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Research Article

Hybrid Algorithm for Finding Common Elements of the Set of Generalized Equilibrium Problems and the Set of Fixed Point Problems of Strictly Pseudocontractive Mapping

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The purpose of this paper is to prove the strong convergence theorem for finding a common element of the set of fixed point problems of strictly pseudocontractive mapping in Hilbert spaces and two sets of generalized equilibrium problems by using the hybrid method.

1. Introduction

Let C be a closed convex subset of a real Hilbert space H, and let $F : C \times C \to \mathbb{R}$ be a bifunction. Recall that the equilibrium problem for a bifunction F is to find $x \in C$ such that

$$F(x, y) \geq 0, \quad \forall y \in C. \tag{1.1}$$

The set of solutions of (1.1) is denoted by $\mathrm{EP}(F)$. Given a mapping $T : C \to H$, let $F(x, y) = \langle Tx, y - x\rangle$ for all $x, y \in C$. Then, $z \in \mathrm{EP}(F)$ if and only if $\langle Tz, y - z\rangle \geq 0$ for all $y \in C$; that is, z is a solution of the variational inequality. Let $A : C \to H$ be a nonlinear mapping. The variational inequality problem is to find a $u \in C$ such that

$$\langle v - u, Au\rangle \geq 0 \tag{1.2}$$

for all $v \in C$. The set of solutions of the variational inequality is denoted by $\mathrm{VI}(C, A)$. Now, we consider the following generalized equilibrium problem:

$$\text{Find } z \in C \text{ such that } F(z, y) + \langle Az, y - z\rangle \geq 0, \quad \forall y \in C. \tag{1.3}$$

The set of $z \in C$ is denoted by $\mathrm{EP}(F, A)$, that is,

$$\mathrm{EP}(F, A) = \{z \in C : F(z, y) + \langle Az, y - z\rangle \geq 0,\ \forall y \in C\}. \tag{1.4}$$

In the case of $A \equiv 0$, $\mathrm{EP}(F, A)$ is denoted by $\mathrm{EP}(F)$. In the case of $F \equiv 0$, $\mathrm{EP}(F, A)$ is also denoted by $\mathrm{VI}(C, A)$. Numerous problems in physics, optimization, variational inequalities, minimax problems, the Nash equilibrium problem in noncooperative games, and economics are reduced to find a solution of (1.3); see, for instance, [1–3].

A mapping A of C into H is called *inverse strongly monotone mapping*, see [4], if there exists a positive real number α such that

$$\langle x - y, Ax - Ay\rangle \geq \alpha\|Ax - Ay\|^2 \tag{1.5}$$

for all $x, y \in C$. The following definition is well known.

Definition 1.1. A mapping $T : C \to C$ is said to be a κ-strict pseudocontraction if there exists $\kappa \in [0, 1)$ such that

$$\|Tx - Ty\|^2 \leq \|x - y\|^2 + \kappa\|(I - T)x - (I - T)y\|^2, \quad \forall x, y \in C. \tag{1.6}$$

A mapping T is called *nonexpansive* if

$$\|Tx - Ty\| \leq \|x - y\| \tag{1.7}$$

for all $x, y \in C$.

We know that κ-strict pseudocontraction includes a class of nonexpansive mappings. If $\kappa = 1$, T is said to be a pseudocontractive *mapping*. T is *strong pseudocontraction* if there exists a positive constant $\lambda \in (0, 1)$ such that $T + \lambda I$ is pseudocontraction. In a real Hilbert space H, (1.6) is equivalent to

$$\langle Tx - Ty, x - y\rangle \leq \|x - y\|^2 - \frac{1 - \kappa}{2}\|(I - T)x - (I - T)y\|^2, \quad \forall x, y \in D(T). \tag{1.8}$$

T is pseudocontraction if and only if

$$\langle Tx - Ty, x - y\rangle \leq \|x - y\|^2, \quad \forall x, y \in D(T). \tag{1.9}$$

Then, T is strong pseudocontraction if there exists positive constant $\lambda \in (0, 1)$

$$\langle Tx - Ty, x - y\rangle \leq (1 - \lambda)\|x - y\|^2, \quad \forall x, y \in D(T). \tag{1.10}$$

The class of κ-strict pseudocontractions falls into the one between classes of nonexpansive mappings, and the pseudocontraction mappings, and the class of strong pseudocontraction mappings is independent of the class of κ-strict pseudocontraction.

We denote by $F(T)$ the set of fixed points of T. If $C \subset H$ is bounded, closed, and convex, and T is a nonexpansive mapping of C into itself, then $F(T)$ is nonempty; for instance, see [5]. Browder and Petryshyn [6] show that if a κ-strict pseudocontraction T has a fixed point in C, then starting with an initial $x_0 \in C$, the sequence $\{x_n\}$ generated by the recursive formula:

$$x_{n+1} = \alpha x_n + (1-\alpha)Tx_n, \tag{1.11}$$

where α is a constant such that $0 < \alpha < 1$, converges weakly to a fixed point of T. Marino and Xu [7] have extended Browder and Petryshyns above-mentioned result by proving that the sequence $\{x_n\}$ generated by the following Manns algorithm [8]:

$$x_{n+1} = \alpha_n x_n + (1-\alpha_n)Tx_n \tag{1.12}$$

converges weakly to a fixed point of T provided the control sequence $\{\alpha_n\}_{n=0}^{\infty}$ satisfies the conditions that $\kappa < \alpha_n < 1$ for all n and $\sum_{n=0}^{\infty}(\alpha_n - \kappa)(1-\alpha_n) = \infty$. In 1974, S. Ishikawa proved the following strong convergence theorem of pseudocontractive mapping.

