Journal of Geodetic Science

• 3(3) • 2013 • 209-218 DOI: 10.2478/jogs-2013-0027 •

Validation of GOCE gravity field models using GPS-leveling data and EGM08: a case study in Brazil

Research Article

V. G. Ferreira[1]*, Y. Zhang[1] and S. R. C. de Freitas[2]

1 *School of Earth Sciences and Engineering, Hohai University, Nanjing, 210098, Jiangsu, China*
2 *Department of Geomatics, Federal University of Paraná, Curitiba, 81531-980, Paraná, Brazil*

Abstract:
Validation of geopotential models derived from *Gravity field and steady-state Ocean Circulation Explorer* (GOCE) observations is a challenging task in regions with less advanced geodetic infrastructure such as Brazil. In order to assess the current performance of these models, 262 GPS-leveling sites, Earth Gravitational Model 2008 (EGM08) and Residual Terrain Model (RTM) are employed. The validation is based on the differences between GPS-leveling and GOCE-derived models. For the former, the spectral content beyond the GOCE-derived models' maximum degree is removed by using EGM08 and RTM. The results indicate that the GOCE-based models: DGM-1S, SPW (Releases 1 and 2), TIM (Releases 1, 2, 3 and 4), and DIR (Releases 2, 3 and 4), at their maximum degrees have a worse performance than EGM08 while DIR-R1 shows an improvement of 11%. Furthermore, from the steepness of the slopes of the root mean square error (RMSE), it is observed that the optimal combination between DIR-R1 and EGM08 occurs at degree 230 (RMSE of 0.201 m). For the satellite-only models, DIR-R3 reduces the RMSE by ~1.4% compared to TIM-R4 at degree 190. These results are important for Brazil where the accuracy of the current geoid model is approximately 0.28 m.

Keywords:
EGM08 • GOCE • GPS-leveling • normal-orthometric heights • omission error • residual terrain model

Received 13-06-2013; accepted 29-08-2013

1. Introduction

It is well known that determination of physically meaningful heights using Global Navigation Satellite System (GNSS) technologies (e.g., Global Positioning System – GPS) is based on a simple operative relation that links GPS-derived ellipsoidal height h and the orthometric height H^o. Here, these heights are counted along the same normal to the ellipsoid, which is also considered normal to the geoid without loss of precision. By applying this permissible simplification, it is possible to write (Jekeli et al. 2009):

$$H^o = h - N \tag{1}$$

where N is the geoid height with respect to the reference ellipsoid (e.g., Geodetic Reference System 1980 – GRS80). Relation (1) also holds for normal height H^n and height anomaly (quasi-geoid height) ζ. The geoid heights (and the height anomalies) are generally determined from gravity anomalies as well as gravity disturbances and deflections of the vertical obtained on or near the Earth's surface. However, in areas where the local geoid lacks agreement (e.g., the Amazon region) with the ellipsoidal height, the question of how to determine the orthometric height (or normal height) by using the presently available data becomes an interesting topic (Shen et al. 2011).

Based on the Earth Gravitational Model 2008 (EGM08), released by the US National Geospatial Agency in April 2008 (Pavlis et al. 2008), it is possible to determine the global geoid (or quasi-geoid). However, in areas with less advanced geodetic infrastructure (e.g.,

*E-mail: vagnergf@hhu.edu.cn

Brazil) this model will not deliver a sufficient high-frequency gravity field signal (Rummel 2013, Gruber et al. 2011). Following Gruber et al. (2011), validation of global geopotential models (GGMs) is very challenging in such regions. Since the public release of EGM08, the number of GGMs has grown with the increase of data collected by satellite missions such as the *Gravity field and steady-state Ocean Circulation Experiment* (GOCE). In this case, some researchers have validated recent GGMs over Brazilian territory (Guimarães et al. 2012, Ferreira and de Freitas 2012, de Matos et al. 2012, Gruber et al. 2012). These validations have been carried out by using GPS-leveling data sets and, in the evaluation carried out by Guimarães et al. (2012), gravity disturbance data. Gruber et al. (2011) mentioned that an important issue to be considered when comparing these quantities is that any gravity field functionally observed on the Earth's surface contains the full signal, whereas the GGMs are limited by their spectral resolution.

