Undergraduate Journal of Mathematical Modeling: One + Two

Volume 12 | 2022 Spring 2022 Article 1

2022

The Effects of Effluent Discharge into the Santa Fé River

João Vito Bezerra Dragone
University of South Florida

Advisors:
Arcadii Grinshpan, Mathematics and Statistics
José Carlos Mierzwa, Poli-USP, Brazil

Problem Suggested By: Mateus Patriani Cardosom

Recommended Citation
Dragone, João Vito Bezerra (2022) "The Effects of Effluent Discharge into the Santa Fé River," *Undergraduate Journal of Mathematical Modeling: One + Two*: Vol. 12: Iss. 2, Article 1.
DOI: https://doi.org/10.5038/2326-3652.12.2.4943
Available at: https://digitalcommons.usf.edu/ujmm/vol12/iss2/1

The Effects of Effluent Discharge into the Santa Fé River

Abstract

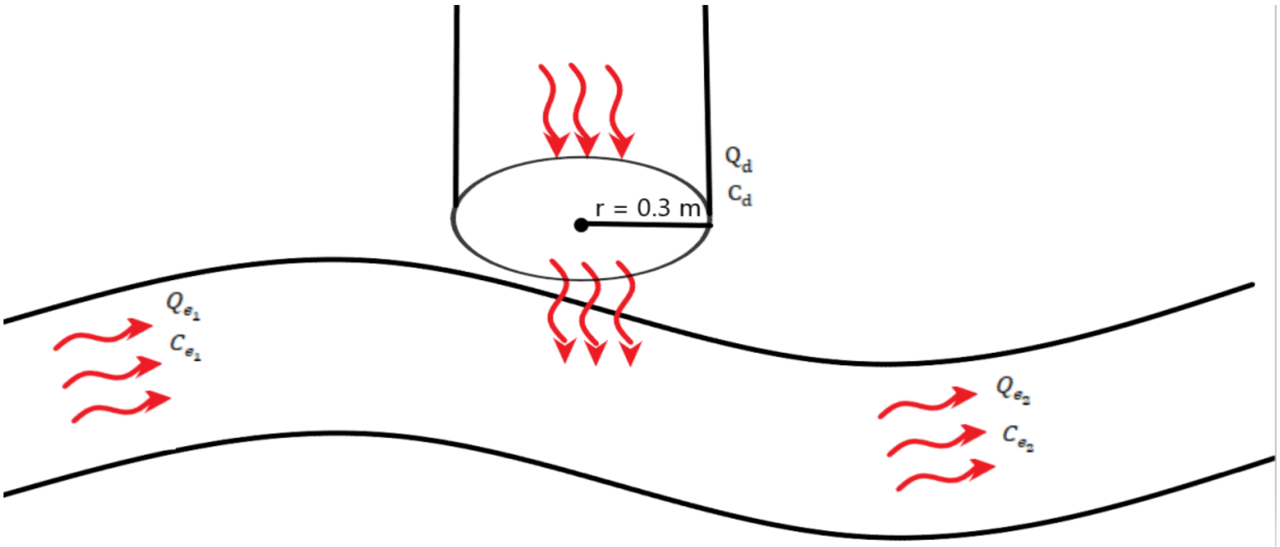
The discharge of effluents into rivers is something that has occurred a lot throughout our history, rivers like the Mississippi River and the Tiete River (São Paulo, Brazil) have been targets for many years. Therefore, in this paper I choose to show, through hard data and some assumptions, the impact of sewage dumping in water bodies, in this case the Santa Fé River, Florida and the reason why we should avoid dumping these effluents into the rivers.

Keywords

effluent, sewage dumping, flow rate

PROBLEM STATEMENT

An effluent is discharged via a cylindrical pipe into the Santa Fe River. According to the USGS (The U.S. Geological Survey) and the Florida Spring Institute data, the average flow rate of the Santa Fe River is known to be 39643 L/s, average temperature of 25 degrees Celsius and DO (dissolved oxygen) concentration of 4.8 mg/L.



a) Given $r = 0.3\ m$ and the velocity profile $U(r) = 1.1(1 - 11r^2)\ \frac{m}{s}$ we calculate the volumetric flow rate of this effluent.

b) Based on the data given and obtained from part (a), we know that the effluent DO concentration is 0 mg/L, Determine the DO concentration of the river immediately after mixing and its oxygen deficit.