Theorem 1.2 (see [9]). *Let C be a convex compact subset of a Hilbert space H, and let $T : C \to C$ be a Lipschitzian pseudocontractive mapping. For any $x_1 \in C$, suppose that the sequence $\{x_n\}$ is defined by*

$$\begin{gathered} y_n = (1-\beta_n)x_n + \beta_n T x_n, \\ x_{n+1} = (1-\alpha_n)x_n + \alpha_n T y_n, \quad \forall n \in \mathbb{N}, \end{gathered} \tag{1.13}$$

where $\{\alpha_n\}$, $\{\beta_n\}$ are two real sequences in $[0,1]$ satisfying

(i) $\alpha_n \le \beta_n$, *for all* $n \in \mathbb{N}$,

(ii) $\lim_{n\to\infty}\beta_n = 0$,

(iii) $\sum_{n=1}^{\infty} \alpha_n\beta_n = \infty$.

Then $\{x_n\}$ converges strongly to a fixed point of T.

In order to prove a strong convergence theorem of Mann algorithm (1.12) associated with strictly pseudocontractive mapping, in 2006, Marino and Xu [7] proved the following theorem for strict pseudocontractive mapping in Hilbert space by using CQ method.

Theorem 1.3 (see [7]). *Let C be a closed convex subset of a Hilbert space H. Let $T : C \to C$ be a κ-strict pseudocontraction for some $0 \leq \kappa < 1$, and assume that the fixed point set $F(T)$ of T is nonempty. Let $\{x_n\}_{n=1}^{\infty}$ be the sequence generated by the following (CQ) algorithm:*

$$\begin{aligned}
&x_1 \in C,\\
&y_n = \alpha_n x_n + (1-\alpha_n)Tx_n,\\
C_n &= \left\{z \in C : \left\|y_n - z\right\|^2 \leq (1-\alpha_n)(\kappa-\alpha_n)\|x_n - Tx_n\|^2\right\},\\
Q_n &= \{z \in C : \langle x_n - z, x_1 - x_n\rangle\},\\
x_{n+1} &= P_{C_n \cap Q_n} x_1.
\end{aligned} \tag{1.14}$$

Assume that the control sequence $\{\alpha_n\}_{n=1}^{\infty}$ is chosen so that $\alpha_n < 1$ for all $n \in \mathbb{N}$. Then $\{x_n\}$ converges strongly to $P_{F(T)}x_1$. Very recently, in 2010, [10] established the hybrid algorithm for Lipschitz pseudocontractive mapping as follows:

$$\begin{aligned}
&\text{For } C_1 = C, \qquad x_1 = P_{C_1}x_1,\\
&y_n = (1-\alpha_n)x_n + \alpha_n T z_n,\\
&z_n = \left(1-\beta_n\right)x_n + \beta_n T x_n,\\
C_{n+1} &= \left\{z \in C_n : \left\|\alpha_n (I-T)y_n\right\|^2 \leq 2\alpha_n \langle x_n - z, (I-T)y_n\rangle \right.\\
&\qquad \left. +2\alpha_n\beta_n L\|x_n - Tx_n\|\left\|y_n - x_n + \alpha_n(I-T)y_n\right\|\right\},\\
&x_{n+1} = P_{C_{n+1}}x_1, \quad \forall n \in \mathbb{N}.
\end{aligned} \tag{1.15}$$

Under suitable conditions of $\{\alpha_n\}$ and $\{\beta_n\}$, they proved that the sequence $\{x_n\}$ defined by (1.15) *converges strongly to $P_{F(T)}x_1$.*

Many authors study the problem for finding a common element of the set of fixed point problem and the set of equilibrium problem in Hilbert spaces, for instance, [2, 3, 11–15]. The motivation of (1.14), (1.15)*, and the research in this direction, we prove the strong convergence theorem for finding solution of the set of fixed points of strictly pseudocontractive mapping and two sets of generalized equilibrium problems by using the hybrid method.*

2. Preliminaries

In order to prove our main results, we need the following lemmas. Let C be closed convex subset of a real Hilbert space H, and let P_C be the metric projection of H onto C; that is, for $x \in H$, $P_C x$ satisfies the property

$$\|x - P_C x\| = \min_{y \in C}\left\|x - y\right\|. \tag{2.1}$$

The following characterizes the projection P_C.

Lemma 2.1 (see [5]). *Given that $x \in H$ and $y \in C$, then $P_C x = y$ if and only if the following inequality holds:*

$$\langle x - y, y - z \rangle \geq 0, \quad \forall z \in C. \tag{2.2}$$

The following lemma is well known.

Lemma 2.2. *Let H be Hilbert space, and let C be a nonempty closed convex subset of H. Let $T : C \to C$ be κ-strictly pseudocontractive, then the fixed point set $F(T)$ of T is closed and convex so that the projection $P_{F(T)}$ is well defined.*

Lemma 2.3 ((demiclosedness principle) (see [16])). *If T is a κ-strict pseudocontraction on closed convex subset C of a real Hilbert space H, then $I - T$ is demiclosed at any point $y \in H$.*

To solve the equilibrium problem for a bifunction $F : C \times C \to \mathbb{R}$, assume that F satisfies the following conditions:

$(A1)$ $F(x,x) = 0$ for all $x \in C$,

$(A2)$ F is monotone, that is, $F(x,y) + F(y,x) \leq 0$, for all $x, y \in C$,

$(A3)$ for all $x, y, z \in C$,

$$\lim_{t \to 0^+} F(tz + (1-t)x, y) \leq F(x,y), \tag{2.3}$$

$(A4)$ for all $x \in C, y \mapsto F(x,y)$ is convex and lower semicontinuous.

The following lemma appears implicitly in [1].

Lemma 2.4 (see [1]). *Let C be a nonempty closed convex subset of H, and let F be a bifunction of $C \times C$ into $\mathbb{R}$ satisfying $(A1)$–$(A4)$. Let $r > 0$, and $x \in H$. Then, there exists $z \in C$ such that*

$$F(z,y) + \frac{1}{r}\langle y - z, z - x \rangle, \tag{2.4}$$

for all $x \in C$.

