; Shankar and Shah, 2003, p.1422; 2008, p.144; Li and Xu, 2008, p.34):

$$WMM = \frac{y_{\max}}{y_{\min}}, \quad (3)$$

This indicator shows the measure of scope of those differences. If the measure equals 1, this means that different regions are perfectly homogeneous. If it is high, a problem with interpretation may be encountered. A high level of the indicator may be explained by considerable changes in the distribution of GDP or may indicate the presence of “outsiders”<sup>1</sup>.

The most frequently used coefficient of imbalance measure in the literature is the coefficient of variation<sup>2</sup>. Most often it is simple or weighted. This measure is standardised and may be used for comparisons among countries and in time.

Simple coefficient of variation can specify a formula (Bird and Tarasov, 2002, p.12; Shankar and Shah, 2003, p.1422; 2008, pp.144-145; Huang and Leung, 2009, p.28; Cowell, 2011, p.155):

$$CV_U = \frac{\sqrt{\sum_i \frac{(y_i - \bar{y}_U)^2}{n}}}{\bar{y}_U}, \quad (4)$$

$$\bar{y}_U = \frac{1}{n} \sum_{i=1}^n y_i, \quad (5)$$

where:  $y_i$  - observed parameter per capita in region  $i$  (for example income per capita),  $n$  - number of regions,  $\bar{y}_U$  - national average of given parameter, non-weighted.

If it equals 0, it means perfect homogeneity. This measure should be used for time-framed comparisons. Comparisons among countries may be problematic since it is sensitive to the number of regions.

Weighted coefficient of variation<sup>3</sup> is defined by the following formula (Bird and Tarasov, 2002, p.12; Shankar and Shah, 2003, p.1423; 2008, pp.145-146):

$$CV_w = \frac{\sqrt{\sum_i (y_i - \bar{y})^2 \frac{p_i}{P}}}{\bar{y}}, \quad (6)$$

<sup>1</sup> These may be city-states, eastern Lands in Germany, low populated poor regions in Canada or low populated rich regions in the USA, e.g. Alaska.

<sup>2</sup> It is a measure of dispersion.

<sup>3</sup> Sometimes it may be encountered in the literature as the Williamson index (*population weighted coefficient of variation*) (Portnov and Felsenstein, 2005, p.49).

$$\bar{y} = \sum_{i=1}^n y_i \frac{p_i}{P}, \quad (7)$$

where:  $p_i$  - population of region  $i$ .

If it equals 0, it means perfect homogeneity. This measure should be used for comparisons among countries since it does not depend on the number of regions but on the percentage of population in the region.

Relative mean deviation<sup>1</sup> is defined by the following formula (Bird and Tarasov, 2002, p.12; Shankar and Shah, 2003, p.1423; 2008, p.146):

$$R_w = \frac{\sum_i |y_i - \bar{y}| \frac{p_i}{P}}{\bar{y}}, \quad (8)$$

This measure allows to avoid the problem of deviation, and hence it may be applied for checking the *CV* results. It may range from 0 to 2, where 0 means perfect homogeneity, and 2 - perfect imbalance. This measure should be used for comparisons among countries since it does not depend on the number of regions.

The Gini index is another frequently used coefficient. It may be non-weighted or weighted. This coefficient is the area of the double field between the Lorenz curve and the diagonal of the unit square.

Unweighted Gini index can be defined by the following formula (Shankar and Shah, 2003, p.1423; 2008, pp.146-147; Li and Xu, 2008, p.34):

$$G_U = \left( \frac{1}{2\bar{y}_U} \right) \frac{1}{n(n-1)} \sum_{i=1}^n \sum_{j=1}^n |y_i - y_j|, \quad (9)$$

where:  $y_j$  - observed parameter per capita in region  $j$ .

0 means perfect homogeneity and 1 - perfect imbalance.

We may also find the index in the following form which is proposed e.g. by Portnov and Felsenstein (2010, p.213), Litchfield (1999) and Cowell (2011, p.155):

$$G_U = \frac{1}{2n^2 \bar{y}} \sum_{i=1}^n \sum_{j=1}^n |y_i - y_j|, \quad (10)$$

Weighted Gini index is defined by the following formula (Shankar and Shah 2003, p.1423; 2008, p.147; Li and Xu, 2008, p.34):

$$G_w = \left( \frac{1}{2\bar{y}} \right) \sum_{i=1}^n \sum_{j=1}^n |y_i - y_j| \frac{p_i p_j}{P^2}, \quad (11)$$

where:  $p_j$  - population of region  $j$ .

