Published in final edited form as:
Res Econ Inequal. 2013 January 1; 21: 157–192.

Reference value sensitivity of measures of unfair health inequality

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Abstract

Most politicians and ethical observers are not interested in pure health inequalities, as they want to distinguish between different causes of health differences. Measures of "unfair" inequality - direct unfairness and the fairness gap, but also the popular standardized concentration index - therefore neutralize the effects of what are considered to be "legitimate" causes of inequality. This neutralization is performed by putting a subset of the explanatory variables at reference values, e.g. their means. We analyze how the inequality ranking of different policies depends on the specific choice of reference values. We show with mortality data from the Netherlands that the problem is empirically relevant and we suggest a statistical method for fixing the reference values.

Introduction

Most politicians and ethical observers are not interested in so-called pure health inequalities. They focus on some specific causes of health inequalities that are considered as especially troublesome from an ethical point of view. The most popular approach within health economics, which is based on the concentration index, is ultimately concerned about the correlation between socioeconomic status and health (see, e.g., O'Donnell et al., 2008; van Doorslaer and Van Ourti, 2011). The more recent approaches of the theory of fair allocation or of inequality of opportunity make an explicit distinction between two sets of variables: on the one hand the so-called "circumstance (or compensation) variables" for which individuals cannot be held responsible and that therefore lead to ethically illegitimate differences, and on the other hand variables for which individuals are held responsible and that therefore generate inequalities which are no cause for ethical concern (Fleurbaey and Schokkaert, 2009, 2012). Focusing on specific causes of inequalities implies, however, that one has to neutralize in one way or another the effects of the other variables underlying health differences. In the case of the concentration index it has become common practice to standardize the health data to control for the effects of demographic variables such as age and gender. In the theory of fair allocation the main purpose is to neutralize the effects of the responsibility variables, so that measured inequality only captures the effects of the circumstance variables. Conditional-egalitarian approaches and egalitarian-equivalent approaches (leading respectively to the notions of direct unfairness and of the fairness gap in Fleurbaey and Schokkaert, 2009) introduce reference values for the responsibility and the circumstance variables respectively. All this raises an obvious question: does the choice of reference values matter for the calculation of unfair inequality?

There is surprisingly little literature on this issue. In the case of the concentration index, it is common practice to fix the reference values in the standardization exercise at their sample means (Gravelle, 2003). This may seem a natural choice, but it remains one specific option for which there is in fact no strong theoretical support. In the same vein, the literature on fair allocation often remains silent about possible justifications for choosing specific reference values. For the fairness gap, Fleurbaey and Schokkaert (2012) suggest to pick as reference for the circumstance variables the values that lead to the best health outcomes, exploiting the intuitive idea that a fair situation corresponds to a state where everyone enjoys the best health prospect. For direct unfairness, no such proposal has been made. Luttens and Van de gaer (2007) have analyzed the issue in the context of optimal income taxation, but their results cannot be directly transferred to the health setting.

From this, one gets the impression that there is some general feeling in the literature that the choice of reference values is rather innocuous, in that it does not influence the ranking of the degree of inequality in different situations. In this paper we analyze whether this optimistic feeling is justified, and we show that it is not. Of course, it is trivially true that different reference values will lead to different numerical outcomes for measured inequality. More interesting is the comparison of two different health distributions within the same society, i.e. for two states of the world with the same underlying model of the relationship between health and other variables. Such a comparison is relevant, e.g., when one wants to evaluate the equity effects of a specific policy. We analyze under which conditions the change in inequity in moving from one to the other distribution has the same sign irrespective of the chosen reference values. These conditions are intuitively easy to understand in the case of direct unfairness (or direct standardization for the concentration index). They are more complicated in the case of the fairness gap (or indirect standardization). We illustrate our results with an empirical analysis of mortality in the Netherlands. We analyze inequity in mortality with the variance and the absolute Gini of direct unfairness and the fairness gap for various ethical positions. We also show the results for the concentration index, with education as the measure of socioeconomic status. It turns out that in some cases the ranking of policies does depend on the chosen reference values.

Since the choice of reference values does matter, it is obvious that we are in need of a good theory for choosing these values, both in the case of the standardized concentration index and *a fortiori* in applications of the theory of fair allocation. The formulation of such a theory is beyond the objectives of this paper, but we will introduce a statistical approach that provides a normative criterion for choosing reference values that is sufficiently simple for empirical work. The exploration of this research track is left for future work.

The structure of our paper is as follows. The next section introduces the different inequity measures and shows where the reference values enter in the calculations. We then derive the general conditions for inequity comparisons to be independent of the chosen reference values. We illustrate our theoretical findings with an empirical analysis of inequity in mortality in the Netherlands. Finally we briefly present a possible way to move forward.

Reference values in the theory of fair allocation and in the standardized concentration index

Let us start with some notation and definitions. We are interested in the distribution of health, denoted m_i.[1] Pure inequality in health as such is not a matter of ethical concern, however. The causes underlying these health differences do matter for the evaluation. In order to keep things simple, we assume that m_i is linearly linked to a set of $K+J$ characteristics of individual i, denoted by $x_i = x_{il}$, $l = 1, \ldots, K+J$:

$$m_i=\alpha+\sum_{l=1}^{K+J}\alpha_l x_{il}+\varepsilon_i \quad (1)$$

where the unexplained part is picked up by an error term ε_i. To simplify matters, we assume that policy makers are not interested in inequality due to this unexplained part ε_i. This means that from now on, we only consider the deterministic part of equation (1), i.e. $m_i^P=\alpha+\sum_l \alpha_l x_{il}$.[2] Our question is how to operationalize different ethical stances with respect to inequity in m_i taking into account that the drivers of health as presented in equation (1) are known.

Different ethical positions can be distinguished on the basis of their view on the ethical justifiability of the different drivers of health differences. They are therefore characterized by a specific subdivision of the vector of characteristics x_i into those elements reflecting individual responsibility r_{ik}, $k = 1, \ldots K$ - for which policy makers hold individuals responsible -, and those for which individuals are not responsible and for which policy makers are willing to compensate, c_{ij}, $j =1, \ldots J$ (and which, if not compensated for, lead to health inequity, i.e. to illegitimate health inequalities). We then write the deterministic part of equation (1) in terms of K responsibility and J circumstance variables

$$m_i^P=\alpha+\sum_{j=1}^{J}\beta_j c_{ij}+\sum_{k=1}^{K}\gamma_k r_{ik}+\sum_{j=1}^{J}\sum_{k=1}^{K}\delta_{jk} c_{ij} r_{ik} \quad (2)$$

where we stick to the linear specification, but allow for interactions between the circumstance and responsibility variables. Indeed, it will turn out that the choice of reference variables is important only when the function explaining m_i is not separable in the set of compensation and responsibility variables, i.e. when at least one δ_{jk} differs from zero. The specification (2) is the simplest possible way to introduce non-separability.[3] We will now

[1]We use this notation because our empirical work will refer to inequality in mortality. However, our theoretical analysis is general in that m_i can stand for all variables that are considered to be relevant for equity purposes, and need not even be a health variable.

[2]We work with the population version of m_i^P in this section. Issues related to estimation (error) are discussed in the empirical part of the paper.

[3]In the context of standardization of the concentration index, Gravelle (2003) uses the term “essential nonlinearities” to capture the same idea.

first explain how reference values enter in the theory of fair allocation and then introduce the concentration index.

The theory of fair allocation

When measuring the inequity of different health distributions, one wants to include the effects of differences in the circumstance variables, but at the same time neutralize the effects of the responsibility variables. The theory of fair allocation tries to combine this double concern. The first concern is captured by the so-called *compensation principle*, stating that any health differences between individuals who are equally responsible are to be seen as unfair. The second concern is summarized in the *reward principle* which prescribes that health differences which are due to responsibility variables should not lead to an increase in measured unfairness.

Fleurbaey and Schokkaert (2009) explain that it is impossible to satisfy both principles together, unless equation (2) is additively separable in the circumstance and responsibility variables. It is instructive to illustrate this incompatibility for the functional form (2). Take first two individuals s and m with equal responsibility variables, i.e. $r_{sk} = r_{mk} = r_k$, $k = 1, \ldots K$. The health difference between these two individuals is given by

$$m_s^P - m_m^P = \sum_{j=1}^{J} \beta_j (c_{sj} - c_{mj}) + \sum_{j=1}^{J} \sum_{k=1}^{K} \delta_{jk} (c_{sj} - c_{mj}) r_k \quad (3)$$

which clearly depends on the values taken by r_k. Any measure that satisfies the compensation principle and that therefore takes up all effects of the differences between c-variables, will be unable to satisfy the reward principle because, due to the presence of the interaction terms, the effects of differences in the circumstance variables are "blown up" by the value of r_k. On the other hand, consider two individuals in equal circumstances, i.e. $c_{pj} = c_{qj} = c_j$, $j = 1, \ldots J$. The health difference between these two is

$$m_p^P - m_q^P = \sum_{k=1}^{K} \gamma_k (r_{pk} - r_{qk}) + \sum_{j=1}^{J} \sum_{k=1}^{K} \delta_{jk} c_j (r_{pk} - r_{qk}) \quad (4)$$

which depends on the value taken by c_j. Any measure that satisfies the reward principle and therefore neglects the inequalities in (4) will at the same time miss the effects of differences in circumstance variables that work through the interaction effect. Note that the problem does not occur if eq. (2) is additively separable in c_i and r_i, i.e. if $\delta_{jk} = 0, \forall j,k$.