Lemma 2.5 (see [11]). *Assume that $F : C \times C \to \mathbb{R}$ satisfies $(A1)$–$(A4)$. For $r > 0$ and $x \in H$, define a mapping $T_r : H \to C$ as follows:*

$$T_r(x) = \left\{ z \in C : F(z,y) + \frac{1}{r}\langle y - z, z - x \rangle \geq 0, \ \forall y \in C \right\}, \tag{2.5}$$

for all $z \in H$. Then, the following hold:

(1) *T_r is single-valued;*

(2) T_r *is firmly nonexpansive, that is,*

$$\left\|T_r(x)-T_r(y)\right\|^2 \leq \left\langle T_r(x)-T_r(y), x-y\right\rangle, \quad \forall x, y \in H, \tag{2.6}$$

(3) $F(T_r) = EP(F)$;

(4) $EP(F)$ *is closed and convex.*

Lemma 2.6 (see [17]). *Let C be a closed convex subset of H. Let $\{x_n\}$ be a sequence in H and $u \in H$. Let $q = P_C u$; if $\{x_n\}$ is such that $\omega(x_n) \subset C$ and satisfy the condition*

$$\|x_n - u\| \leq \left\|u-q\right\|, \quad \forall n \in \mathbb{N}, \tag{2.7}$$

then $x_n \to q$, as $n \to \infty$.

Lemma 2.7 (see [7]). *For a real Hilbert space H, the following identities hold: if $\{x_n\}$ is a sequence in H weak convergence to z, then*

$$\limsup_{n\to\infty}\left\|x_n-y\right\|^2 = \limsup_{n\to\infty}\|x_n-z\|^2 + \left\|z-y\right\|^2, \tag{2.8}$$

for all $y \in H$.

3. Main Result

Theorem 3.1. *Let C be a nonempty closed convex subset of a Hilbert space H. Let F and G be bifunctions from $C\times C$ into $\mathbb{R}$ satisfying (A_1)–(A_4), respectively. Let $A : C \to H$ be an α-inverse strongly monotone mapping, and let $B : C \to H$ be a β-inverse strongly monotone mapping. Let $T : C \to C$ be a κ-strict pseudocontraction mapping with $\mathfrak{F} = F(T)\cap EP(F,A)\cap EP(G,B) \neq \emptyset$. Let $\{x_n\}$ be a sequence generated by $x_1 \in C = C_1$ and*

$$\begin{gathered}
F(u_n,u) + (Ax_n, u-u_n) + \frac{1}{r_n}\langle u-u_n, u_n-x_n\rangle \geq 0, \quad \forall u \in C,\\
G(v_n,v) + (Bx_n, v-v_n) + \frac{1}{s_n}\langle v-v_n, v_n-x_n\rangle \geq 0, \quad \forall v \in C,\\
z_n = \delta_n u_n + (1-\delta_n)v_n,\\
y_n = \alpha_n z_n + (1-\alpha_n)Tz_n,\\
C_{n+1} = \left\{z \in C_n : \left\|y_n - z\right\| \leq \|x_n - z\|\right\},\\
x_{n+1} = P_{C_{n+1}}x_1, \quad \forall n \geq 1,
\end{gathered} \tag{3.1}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is sequence in $[0,1]$, $r_n \in [a,b] \subset (0,2\alpha)$, and $s_n \subset [c,d] \subset (0,2\beta)$ satisfy the following conditions:

(i) $\lim_{n\to\infty}\delta_n = \delta \in (0,1)$,

(ii) $0 \le \kappa \le \alpha_n < 1$, *for all* $n \ge 1$.

Then x_n *converges strongly to* $P_{\mathfrak{F}} x_1$.

Proof. First, we show that $(I - r_n A)$ is nonexpansive. Let $x, y \in C$. Since A is α-inverse strongly monotone mapping and $r_n < 2\alpha$, we have

$$\begin{aligned}
\left\|(I - r_n A)x - (I - r_n A)y\right\|^2 &= \left\|x - y - r_n(Ax - Ay)\right\|^2 \\
&= \left\|x - y\right\|^2 - 2r_n\langle x - y, Ax - Ay\rangle + r_n^2\left\|Ax - Ay\right\|^2 \\
&\le \left\|x - y\right\|^2 - 2\alpha r_n\left\|Ax - Ay\right\|^2 + r_n^2\left\|Ax - Ay\right\|^2 \\
&= \left\|x - y\right\|^2 + r_n(r_n - 2\alpha)\left\|Ax - Ay\right\|^2 \\
&\le \left\|x - y\right\|^2.
\end{aligned} \tag{3.2}$$

Thus $(I - r_n A)$ is nonexpansive, so are $I - s_n B$, $T_{r_n}(I - r_n A)$, and $T_{s_n}(I - s_n B)$. Since

$$F(u_n, u) + \langle Ax_n, u - u_n\rangle + \frac{1}{r_n}\langle u - u_n, u_n - x_n\rangle \ge 0, \quad \forall u \in C, \tag{3.3}$$

then we have

$$F(u_n, u) + \frac{1}{r_n}\langle u - u_n, u_n - (I - r_n A)x_n\rangle \ge 0. \tag{3.4}$$

By Lemma 2.5, we have $u_n = T_{r_n}(I - r_n A)x_n$. By the same argument as above, we conclude that $v_n = T_{s_n}(I - s_n B)x_n$.

Let $z \in \mathfrak{F}$. Then $F(z, y) + \langle y - z, Az\rangle \ge 0$ and $G(z, y) + \langle y - z, Bz\rangle \ge 0$. Hence

$$\begin{aligned}
F(z, y) + \frac{1}{r_n}\langle y - z, z - z + r_n Az\rangle \ge 0, \\
G(z, y) + \frac{1}{s_n}\langle y - z, z - z + s_n Bz\rangle \ge 0.
\end{aligned} \tag{3.5}$$

Again by Lemma 2.5, we have $z = T_{r_n}(z - r_n Az) = T_{s_n}(z - s_n Bz)$. By nonexpansiveness of $T_{r_n}(I - r_n A)$ and $T_{s_n}(I - s_n B)$, we have

$$\begin{aligned}
\|u_n - z\| &= \|T_{r_n}(I - r_n A)x_n - T_{r_n}(I - r_n A)z\| \\
&\le \|x_n - z\|, \\
\|v_n - z\| &= \|T_{s_n}(I - s_n A)x_n - T_{s_n}(I - s_n A)z\| \\
&\le \|x_n - z\|.
\end{aligned} \tag{3.6}$$