<sup>1</sup> It is also a measure of dispersion.

The value may range from 0 to 1-( $\pi_i/P$ ), where 0 means perfect homogeneity and 1-( $\pi_i/P$ ) - perfect imbalance.

Theil index, based on the measure of entropy, may be defined by the formulas which were proposed by Bird and Tarasov (2002, p.12):

$$T_{B,T} = \sum_i \frac{y_i}{\bar{y}} \cdot \frac{p_i}{P} \log \left( \frac{y_i}{\bar{y}} \right), \quad (12)$$

Shankar and Shah (2003, p.1423; 2008, p.147) in their works proposed the following formula:

$$T_{S,S} = \sum_i x_i \log \left( \frac{x_i}{q_i} \right), \quad (13)$$

where:  $x_i$  - share of region  $i$  in a given parameter (for example share in GDP),  $q_i$  - population share of region  $i$ .

Cowell (2011, p.155) and Litchfield (1999) presented the following formula for the Theil index:

$$T_C = \frac{1}{n} \sum_{i=1}^n \frac{y_i}{\bar{y}} \log \left( \frac{y_i}{\bar{y}} \right), \quad (14)$$

The value of this measure may range from 0 to  $\log(n)$ , where 0 means perfect homogeneity. The  $\log(n)$  value will appear when the entire value of the defined parameter is concentrated in one region. This measure shows global inequalities at a given point of time. This measure is sometimes subject to decomposition for the arithmetic and analytic purposes.

The Hoover coefficient is defined by the following formula (Portnov and Felsenstein, 2010, p.213):

$$H = \frac{1}{2} \sum_{i=1}^n \left| \frac{p_i}{P} \frac{y_i}{\bar{y}} - \frac{p_i}{P} \right|, \quad (15)$$

The Coulter coefficient is defined by the following formula (Portnov and Felsenstein, 2010, p.213):

$$C = \left[ \frac{1}{2} \sum_{i=1}^n \left( \frac{p_i}{P} \frac{y_i}{\bar{y}} - \frac{p_i}{P} \right)^2 \right]^{\frac{1}{2}}, \quad (16)$$

Attempts to measure horizontal imbalance and the impact of equalisation transfers have been made by simple measures of dispersion or concentration. The subject literature does not describe many effective techniques of inequality measurement. Some attempts have been made to deploy two statistics concepts which are useful for considering the

dynamics of regional inequalities<sup>1</sup>. They are dynamic measures of beta ( $\beta$ ) convergence<sup>2</sup> and sigma ( $\sigma$ ) convergence<sup>3</sup> based on the convergence hypothesis or the divergence hypothesis (Shankar and Shah, 2003, pp.1423-1425; 2008, pp.147-149; Rey and Janikas, 2005, p.159; Villaverde and Maza, 2009, pp.8-9). Making up for the distance in income of relatively poorer regions by faster growth is called beta convergence, while decreasing the regional dispersion in income in time is referred to as sigma convergence (Shankar and Shah, 2003, p.1424; 2008, p.149). This measure was presented by (see more in Kowalik (2012, pp.289-292; 2014, pp.143-144)):

Villaverde and Maza (2009, p.8) shows that:

$$\left(\frac{1}{T}\right) \log\left(\frac{y_{iT}}{y_{i0}}\right) = c - \left(\frac{1 - e^{-bT}}{T}\right) \log(y_{i0}) + \mu_i, \quad (17)$$

where:  $\beta = -\left(\frac{1 - e^{-bT}}{T}\right)$ ;  $b = -\log[1 - \beta T]/T$ ,  $c$  - constant term,  $b$  - speed of

convergence,  $y_i$  - per capita income of observation  $i$ ,  $T$  - sample period,  $\mu_i$  - error term.

If the coefficient of beta regression is negative, this means that weak economies develop faster than the richer ones. The classical equation of conditional beta convergence may be presented by the following formula (Villaverde and Maza, 2009, p.9):

$$\left(\frac{1}{T}\right) \log\left(\frac{y_{iT}}{y_{i0}}\right) = c + \beta \log(y_{i0}) + \gamma X_i' + \mu_i, \quad (18)$$

where:  $X_i'$  - vector of conditioning variables.