The theory then has proposed partial solutions, which satisfy only one of the two principles (see Fleurbaey and Schokkaert, 2009). These partial solutions define first an individual's *contribution* to overall inequality, and then calculate overall inequity as the inequality in these individual measures. The first (conditional egalitarian) approach stresses *reward* by fixing the responsibility variables in equation (2) to a reference value $(\tilde{r})$. The resulting expression is called direct unfairness (du_i):

$$du_i(\tilde{r}_k)=\alpha+\sum_{k=1}^{K}\gamma_k\tilde{r}_k+\sum_{j=1}^{J}\left(\beta_j+\sum_{k=1}^{K}\delta_{jk}\tilde{r}_k\right)c_{ij} \quad (5)$$

It is immediately clear that inequality in *du* will only reflect differences in the *c* variables, since $\tilde{r}$ is fixed by construction. The multiplicative effects between actual *c* and *r* variables are not taken up in the measure of direct unfairness. The alternative approach stresses the *compensation* principle by comparing the actual health situation of each individual with the health he would obtain in a fair situation (i.e. where the influence of the circumstance variables would be ruled out). This leads to the fairness gap (fg_i) which is obtained by taking the difference between m_i^P and the hypothetical health level that would apply when circumstance is fixed at a fair reference value ($\tilde{c}$):

$$fg_i(\tilde{c}_j)=\sum_{j=1}^{J}\left(\beta_j+\sum_{k=1}^{K}\delta_{jk}r_{ik}\right)(c_{ij}-\tilde{c}_j) \quad (6)$$

As eq. (6) shows, the multiplicative effects now do enter the definition of the fairness gap. Both c_{ij} and r_{ik} appear in the fairness gap while only c_{ij} enters in direct unfairness. When equation (2) is additively separable, i.e. $\delta_{jk}=0, \forall j,k$, direct unfairness is equal to the fairness gap up to a constant, i.e. $du_i(\tilde{r}_k)=fg_i(\tilde{c}_j)+\alpha+\sum_{k=1}^{K}\gamma_k\tilde{r}_k+\sum_{j=1}^{J}\beta_j\tilde{c}_j$. When additive separability does not hold, however, the reference values $\tilde{r}_k, \tilde{c}_j$ are going to matter.

To measure overall inequity in the health distribution, one then calculates the inequality in the individual values of direct unfairness (5) or the fairness gaps (6). In line with the additive approach in equations (2)–(6) and as we want direct unfairness and the fairness gap approach to give the same results when there are no interaction effects and both the reward and the compensation principles are satisfied, it is preferable to use an absolute inequality measure. In this paper, we opt for two common measures of inequality. The first is the absolute Gini index (Yitzhaki, 1983), which can be written as

$$AG(y_i)=\frac{2}{n^2}\sum_{i=1}^{n}z_i^y y_i \quad (7)$$

where $y_i = du_i(\tilde{r}_k)$ or $fg_i(\tilde{c}_j)$; and $z_i^y=\frac{2i-(n+1)}{2}$ with the individuals i=1, …, n being ranked from low to high y_i. The second is the variance which has been proposed for measuring absolute inequality of bounded variables (Lambert and Zheng, 2011), i.e.

$$VAR(y_i)=\frac{1}{n}\sum_{i=1}^{n}(y_i-\bar{y})^2 \quad (8)$$

The concentration index

The concentration index (CI) approach takes a more restricted view on equity, in that it is only interested in the extent to which health inequalities are linked to an indicator of socioeconomic status. The absolute CI is very similar to the absolute Gini introduced in equation (7), except that individuals are ranked by their socioeconomic status (SES) (Wagstaff et al., 1991):

$$AC(y_i, SES_i) = \frac{2}{n^2} \sum_{i=1}^{n} z_i^{SES} y_i \quad (9)$$

In its simplest form, the CI approach is only interested in the extent of the association between SES and health inequalities - i.e. $y_i = m_i$ - and is thus distinctly different from the approaches based on the fairness gap and direct unfairness. However, more subtle approaches, that standardize the health variables for differences in demographic variables have been developed in the CI approach (Van Doorslaer et al., 1997; O'Donnell et al., 2008). This standardization implies that health differences due to age and gender are not considered as problematic. In a certain sense the concentration index approach then treats the standardizing variables in the same way as the responsibility/compensation variables are treated in the fair allocation approach.[4]

Direct standardization resembles direct unfairness as it puts the standardizing ("responsibility") variables to a reference (mostly mean) value. When the direct unfairness measure considers only SES as the circumstance variable ($J = 1$) and when $\beta_1 + \sum_{k=1}^{K} \delta_{1k} \tilde{r}_k > 0$, eq. (7) for direct unfairness will give the same results as (9), since $z_i^{du} = z_i^{SES}, \forall i$, i.e. the rank of direct unfairness then equals the rank of the SES variable (see equation (5)). When $\beta_1 + \sum_{k=1}^{K} \delta_{1k} \tilde{r}_k < 0$, the ranking z_i^{du} is opposite to the ranking z_i^{SES}. The concentration index will then have the same magnitude, but with opposite sign.[5]

Indirect standardization resembles the fairness gap as it also compares actual m_i^P with a "standardized" fair situation. However, even if only SES is taken as circumstance variable, the absolute concentration index will in general deviate from the absolute Gini of the fairness gaps, since the ranking of the fairness gap does not in general coincide with the ranking of SES (see equation (6)).

Reference-value-consistent inequality comparisons

The previous section has made clear that the values taken by the inequity measures will depend on the choice of the reference values. As such, this is of course a trivial result. In this section we will focus on a more interesting question. Compare two states of the world for

[4]In the context of equity in health care use, Van de Poel et al. (2012) estimate different health care equations for different groups in the population. This can be written as in eq. (2) if one introduces dummy variables that take the value 1 if the individual belongs to a specific group. They then introduce in the standardization procedure a notion of vertical equity based on the assumption that the "adequate" level of health care is that given to the richest urban groups in the population.

[5]This is because the absolute concentration index belongs to the class of symmetric indices defined by Erreygers et al. (2012). The 'symmetry' property implies that turning the world upside down (i.e. the poorest becomes the richest, the second poorest the second richest, and so on) leaves the magnitude of inequality unchanged, but reverses its sign.

which eq. (2) holds, but with possibly different values for the regressors. Suppose we measure the inequity in both situations. Does the (in)equity ranking of the situations depend on the chosen reference values? A natural interpretation is the comparison of a baseline situation with the (simulated) state that would result as the consequence of a policy measure, such as e.g. increasing the education level of the population. This is the interpretation that we will favor in this section, but other applications are also possible, as long as eq. (2) holds. While we assume that the actual and simulated states of the world refer to the same population, all our results also hold for comparisons between different populations. However, they cannot be used for evaluating a policy that changes the coefficients of equation (2) rather than the values of its regressors, or for comparisons between countries for which the coefficients in equation (2) differ. We first look at direct unfairness, then at the fairness gap and finally at the concentration index.

Direct unfairness

We consider the comparison between two states of the world summarized by the vectors of the individual values of direct unfairness, i.e. $du_i(\tilde{r}_k)$ and $du_i^S(\tilde{r}_k)$ where *S* refers to 'simulated'. The question at hand is whether the inequality ordering of the absolute Gini and the variance depend on the chosen reference values.

Absolute Gini—In case of the absolute Gini, we want to know whether the sign of $AG[du_i^S(\tilde{r}_k)] - AG[du_i(\tilde{r}_k)]$ depends on $\tilde{r}_k$. The expression one obtains is:[6]

$$AG\left[du_i^S(\tilde{r}_k)\right] - AG[du_i(\tilde{r}_k)] == \frac{2}{n^2}\sum_{i=1}^{n}\left\{z_i^{du^S(\tilde{r}_k)}\left[\alpha + \sum_{k=1}^{K}\gamma_k\tilde{r}_k + \sum_{j=1}^{J}\left(\beta_j + \sum_{k=1}^{K}\delta_{jk}\tilde{r}_k\right)c_{ij}^S - z_i^{du(\tilde{r}_k)}\left[\alpha + \sum_{k=1}^{K}\gamma_k\tilde{r}_k + \sum_{j=1}^{J}\left(\beta_j + \sum_{k=1}^{K}\delta_{jk}\tilde{r}_k\right)c_{ij}\right] = \sum_{j=1}^{J}\left(\beta_j + \sum_{k=1}^{K}\delta_{jk}\tilde{r}_k\right)\left\{AC\left[c_{ij}^S, du_i^s(\tilde{r}_k)\right] - AC[c_{ij}, du_i(\tilde{r}_k)]\right\} \quad (10)$$

[6]In deriving equation (10), we have used that the absolute concentration index of a constant is zero, i.e. $AC[\alpha, du_i(\tilde{r}_k)] = AC[\alpha, du_i^S(\tilde{r}_k)] = AC[\gamma_k\tilde{r}_k, du_i(\tilde{r}_k)] = AC[\gamma_k\tilde{r}_k, du_i^S(\tilde{r}_k)] = 0$.