MOTIVATION

The discharge of untreated liquid effluents from industry and sewage into rivers, lakes, and streams causes a serious imbalance in the aquatic ecosystem and health problems for humans. Domestic sewage, for example, consumes oxygen in its decomposition process, causing the death of fish and other living beings in the environment. Furthermore, microorganisms from animal and human fecal matter can cause disease if there is no treatment at the disposal sites. Hence, when we perform the mass balance, we obtain the data how these emissions pass or fail to pass the quality standard determined, then we regulate and remove the damage to the environment. The mass balance process is originated according to Antoine Lavoisier's Law of Mass Conservation, which states that mass can neither be created nor destroyed. Thus, the accumulated mass must be equal to the incoming mass minus the outgoing mass, however, to perform this operation it is necessary to have a control volume, because in several cases there are conversion processes of matter through chemical reactions, which transform the equation into:

$$M_T = M_I - M_0 \pm M_c \ (1.1)$$

In other words, we can say that the mass balance consists of the variation of the amount of matter in the system with time, which is:

$$\frac{dM}{dt} = M_I - M_0 \pm M_c \ (1.2)$$

When dealing with environmental processes, we use the flow rates and concentrations involved in the system. In our example, we define the mass balance of effluents discharged into bodies of water by:

$$Q_{e_1}.C_{e_1} + Q_d.C_d = Q_{e_2}.C_{e_2} \ (1.3)$$

For our problem, I opt for DO (dissolved oxygen) concentration, because knowing the oxygen concentration in an environment we can determine if there is life. Therefore, it is necessary to point out some concepts about this. When effluents are discharged into the river, there is an increase in nutrients and organic matter, which leads to a decrease in DO due to the increased oxygen consumption of the microorganisms present. For this reason, BOD (Biochemical oxygen demand) is used, which indicates the amount of DO required by aerobic biological organisms to break down this increased organic matter.

To be able to perform these calculations, it is necessary to use the Streeter-Phelps model, but since my goal is just to find the oxygen deficit after mixing, I will use it:

$$D_0 \ = \ C_s - \ C_{e_2} \ (1.4)$$

Where C_s is nothing more than the oxygen saturation concentration in the river, determined by factors such as temperature, salinity, and atmospheric pressure. We use the values from TABLE 1 as a parameter in the equations:

DOI: https://doi.org/10.5038/2326-3652.12.2.4943

TABLE 1

Temperature-Oxygen Solubility Relationship	
Temperature (ºC)	Oxygen Solubility (mg/L)
0	14.6
5	12.8
10	11.3
15	10.2
20	9.2
25	8.6
100	0

Oxygen solubility at different water temperatures.
From(https://serc.carleton.edu/microbelife/research_methods/environ_sampling/oxygen.html)

Mathematical Description and Solution Approach

We know that:

$$Q = v.A \quad (2.1)$$

$$A = \pi.r^2 \quad (2.2)$$

$$dA = 2\pi r.dr \quad (2.3)$$

But we only use equation 2.1 when we have constant velocity, which is not the case, so we adopt it:

$$Q_d = \int v\, dA$$

Putting the values into the equation and adjusting for the given problem we have that:

$$Q_d = \int_0^{0.3} 1.1(1 - 11r^2)\, dA = \int_0^{0.3} 1.1(1 - 11r^2).2\pi r\, dr \ =$$

$$\left(-\frac{\pi\,(11\,\mathrm{r}^2\ -\ 1)^2}{20}\right)|_{(\mathrm{r}=0.3)} = -\frac{\pi}{200000}$$

$$\left(-\frac{\pi(11r^2-1)^2}{20}\right)|_{(r=0)} = -\frac{\pi}{20}$$

$$\int_0^{0.3}\left(\pi r\left(\frac{11}{5}-\frac{121r^2}{5}\right)\right)dr = \left(-\frac{\pi(11r^2-1)^2}{20}\right)|_{(r=.3)} - \left(-\frac{\pi(11r^2-1)^2}{20}\right)|_{(r=0)} = \frac{9999\pi}{200000} \cong 157\ \frac{L}{s}$$