The truncation of the spherical harmonic expansion at maximum degree produces an omission error due to the neglected part of the gravity field (Torge and Müller 2012). This omission error of EGM08, for example, may be reduced considerably by using the Residual Terrain Model (RTM) approach in an augmentation of the gravitational information beyond EGM08's maximum degree (Pavlis et al. 2012). This novel approach, proposed by Hirt et al. (2010), uses a high-resolution RTM to improve EGM08-derived height anomalies in areas without sufficient regional gravity data coverage. For the case of the GOCE-derived GGMs at their maximum degree of expansion, the omission error can be estimated by means of the EGM08, as proposed by Gruber et al. (2011).

The aim of the present work is to examine the GOCE's state-of-the-art models in order to observe the advances in the modeling of GGMs and their strengths at various degrees of expansion. The methodologies published in Gruber et al. (2011) and Hirt (2013) are used to improve the estimation of omission errors by employing the EGM08 and RTM gravity forward modeling with basis in topography/bathymetry data. The results of the comparisons are encouraging and demonstrate that the disagreement between models is large on a regional-based evaluation, as seen from the example of the Southern Brazil. With regard to the RTM forward modeling, it failed to deliver significant improvement on the short-scale gravity field. Only approximately 46% of the total 262 EGM08-derived height anomalies at the GPS-leveling points were improved.

2. Material and methods

2.1. Test area and applied GPS-leveling data set

A large area covering the three states of São Paulo, Paraná and Santa Catarina in Southern Brazil, with elevations from 0 to 1,600 meters was selected as the test area. This was chosen because 262 GPS-leveling benchmarks, distributed randomly over the target area (Fig. 1, left panel), were available for comparison purposes. The accuracy of ellipsoidal heights is about 0.0494 m in terms of mean value and the maximum value is 0.1391 m. The heights of the Brazilian Height System (BHS) were obtained by using spirit leveling as a static system, and reduced for the effect of gravity by the normal gravity field as (Heck 2003, p. 295):

$$noc_{AB} = -\frac{f^*}{R}\sum_{A}^{B} H_A \sin(2\varphi_{av})\cos\alpha_{AB}\delta s_{AB} + \ldots \qquad (2)$$

where $f^* = \frac{\gamma_b - \gamma_a}{\gamma_a}$ is the gravity flattening obtained from the normal gravity values at the equator (γ_a) and the pole (γ_b); R is the radius of the Earth; H_A is the height at the start point; φ_{av}, α and δs are the average latitude, azimuth and the horizontal distance respectively, between the points A and B. Thus, from the practical point of view, the BHS could be considered as a *normal-orthometric height system*.

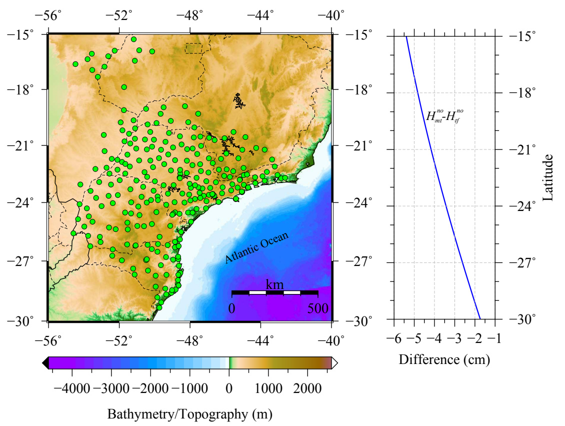

Figure 1. Distribution of 262 GPS-leveling points over the study area in Southern Brazil (left panel). The graphical scale is related to the parallel −22.5°. The graphic (right panel) shows the differences between the heights in the mean-tide (*mt*) and tide-free (*tf*) systems at a latitudinal profile over the study area calculated by Eq. (4).