The interpretation of eq. (10) is relatively straightforward. It decomposes the difference between the two values of the absolute Gini as a sum of components related to the different circumstance variables. For each variable the first term $(\beta_j+\sum_{k=1}^{K}\delta_{jk}\tilde{r}_k)$ gives its overall effect on health (with the responsibility variables set at their reference values). Somewhat loosely formulated, one can interpret the second term $\left\{AC\left[c_{ij}^{S}, du_i^{S}(\tilde{r}_k)\right] - AC\left[c_{ij}, du_i(\tilde{r}_k)\right]\right\}$ as an indication of the correlation between the variable and the overall measure $du_i(\tilde{r}_k)$: If that correlation is larger in the simulated situation and the effect of the variable is positive, the absolute Gini of direct unfairness increases.

Specific conclusions can be drawn for the easy but relevant case where there is only one circumstance variable, i.e. $J = 1$. In this case, equation (10) simplifies to

$$\left(\beta_1+\sum_{k=1}^{K}\delta_{1k}\tilde{r}_k\right)\left\{AC\left[c_{i1}^{S}, du_i^{s}(\tilde{r}_k)\right] - AC[c_{i1}, du_i(\tilde{r}_k)]\right\} \quad (11)$$

This expression will be positive (negative) when the terms $\beta_1+\sum_{k=1}^{K}\delta_{1k}\tilde{r}_k$ and $AC[c_{i1}^{S}, du_i^{s}(\tilde{r}_k)] - AC[c_{i1}, du_i(\tilde{r}_k)]$ have the same (opposite) sign. Since there is only one circumstance variable, the ranking of du_i and c_{i1} will be the same when $\beta_1+\sum_{k=1}^{K}\delta_{1k}\tilde{r}_k>0$, allowing us to rewrite the second term as $AG(c_{i1}^{S}) - AG(c_{i1})$, i.e. the difference in absolute inequality in the circumstance variable in both states of the world. When $\beta_1+\sum_{k=1}^{K}\delta_{1k}\tilde{r}_k<0$, the ranking of du_i and c_{i1} will reverse and turn 'upside down', and this will also change the sign, but not the magnitude of the second term. This all means that the *sign* of equation (10) is independent of the value of $\tilde{r}_k$, but interestingly also from the values of β_1 and δ_{1k}, and ultimately only depends on the absolute inequality in the circumstance variable.

When there is more than one circumstance variable, i.e. $J \geq 2$, we cannot make any predictions in general since the effect of one circumstance variable may be outweighed by that of another. We cannot even derive general conclusions about the signs of the separate terms for each circumstance variable, since the ranking of direct unfairness depends on all the circumstance variables at the same time. The dependency of the difference in the absolute Gini indices on the chosen reference values then becomes an empirical problem, and there might be more than one unique set of reference values for which equation (10) turns zero, i.e. the difference between absolute Gini indices need not be a monotonous function of the set of reference values.

Variance—We can develop a similar reasoning for the variance. The difference between the variance of direct unfairness under the simulated and the actual situation will not depend on the reference value when the sign of

$$VAR\left[du_i^S(\tilde{r}_k)\right] - VAR[du_i(\tilde{r}_k)] = \frac{1}{n}\sum_{i=1}^{n}\left\{\left[\sum_{j=1}^{J}\left(\beta_j+\sum_{k=1}^{K}\delta_{jk}\tilde{r}_k\right)\left(c_{ij}^S-\overline{c_j^S}\right)\right]^2-\left[\sum_{j=1}^{J}\left(\beta_j+\sum_{k=1}^{K}\delta_{jk}\tilde{r}_k\right)\left(c_{ij}-\overline{c_j}\right)\right]^2\right\} = \sum_{j=1}^{J}\left\{\left(\beta_j+\sum_{k=1}^{K}\delta_{jk}\tilde{r}\right. - VAR(c_{ij})] + 2\sum_{l>j}^{J}(\beta_j + \sum_{k=1}^{K}\delta_{jk}\tilde{r}_k\,(\beta_l + \sum_{k=1}^{K}\delta_{lk}\tilde{r}_k\left.\right)\left[COV\left(c_{ij}^S, c_{il}^S\right) - COV(c_{ij}, c_{il})]\right\} \quad (12)$$

is fixed. The expression in equation (12) has a similar structure as the equivalent expression for the absolute Gini in equation (10). It again decomposes the change in the variance as the sum of J components, each related to one circumstance variable. The first term shows that the variance of direct unfairness will increase if the variance of the c variable increases. In addition, the change in the association between the circumstance variables also matters (through the covariance terms). Intuitively, this association captures whether high/low values of circumstance variables are more likely to be clustered among the same individuals. It is weighted by $(\beta_j+\sum_{k=1}^{K}\delta_{jk}\tilde{r}_k)(\beta_l+\sum_{k=1}^{K}\delta_{lk}\tilde{r}_k)$, i.e. the product of the overall health effects of the variables j and l (with the responsibility variables put at their reference values). If these effects have the same sign for two circumstance variables, an increase in their covariance increases the variance of direct unfairness. If these effects have opposite signs, an increase in the covariance decreases the variance of direct unfairness. Again, all this is intuitively understandable and to some extent plays the same role in eq. (12) as was played by the absolute concentration indices in eq. (10) for the absolute Gini.

If there is more than one circumstance variable, it is difficult to derive from eq. (12) general conclusions about the influence of the chosen reference values on the variance of direct unfairness, and there need not be a unique set of reference values for which equation (12) equals zero. However, when there is only one circumstance variable, the second term consisting of the sum of pairwise covariances drops out. The sign of the difference between the variance of direct unfairness under the simulated and actual direct unfairness is in this case only determined by the difference in the variances of the circumstance variables. This is completely analogous to the result for the absolute Gini. Here also, the choice of the reference values (and the value of the coefficients in equation (2)) do not influence the ranking of the values for the variance if there is only one circumstance variable.

The fairness gap

In this section, we repeat the analysis of the previous subsection for the fairness gap. In general, the expressions are more complicated functions of the reference values of the circumstance variables $\tilde{c_j}$. This is explained by the fact that both c_{ij} and r_{ik} appear in the expression for the fairness gap (see equation (6)).

Absolute Gini—The difference between the absolute Gini's of the simulated and actual fairness gaps equals:

$$AG\left[fg_i^S(\tilde{c}_j)\right] - AG[fg_i(\tilde{c}_j)] = \frac{2}{n^2}\sum_{i=1}^{n}\left\{z_i^{fg^S(\tilde{c}_j)}\left[\sum_{j=1}^{J}(\beta_j + \sum_{k=1}^{K}\delta_{jk}r_{ik}^S(c_{ij}^S - \tilde{c}_j)] - z_i^{fg(\tilde{c}_j)}\left[\sum_{j=1}^{J}(\beta_j + \sum_{k=1}^{K}\delta_{jk}r_{ik}(c_{ij} - \tilde{c}_j)]\right\} = \sum_{j=1}^{J}\left\{\beta_j\left\{AC\left[c_{ij}^S, fg^S(\tilde{c}_j)\right] - AC[c_{ij}, fg(\tilde{c}_j)]\right\} + \sum_{k=1}^{K}\delta_{jk}\left\{AC[r_{ik}^S\left(c_{ij}^S - \tilde{c}_j\right), fg_i^s(\tilde{c}_j)] - AC[r_{ik}(c_{ij} - \tilde{c}_j), fg_i(\tilde{c}_j)]\right\}\right\} \quad (13)$$

Eq. (13) is a sum of J components, one for each circumstance variable. The first term in each of these components is very similar to the corresponding expression for direct unfairness, as it is the change in the concentration index of the circumstance variable (now ranked by the fairness gap) weighted by the health impact β_j. The second term is new and captures the effect of the interaction terms in eq. (2). Remember that the fairness gap, contrary to direct unfairness, includes these interactions in the illegitimate part of inequality. For all interaction terms with $\delta_{jk} > 0$ the absolute Gini of the fairness gap increases with the absolute concentration index (with the fairness gap as the ranking variable) of the responsibility variables weighted by the deviation between the circumstance variable and its reference value.

Contrary to the case of direct unfairness, the sign of equation (13) is not independent of the reference value $\tilde{c_j}$ when there is only one circumstance variable. Indeed, as eq. (13) shows, the change in ranking depends even in this simple case on the value of the responsibility variables r_{ik}. Even the sign of the first term cannot be predicted *a priori*. While in the case of direct unfairness the ranking of direct unfairness and the ranking of the circumstance variable coincide or are 'upside down' after changing the reference value, this is not true for the fairness gap, which also depends on the responsibility variables (see eq. (6)).

Variance—For the variance, we obtain:

$$VAR\left[fg_i^S(\tilde{c}_j)\right]-VAR[fg_i(\tilde{c}_j)] = \frac{1}{n}\sum_{i=1}^{n}\left\{\left[\sum_{j=1}^{J}\left[\beta_j\left(c_{ij}^S-\overline{c_j^S}\right)+\sum_{k=1}^{K}\delta_{jk}\left(r_{ik}^S c_{ij}^S-\left(\frac{1}{n}\sum_{i=1}^{n}r_{ik}^S c_{ij}^S\right)\right)-\tilde{c}_j\sum_{k=1}^{K}\delta_{jk}\left(r_{ik}^S-\overline{r_k^S}\right)\right]\right]^2\right. \quad (14)$$

$$\left.-\left[\sum_{j=1}^{J}\left[\beta_j(c_{ij}-\overline{c_j})+\sum_{k=1}^{K}\delta_{jk}\left(r_{ik}c_{ij}-\left(\frac{1}{n}\sum_{i=1}^{n}r_{ik}c_{ij}\right)\right)-\tilde{c}_j\sum_{k=1}^{K}\delta_{jk}(r_{ik}-\overline{r_k})\right]\right]^2\right\} \quad (15)$$

Again this formula is different, but has a similar structure as the formula for the variance of direct unfairness. When there are no interactions between the circumstance and responsibility variables, both expressions are similar. In general, the change in the variance as the result of a policy change, depends in a complicated way on the interplay between the different variables, including those that are fixed at reference values.

