Analytical Solutions for Two Ferromagnetic Nanoparticles Immersed in a Magnetic Field

Mathematical Model in Bispherical Coordinates

Synthesis Lectures on Electrical Engineering

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ISBN: 978-3-031-00891-7 paperback
ISBN: 978-3-031-02019-3 ebook

DOI 10.1007/978-3-031-02019-3

A Publication in the Springer series
SYNTHESIS LECTURES ON ELECTRICAL ENGINEERING

Lecture #5
Series Editor: Richard C. Dorf, *University of California, Davis*
Series ISSN
Print 1559-811X Electronic 1559-8128

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SYNTHESIS LECTURES ON ELECTRICAL ENGINEERING #5

ABSTRACT

The investigation of the behavior of ferromagnetic particles in an external magnetic field is important for use in a wide range of applications in magnetostatics problems, from biomedicine to engineering. To the best of the author's knowledge, the systematic analysis for this kind of investigation is not available in the current literature. Therefore, this book contributes a complete solution for investigating the behavior of two ferromagnetic spherical particles, immersed in a uniform magnetic field, by obtaining exact mathematical models on a boundary value problem. While there are a vast number of common numerical and analytical methods for solving boundary value problems in the literature, the rapidly growing complexity of these solutions causes increase usage of the computer tools in practical cases.

We analytically solve the boundary value problem by using a special technique called a *bispherical* coordinates system and the numerical computations were obtained by a computer tool. In addition to these details, we will present step-by-step instructions with simple explanations throughout the book, in an effort to act as inspiration in the reader's own modeling for relevant applications in science and engineering. On the other hand, the resulting analytical expressions will constitute benchmark solutions for specified geometric arrangements, which are beneficial for determining the validity of other relevant numerical techniques. The generated results are analyzed quantitatively as well as qualitatively in various approaches. Moreover, the methodology of this book can be adopted for real-world applications in the fields of ferrohydrodynamics, applied electromagnetics, fluid dynamics, electrical engineering, and so forth. Higher-level university students, academics, engineers, scientists, and researchers involved in the aforementioned fields are the intended audience for this book.

KEYWORDS

analytical solutions, bispherical coordinates, ferromagnetic nanoparticles, Laplace equation, Legendre polynomials, mathematical model

To my family and teachers.

Contents

Preface

This book builds on fundamental principles in classical electromagnetics and is suitable for self-study. It is assumed that the reader is comfortable with elementary geometrical analysis, including the idea of the general solution of the Laplace equation; otherwise, the text is self-supportive. In addition, the corresponding exercises in each chapter will be beneficial to the reader. The first two chapters provide an introduction and brief description on most common numerical and analytical methods available in the literature. Chapters 3 and 4 are committed to obtaining the relevant sub-models and main models using bispherical coordinates for the selected boundary value problem of the book. Then, in Chapter 5, the numerical results are analyzed qualitatively as well as quantitatively from the models obtained. The conclusions of this study are provided in Chapter 6, and, finally, the solutions for the selected exercises are included in Chapter 7.

The hallmark of this book is its clear and easy-to-understand methodology. In addition, detailed, step-by-step guidelines are provided to emphasize the art of mathematical modeling in computational electromagnetics. This book also offers more thorough, complete, and solid information on specific boundary value problems, which apply in many applications in various areas of magnetostatics research, and have a common mathematical formation in applied electromagnetics. When working toward the solution of a problem, it helps to know the answer. In this book we illustrate the beauty of hidden computations on a boundary value problem with step-by-step instructions for building benchmark solutions for magnetic fields in the presence of ferromagnetic objects. The methodology of this book can be used to obtain the solutions for other relevant applications in magnetostatics problems.

Gehan Anthonys
January 2018

Acknowledgments

I would like to thank Professor I.M.R. Ciric attached to the Department of Electrical and Computer Engineering at the University of Manitoba, Canada for his helpful discussions in the early stages of this study.

Gehan Anthonys
January 2018

Symbols

a	Dimension of the length for bispherical coordinates (m)
$\mathbf{B}$	Magnetic flux density (T)
B_n	Normal component of the magnetic flux density
B_r, B_θ, B_φ	Components of flux density in spherical coordinates
B_β, B_η, B_ϕ	Components of flux density in bispherical coordinates
C, C', D, D'	Series coefficients for the outside quantities
E, E'	Series coefficients for the inside quantities
f	Scalar magnetic potential (A)
$\mathbf{H_0}$	External magnetic field intensity (Am^{-1})
$\mathbf{H}$	Magnetic field intensity (Am^{-1})
H_r, H_θ, H_φ	Components of field intensity in spherical coordinates
H_β, H_η, H_ϕ	Components of field intensity in bispherical coordinates
N	Positive integer
$\mathbf{p}$	A point inside the particle
$\mathbf{P}$	A point outside the particle
$P_n(\alpha)$	Legendre polynomials of first kind, degree n, and argument α
$P_n'(\alpha)$	First derivatives of Legendre polynomials with respect to argument α
$P_n^m(\alpha)$	Associated Legendre functions of first kind, degree n, order m
$P_n^{m\prime}(\alpha)$	First derivatives of associated Legendre functions
r_i	Distance from center of particle i to a selected point (m)
(r, θ, φ)	Spherical coordinates system
$\hat{r}, \hat{\theta}, \hat{\varphi}$	Unit vectors for the spherical coordinates

R_i	Radius of the particle i
(x, y, z)	Cartesian coordinates system
$\hat{x}, \hat{y}, \hat{z}$	Unit vectors for Cartesian coordinates
α	An argument between -1 and 1
(β, η, ϕ)	Bispherical coordinates system
$\hat{\beta}, \hat{\eta}, \hat{\phi}$	Unit vectors for the bispherical coordinates
Φ	Scalar magnetic potential outside the particles
Ψ	Scalar magnetic potential inside the particle
μ_0	Permeability of the medium outside
Γ, Λ, ζ	Defined by the equations