By (3.6), we have

$$\|z_n - z\| \leq \|x_n - z\|. \tag{3.7}$$

Next, we show that C_n is closed and convex for every $n \in \mathbb{N}$. It is obvious that C_n is closed. In fact, we know that, for $z \in C_n$,

$$\|y_n - z\| \leq \|x_n - z\| \quad \text{is equivalent to } \|y_n - x_n\|^2 + 2\langle y_n - x_n, x_n - z\rangle \leq 0. \tag{3.8}$$

So, we have that for all $z_1, z_2 \in C_n$ and $t \in (0,1)$, it follows that

$$\begin{aligned} &\|y_n - x_n\|^2 + 2\langle y_n - x_n, x_n - (tz_1 + (1-t)z_2)\rangle \\ &\quad = t\left(2\langle y_n - x_n, x_n - z_1\rangle + \|y_n - x_n\|^2\right) \\ &\qquad + (1-t)\left(2\langle y_n - x_n, x_n - z_2\rangle + \|y_n - x_n\|^2\right) \\ &\quad \leq 0. \end{aligned} \tag{3.9}$$

Then, we have that C_n is convex. By Lemmas 2.5 and 2.2, we conclude that $\mathfrak{F}$ is closed and convex. This implies that $P_{\mathfrak{F}}$ is well defined. Next, we show that $\mathfrak{F} \subset C_n$ for every $n \in \mathbb{N}$.

Taking $p \in \mathfrak{F}$, we have

$$\begin{aligned} \|y_n - p\|^2 &= \|\alpha_n(z_n - p) + (1-\alpha_n)(Tz_n - p)\|^2 \\ &= \alpha_n\|z_n - p\|^2 + (1-\alpha_n)\|Tz_n - p\|^2 - \alpha_n(1-\alpha_n)\|z_n - Tz_n\|^2 \\ &\leq \alpha_n\|z_n - p\|^2 + (1-\alpha_n)\left(\|z_n - p\|^2 + \kappa\|(I-T)z_n - (I-T)p\|^2\right) \\ &\quad - \alpha_n(1-\alpha_n)\|z_n - Tz_n\|^2 \\ &= \alpha_n\|z_n - p\|^2 + (1-\alpha_n)\|z_n - p\|^2 + \kappa(1-\alpha_n)\|z_n - Tz_n\|^2 \\ &\quad - \alpha_n(1-\alpha_n)\|z_n - Tz_n\|^2 \\ &= \|z_n - p\|^2 + (\kappa - \alpha_n)(1-\alpha_n)\|z_n - Tz_n\|^2 \\ &\leq \|z_n - p\|^2 \\ &\leq \|x_n - p\|^2. \end{aligned} \tag{3.10}$$

It follows that $p \in C_n$. Then, we have $\mathfrak{F} \subset C_n$, for all $n \in \mathbb{N}$. Since $x_n = P_{C_n}x_1$, for every $w \in C_n$, we have

$$\|x_n - x_1\| \leq \|w - x_1\|, \quad \forall n \in \mathbb{N}. \tag{3.11}$$

In particular, we have

$$\|x_n - x_1\| \le \|P_{\mathfrak{F}} x_1 - x_1\|. \tag{3.12}$$

By (3.11), we have that $\{x_n\}$ is bounded, so are $\{u_n\}, \{v_n\}, \{z_n\}, \{y_n\}$. Since $x_{n+1} = P_{C_{n+1}} x_1 \in C_{n+1} \subset C_n$ and $x_n = P_{C_n} x_1$, we have

$$\begin{aligned} 0 &\le \langle x_1 - x_n, x_n - x_{n+1} \rangle \\ &= \langle x_1 - x_n, x_n - x_1 + x_1 - x_{n+1} \rangle \\ &\le -\|x_n - x_1\|^2 + \|x_n - x_1\| \|x_1 - x_{n+1}\|. \end{aligned} \tag{3.13}$$

It is implied that

$$\|x_n - x_1\| \le \|x_{n+1} - x_1\|. \tag{3.14}$$

Hence, we have that $\lim_{n\to\infty} \|x_n - x_1\|$ exists. Since

$$\begin{aligned} \|x_n - x_{n+1}\|^2 &= \|x_n - x_1 + x_1 - x_{n+1}\|^2 \\ &= \|x_n - x_1\|^2 + 2\langle x_n - x_1, x_1 - x_{n+1} \rangle + \|x_1 - x_{n+1}\|^2 \\ &= \|x_n - x_1\|^2 + 2\langle x_n - x_1, x_1 - x_n + x_n - x_{n+1} \rangle + \|x_1 - x_{n+1}\|^2 \\ &= \|x_n - x_1\|^2 - 2\|x_n - x_1\|^2 + 2\langle x_n - x_1, x_n - x_{n+1} \rangle + \|x_1 - x_{n+1}\|^2 \\ &\le \|x_1 - x_{n+1}\|^2 - \|x_n - x_1\|^2, \end{aligned} \tag{3.15}$$

it is implied that

$$\lim_{n\to\infty} \|x_n - x_{n+1}\| = 0. \tag{3.16}$$

Since $x_{n+1} = P_{C_{n+1}} x_1 \in C_{n+1}$, we have

$$\|y_n - x_{n+1}\| \le \|x_n - x_{n+1}\|, \tag{3.17}$$

And by (3.16), we have

$$\lim_{n\to\infty} \|y_n - x_{n+1}\| = 0. \tag{3.18}$$

Since

$$\|y_n - x_n\| \le \|y_n - x_{n+1}\| + \|x_{n+1} - x_n\|, \tag{3.19}$$

by (3.16) and (3.18), we have

$$\lim_{n\to\infty}\|y_n - x_n\| = 0. \tag{3.20}$$

Next, we show that

$$\lim_{n\to\infty}\|u_n - x_n\| = 0, \quad \lim_{n\to\infty}\|v_n - x_n\| = 0. \tag{3.21}$$