The standardized concentration index

We finally derive similar expressions for the standardized concentration index, i.e. the traditional approach in health economics. The concentration index (9) focuses on socioeconomic health inequality, i.e. on the relationship between health and socioeconomic status. It is natural to interpret this in our more general setting, but with SES_i as the ranking variable, which might (or not) be one of the elements in the vector c_{ij}. In the simplest approach, where $y_i = m_i$, there is no need to choose any reference values. However, as soon as one standardizes the health variable, reference values enter the picture again. It is common practice in the literature to fix the reference value to its mean value, but we provide here a more general treatment that allows to fix the reference value at any value. We denote the standardizing variables by r_{ik}, since they play the same role as the responsibility variables in the fair allocation approach.

Directly standardized health variables are then given by eq. (5), i.e. they correspond to the concept of direct unfairness. A change in policy leads to the following change in the concentration index:

$$AC\left[du_i^S(\tilde{r}_k), SES_i^S\right] - AC[du_i(\tilde{r}_k), SES_i] = \frac{2}{n^2}\sum_{i=1}^{n}\left\{z_i^{SES^S}\left[\alpha+\sum_{k=1}^{K}\gamma_k\tilde{r}_k+\sum_{j=1}^{J}\left(\beta_j+\sum_{k=1}^{K}\delta_{jk}\tilde{r}_k\right)c_{ij}^S\right] - z_i^{SES}\left[\alpha+\sum_{k=1}^{K}\gamma_k\tilde{r}_k+\sum_{j=1}^{J}\left(\beta_j+\sum_{k=1}^{K}\delta_{jk}\tilde{r}_k\right)c_{ij}\right]\right\} = \sum_{j=1}^{J}\left(\beta_j+\sum_{k=1}^{K}\delta_{jk}\tilde{r}_k\right)\left[AC\left(c_{ij}^S, SES_i^s\right) - AC(c_{ij}, SES_i)\right] \quad (16)$$

It is useful to compare this expression with the analogous expression (11) for the absolute Gini of direct unfairness. These expressions are similar except for the ranking variables in the absolute concentration indices. Hence, when SES_i is the sole circumstance variable, i.e. $J = 1$ and $c_{i1} = SES_i$ both approaches are identical when $\beta_1+\sum_{k=1}^{K}\delta_{1k}\tilde{r}_k>0$.[7] Yet, contrary to the case of direct unfairness, the sign of eq. (16) does change with $\beta_1+\sum_{k=1}^{K}\delta_{1k}\tilde{r}_k$, and therefore may depend on the choice of the reference value $\tilde{r_k}$. Picking the mean value, even in the case where SES_i is the sole circumstance variable, is therefore not innocuous. Yet, it is important to remember that the sign of $\beta_1+\sum_{k=1}^{K}\delta_{1k}\tilde{r}_k$ determines whether the concentration index is positive or negative. An increase in the absolute Gini of socioeconomic status will therefore always increase the *absolute* value of the concentration index, i.e. the degree of inequity. When $J \geq 2$, we cannot make any predictions in general.

Indirect standardization leads to an expression that is similar to the absolute Gini of the fairness gap (13):

[7]When SES_i is the only circumstance variable, $AC(c_{i1}^S, SES_i^s) - AC(c_{i1}, SES_i) = AG(SES_i^s) - AG(SES_i)$.

$$AC\left[fg_i^S(\tilde{c}), SES_i^S\right] - AC[fg_i(\tilde{c}), SES_i] = \frac{2}{n^2}\sum_{i=1}^{n}\left\{z_i^{SES^S} \times \left[\sum_{j=1}^{J}(\beta_j + \sum_{k=1}^{K}\delta_{jk}r_{ik}^S\left(c_{ij}^S - \tilde{c}_j - z_i^{SES}\left[\sum_{j=1}^{J}(\beta_j + \sum_{k=1}^{K}\delta_{jk}r_{ik}(c_{ij} - \tilde{c}_j)\right]\right\} = \sum_{j=1}^{J}\beta_j\left\{AC\left[c_{ij}^S, SES_i^S\right] - AC[c_{ij}, SES_i]\right\} + \sum_{j=1}^{J}\sum_{k=1}^{K}\delta_{jk}\left\{AC\left[r_{ik}^S\left(c_{ij}^S - \tilde{c}_j\right), SES_i^S\right] - AC[r_{ik}(c_{ij} - \tilde{c}_j), SES_i]\right\}. \tag{17}$$

For the same reasons as before, i.e. the fact that indirect standardization takes into account the interaction effects between socioeconomic status and the standardizing variables, it is not straightforward to derive general conclusions from (17). The dependency of the equity evaluation of different policies on the choice of reference values is contingent on the actual structure of the data and can only be determined through empirical analysis.

Summary

The goal of this section was to understand whether inequality comparisons of direct unfairness and the fairness gap - based on the absolute Gini and the variance - are independent of the choice of the reference value. We also compared these results with the more traditional concentration index approach which uses indirect and direct standardization. Our most important findings are:

1. It is easier to find attractive interpretations for measures of direct unfairness than for measures with the fairness gap. This is the obvious consequence of the fact that direct unfairness is only a function of c_{ij}, while the fairness gap depends on both c_{ij} and r_{ij}.
2. The absolute Gini for direct unfairness summarizes the difference between two distributions as a weighted sum of the differences between the concentration indices for the individual circumstance variables (ranked by direct unfairness),

where the weights refer to their effect on health at the reference values for the responsibility variables. The variance applied to direct unfairness results in an expression containing the variances of and the covariances between the circumstance variables.

3. When there is only one circumstance variable, we get a strong result for the case of direct unfairness. The rankings of the absolute Gini and the variance for different policies are independent of the chosen reference values, and, maybe more surprising, are even independent of the coefficient estimates in equation (2).

4. No general results can be derived for direct unfairness if there is more than one circumstance variable or for the fairness gap, whatever the number of circumstance variables. When equation (2) and the distributions of all responsibility and circumstance variables are known, the results in this section illustrate the different forces at work, but the overall conclusions will be data-dependent. When the inequality comparison is found to depend on the reference values, one cannot guarantee a unique set of reference values for which inequality in the 'simulated' and baseline case is identical.

5. The approach based on the concentration index with direct standardization is to some extent analogous to direct unfairness when SES is the only circumstance variable, while indirect standardization leads to expressions which bear some resemblance to those for the fairness gap. The common assumption of picking the average value for the reference value in the CI approach is not innocuous, as the choice of reference value may influence the results, even in the case of direct standardization.

Empirical illustration

We illustrate the results of the previous section with data on mortality in the Netherlands. Each individual is followed over a period of 10 years and we observe some of their initial characteristics x_{il} (age, gender, lifestyles, and education), and whether they have died by the end of the 10 year follow-up period. In the remainder of this section, we discuss the data, our empirical model for mortality, the resulting health inequality estimates, and the role of the reference values when comparing inequality between a baseline and two 'simulated' situations.

Data

We use data from the Dutch cross-sectional survey on living conditions in 1998, 1999 and 2000. These surveys have been linked to the Dutch cause-of-death registry, and we use this linkage to reconstruct 10-year survival for all individuals, i.e. for each individual observed in period t = 1998, 1999, 2000, we check whether this individual is deceased ten years later (t +10). We dropped individuals younger than 40 at the time of survey as they represent only about 3 percent of those who died by 2010, and since their causes of death are generally unrelated to the lifestyles we consider.

We link survival during the 10-year follow up period to demographics, education, and unhealthy lifestyles in the baseline cross-sections 1998, 1999 and 2000. We consider four indicators of unhealthy lifestyles, whether individual: i) is a smoker, ii) does not exercise (or exercises less than 1 hour per week), iii) is underweight (i.e., if BMI<18.5) and iv) is obese (i.e., if BMI>30). Education is summarized with a binary variable measuring whether individuals had at most primary education, and we define demographics as the set of interactions between gender and the age categories 40–59, 60–74 and 75+ (females between 40–59 are the reference category). We present descriptive statistics in table 1.

Mortality model

Our goal is to estimate equation (2) with the simplest model available that still allows illustration of the results derived in the last section. For a more sophisticated mortality model with the same data, we refer to our earlier work (García-Gómez et al., 2012). Some coefficient estimates are most likely driven by reverse causality, but we do not require causal estimates to illustrate our methods. We return to this point later.

We use a linear probability model with dummy explanatory variables, and initially allowed for all two-way interactions between education, demographics and unhealthy lifestyles. In our final model we have removed the interaction between the 4 lifestyles and low education as these are jointly insignificant. We also find that a majority of the individual estimates turn out insignificant even though all sets of included interactions are jointly significant.