The BHS does not have a complete physical meaning, i.e., a defined reference surface does not exist, and the heterogeneities of the Earth's crust are not considered. The normal-orthometric heights are not capable of supporting the physical height determination departing from the ellipsoidal heights obtained with GPS, as suggested by relation (1). The difference between the ellipsoidal height (h) and normal-orthometric height (H^{no}) over generic benchmarks yields:

$$\eta = h - H^{no}. \qquad (3)$$

We decided to call the term η in (3) the "normal-geoid height" by analogy with the normal-orthometric height, i.e., an orthometric height defined on the basis of the normal gravity field. An experiment carried out by Ferreira et al. (2011), taking into account different approaches for the geoid to quasi-geoid separation, found

that in Southern Brazil, the term η ("normal-geoid height") fits a little better to ζ (height anomaly) than to N (geoid height).
The original ellipsoidal heights derived from the GPS measurements refer in principle to a tide-free (tf) system in terms of the treatment of the permanent tide effect (Poutanen et al. 1996). However, as no tidal corrections were applied originally to the field measurements, and the results of the leveling work performed by the national mapping agency (*Instituto Brasileiro de Geografia e Estatística* – IBGE) in Brazil, the available normal-orthometric heights refer, in principle, to a mean-tide (mt) system. For our subsequent analysis in Section 3, these values were transformed to the tide-free (tf) system by using the formula (Tenzer et al. 2010):

$$H_{tf}^{no} = H_{mt}^{no} + \left\{(1+k-h)\left[-0.198\left(\frac{3}{2}\sin^2\bar{\varphi}-\frac{1}{2}\right)\right]\right\} \tag{4}$$

where k and h are the tidal Love numbers and their values are 0.3 and 0.62 respectively, and $\bar{\varphi}$ is the geocentric latitude. This was necessary because the GPS and the applied GGMs are related to a tide-free system. The computed differences of the heights defined in these two tide systems (mean-tide and tide-free) at the points of the GPS-leveling in the study region vary from approximately −1.7 cm to approximately −5.4 cm, see Fig. 1 (right panel).

2.2. Global geopotential model based height anomaly

The disturbing potential T in spherical harmonics is (Torge and Müller 2012, p. 224):

$$T = T_0 + \frac{GM_{GGM}}{r}\sum_{n=2}^{n_{max}}\left(\frac{a_{GGM}}{r}\right)^n\sum_{m=0}^{n}(\bar{C}_{nm}^{T}\cos m\lambda + \bar{S}_{nm}^{T}\sin m\lambda)\bar{P}_{nm}(\cos\vartheta) \tag{5}$$

where n is the degree and m is the order of the harmonic coefficients and n_{max} indicates the maximum degree of the series expansions; GM_{GGM} is the geocentric gravitational constant for the GGM; a_{GGM} is the semi-major axis for the GGM; and $\bar{P}_{nm}(cos\vartheta)$ are the fully normalized associated Legendre's functions. The spherical polar coordinates (r, ϑ, λ) of the computation point are the radius, geocentric co-latitude, and longitude. These are computed from the geodetic coordinates (φ, λ, h) for each analysis point. The terms $\bar{C}_{nm}^{T}$ and $\bar{S}_{nm}^{T}$ are the residual harmonic coefficients obtained from differences between the coefficients of the actual and normal gravity field (i.e., coefficient of the disturbing potential). The term $\bar{C}_{nm}^{T}$, which denotes that the even zonal harmonics of the reference ellipsoid (e.g., GRS80) are removed from the GGM's coefficients $\bar{C}_{nm}$, is computed by (Barthelmes 2013, p. 19):

$$\bar{C}_{nm}^{T} = \bar{C}_{n0}^{GGM} - \frac{GM_{GRS}}{GM_{GGM}}\left(\frac{a_{GRS}}{a_{GGM}}\right)^n \bar{C}_{n0}^{GRS} \tag{6}$$

where the GM_{GRS} is the geocentric gravitational constant for the related Geodetic Reference System (GRS) and a_{GRS} is the semi-major axis for the GRS. Because $\bar{S}_{n0}$ coefficients do not exist, $\bar{S}_{nm}^{T} = \bar{S}_{nm}^{GGM}$ (cf. Barthelmes 2013).
The zero-degree term $T_0(r)$ in (5) is given by:

$$T_0(r) = \frac{GM_{GGM} - GM_{GRS}}{r} - (W_0 - U_0)\,. \tag{7}$$

An important topic is related to the choice of the W_0 value which has been the subject of several studies and several discussions (e.g., Burša et al. 2006 and Sanchez 2007). We adopted the geopotential value $W_0 = 62{,}636{,}856.0\ \mathrm{m^2s^{-2}}$ for the definition of the World Height System, following Burša et al. (2004). The normal potential value on the GRS80 ellipsoid is $U_0 = 62{,}636{,}860.850\ \mathrm{m^2s^{-2}}$ (Moritz 1980).
The height anomaly related to a global vertical datum with basis in the disturbing potential, is obtained by applying Bruns's formula:

$$\zeta_P = \frac{T_P}{\gamma_Q}. \tag{8}$$

The normal gravity γ_Q is computed in the point at the telluroid by using (Hofmann-Wellenhof and Moritz 2006, p. 82):

$$\gamma_Q = \gamma\left[1 - \frac{2}{a}(1+f+m-2f\sin^2\varphi)H^n + 3\left(\frac{H^n}{a}\right)^2\right] \tag{9}$$

where H^n is the normal height, $f = \frac{a-b}{a}$ is the geometric flattening, the term m is computed by $m = \frac{\omega^2 a^2 b}{GM}$, where ω is the angular velocity of the Earth, and γ is the normal gravity on the ellipsoid given by Somigliana's formula (Hofmann-Wellenhof and Moritz 2006, p. 72):

$$\gamma = \frac{a\gamma_a\cos^2\varphi + b\gamma_b\sin^2\varphi}{\sqrt{a^2\cos^2\varphi + b^2\sin^2\varphi}}. \tag{10}$$

All numerical constants related to the GRS80 can be found in Moritz (1980).

Note that the height anomaly (8) is dependent of the normal gravity (9) computed at the telluroid and the latter, among other variables, is dependent of the normal height (H^n). However, the normal heights are unknown at the GPS-leveling points (Fig. 1). In this case, an iterative solution is necessary starting with $H^n = h$. Using the geodetic coordinates (27°59′17.09385″, 86°55′30.75851″, 8821.094 m) of the snow peak of Mount Everest (worst case in terms of elevation) available in Shen et al. (2011) only one iteration was necessary with a tolerance of 0.0005 m, given $\zeta = -26.0894$ m (without taking into account the zero-degree term as in (5)). The value given by Table 4 of Shen et al. (2011) is -26.09 using a second order approximation. Equations (5 – 10) were carried out in the MATLAB® programming environment and the routines are available on request from the authors.

2.3. GOCE geopotential models

Several GGMs were used for this research: the EGM08 (Pavlis et al. 2008); the Delft Gravity Model, Release 1, Satellite-only (DGM-1S) (Hashemi-Farahani et al. 2013); the direct approach (DIR Releases 1, 2, 3 and 4) (Bruinsma et al. 2010, 2013); the time-wise approach (TIM Releases 1, 2, 3 and 4) (Pail et al. 2010, 2011); and the space-wise approach (SPW Releases 1 and 2) (Migliaccio et al. 2010, 2011). These models are available at the International Centre for Global Earth Models (ICGEM) belonging to the International Gravity Field Service (IGFS) of the International Association of Geodesy (IAG). A summary of the used GGMs is shown in Table 1.

The reason behind the selection of these models for the numerical analysis was the intention to observe the contributions of the GOCE mission. Such comparisons can be performed by using relative methods (comparison between models) if qualified and independent ground-based geoid information is available. The important point to be taken into consideration for ground-based comparison is that GPS-leveling data represents the full spectrum of the geoid (quasi-geoid) signal, whereas a global model is limited by its spectral resolution (Ustun and Abbak 2010). However, for the evaluation of derived geopotential models one needs to know the impact of the omission error on the results.

In order to evaluate the GOCE-derived geopotential models in the quasi-geoid function by means of GPS-leveling, we applied the methodology published in Gruber et al. (2012):

$$\delta\zeta_P = \eta_P - \left(\zeta_P^{LWL} + \zeta_P^{SWL}\right) \tag{11}$$

where η_P is the "normal-geoid height" (cf. Eq. (3)) in its full signal, ζ_P^{LWL} is the long wavelength part of the height anomaly (computed height anomaly from the GOCE models), and ζ_P^{SWL} is the short wavelength part of the height anomaly (residual height anomaly signal not represented by the GOCE models, i.e., omission error). For the estimation of the short wavelength part (ζ_P^{SWL}), we used the EGM08 and the omission error of the EGM08 was estimated over the study area as proposed by Hirt et al. (2013).