Let $p \in \mathfrak{F}$, by (3.10) and (3.7), we have

$$\begin{aligned}
\|y_n - p\|^2 &= \|\alpha_n(z_n - p) + (1-\alpha_n)(Tz_n - p)\|^2 \\
&= \alpha_n\|z_n - p\|^2 + (1-\alpha_n)\|Tz_n - p\|^2 - \alpha_n(1-\alpha_n)\|z_n - Tz_n\|^2 \\
&\le \alpha_n\|z_n - p\|^2 + (1-\alpha_n)\left(\|z_n - p\|^2 + \kappa\|(I-T)z_n - (I-T)p\|^2\right) \\
&\quad - \alpha_n(1-\alpha_n)\|z_n - Tz_n\|^2 \\
&= \alpha_n\|z_n - p\|^2 + (1-\alpha_n)\|z_n - p\|^2 + \kappa(1-\alpha_n)\|z_n - Tz_n\|^2 \\
&\quad - \alpha_n(1-\alpha_n)\|z_n - Tz_n\|^2 \\
&= \alpha_n\|z_n - p\|^2 + (1-\alpha_n)\|z_n - p\|^2 + (\kappa-\alpha_n)(1-\alpha_n)\|z_n - Tz_n\|^2 \\
&\le \alpha_n\|x_n - p\|^2 + (1-\alpha_n)\ \|z_n - p\|^2 \\
&\le \alpha_n\|x_n - p\|^2 + (1-\alpha_n)\left(\delta_n\|u_n - p\|^2 + (1-\delta_n)\|v_n - p\|^2\right).
\end{aligned} \tag{3.22}$$

Since $u_n = T_{r_n}(I - r_nA)x_n$, $p = T_{r_n}(I - r_nA)p$, we have

$$\begin{aligned}
\|u_n - p\|^2 &= \|T_{r_n}(I - r_nA)x_n - T_{r_n}(I - r_nA)p\|^2 \\
&\le \|(I - r_nA)x_n - (I - r_nA)p\|^2 \\
&= \|x_n - r_nAx_n - p + r_nAp\|^2 \\
&= \|x_n - p - r_n(Ax_n - Ap)\|^2 \\
&= \|x_n - p\|^2 + r_n^2\|Ax_n - Ap\|^2 - 2r_n\langle x_n - p, Ax_n - Ap\rangle \\
&\le \|x_n - p\|^2 + r_n^2\|Ax_n - Ap\|^2 - 2r_n\alpha\|Ax_n - Ap\|^2 \\
&= \|x_n - p\|^2 + r_n(r_n - 2\alpha)\|Ax_n - Ap\|^2.
\end{aligned} \tag{3.23}$$

Since $v_n = T_{s_n}(I - s_nB)x_n$, $p = T_{s_n}(I - s_nB)p$, we have

$$\begin{aligned}
\|v_n - p\|^2 &= \|T_{s_n}(I - s_nB)x_n - T_{s_n}(I - s_nB)p\|^2 \\
&\leq \|(I - s_nB)x_n - (I - s_nB)p\|^2 \\
&= \|x_n - s_nBx_n - p + s_nBp\|^2 \\
&= \|x_n - p - s_n(Bx_n - Bp)\|^2 \\
&= \|x_n - p\|^2 + s_n^2\|Bx_n - Bp\|^2 - 2s_n\langle x_n - p, Bx_n - Bp\rangle \\
&\leq \|x_n - p\|^2 + s_n^2\|Bx_n - Bp\|^2 - 2s_n\beta\|Bx_n - Bp\|^2 \\
&= \|x_n - p\|^2 + s_n(s_n - 2\beta)\|Bx_n - Bp\|^2.
\end{aligned} \tag{3.24}$$

Substituting (3.23) and (3.24) into (3.22),

$$\begin{aligned}
\|y_n - p\|^2 &\leq \alpha_n\|x_n - p\|^2 + (1 - \alpha_n)\left(\delta_n\|u_n - p\|^2 + (1 - \delta_n)\|v_n - p\|^2\right) \\
&\leq \alpha_n\|x_n - p\|^2 \\
&\quad + (1 - \alpha_n)\Big(\delta_n\left(\|x_n - p\|^2 + r_n(r_n - 2\alpha)\|Ax_n - Ap\|^2\right) \\
&\qquad\qquad + (1 - \delta_n)\left(\|x_n - p\|^2 + s_n(s_n - 2\beta)\|Bx_n - Bp\|^2\right)\Big) \\
&= \alpha_n\|x_n - p\|^2 \\
&\quad + (1 - \alpha_n)\Big(\delta_n\|x_n - p\|^2 + \delta_nr_n(r_n - 2\alpha)\|Ax_n - Ap\|^2 \\
&\qquad\qquad + (1 - \delta_n)\|x_n - p\|^2 + s_n(1 - \delta_n)(s_n - 2\beta)\|Bx_n - Bp\|^2\Big) \\
&= \alpha_n\|x_n - p\|^2 + (1 - \alpha_n) \\
&\quad \times \left(\|x_n - p\|^2 + \delta_nr_n(r_n - 2\alpha)\|Ax_n - Ap\|^2 + s_n(1 - \delta_n)(s_n - 2\beta)\|Bx_n - Bp\|^2\right) \\
&= \alpha_n\|x_n - p\|^2 + (1 - \alpha_n)\|x_n - p\|^2 + (1 - \alpha_n)\delta_nr_n(r_n - 2\alpha)\|Ax_n - Ap\|^2 \\
&\quad + s_n(1 - \alpha_n)(1 - \delta_n)(s_n - 2\beta)\|Bx_n - Bp\|^2 \\
&= \|x_n - p\|^2 + (1 - \alpha_n)\delta_nr_n(r_n - 2\alpha)\|Ax_n - Ap\|^2 \\
&\quad + s_n(1 - \alpha_n)(1 - \delta_n)(s_n - 2\beta)\|Bx_n - Bp\|^2.
\end{aligned} \tag{3.25}$$