Our estimation results are presented in table 2. We find that 10-year mortality increases dramatically with age, and is higher for males (except in the 40–59 age group). Among the reference group of women 40–59 years, those with lower education have a slightly larger (and borderline significant) probability to die within 10 years, and those with unhealthy lifestyles experience a higher risk of mortality (although obesity and underweight are not individually statistically significant). In terms of the interaction effects, we find that the age gradient in mortality is stronger for (a) the lower educated, (b) those not exercising, (c) those smoking[8], (d) those with underweight and the obese (although the effect is not very strong for the obese). Remember that these interaction effects are essential for our purpose: without interaction effects between circumstance and responsibility variables, direct unfairness and the fairness gap are identical and the choice of the reference values becomes irrelevant.

In the following sections, we use the estimates in table 2 to calculate health inequality indices. We assume that policy makers are interested in ex-ante mortality differences, implying that we can neglect the inequality due to the unexplained part ε_i. This means that we work with the deterministic part of equation (1), i.e. $m_i^P = \alpha + \sum_l \alpha_l x_{il}$, and the variable of interest is no longer binary but belongs to the unit interval.[9]

Health inequity for different ethical stances

As was mentioned before, different health inequity measures ultimately reflect different ethical positions. These positions can be differentiated along three dimensions. First, if one

[8]The oldest age group is an exception. This is probably due to selection effects such as the healthy survivor effect.

[9]The potential concern about predicted mortality being outside the unit interval is not very important for our analysis since we will consider absolute inequality. Nevertheless and reassuringly, no individual has a predicted mortality below 0, and only 2 individuals (out of 12,527) have a predicted mortality higher than 1.

opts for the concentration index one is only interested in the relationship between health and socioeconomic status. Approaches in the theory of fair allocation take a broader perspective on inequality. Second, in the latter approaches one cannot satisfy compensation and reward at the same time. Direct unfairness measures satisfy the reward principle, but they do not satisfy compensation. Measures based on the fairness gap satisfy compensation at the cost of including some effects of responsibility differences in the measure of inequity. Third, one has to take a decision on which variables should be seen as part of circumstances and which should be seen as related to responsibility. In this respect, we will consider in this section four different possibilities as summarized in Table 3.[10] In the first option one considers all variables in eq. (2) as leading to illegitimate health inequalities (row 'All illegitimate'): inequity then coincides with "pure" health inequalities. The second option is most in the spirit of the theories of fair allocation or, for that matter, equality of opportunity (row 'Control/Preference'): only differences that follow from the own choices of the respondents are legitimate, all other variables (including age and gender) belong to circumstances. A third option starts from the popular argument that only health inequalities that can be influenced by policy should be a cause of ethical concern and that health differences according to age and gender are largely unavoidable (row 'Standardization'). Illegitimate inequalities then follow from all the other variables. A last option includes lifestyles among the legitimate sources of inequalities (row 'SES-inequality'). Only education then remains as an "illegitimate" source of health inequalities.

The results for the various approaches in the (observed) baseline situation are shown in Table 4. In each case, we calculated the absolute Gini and the variance for three possible sets of reference values. The "best" values are those that give the best health results (the lowest mortality risks): these are the youngest females without lowest education and with healthy lifestyles (normal weight, non smoking and exercising). The "worst" health results are achieved by the smoking oldest underweighted males with the lowest level of education and without exercise. We also calculate all measures with the "mean" values for the neutralized variables as the reference as this is the most common approach in health economics (Gravelle, 2003).

Finally, we calculate the concentration index with education as the ranking variable. In the "standardization" case, the concentration index is standardized for age and gender. This is the standard approach in health economics. In the "SES-inequality" case, we also standardize for lifestyle differences. As described before, in the context of the concentration index, "direct unfairness" and "fairness gap" correspond to direct and indirect standardization respectively.

The first columns in Table 4 show the results for "pure" inequality in mortality. In that case the choice of reference values is inconsequential. In all other cases, different reference values lead to different results for the measures of inequity. This is of course trivial, yet it is still worth pointing out that this general finding also holds for the concentration index. As

[10]See García-Gómez et al. (2012) for a more detailed discussion of these different positions. The philosophical literature on responsibility-sensitive egalitarianism makes a distinction between approaches in which people are held responsible for their "preferences" (even if these are not chosen by them) and approaches in which they are held responsible for variables which are under their "control". In the simple model we use in this paper it is not really possible to distinguish these two approaches.

was shown before, the concentration index is equal to the absolute Gini for direct unfairness if we take education as the only circumstance variable. The sign is different, however (compare in the column "SES inequality" the rows referring to the absolute Gini with those referring to the CI). The concentration index makes a distinction between health concentrated among individuals of low or high socioeconomic status. In our case the negative sign reflects the fact that there is concentration of mortality among the lower educated. In the fair allocation approach, the inequality measures can only take positive values.

Although it is not the main point of this paper to compare the results for the different ethical stances, it is interesting to note the importance of age and gender. While this is not at all surprising in the light of the estimation results in Table 2, the implications for the inequity measures are still striking. Including age and gender among the "legitimate" sources of health inequalities (as in the standardization and SES-approaches) leads to a sharp decrease in the measured inequity.

Fleurbaey and Schokkaert (2012) have argued that in the setting of the fairness gap, it might be most plausible to pick the set of reference values that leads to the lowest mortality risk, the intuition being that when comparing actual mortality with mortality in a fair situation, one should consider the fair situation as the situation where mortality risk due to the circumstance variables is lowest. This choice of reference values, as compared to the "worst" and "mean" case, has a limited impact on the inequity estimates for the fairness gap within the control/preference approach, but a larger impact in the other approaches.

It is tempting to take "pure health inequalities" as a kind of benchmark and then interpret all the other results in comparison to that benchmark. It is also tempting to think that the approach of the fairness gap which satisfies compensation should lead to a larger inequity than the direct unfairness approach since the fairness gap also includes some parts of the responsibility differences in the illegitimate part of inequality. However, the dependency of the results on the choice of reference values may go against these intuitions. Consider the results for direct unfairness in the preference/control approach. In the case of the "worst" reference values direct unfairness gives a higher value than the fairness gap (both for the absolute Gini and for the variance). Of course, the variables that are put at their "worst" value are different in the two cases: for direct unfairness the lifestyles are neutralized, for the fairness gap reference values are picked for the circumstances (age-gender and education). More strikingly, for these worst reference values the inequity as measured by the direct unfairness approach is larger than the pure health inequality. While this may seem surprising at first sight, it is easily understood. The direct unfairness approach with "worst" reference values measures inequality in the hypothetical situation where all individuals are underweight, are smoking and do not exercise. The interaction effects between these variables and the age-gender dummies are significant and show a clear pattern, in general becoming larger for the older respondents.

This result suggests that it is misleading to compare the numerical values in Table 4 as such if they refer to different ethical concepts or different reference values. That is why we have focused in the previous section on comparisons between different states of the world for a given ethical concept and for a fixed choice of reference values. We now turn to this issue with our empirical data.

Reference value-consistent inequality comparisons

In order to illustrate how reference values might affect inequality comparisons for a given ethical stance and a given inequity measure, we compare the degree of health equity in different counterfactual situations. In addition to the baseline, we consider an “education” scenario, in which everyone obtains at least a lower vocational or a lower secondary education degree, and an “exercise” scenario in which all individuals exercise more than 1 hour per week. Both scenarios are simulated by calculating the predicted probability to die from our estimated mortality model with the regressors lowedu and nosport put to zero for all.[11] Our aim is to illustrate that the choice of the reference value may have important implications when measuring inequity in (opportunity of) health.

Table 5 reveals how reference values might impact on inequality comparisons. For each of the three scenarios (baseline, education, exercise) we calculate for the different ethical stances the health inequality according to the different measures that have been introduced before. We do this for all possible reference values. When there is only one reference variable (e.g. in the SES ethical stance with the fairness gap), this is relatively straightforward, but when there is more than one variable to be fixed at its reference value, this becomes more complicated. However, since we have only dummy variables in our mortality model, the number of potential 0/1 combinations of reference values is finite, and we can easily calculate the inequality for all potential combinations. We then calculate for each of these sets of reference values the differences in the resulting inequality values for two scenarios. Table 5 shows the largest and the smallest of these differences. If the inequality differences between two scenarios are always positive (e.g. in the “All illegitimate” ethical stance), then the second policy scenario would be preferred as its inequity is lower. For example, we find that a policy that achieves that all individuals exercise more than 1 hour per week attains always the lower inequity in the “All illegitimate” ethical stance. When these inequality differencse change sign, the inequality ranking of these two scenarios is not robust to the choice of reference value. We have reported these cases in bold. For example, notice that in the “Standardization” ethical stance for all inequality measures inequity in mortality is lower for the “Education” policy reform compared to the “Exercise” policy reform for some reference values (the minimum value of the inequality difference between the Education and Exercise policy scenario is negative), while the opposite is true for another set of reference values (the maximum value of the inequality difference between the Education and Exercise policy scenario is positive).