Shankar and Shah (2003, p.1424) estimated the growth equation basing it on the log-linear convergence around the state of equilibrium of the Solow growth model:

$$\frac{1}{T} \ln\left(\frac{y_{i,t}}{y_{i,t-T}}\right) = \alpha_i - \left(\frac{1 - e^{-\beta T}}{T}\right) \ln(y_{i,t-T}) + \mu_{i,t}, \quad (19)$$

where:  $y_{i,t}$  - per capita regional GDP of the region  $i$  at time  $t$ ,  $y_{i,t-T}$  - per capita regional GDP of the region  $i$  at the beginning of the time interval under study,  $\mu$  - error term.

Subsequently it was converted to this form (Shankar and Shah, 2003, p.1424):

$$\ln(y_{i,t}) - \ln(y_{i,t-T}) = \alpha_i + \beta \ln(y_{i,t-T}) + \mu_{i,t}, \quad (20)$$

Próchniak and Rapacki estimated the following regression equation (2007, p.43):

<sup>1</sup> Hence their names - dynamic measures.

<sup>2</sup> And their modifications. It assumes that countries with a lower initial income level are characterised by a greater pace of growth than the initially richer countries, which finally leads to the equalisation of income per capita.

<sup>3</sup> It refers to the measurement of diversity countries, affluence. Sigma convergence is referred to in the case of decreasing values of the adopted indicator in successive moments of the defined time.

$$\frac{1}{T}(\ln y(T) - \ln y(0)) = \alpha_0 - \alpha_1 \ln y(0), \quad (21)$$

where:  $y(T)$  - regional GDP per capita in the end year,  $y(0)$  - regional GDP per capita in the initial year.

The left side of the above formula determines economic growth rate. The first variable on the right side of  $\ln y(0)$  represents the initial level of regional GDP, and hence the  $\alpha_1$  parameter informs about the occurrence of real  $\beta$ -convergence. Such a convergence occurs where  $\alpha_1$  is negative and statistically significant.  $\beta$ -value can be determined in the following way (Próchniak and Rapacki, 2007, p.44; Rapacki and Próchniak, 2009, p.3; Próchniak and Witkowski, 2012, p.28):

$$\beta = -\frac{1}{T} \ln(1 + \alpha_1 T), \quad (22)$$

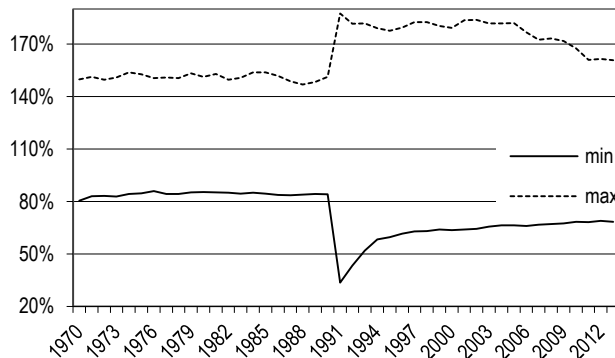
It is very similar to the formula proposed by Villaverde and Maza (2009, p.10), who defined the rate of convergence as  $b = -\log[1 - \beta T]/T$ .

## Research results

The calculations were based on statistical data obtained from the Federal Statistical Office of Germany regarding population (StatBA, 2014) and statistical data obtained from the Baden-Württemberg Statistical Office regarding the regional GDP for 11 federal states in the period 1970-1990 (VGR der Länder, 2007) and 16 federal states in the period 1991-2013 (VGR der Länder, 2014).

The inequality measures are presented for unified Germany (1991-2013) as well as for the states that were part of the Federal Republic of Germany prior to unification (1970-1990). As expected, the horizontal imbalance in the FRG was lower than in unified Germany (Figures 1-6, Table 1).

FIGURE 1. COEFFICIENT OF MINIMUM (MAXIMUM) AS PERCENT OF NATIONAL AVERAGE (1970-2013)

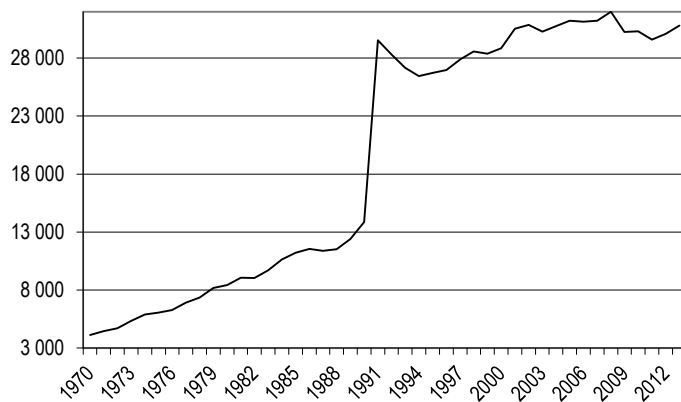


Source: Own calculations.