It is implied that

$$
\begin{aligned}
(1-\alpha_n)\delta_n r_n(2\alpha - r_n)\|Ax_n - Ap\|^2 &\le \|x_n - p\|^2 - \|y_n - p\|^2 \\
&\quad + s_n(1-\alpha_n)(1-\delta_n)(s_n - 2\beta)\|Bx_n - Bp\|^2 \\
&\le (\|x_n - p\| + \|y_n - p\|)\|x_n - y_n\|.
\end{aligned}
\tag{3.26}
$$

By (3.20) and condition (i), we have

$$
\lim_{n\to\infty}\|Ax_n - Ap\| = 0. \tag{3.27}
$$

By using the same method as (3.27), we have

$$
\lim_{n\to\infty}\|Bx_n - Bp\| = 0. \tag{3.28}
$$

By Lemma 2.5 and firm nonexpansiveness of T_{r_n}, we have

$$
\begin{aligned}
\|u_n - p\|^2 &= \|T_{r_n}(I - r_nA)x_n - T_{r_n}(I - r_nA)p\|^2 \\
&\le \langle (I - r_nA)x_n - (I - r_nA)p, u_n - p\rangle \\
&= \frac{1}{2}\Big(\|(I - r_nA)x_n - (I - r_nA)p\|^2 + \|u_n - p\|^2 \\
&\qquad -\|(I - r_nA)x_n - (I - r_nA)p - (u_n - p)\|^2\Big) \\
&= \frac{1}{2}\Big(\|(I - r_nA)x_n - (I - r_nA)p\|^2 + \|u_n - p\|^2 \\
&\qquad -\|x_n - u_n - r_n(Ax_n - Ap)\|^2\Big) \\
&= \frac{1}{2}\Big(\|(I - r_nA)x_n - (I - r_nA)p\|^2 + \|u_n - p\|^2 \\
&\qquad -\Big(\|x_n - u_n\|^2 + r_n^2\|Ax_n - Ap\|^2 - 2r_n\langle x_n - u_n, Ax_n - Ap\rangle\Big)\Big) \\
&\le \frac{1}{2}\Big(\|x_n - p\|^2 + \|u_n - p\|^2 - \|x_n - u_n\|^2 - r_n^2\|Ax_n - Ap\|^2 \\
&\qquad +2r_n\langle x_n - u_n, Ax_n - Ap\rangle\Big).
\end{aligned}
\tag{3.29}
$$

By (3.29), it is implied that

$$\begin{aligned}\|u_n - p\|^2 &\le \|x_n - p\|^2 - \|x_n - u_n\|^2 - r_n^2\|Ax_n - Ap\|^2 \\ &\quad + 2r_n\langle x_n - u_n, Ax_n - Ap\rangle \\ &\le \|x_n - p\|^2 - \|x_n - u_n\|^2 - r_n^2\|Ax_n - Ap\|^2 \\ &\quad + 2r_n\|x_n - u_n\|\|Ax_n - Ap\|.\end{aligned} \tag{3.30}$$

Again, by Lemma 2.5 and firm nonexpansiveness of T_{s_n}, we have

$$\begin{aligned}\|v_n - p\|^2 &= \|T_{s_n}(I - s_nB)x_n - T_{s_n}(I - s_nA)p\|^2 \\ &\le \langle (I - s_nB)x_n - (I - s_nB)p, v_n - p\rangle \\ &= \frac{1}{2}\Big(\|(I - s_nB)x_n - (I - s_nB)p\|^2 + \|v_n - p\|^2 \\ &\qquad -\|(I - s_nB)x_n - (I - s_nB)p - (v_n - p)\|^2\Big) \\ &= \frac{1}{2}\Big(\|(I - s_nB)x_n - (I - s_nB)p\|^2 + \|v_n - p\|^2 - \|x_n - v_n - s_n(Bx_n - Bp)\|^2\Big) \\ &= \frac{1}{2}\Big(\|(I - s_nB)x_n - (I - s_nB)p\|^2 + \|v_n - p\|^2 \\ &\qquad -\Big(\|x_n - v_n\|^2 + s_n^2\|Bx_n - Bp\|^2 - 2s_n\langle x_n - v_n, Bx_n - Bp\rangle\Big)\Big) \\ &\le \frac{1}{2}\Big(\|x_n - p\|^2 + \|v_n - p\|^2 - \|x_n - v_n\|^2 - s_n^2\|Bx_n - Bp\|^2 \\ &\quad +2s_n\langle x_n - v_n, Bx_n - Bp\rangle\Big).\end{aligned} \tag{3.31}$$

By (3.31), it is implied that

$$\begin{aligned}\|v_n - p\|^2 &\le \|x_n - p\|^2 - \|x_n - v_n\|^2 - s_n^2\|Bx_n - Bp\|^2 + 2s_n\langle x_n - v_n, Bx_n - Bp\rangle \\ &\le \|x_n - p\|^2 - \|x_n - v_n\|^2 - s_n^2\|Bx_n - Bp\|^2 + 2s_n\|x_n - v_n\|\ \|Bx_n - Bp\|.\end{aligned} \tag{3.32}$$