As predicted by the analysis in the previous section, we find that the reference values do not matter for the case of pure health inequalities. This happens since under “all illegitimate”, there are no interactions between the compensation and responsibility variables. We also

[11]It is dangerous to interpret these counterfactual situations as resulting from simulated “policy changes”, since our mortality model most likely suffers from reverse causality and other endogeneity issues. In this respect, our approach is perfectly comparable to the practice in most papers on inequity measurement in health, including the literature that intends to decompose inequalities in health into its contributing factors (see e.g. Wagstaff et al. (2003). In the huge literature on health inequity measurement, there are - as far as we know - only a few counterexamples that more carefully look at underlying endogeneity issues (such as Jones et al. 2012).

confirm that the reference value does not matter for the robustness of the inequality comparison when there is only one circumstance variable in the case of direct unfairness (see variance and absolute Gini for the SES scenario). Remember that in this case the change in the absolute Gini (the variance) of direct unfairness only depends on the change in the absolute Gini (the variance) of the single circumstance variable (here education). Therefore the scenario “exercise” has the same inequality as the baseline, as it does not change the distribution of education.[12] It might seem more striking that the “exercise” scenario does not differ from the baseline in the case of the fairness gap, but it is not. Equations (13) and (15) show that the “exercise” scenario would only be different for the fairness gap if there were an interaction between education and exercise in our mortality model (see table 2). We also find - like in table 4 - that the results for the concentration index with direct standardization are equal in absolute value to those for the absolute Gini under direct unfairness.

When there is more than one compensation variable, also direct unfairness may depend on the reference value and this happens in our case for the ethical stance ”standardization” when comparing the education with the public health scenario. More generally, most sign reversals of the inequality comparisons are found for this ”standardization” stance, in particular when we use the fairness gap.

In order to further improve the understanding of the results from the previous section, we provide some additional illustrations. Panel A of Table 6 illustrates the variance of direct unfairness in case of the ethical stance “standardization” for the comparison of the educational and the public health scenarios. Panel B provides an example for the absolute Gini of direct unfairness in case of the control approach. Similar examples can be provided for the fairness gap, but are left out for reasons of space and since these expressions are more complicated to interpret.

Let us start with the variance of direct unfairness under the ethical stance “standardization” (panel A in table 6). The first term of equation (12) (see row “sum term 1”) consists of a weighted sum of differences of variances, and only the weights depend on the reference value, i.e. via $\sum_{k=1}^{K}\delta_{jk}\tilde{r}_k$. The variances of the lifestyles “smoke”, “underweight”, and “obese” are identical in the exercise and education scenarios. The variances of education and exercise become zero in the “education” and “exercise” scenarios respectively. They therefore have an opposite effect on the difference between the inequality in the two scenarios, i.e. the difference between the variances of education is −0.1904 and that of nosport is slightly larger in absolute terms (0.2481). The reference value will matter when it changes the relative weight of these two variances, and that is what happens in panel A. When we take women of 75 and older as the reference category, the full health impact of “nosport” (0.0155+0.1451) dominates that of “lowedu” (0.0141+0.02090) and thus we observe that inequality is larger under the education scenario. When we take women between 60 and 74 as the reference category, we come to the opposite conclusion. These

[12]A similar argument explains why the variance and absolute Gini for direct unfairness under “control” are similar for the exercise and baseline scenarios.

findings are also intuitively plausible (see table 2): the mortality reduction of doing sports is much larger for women aged 75+ compared to those between 60 and 75, and thus at the older age range, the inequality in exercise dominates. Equation (12) also contains a second term, which is determined by the change in the clustering of the circumstance variables within individuals (weighted by their total health impact). We find that this second term is much smaller compared to the first one under both reference categories. This is plausible as our education and exercise scenarios are not targeted to reduce the within-individual clustering of the circumstance variables.[13]

In panel B of table 6, we consider the absolute Gini applied to direct unfairness in the control approach. In this case there is no sign reversal in Table 5. We present the results for two sets of reference values: one where lifestyles are fixed at unhealthy levels ("smoke", "underweight", "nosport"), and another where these are fixed at healthy levels ("nosmoke", "normal", "sport"). In both cases, the absolute concentration indices for education and its interactions with the demographic variables equal zero under the education scenario since everyone has now the same education level (i.e. "loweduc" = 0).[14] Also in both cases, we find that the contributions of the demographic variables are zero (rows dM6074-dM75m, column eq. (10)) because their absolute concentration indices are identical in the exercise and education scenario, except for the youngest males.[15] The only sizeable contribution to the difference in the absolute Gini of direct unfairness therefore is due to the absolute concentration indices of education (and its interactions with the demographic variables) in the exercise scenario (see rows "lowedu"-"dF75*lowedu"). The upshot of all this is that the change in absolute Gini indices is driven by what happens to education, while the exercise scenario plays no role as exercise is a variable for which individuals are held responsible in the control approach. It is thus no surprise that the education scenario has a lower inequality with direct unfairness under this ethical stance, independent of the chosen reference values.

Choosing the reference value when it influences inequality comparisons

In the previous sections, we have analyzed how inequality comparisons based on the absolute Gini and the variance (and the concentration index) depend on the reference values. One could draw from this the pessimistic conclusion that in these cases it is in principle impossible to rank states of the world unambiguously. However, a more constructive approach can start from the idea that the choice of reference values is not arbitrary, i.e. that

[13]This second term consists of a weighted sum of covariance differences (see eq. (12)). Only the weights depend on the reference category. All the covariances with "lowedu" in the education scenario, and all the covariances with "nosport" in the exercise scenario drop out since these variables are zero under the respective scenarios. The weighted sums of the remaining non-zero covariances, i.e. those between "lowedu" and "under", "obese" and "smoke" in the exercise scenario, and those between "nosport" and "under", "obese" and "smoke" in the education scenario are small since the covariances and their weights are small, and since the weights are not strongly influenced by the reference categories.

[14]Interactions among circumstance variables are treated in our linear probability model as just another circumstance variable.

[15]The contribution of "dM4059" is non-zero (albeit small) since the absolute concentration indices are not identical under both scenarios. Since "dM4059" is identical in the education and exercise scenario, this derives from a difference in the ranking variable between both scenarios, i.e. direct unfairness is different. This is the case since the coefficient of "dM4059" (see column β_j) is much smaller compared to the coefficient of "lowedu", and hence the ranking in terms of direct unfairness of the youngest males differs in both scenarios. This does not happen for the other demographic variables since their coefficients (and the differences between each of these coefficients) are much larger than that of "lowedu", and thus completely dominate direct unfairness. In other words, the ranking in terms of direct unfairness for the other demographic groups is similar under both scenarios.

not all reference values are equally good. Ideally, this would call for the construction of a *theory* on how to choose the "best" reference values. In this section we offer a cursory sketch of one possible direction to think about this issue. In the case of direct unfairness, reward is satisfied, and we will propose a value for the reference variable that minimizes the violation of compensation. In the case of the fairness gap, we minimize the violation of the reward principle. For this purpose, we introduce a statistical approach that is sufficiently simple for empirical work, but we leave the empirical exploration for future work.

Direct unfairness: minimizing the violation of compensation

We know that direct unfairness satisfies the reward principle, i.e. that the resulting measures of inequality do not depend on the value of the responsibility variables. However, it violates the compensation condition that when two individuals share their responsibility variables, the measure of inequality should be equal to the mortality difference between them. We propose to choose the reference values in such a way that the deviation from this ideal situation is as small as possible.

To be more concrete, assume that there are *M* (mutually exclusive) subgroups *m* = 1,2, …, *M* of individuals that share the same value for the responsibility variables, i.e. for all *i* within each subgroup $m, r_{ik}=r_k^m, \forall k=1,\ldots,K$. Let us first consider one such group *m*. We then should ideally have that ∀*i*,*l* in group *m*; i.e. with $r_{ik}=r_{lk}=r_k^m, \forall k=1,2,\ldots,K \Rightarrow du_i(\tilde{r})-du_l(\tilde{r})=m_i^P-m_l^P$. This implies that we should minimize:

$$\sum_{j=1}^{J}\left(\beta_j+\sum_{k=1}^{K}\delta_{jk}\tilde{r}_k\right)c_{ij}-\sum_{j=1}^{J}\left(\beta_j+\sum_{k=1}^{K}\delta_{jk}\tilde{r}_k\right)c_{lj}-\sum_{j=1}^{J}(\beta_j+\sum_{k=1}^{K}\delta_{jk}r_{ik}c_{ij}+\sum_{j=1}^{J}\left(\beta_j+\sum_{k=1}^{K}\delta_{jk}r_{lk}\right)c_{lj}=\sum_{j=1}^{J}\sum_{k=1}^{K}\delta_{jk}c_{ij}(\tilde{r}_k-r_k^m)-\sum_{j=1}^{J}\sum_{k=1}^{K}\delta_{jk}c_{lj}(\tilde{r}_k-r_k^m) \quad (18)$$

Equation (18) is key to understanding our minimization approach. It shows that we must choose each $\tilde{r_k}$ such that the deviation with the actual value of the responsibility variable is minimized when the difference in the circumstance variables (weighted with the interaction coefficient) is large. When the difference between the latter is small, the value of the reference variables is not that important.

When we focus on one group only (e.g. because we care for normative reasons only about this group), equation (18) shows that we can always satisfy compensation for this group by choosing $\tilde{r}_k = r_k^m, \forall k = 1, \ldots, K$. When $J = K = 1$, this is the only solution, but when $K \geq 2$, there might be more than one set of values $\tilde{r_1}, \tilde{r_2}, \ldots, \tilde{r_K}$ for which equation (18) is satisfied. However, our proposal of picking $\tilde{r}_k = r_k^m, \forall k = 1, \ldots, K$ is the only solution that will always work independently of all possible values of the circumstance variables and the coefficients δ_{jk}.