The observed regional diversity was weaker before the reunification than it has been since 1990. Despite the increasing gap between the best and the worst federal state (Figure 2), the maximum-to-minimum ratio remained at an almost unchanged level of circa 1.8 (Figure 3). The maximum-to-minimum ratio as % of the national average also remained at a similar level: it equalled 84% average and 151%, respectively, in the period 1970-1990 (Figure 1). Other ratios also indicate a slight horizontal imbalance in the period before 1990, which did not change over time (see Figures 4-6).

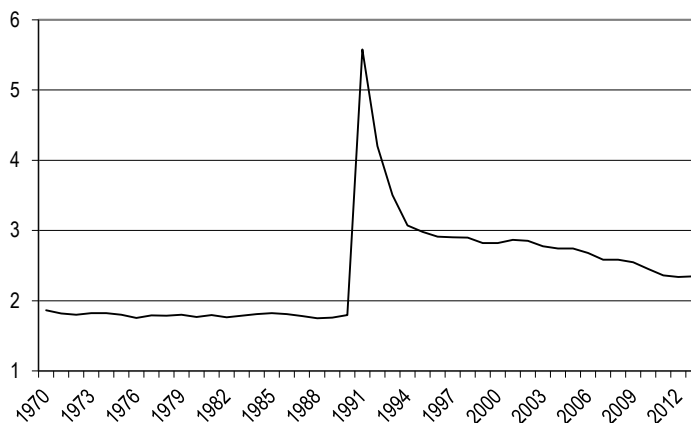
After the German reunification, increasing disproportions among federal states can be noticed: mainly among the “old” and “new” länder (Table 1). The difference between the minimum and maximum ratios as % of the national average being on average 67% until 1990 and increased in 1991 up to nearly 154%. In the initial four years, the difference was reduced to 120%. The twelve successive years allowed the reduction of the difference to 111%, and the recent seven years - to 92.3% (Figure 1).

FIGURE 2. RANGE (1970-2013)



Source: Own calculations.

FIGURE 2. MAXIMUM-TO-MINIMUM RATIO (1970-2013)



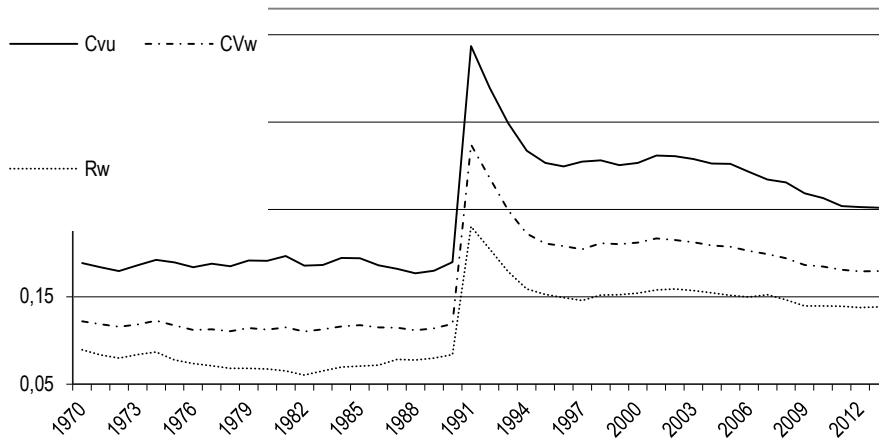
Source: Own calculations.



The difference between the richest and the poorest federal state was slightly reduced but after eleven years it exceeded the 1991 value again and since then has remained at a roughly the same level (Figure 2).

The maximum-to-minimum ratio, which was 5.578 in 1991, was reduced to the level of 2.5, with the further decreasing tendency (Figure 3). Other ratios also demonstrate a slow reduction of disproportions between federal states (Figures 4-6).