Substituting (3.30) and (3.32) into (3.22), we have

$$
\begin{aligned}
\|y_n - p\|^2 &\le \alpha_n \|x_n - p\|^2 + (1-\alpha_n)\Big(\delta_n \|u_n - p\|^2 + (1-\delta_n)\|v_n - p\|^2\Big) \\
&\le \alpha_n \|x_n - p\|^2 + (1-\alpha_n) \\
&\quad \times \Big(\delta_n\Big(\|x_n - p\|^2 - \|x_n - u_n\|^2 - r_n^2\|Ax_n - Ap\|^2 + 2r_n\|x_n - u_n\|\|Ax_n - Ap\|\Big) \\
&\qquad + (1-\delta_n)\Big(\|x_n - p\|^2 - \|x_n - v_n\|^2 - s_n^2\|Bx_n - Bp\|^2 \\
&\qquad\qquad + 2s_n\|x_n - v_n\|\|Bx_n - Bp\|\Big)\Big) \\
&\le \alpha_n \|x_n - p\|^2 \\
&\quad + (1-\alpha_n)\Big(\delta_n\Big(\|x_n - p\|^2 - \|x_n - u_n\|^2 + 2r_n\|x_n - u_n\|\|Ax_n - Ap\|\Big) \\
&\qquad + (1-\delta_n)\Big(\|x_n - p\|^2 - \|x_n - v_n\|^2 + 2s_n\|x_n - v_n\|\|Bx_n - Bp\|\Big)\Big) \\
&= \alpha_n \|x_n - p\|^2 \\
&\quad + (1-\alpha_n)\Big(\delta_n\|x_n - p\|^2 - \delta_n\|x_n - u_n\|^2 + 2\delta_n r_n\|x_n - u_n\|\|Ax_n - Ap\| + (1-\delta_n) \\
&\qquad \times \|x_n - p\|^2 - (1-\delta_n)\|x_n - v_n\|^2 + 2(1-\delta_n)s_n\|x_n - v_n\|\|Bx_n - Bp\|\Big) \\
&= \alpha_n \|x_n - p\|^2 + (1-\alpha_n) \\
&\quad \times \Big(\|x_n - p\|^2 - \delta_n\|x_n - u_n\|^2 + 2\delta_n r_n\|x_n - u_n\|\|Ax_n - Ap\| - (1-\delta_n)\|x_n - v_n\|^2 \\
&\qquad + 2(1-\delta_n)s_n\|x_n - v_n\|\|Bx_n - Bp\|\Big) \\
&\le \|x_n - p\|^2 - (1-\alpha_n)\delta_n\|x_n - u_n\|^2 + 2\delta_n r_n\|x_n - u_n\|\|Ax_n - Ap\| \\
&\quad - (1-\alpha_n)(1-\delta_n)\|x_n - v_n\|^2 + 2(1-\delta_n)s_n\|x_n - v_n\|\|Bx_n - Bp\|,
\end{aligned}
\tag{3.33}
$$

which implies that

$$
\begin{aligned}
(1-\alpha_n)\delta_n\|x_n - u_n\|^2 &\le \|x_n - p\|^2 - \|y_n - p\|^2 + 2\delta_n r_n\|x_n - u_n\|\|Ax_n - Ap\| \\
&\quad - (1-\alpha_n)(1-\delta_n)\|x_n - v_n\|^2 + 2(1-\delta_n)s_n\|x_n - v_n\|\|Bx_n - Bp\| \\
&\le \|x_n - y_n\|(\|x_n - p\| + \|y_n - p\|) + 2\delta_n r_n\|x_n - u_n\|\|Ax_n - Ap\| \\
&\quad - (1-\alpha_n)(1-\delta_n)\|x_n - v_n\|^2 + 2(1-\delta_n)s_n\|x_n - v_n\|\|Bx_n - Bp\| \\
&\le \|x_n - y_n\|(\|x_n - p\| + \|y_n - p\|) + 2\delta_n r_n\|x_n - u_n\|\|Ax_n - Ap\| \\
&\quad + 2(1-\delta_n)s_n\|x_n - v_n\|\|Bx_n - Bp\|,
\end{aligned}
\tag{3.34}
$$

and by (3.27), (3.28), (3.20), and conditions (i), (ii), we have

$$\lim_{n\to\infty} \|x_n - u_n\| = 0. \tag{3.35}$$

By using the same method as (3.35), we have

$$\lim_{n\to\infty} \|x_n - v_n\| = 0. \tag{3.36}$$

Since

$$\|z_n - x_n\| \le \delta_n \|u_n - x_n\| + (1-\delta_n)\|v_n - x_n\|, \tag{3.37}$$

from (3.35), (3.36), and condition (i), we have

$$\lim_{n\to\infty} \|z_n - x_n\| = 0. \tag{3.38}$$

By (3.20) and (3.38), we have

$$\lim_{n\to\infty} \|y_n - z_n\| = 0. \tag{3.39}$$

Since

$$y_n - z_n = (1-\alpha_n)(Tz_n - z_n), \tag{3.40}$$

from (3.39) and condition (ii), we have

$$\lim_{n\to\infty} \|Tz_n - z_n\| = 0. \tag{3.41}$$

Let $\omega(x_n)$ be the set of all weaks ω-limit of $\{x_n\}$. We will show that $\omega(x_n) \subset \mathfrak{F}$. Since $\{x_n\}$ is bounded, then $\omega(x_n) \neq \emptyset$. Letting $q \in \omega(x_n)$, there exists a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ converging to q. By (3.35), we have $u_{n_i} \rightharpoonup q$ as $i \to \infty$. Since $u_n = T_{r_n}(I - r_n A)x_n$, for any $y \in C$, we have

$$F(u_n, y) + \langle Ax_n, y - u_n\rangle + \frac{1}{r_n}\langle y - u_n, u_n - x_n\rangle \ge 0. \tag{3.42}$$

From $(A2)$, we have

$$\langle Ax_n, y - u_n\rangle + \frac{1}{r_n}\langle y - u_n, u_n - x_n\rangle \ge F(y, u_n). \tag{3.43}$$

This implies that

$$\langle Ax_{n_i}, y - u_{n_i} \rangle + \frac{1}{r_{n_i}} \langle y - u_{n_i}, u_{n_i} - x_{n_i} \rangle \geq F(y, u_{n_i}). \tag{3.44}$$