When we want to take into account more than one group of individuals sharing the same values for the responsibility variables, we propose a separate solution for the variance and the absolute Gini. In the case of the variance, it is natural to use the (weighted) within-group variance of the left hand side of equation (18) as our objective function. In other words, one should minimize:

$$\sum_{m=1}^{M}\frac{n_m}{n}\left\{\frac{1}{n_m}\sum_{i\in m}\left[\left(\sum_{j=1}^{J}\sum_{k=1}^{K}\delta_{jk}c_{ij}(\tilde{r}_k - r_{ik})\right) - \left(\frac{1}{n_m}\sum_{i\in m}\sum_{j=1}^{J}\sum_{k=1}^{K}\delta_{jk}c_{ij}(\tilde{r}_k - r_{ik})\right)\right]^2\right\} = \sum_{m=1}^{M}\frac{n_m}{n}\left\{\frac{1}{n_m}\sum_{i\in m}\left[\sum_{k=1}^{K}(\tilde{r}_k - r_k^m)\sum_{j=1}^{J}\delta_{jk}(c_{ij}\right.\right.$$

with respect to $\tilde{r_k}$, $k = 1, \ldots, K$, where n_m and $\overline{c_{mj}}$ correspond to respectively the population size and the average of c_{ij} of subgroup m.[16] Equation (19) corresponds to a least squares framework, and can thus be related to the sum of squared residuals in a linear OLS regression.[17] Note that eq. (19) assumes that we weigh each subgroup m with its population share while minimizing the within-variance. This might be found restrictive from a normative perspective and can be generalized to a weighted least squares procedure by replacing the population shares by any weighting scheme that sums to 1. Focusing on one group about which one is most concerned from an ethical point of view, is then just a special case of this more general approach.

A similar argument can be developed for the absolute Gini index (and the concentration index). Two complications arise though. First, the absolute Gini is not subgroup decomposable. This means that one has to choose between two suboptimal options: either one considers only the within absolute Gini, or one considers the sum of the within absolute Gini and the reranking term. Second, although it is possible to minimize by applying a

[16]Note that the variance is subgroup decomposable into between- and within-group variance. Here we only consider the within group variance of the M mutually exclusive subgroups, since the between-group variance is irrelevant from the perspective of the compensation axiom.

[17]One can obtain the reference values that minimize the violation of compensation by regressing $-\sum_{j=1}^{J}\sum_{k=1}^{K}\delta_{jk}(c_{ij} - \overline{c_{mj}})r_{mk}$ on $\sum_{j=1}^{J}\delta_{j1}(c_{ij} - \overline{c_{mj}}), \sum_{j=1}^{J}\delta_{j2}(c_{ij} - \overline{c_{mj}}), \ldots, \sum_{j=1}^{J}\delta_{jK}(c_{ij} - \overline{c_{mj}})$ with an OLS regression without constant. The coefficients obtained from this regression should be used as reference values. Note that this only holds when the responsibility variables are continuous and have cardinal measurement properties. In case one uses dummy variables (or categorical values), one should do a constrained OLS regression imposing that (sets) of coefficients should equal 0 or 1.

minimization algorithm, there is in the Gini case no easy correspondence to a regression framework.

The fairness gap: minimizing the violation of reward

The fairness gap satisfies the *compensation principle*, which prescribes that any difference in the probability of death is measured as unfair for individuals that are equally responsible. However, the fairness gap does not satisfy the reward principle which prescribes that a measure of inequality should not depend on the value of the responsibility variables (see eq. (6)). To minimize the deviations from this reward principle, we suggest an approach which is conceptually similar to the approach in the previous subsection.

We now consider different subgroups $g=1, \ldots, G$, consisting of individuals with the same values for the compensation variables. Reward will hold when the fairness gaps for two individuals $i \neq l$ within one such group are equal. In other words, $\forall i \neq l$ in group g with $c_{ij} = c_{lj} = c_j^g$, ideally $fg_i(\tilde{c}) = fg_l(\tilde{c})$. Minimizing the deviations from this ideal implies choosing $\tilde{c}$ in such a way that

$\sum_{j=1}^{J}[\beta_j c_{ij}+\sum_{k=1}^{K}\delta_{jk}r_{ik}(c_{ij}-\tilde{c}_j)]-\sum_{j=1}^{J}[\beta_j c_{lj}+\sum_{k=1}^{K}\delta_{jk}r_{lk}(c_{lj}-\tilde{c}_j)]$ is minimized. The logic resembles the discussion of direct unfairness - the value of the reference variables will matter mostly when the responsibility variables differ - but there is also an overall scale effect due to the sum of the responsibility variables $\beta_j c_{ij}$.

Similar to the previous section, we propose for the variance a solution based on the within-variance so that one obtains a minimization problem that resembles a least squares framework.[18] The approach can be also generalized by allowing for 'importance' weights for each group g. One can ascertain a unique solution when one gives priority to one single group (i.e. the weights of all other groups are zero) and when $J = K = 1$ (although also here picking the reference values equal to the actual values of the circumstance variables seems most sensible in case of giving priority to only one group when $J \geq 2$ and/or $K \geq 2$).

In case of the absolute Gini index (and the concentration index), the same complications arise as in case of direct unfairness, but in principle it is possible to obtain an objective function and derive the reference values.

Conclusion

The choice of reference values does matter when one wants to evaluate the (in)equity in different situations. This conclusion holds definitely when one applies the measures of direct unfairness and the fairness gap that have recently been proposed in the theory of fair allocation. While the concentration index as such does not require the choice of reference

[18]One should run an OLS model without constant of $\sum_{j=1}^{J}\left\{\beta_j(c_{ij}-\overline{c_{gj}})+\sum_{k=1}^{K}\delta_{jk}\left[r_{ik}c_{ij}-\left(\frac{1}{n_g}\sum_{i\in g}r_{ik}c_{ij}\right)\right]\right\}$ upon $\sum_{k=1}^{K}\delta_{1k}(r_{ik}-\overline{r_{gk}}), \sum_{k=1}^{K}\delta_{2k}(r_{ik}-\overline{r_{gk}}), \ldots, \sum_{k=1}^{K}\delta_{Jk}(r_{ik}-\overline{r_{gk}})$. The J coefficients provide the reference values. Constrained regression is required for dummy and categorical variables.

values, the standard approach in economics in which one first standardizes the health variable does. Indeed, its results may depend on the choice of the reference values in the standardization exercise. It is therefore surprising that this issue is largely neglected in the empirical literature and that there is little theoretical work on the topic.

Our approach has obvious limitations. We worked with a simple linear specification, where the essential nonlinearities were modeled as simple interaction terms between responsibility and circumstance variables. Moreover, we focused on the comparison of two situations for which it can be assumed that the underlying explanatory model of health remains the same. To extend the applicability of our results, one should go beyond these assumptions. This would require decomposing the differences in the inequality measures using methods that are extensions of the Oaxaca approach (see, e.g. Fortin et al., 2011).

Our results strongly suggest that we need a theory on how to choose the reference values. A general theory would probably imply the introduction of additional ethical considerations. One might try to elaborate the simple suggestion put forward by Fleurbaey and Schokkaert (2012), that from an ethical point of view it is natural to take as reference the best values of the circumstance variables.[19] In this paper, we did not at all have the ambition to construct such a general theory. However, we suggested a statistical approach that could be used in the future to pick the reference values for direct unfairness and the fairness gap. While simple, our proposal has the obvious disadvantage that it makes the choice of reference values depend on the specific empirical features of the issue to be analyzed.

Acknowledgments

Pilar García-Gómez is a Postdoctoral Fellows of the Netherlands Organization for Scientific Research – Innovational Research Incentives Scheme –Veni. Tom Van Ourti acknowledges support from the National Institute on Ageing, under grant R01AG037398. We thank the Netherlands Central Bureau of Statistics for access to the linked data sets used for this research (POLS Gezondheid 1998–2000, DO 1999–2010). We have benefited from the comments and suggestions of Owen O'Donnell. The usual caveats apply and all remaining errors are our responsibility.