2.4. Residual Terrain Model (RTM)

The terrain effects are calculated using the residual terrain model (RTM) to better account for the short wavelengths (small features) of the Earth's gravity field (Forsberg 1984). Following Hirt et al. (2013), the RTM technique is capable of modeling major parts of the EGM08's signal omission error; nevertheless, it does not augment the EGM08's spectral content. The RTM-based height anomalies (ζ_{RTM}) can be calculated by:

$$\zeta_{RTM} = \frac{T_{RTM}}{\gamma_Q} \tag{12}$$

where γ_Q is the normal gravity on the telluroid as in Eq. (8). The potential T_{RTM} can be calculated by using Eq. (4) published in Nagy et al. (2000) and available in the TC program (Forsberg 1984).

The RTM-based height anomalies (ζ_{RTM}) can be computed by using a merged digital terrain model (DTM) from SRTM V4.1 topography (Jarvis et al. 2008) with a digital bathymetry model (DBM) from SRTM30plus (Becker et al. 2009). The 7.5 arc-seconds merged DTM/DBM is converted to rock-equivalent topography in order to distinguish between different density values (Hirt 2013). Thus, the ocean water masses are compressed into rock masses by reducing the ocean depths:

$$H^{RET} = H\left(1 - \frac{\rho_w}{\rho}\right) \tag{13}$$

where ρ_w is the water density (1030 kg m^{-3}) and ρ is the density of crust (2670 kg m^{-3}). Equation (13) can be applied for all $H < 0$ providing in this way the SRTM-RC. For all details about this procedure, we recommend Hirt (2013). The reference model for calculating the residual topography is the spherical harmonic model of the Rock-Equivalent Topography model (RET2012) (Hirt et al. 2012). The RET2012's spherical harmonic coefficients are complete up to degree and order (d/o) 2160, and the water masses of the oceans and major lakes, as well as the ice sheets, are compressed into layers equivalent to topographic rock of 2670 kg m^{-3}.

Short-wavelength errors are introduced to the ζ_{RTM} components due to the spacing of the DTM/DBM models. For more details about the DTM/DBM's resolution requirements, see the discussion on aliasing effects on the geoid by Tziavos et al. (2009). The RTM forward modeling may be able to augment the EGM08's performance beyond the 5 arc-minute resolution by employing a DTM/DBM with a resolution of 7.5 arc-seconds. For the present study, we used the RTM-based height anomalies above EGM08's maximum spectral content obtained through the kind courtesy of Christian Hirt. Over the GPS-leveling stations, the RTM effects in height anomalies have a mean value of -0.002 m, maximum value of 0.029 m, minimum value of -0.089, and a standard deviation (SD) of 0.013 m.

Table 1. Global gravity field models used in this study.

<table>
<tr><th>Name</th><th>Maximum d/o</th><th>Data</th><th>References</th></tr>
<tr><td>EGM08</td><td>2190/2158</td><td>GRACE, gravity anomalies and satellite altimetry</td><td>Pavlis et al. (2008)</td></tr>
<tr><td>DGM-1S</td><td>250</td><td>GOCE and GRACE</td><td>Hashemi-Farahani et al. (2012)</td></tr>
<tr><td>DIR-R1</td><td>240</td><td>GOCE and EIGEN-5C</td><td rowspan="2">Bruinsma et al. (2010)</td></tr>
<tr><td>DIR-R2</td><td>240</td><td>GOCE</td></tr>
<tr><td>DIR-R3</td><td>240</td><td rowspan="2">GOCE, GRACE and LAGEOS</td><td>Bruinsma et al. (2010)</td></tr>
<tr><td>DIR-R4</td><td>260</td><td>Bruinsma et al. (2013)</td></tr>
<tr><td>TIM-R1</td><td>224</td><td rowspan="4">GOCE</td><td>Pail et al. (2010)</td></tr>
<tr><td>TIM-R2</td><td>250</td><td rowspan="3">Pail et al. (2011)</td></tr>
<tr><td>TIM-R3</td><td>250</td></tr>
<tr><td>TIM-R4</td><td>250</td></tr>
<tr><td>SPW-R1</td><td>210</td><td rowspan="2">GOCE</td><td>Migliaccio et al. (2010)</td></tr>
<tr><td>SPW-R2</td><td>240</td><td>Migliaccio et al. (2011)</td></tr>
</table>

3. Results and discussion

As a first comparison, the spectra of the signals for the twelve models under consideration were calculated in terms of the square root of the geoid height (Fig. 2). As mentioned in Ustun and Abbak (2010), the difference between the satellite-only models can be observed by their signal spectrum. The satellite signal powers can be evaluated by taking as reference the EGM08, which has full resolution up to d/o 2190/2158. Thus, from Fig. 2 it is possible to observe that the satellite-only geopotential models perform well in the determination of the long wavelength functional of the gravity field (up to degree 180). Furthermore, DIR-R4 and TIM-R4 are very close to the EGM08 up to degree 200, after which they start to lose power. EGM08 model originated from the GRACE satellite-only model up to degree 180, between degree 181 up to 2160, the solution is based on terrestrial gravity when available (e.g., gravity anomalies), Satellite Altimetry data, and digital terrain models.