Put $z_t = ty + (1-t)q$ for all $t \in (0,1]$ and $y \in C$. Then, we have $z_t \in C$. So, from (3.44), we have

$$\begin{aligned} \langle z_t - u_{n_i}, Az_t \rangle \geq{}& \langle z_t - u_{n_i}, Az_t \rangle - \langle z_t - u_{n_i}, Ax_{n_i} \rangle - \left\langle z_t - u_{n_i}, \frac{u_{n_i} - x_{n_i}}{r_{n_i}} \right\rangle + F(z_t, u_{n_i}) \\ ={}& \langle z_t - u_{n_i}, Az_t - Au_{n_i} \rangle + \langle z_t - u_{n_i}, Au_{n_i} - Ax_{n_i} \rangle - \left\langle z_t - u_{n_i}, \frac{u_{n_i} - x_{n_i}}{r_{n_i}} \right\rangle \\ &+ F(z_t, u_{n_i}). \end{aligned} \tag{3.45}$$

Since $\|u_{n_i} - x_{n_i}\| \to 0$, we have $\|Au_{n_i} - Ax_{n_i}\| \to 0$. Further, from monotonicity of A, we have $\langle z_t - u_{n_i}, Az_t - Au_{n_i} \rangle \geq 0$. So, we have

$$\langle z_t - q, Az_t \rangle \geq F(z_t, q) \quad \text{as } i \longrightarrow \infty. \tag{3.46}$$

From $(A1)$, $(A4)$, and (3.46), we also have

$$\begin{aligned} 0 = F(z_t, z_t) &\leq tF(z_t, y) + (1-t)F(z_t, q) \\ &\leq tF(z_t, y) + (1-t)\langle z_t - q, Az_t \rangle \\ &= tF(z_t, y) + (1-t)t\langle y - q, Az_t \rangle. \end{aligned} \tag{3.47}$$

Thus

$$0 \leq F(z_t, y) + (1-t)\langle y - q, Az_t \rangle. \tag{3.48}$$

Letting $t \to 0$, we have, for each $y \in C$,

$$0 \leq F(q, y) + \langle y - q, Aq \rangle. \tag{3.49}$$

This implies that

$$q \in \mathrm{EP}(F, A). \tag{3.50}$$

From (3.36), we have $v_{ni} \rightharpoonup q$. Since $v_n = T_{s_n}(I - s_n B)x_n$, for any $y \in C$, we have

$$G(v_n, y) + \langle Bx_n, y - v_n \rangle + \frac{1}{s_n} \langle y - v_n, v_n - x_n \rangle \geq 0. \tag{3.51}$$

By using the same method as (3.50), we have

$$q \in \mathrm{EP}(G, B). \tag{3.52}$$

Since $x_{n_i} \rightharpoonup q$ as $i \to \infty$ and (3.38), we have $z_{n_i} \rightharpoonup q$ as $i \to \infty$. By Lemma 2.3, $I - T$ is demiclosed at zero, and by (3.41), we have

$$q \in F(T). \tag{3.53}$$

From (3.50), (3.52), and (3.53), we have $q \in \mathfrak{F}$. Hence $\omega(x_n) \subset \mathfrak{F}$. Therefore, by (3.12) and Lemma 2.6, we have that $\{x_n\}$ converges strongly to $P_{\mathfrak{F}} x_1$. The proof is completed. □

4. Applications

By using our main result, we have the following results in Hilbert spaces.

Theorem 4.1. *Let C be a nonempty closed convex subset of a Hilbert space H. Let F and G be bifunctions from $C \times C$ into $\mathbb{R}$ satisfying (A_1)–(A_4), respectively. Let $T : C \to C$ be a κ-strict pseudocontraction mapping with $\mathfrak{F} = F(T) \cap EP(F) \cap EP(G) \neq \emptyset$. Let $\{x_n\}$ be a sequence generated by $x_1 \in C = C_1$ and*

$$\begin{aligned}
F(u_n, u) + \frac{1}{r_n}\langle u - u_n, u_n - x_n\rangle &\geq 0, \quad \forall u \in C,\\
G(v_n, v) + \frac{1}{s_n}\langle v - v_n, v_n - x_n\rangle &\geq 0, \quad \forall v \in C,\\
z_n &= \delta_n u_n + (1 - \delta_n) v_n,\\
y_n &= \alpha_n z_n + (1 - \alpha_n) T z_n,\\
C_{n+1} &= \{z \in C_n : \|y_n - z\| \leq \|x_n - z\|\},\\
x_{n+1} &= P_{C_{n+1}} x_1, \quad \forall n \geq 1,
\end{aligned} \tag{4.1}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is sequence in $[0,1]$, $r_n \in [a,b]$, and $s_n \subset [c,d]$ satisfy the following conditions:

(i) $\lim_{n\to\infty} \delta_n = \delta \in (0,1)$,

(ii) $0 \leq \kappa \leq \alpha_n < 1$, *for all* $n \geq 1$.

Then x_n converges strongly to $P_{\mathfrak{F}} x_1$.

Proof. Putting $A \equiv B \equiv 0$ in Theorem 3.1, we have the desired conclusions. □

Theorem 4.2. *Let C be a nonempty closed convex subset of a Hilbert space H. Let F be bifunctions from $C \times C$ into $\mathbb{R}$ satisfying (A_1)–(A_4), respectively. Let $A : C \to H$ be an α-inverse strongly monotone mapping, and let $\{x_n\}$ be a sequence generated by $x_1 \in C = C_1$ and*

$$\begin{gathered}
F(u_n, u) + (Ax_n, u - u_n) + \frac{1}{r_n}\langle u - u_n, u_n - x_n\rangle \geq 0, \quad \forall u \in C, \\
y_n = \alpha_n u_n + (1 - \alpha_n) T u_n, \\
C_{n+1} = \{z \in C_n : \|y_n - z\| \leq \|x_n - z\|\}, \\
x_{n+1} = P_{C_{n+1}} x_1, \quad \forall n \geq 1,
\end{gathered} \tag{4.2}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is sequence in $[0,1]$, $r_n \in [a,b] \subset (0, 2\alpha)$, and $0 \leq \kappa \leq \alpha_n < 1$, for all $n \geq 1$. Then x_n converges strongly to $P_{\mathfrak{F}} x_1$.

Proof. Putting $G \equiv F$, $A \equiv B$, and $u_n = v_n$, for all $n \geq 1$, in Theorem 3.1, we have the desired conclusions. □

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