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Table 1

Variable description

	Description	Mean
dead	1: when the individual died between t+1 and t+10, 0 otherwise	0.162
dM4059	1: when male and age between 40 and 59; 0 otherwise	0.297
dF4059	1: when female and age between 40 and 59; 0 otherwise*	0.320
dM6074	1: when male and age between 60 and 74; 0 otherwise	0.135
dF6074	1: when female and age between 60 and 74; 0 otherwise	0.139
dM75m	1: when male and age above 74; 0 otherwise	0.046
dF75m	1: when female and age above 74; 0 otherwise	0.063
lowedu	1: when at most primary education; 0 otherwise	0.256
smoke	1: when a current smoker; 0 otherwise	0.295
underweight	1: when BMI<18.5; 0 otherwise	0.012
obese	1: when BMI>30; 0 otherwise	0.123
nosport	1: exercising less than 1 hour per week; 0 otherwise	0.544

Note:

*: reference category in the mortality model

Table 2

Estimates of 10-year mortality follow up in the Netherlands, 1998–2010 (*n* = 12;527)

	OLS coefficient	p-value
dM4059	0.0002	0.979
dM6074	0.1487	0.000
dF6074	0.0471	0.000
dM75m	0.6155	0.000
dF75m	0.4625	0.000
lowedu	0.0141	0.110
smoke	0.0285	0.000
underweight	0.0520	0.131
obese	0.0104	0.308
nosport	0.0155	0.015
constant	0.0166	0.000
dM4059 * lowedu	0.0249	0.120
dM6074 * lowedu	0.0731	0.006
dF6074 * lowedu	0.0733	0.000
dM75m * lowedu	0.0616	0.103
dF75m * lowedu	0.0290	0.418
dM4059 * nosport	0.0188	0.057
dM6074 * nosport	0.0866	0.000
dF6074 * nosport	0.0393	0.030
dM75m * nosport	0.0522	0.257
dF75m * nosport	0.1451	0.001
dM4059 * smoke	0.0124	0.276
dM6074 * smoke	0.0750	0.004
dF6074 * smoke	0.0711	0.015
dM75m * smoke	−0.0020	0.965
dF75m * smoke	−0.0845	0.226
dM4059 * underweight	−0.0388	0.647
dM6074 * underweight	0.1536	0.283
dF6074 * underweight	0.1736	0.067
dM75m * underweight	0.2098	0.000
dF75m * underweight	0.1048	0.260
dM4059 * obese	−0.0009	0.956
dM6074 * obese	0.0304	0.470
dF6074 * obese	0.0369	0.179
dM75m * obese	0.0711	0.270
dF75m * obese	−0.0206	0.648

Table 3

Ethical stances

	responsibility	**circumstance**
All illegitimate		age-gender, education, lifestyles
Control/Preference	lifestyles	age-gender, education
Standardization	age-gender	lifestyles, education
SES-inequality	age-gender, lifestyles	education

Table 4

Measures of inequity in mortality with different ethical stances and reference values

		All illegitimate		Control/Preference		Standardization		SES-inequality	
	Reference values	**DU**	**FG**	**DU**	**FG**	**DU**	**FG**	**DU**	**FG**
Variance	best	0.0413	0.0413	0.0300	0.0404	0.0004	0.0048	0.0000	0.0008
	worst	0.0413	0.0413	0.0757	0.0386	0.0048	0.0199	0.0011	0.0010
	mean	0.0413	0.0413	0.0358	0.0392	0.0021	0.0033	0.0004	0.0007
Absolute Gini	best	0.0956	0.0956	0.0746	0.0923	0.0106	0.0357	0.0027	0.0113
	worst	0.0956	0.0956	0.1384	0.0941	0.0374	0.0739	0.0144	0.0167
	mean	0.0956	0.0956	0.0875	0.0923	0.0250	0.0304	0.0088	0.0127
CI	best					−0.0037	−0.0159	−0.0027	−0.0103
	worst					−0.0190	−0.0018	−0.0144	−0.0083
	mean					−0.0116	−0.0132	−0.0088	−0.0098

Note: DU = direct unfairness, FG = fairness gap. In the case of the concentration index (CI) the columns “DU” and “FG” correspond to direct and indirect standardization respectively.

Table 5

Difference in inequity between pairwise scenario comparisons calculated at different reference values

	Variance				Absolute Gini				Concentration index			
	Direct unfairness		Fairness gap		Direct unfairness		Fairness gap		Direct standardization		Indirect standardization	
Comparison	**min**	**max**	**min**	**max**	**min**	**max**	**min**	**max**	**min**	**max**	**min**	**max**
All illegitimate												
Baseline-Education	0.0039	0.0039	0.0039	0.0039	0.0071	0.0071	0.0071	0.0071				
Baseline-Exercise	0.0104	0.0104	0.0104	0.0104	0.0152	0.0152	0.0152	0.0152				
Education-Exercise	0.0065	0.0065	0.0065	0.0065	0.0081	0.0081	0.0081	0.0081				
Preference/Control												
Baseline-Education	0.0029	0.0062	0.0031	0.0038	0.0068	0.0077	0.0050	0.0073				
Baseline-Exercise	0.0000	0.0000	0.0063	0.0096	0.0000	0.0101	0.0124	0.0142				
Education-Exercise	−0.0062	−0.0029	0.0032	0.0058	−0.0077	−0.0068	0.0056	0.0092				
Standardization												
Baseline-Education	0.0001	0.0024	**−0.0022**	**0.0017**	0.0012	0.0092	**−0.0039**	**0.0074**	−0.0167	−0.0027	−0.1025	−0.1025
Baseline-Exercise	0.0001	0.0065	**−0.0039**	**0.0031**	0.0021	0.0316	**−0.0074**	**0.0137**	−0.0067	−0.0007	−0.0044	−0.0044
Education-Exercise	**−0.0006**	**0.0056**	**−0.0021**	**0.0021**	−0.0015	**0.0277**	**−0.0054**	**0.0064**	0.0015	0.0144	0.0059	0.0059
SES												
Baseline-Education	0.0000	0.0015	0.0001	0.0008	0.0027	0.0167	0.0009	0.0113	−0.0167	−0.0027	−0.0103	−0.0103
Baseline-Exercise	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Education-Exercise	−0.0015	−0.0000	−0.0008	−0.0001	−0.0167	−0.0027	−0.0113	−0.0009	0.0027	0.0167	0.0103	0.0103

Note: For all ethical stances, the differences in the inequity measures between the two counterfactual situations (in the first column) were calculated for all possible sets of reference values. The table gives the minima and maxima for these differences. If maximum and minimum values of the difference have different sign, this points to a reversal in the inequity ranking of the counterfactual situations induced by the choice of reference values. These cases of rank reversal are indicated in bold.

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Table 6

Detailed illustration of the impact of different reference values on pairwise inequity comparisons: two examples

Panel A: Equation (12); variance; DU; standardization; education versus exercise

				$\tilde{r_k}$=**dF75m**		$\tilde{r_k}$=**dF6074**	
circumstance	β_j	$VAR\left(c_{ij}^{educ}\right)$	$VAR\left(c_{ij}^{exer}\right)$	$\sum_{k=1}^{K}\delta_{jk}\tilde{r}_k$	**term 1**$^{\mu}$	$\sum_{k=1}^{K}\delta_{jk}\tilde{r}_k$	**term 1**$^{\mu}$
lowedu	0.0141	0.0000	0.1904	0.0290	−0.0004	0.0733	−0.0015
smoke	0.0285	0.2079	0.2079	−0.0845	0	0.0711	0
underweight	0.0520	0.0120	0.0120	0.1048	0	0.1736	0
obese	0.0104	0.1082	0.1082	−0.0206	0	0.0369	0
nosport	0.0155	0.2481	0.0000	0.1451	0.0064	0.0393	0.0007
sum term 1					0.0060		−0.0007
sum term 2$^{£}$					−0.0004		0.0001
diff var*					0.0056		−0.0006

Panel B: Equation (10); absolute Gini; DU; control; education versus exercise

		$\tilde{r_k}$=**smoke,underweight,nosport**				$\tilde{r_k}$=**nosmoke,normal,sport**			
circumstance	β_j	$\sum_{k=1}^{K}\delta_{jk}\tilde{r}_k$	AC^{educ}	AC^{exer}	**eq. (10)**	$\sum_{k=1}^{K}\delta_{jk}\tilde{r}_k$	AC^{educ}	AC^{exer}	**eq. (10)**
dM4059	0.0002	−0.0076	−0.2088	−0.1811	0.0002	0	−0.0186	−0.0510	0.0000
dM6074	0.1487	0.3152	0.0875	0.0875	0.0000	0	0.0875	0.0875	0.0000
dF6074	0.0471	0.2839	0.0520	0.0520	0.0000	0	0.0520	0.0520	0.0000
dM75m	0.6155	0.2600	0.0436	0.0436	0.0000	0	0.0436	0.0436	0.0000
dF75m	0.4625	0.1654	0.0531	0.0531	0.0000	0	0.0531	0.0531	0.0000
lowedu	0.0141			0.1157	−0.0016			0.1157	−0.0016
dM4059 * lowedu	0.0249			0.0083	−0.0002			0.0083	−0.0002
dM6074 * lowedu	0.0731		0	0.0255	−0.0019		0	0.0255	−0.0019
dF6074 * lowedu	0.0733			0.0275	−0.0020			0.0275	−0.0020

dM75 * lowmedu	0.0616	0.0171	−0.0011	0.0171	−0.0011
dF75 * lowedu	0.0290	0.0319	−0.0009	0.0319	−0.0009
sum eq. (10)			−0.0075		−0.0077

Note:

μ: $(\beta_j+\sum_{k=1}^{K}\delta_{jk}\tilde{r}_k)^2\left[VAR\left(c_{ij}^{educ}\right)-VAR\left(c_{ij}^{exer}\right)\right]$.

£: sum term2$=\sum_{j=1}^{J}\left\{2\sum_{l>j}^{J}(\beta_j+\sum_{k=1}^{K}\delta_{jk}\tilde{r}_k)(\beta_l+\sum_{k=1}^{K}\delta_{lk}\tilde{r}_k)\left[COV\left(c_{ij}^{educ},c_{il}^{educ}\right)-COV\left(c_{ij}^{exer},c_{il}^{exer}\right)\right]\right\}$.

*: diff var$=VAR(du_i^{educ})-VAR(du_i^{exer})$.

$AC^x=AC\left(c_{ij}^x;du_i^x(\tilde{r}_k)\right)$