In Fig. 2 we can also see the relative gain based on a ratio between two error values, for details see Eq. (11) from Ustun and Abbak (2010). The gains were calculated by using the formal errors of the TIM, DIR, and SPW by taking EGM08 as reference. The coefficients estimated from GOCE provide an improvement (gain bigger than 1) at the spectral band of 50–180 for TIM-R4 and 80–150 for SPW-R2 with regard to EGM08's coefficients. However, for the DIR-R4 (DIR-R3), the coefficients estimated by using data from GRACE, GOCE, and LAGEOS provide an improvement at the spectral band of 2–210 (2-190). The contributions of LAGEOS and GRACE data make DIR Releases 3 and 4 superior to other models in terms of gain. Taking into account the gain and the degree of variance, we can conclude that DIR Releases 3 and 4 and TIM-R4 are able to provide an enhanced signal over that presented in EGM08 at the degree and order less than 180. The peak in the DIR-R4 formal error at degree 55 is due to the transition in the GRACE normal equations from release 2 of *Groupe de Recherche de Géodésie Spatiale* (GRGS) to release 5 of GeoForschungsZentrum (GFZ) (Bruinsma et al. 2013). This is because the error in the GRACE release 5 normal equations from GFZ is significantly smaller.

The "height anomaly" differences according to (11) were computed for all 262 GPS-leveling points (Fig. 1) by three approaches. In the first approach, we considered that the omission errors of the GGMs are negligible; in the second, we estimated the omission errors by using EGM08; and in the third, we estimated the omission error by using EGM08 and RTM (Table 2). It can be noted that the effect of the omission errors affected the mean value slightly, while improving the SD significantly. Compare, for example, the results obtained for the TIM-R4; the difference between the mean values from the first two approaches is -5.3 cm, which is equal to the mean value of the omission error estimated by EGM08 (251-2190) over the GPS-leveling sites. This means that the omission error does not cancel out over the GPS-leveling points' distribution; thus, the estimated omission error should be taken into account. The errors due to the leveling networks, inaccuracies within the GPS surveying, and the commission error of the GGMs were not considered here.

Table 2 shows that DIR-R1, in its maximum d/o (240), combined with EGM08 (241-2190) presents the smallest RMSE (0.202 m). The DIR-R1 model was constructed with *a-priori* information from EIGEN-5C, and therefore it is considered a combined model. Furthermore, ground data used in DIR-R1 is the same as in EGM08, which is why they are more similar. However, it is noted that EGM08 seems to perform better than the other combination models (GOCE-based and EGM08) in terms of mean value, SD and RMSE (Table 2) over the whole study area. Concerning the DIR-R1 without taking into account the omission error estimated by EGM08, the RMSE is 0.345 m which is comparable with the RMSE (0.34 m) provided in Table 3 of Guimarães et al. (2012). On average, the models DIR Releases 2 and 3 (240), TIM-R1 (224), TIM-R2 (250), and SPW-R2 (240), show a difference of the RMSEs of ~1 cm between