FIGURE 3. SIMPLE AND WEIGHTED COEFFICIENT OF VARIATION, RELATIVE MEAN DEVIATION (1970-2013)



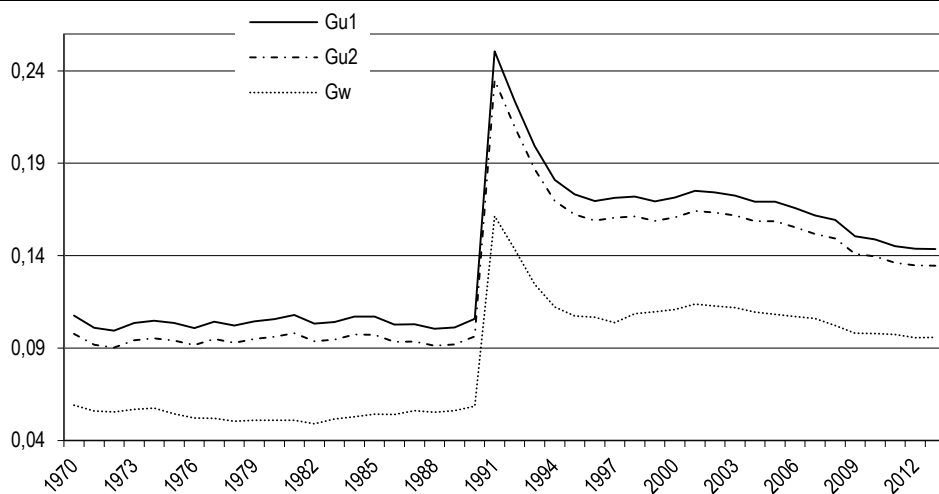
Source: Own calculations.

The dynamic measure of horizontal fiscal imbalances, namely sigma, can be determined on the base of the simple and weighted coefficient of variation (Figure 4):

- in the period 1970-1972 convergence was noticeable,
- in the period 1972-1974 divergence occurred,
- in the period 1974-1976 convergence was present again,
- in the period 1976-1982 convergence and divergence were observed alternately, and hence it can be assumed that sigma was constant,
- in the period 1982-1985 divergence occurred,
- in the period 1985-1988 convergence was present,
- in the period 1988-1990 - divergence,
- in the period 1991-1997 strong convergence occurred, particularly at the beginning of the period,
- in the period 1998-2000 sigma was constant,
- in the period 2001-2013 convergence occurred again.

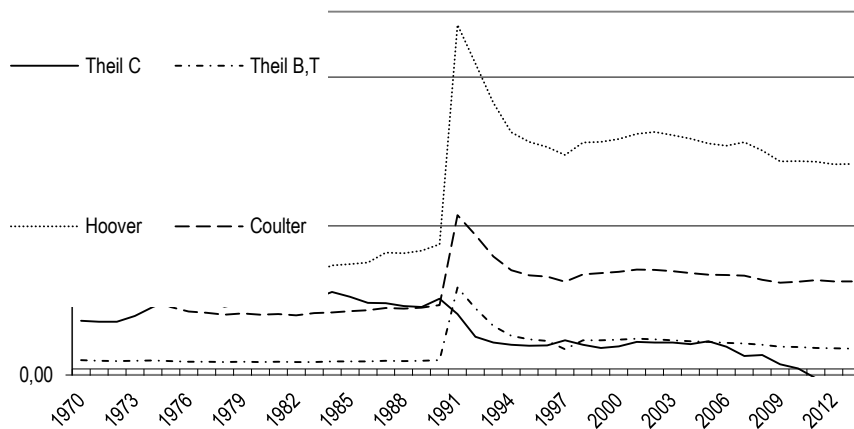
Weighted values are lower than non-weighted ones, thus proving that the regions with the highest regional GDP p.c. have a low population.

FIGURE 4. UNWEIGHTED AND WEIGHTED GINI INDEX (1970-2013)



Source: Own calculations.

FIGURE 5. THEIL INDEX, HOOVER AND COULTER COEFFICIENT (1970-2013)



Source: Own calculations.