Now that part (a) has been completed, we can show all the values we have as:

$$Q_d\ =\ 157\ \frac{L}{s}$$

$$Q_{e_1}\ =\ 39643\ \frac{L}{s}$$

$$Q_{e_2}\ =\ ?$$

$$C_d\ =\ 0\frac{mg}{L}$$

$$C_{e_1}\ =\ 4.8\frac{mg}{L}$$

$$C_{e_2}\ = ?$$

$$C_s\ =\ 8.6\ \frac{mg}{L}$$

In order to obtain the value of the effluent concentration, it is necessary to obtain the value of the flow rate after the effluent has been discharged where by reformulating we get the equation 1.3, because we know that the river flow rate after the effluent discharge is given by the river flow rate at the point before the discharge plus the flow rate of the discharged effluent:

$$Q_{e_2}\ =\ Q_{e_1}+Q_d\ =\ 39643\ +\ 157\ =\ 39800\frac{L}{s}$$

Now that we have found the flow rate of the mixture, we can use equation 1.3 and substitute the values:

$$C_{e_2}\ = \frac{Q_{e_1}.\,C_{e_1}+Q_d.\,C_d}{Q_{e_2}}\ \cong\ 4.78\frac{mg}{L}$$

Thus, we have that the oxygen deficit after the effluent is discharged is:

https://digitalcommons.usf.edu/ujmm/vol12/iss2/1
DOI: https://doi.org/10.5038/2326-3652.12.2.4943

$$D_0 = C_s - C_{e_2} = 3.82\frac{mg}{L}$$

Discussion

With the present results, we can see that even a small discharge of sewage into a large river like the Santa Fe River can decrease the DO of the river. It is worth noting that we do not consider several crucial factors such as the BDO levels of the effluent and the river as well as the distance traveled and the altitude of the environment. It is therefore especially important to understand how damaging it is to marine life and even to our health to discharge effluent into rivers. As a comparison, the minimum DO required in an environment for life to exist is approximately 4 mg/L and analyzing the DO of the Santa Fe River we see how close it is to this limit. The calculations performed are only a small part of water and wastewater treatment. So, I hope that this project will help us, human beings, understand how important it is to treat sewage and stop polluting our environment.

Conclusion and Recommendations

Based on the data collected, we can see why it is so important to treat these effluents before we dump them into the river. According to data from the U.S. Environmental Protection Agency (EPA), the United States has set aside more than $50 billion for national improvements in water and wastewater treatment. It is of utmost importance that we understand that water and sewage treatment is a basic right that every human being should have. The intention of this project is to show the impact of just one effluent in the Santa Fe River, which causes a decrease, even if symbolic, of the DO of the river. To make this study more efficient, I recommend adopting all the data used to calculate the concentration of DO and conducting this research over a longer period.

NOMENCLATURES

Variable	**Meaning**
Q	Flow rate
v	Velocity
A	Area
M_T	Total mass
M_I	Incoming mass
M_0	Outgoing mass
M_c	Converted mass
Q_{e_1}	River Flow rate
C_{e_1}	River DO concentration
Q_d	Effluent Flow rate
C_d	Effluent DO concentration
Q_{e_2}	Mixed Flow rate
C_{e_2}	Mixed DO concentration
C_s	DO Saturation
D_0	oxygen deficit
$U(r)$	Velocity profile
r	radius

References

O. B, Akpor. *Wastewater Effluent Discharge: Effects and Treatment Processes - IPCBEE*. http://ipcbee.com/vol20/16-ICBEE2011E20001.pdf.

Mierzwa, Jose Carlos. *Leis Da Conservação Da Massa e Da Energia*.

Engineers Edge, LLC. "Fluid Volumetric Flow Rate Equation." *Engineers Edge - Engineering, Design and Manufacturing Solutions*, https://www.engineersedge.com/fluid_flow/volumeetric_flow_rate.htm.

USGS Current Conditions for USGS 02322500 Santa Fe River near Fort White, Fla.., https://waterdata.usgs.gov/usa/nwis/uv?site_no=02322500.

Santa Fe River and Springs Environmental Analysis. https://floridaspringsinstitute.org/wp-content/uploads/2019/04/SFR_Phase2_Quarter2_Final.pdf.

Dissolved Oxygen (DO) - Gov. https://www.enr.gov.nt.ca/sites/enr/files/dissolved_oxygen.pdf.

Bezerra, Iury Steiner de Oliveira, et al. "Autodepuração De Cursos d´Água: Um Programa De Modelagem Streeter Phelps Com Calibração Automática E Correção De Anaerobiose." *Rem: Revista Escola De Minas*, Escola De Minas, 1 June 2008, https://www.scielo.br/j/rem/a/9pcpPHzzQddPhWQpMP7RqjP/?lang=pt.

"Dissolved Oxygen." *NIWA*, 6 Oct. 2020, https://niwa.co.nz/our-science/freshwater/tools/kaitiaki_tools/impacts/dissolved-oxygen.

EPA, Environmental Protection Agency, https://www.epa.gov/infrastructure/water-infrastructure-investments.