TABLE 1. SUMMARY TABLE ON MEASURES OF HORIZONTAL FISCAL IMBALANCE IN GERMANY BASED ON PER CAPITA REGIONAL GDP

	Figures are calculated values for the first year of series	Figures are calculated values for the last year of series	Overall range of values over corresponding period	Figures are calculated values for the first year of series	Figures are calculated values for the last year of series	Overall range of values over corresponding period
Number of regions (states)	11			16		
Data	1970-1990			1991-2013		
Number of observations	21			23		
min/max as % of average	80.39-149.87	84.09-151.19	80.39-153.92	33.62-187.53	68.41-160.73	33.62-187.53
R	4 131	13 861	4 131-13 961	29 530	30 794	26 459-31 982
WMM	1.864	1.798	1.751-1.864	5.578	2.350	2.341-5.578
Cvu	0.188	0.189	0.177-0.197	0.437	0.252	0.252-0.437
CVw	0.122	0.119	0.110-0.123	0.324	0.179	0.179-0.324
Rw	0.089	0.084	0.060-0.089	0.230	0.138	0.138-0.230
Gu1	0.108	0.106	0.099-0.108	0.251	0.144	0.144-0.251
Gu2	0.098	0.096	0.090-0.098	0.235	0.135	0.135-0.235
Gw	0.059	0.059	0.049-0.059	0.161	0.096	0.096-0.161
Theil C	0.016	0.024	0.016-0.026	0.018	-0.004	-0.004-0.018
Theil B,T	0.003	0.003	0.002-0.003	0.027	0.007	0.007-0.027
Hoover	0.045	0.042	0.030-0.045	0.116	0.069	0.069-0.116
Coulter	0.023	0.021	0.018-0.023	0.052	0.029	0.029-0.052

Source: Own calculations.

A dynamic measure, namely beta, shows that convergence appeared in most of the periods in FRG - both in the period before and after the reunification. The period of 1980-1991 is an exception here, when only a slight divergence occurred, which corresponds to sigma indications (Table 2).

TABLE 2.  $\beta$ -CONVERGENCE IN GERMANY

Observation period	Independent variable	Number of regions	Speed of convergence	Beta convergence	R <sup>2</sup>
1991-2013	ln(1991PKBp.c.)	16	3.86%	yes	0.92081
1991-1995	ln(1991PKBp.c.)	16	12.67%	yes	0.91594
1991-2000	ln(1991PKBp.c.)	16	6.89%	yes	0.89103
1991-2007	ln(1991PKBp.c.)	16	4.68%	yes	0.88369
1995-2013	ln(1995PKBp.c.)	16	1.23%	yes	0.57165
2000-2013	ln(2000PKBp.c.)	16	1.32%	yes	0.68221
2007-2013	ln(2007PKBp.c.)	16	1.51%	yes	0.58471
1970-1991	ln(1970PKBp.c.)	11	0.08%	yes	0.00104
1970-1980	ln(1970PKBp.c.)	11	0.41%	yes	0.05874
1980-1991	ln(1980PKBp.c.)	11	-0.44%	no	0.01215
1970-2013*	ln(1970PKBp.c.)	11	0.23%	yes	0.02570
1991-2013**	ln(1991PKBp.c.)	5	8.20%	yes	0.88656

Note: \* - for the "old" länder, \*\* - for the "new" länder.

Source: Own calculations.

The highest convergence was noticed in the period of 1991-1995. In the entire post-reunification period, in turn, convergence remains at the level of 3.86%, and the coefficient of determination also stays at an extremely high level of 92.1% (Table 2), which means that all federal states reduce the distance to the state of long-term equilibrium at the rate of 3.86% per annum. This shows that reducing the distance by half with respect to the common state of long-term equilibrium requires nearly 18 years.

## Conclusion

Convergence of regions is currently one of the most frequently addressed research problems, in particular in the context of equalising inequalities among the European Union member states. Despite the growing interest of research on convergence with respect to regions, the measurement of fiscal inequalities among and inside regions is rarely analysed.

The observed regional diversity was weaker before the reunification than it has been since 1990 despite the increasing gap between the best and the worst federal state. After the German reunification, increasing disproportions among federal states can be noticed: mainly among the “old” and “new” *länder*. A dynamic measure, namely beta, shows that there was convergence for the majority of the periods. An exception occurred between 1980 and 1991. The highest convergence was noticed in the period 1991-1995. In the entire post-reunification period, in turn, convergence remains at the level of 3.86%, which means that all federal states reduce the distance to the state of long-term equilibrium at the rate of 3.86% per annum.

The results indicate that horizontal equalization system, both the federal and the *länder* serves its purpose. Please note that horizontal differences cannot be entirely eliminated, especially in the federal states.

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