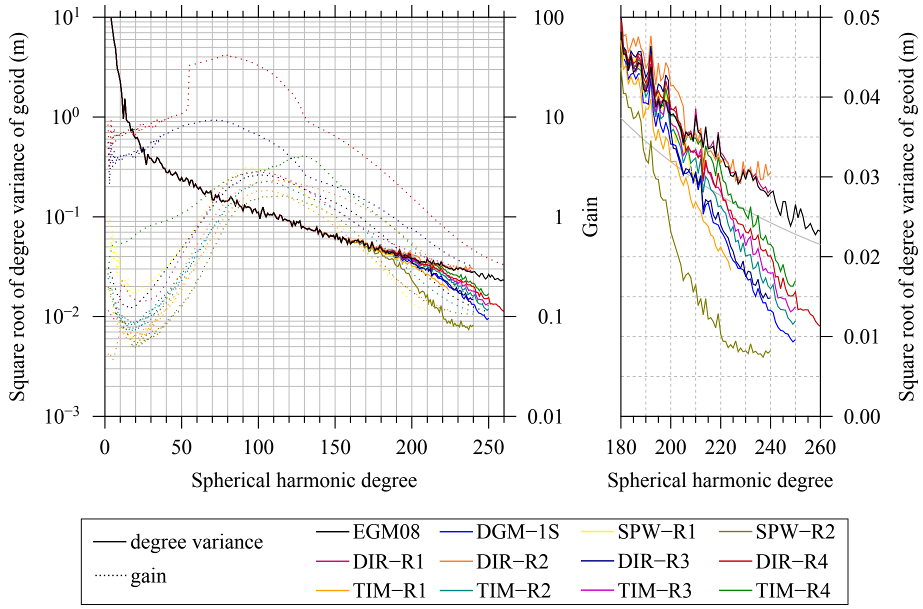

Figure 2. Square root of signal degree variances in terms of geoid heights (solid lines) for EGM08 and GOCE-based models, as well as gains of geopotential models (dotted lines) with respect to EGM08.

those present in Table 3 of Guimarães et al. (2012). However, the models SPW-R1 (210) and TIM-R3 (250) present the large RMSE differences (~3 cm) between both studies. In Table 1 of de Matos et al. (2012) an evaluation of the DIR-R2 is presented, the RMSE value is 0.41 m. The differences of the RMSEs can be attributed to the accuracy, number and distribution of GPS-leveling stations, the tide systems, the functional of the geopotential (here quasi-geoid and there geoid), and the zero-degree term.

In order to investigate more carefully the omission errors estimated by EGM08, we decided to evaluate the EGM08 at d/o 720. This selection is not random. The choice of applying the d/o 720 is to take into account the "fill-in" technique adopted in the EGM08 solution for regions with poor gravity data set coverage, such as the area of study under consideration, see details in Pavlis et al. (2012). Table 2 shows the numerical statistics for the selected spectral window of the EGM08. A slight difference between the EGM08 up to d/o 720 and its maximum degree shows that EGM08's spherical harmonic coefficients between d/o 721 up to 2190 cause a deterioration of the RMSE by 0.7% (Table 2). The "fill-in" technique seems to be unable to improve the EGM08 beyond d/o 720 over the study area. This suggests that we need to estimate a correction for the omission errors from other sources (e.g., gravity anomalies).

Table 2 also shows that adding the RTM-based height anomalies to the EGM08-derived height anomalies at EGM08's maximum degree enlarges this value to 0.225 cm, which corresponds to a deterioration of approximately 0.3%. A further indicator of the RTM performance is the reduction rate residual errors r, which may be expressed as (Hirt et al. 2013):

$$r = \left|\eta^{GPS} - \zeta^{EGM08}\right| - \left|\eta^{GPS} - \zeta^{EGM08+RTM}\right| \qquad (14)$$

where η is the "normal-geoid height" from (3). The Fig. 3 shows this indicator of improvement ($r > 0$) or deterioration ($r < 0$) for the 262 GPS-leveling stations. The application of RTM improved the agreement between EGM08 and GPS-leveling in approximately 46% of the total points. Furthermore, there is no significant correlation between the improvement/deterioration and the topography.

As mentioned by Gruber et al. (2011), from the steepness of the slopes of the RMSE values, it is possible to identify at what degree a model starts to lose power. Figure 4 shows the RMSEs of the height anomaly differences for the degrees varying 10 by 10 and starting from 10 up to the GOCE-based GGMs' maximum degree. The spherical harmonic coefficients of each GOCE-based model from the truncated d/o were replaced by the EGM08's coefficients up to d/o 2190/2158. For the SPW Releases 1 and 2, TIM all releases, DIR releases 2-4 and DGM-1S models, they start to lose signal between the spectral bands of 190–230. A good agreement between TIM-R4 and DIR-R4 can be observed from degree 80 up to 250; therefore, DIR-R4 seems to perform better than TIM-R4 at low degree (10-70). A significant difference in behavior can be observed for the combined DGM-1S and DIR and TIM Releases 3 and 4 models. Overall, all GOCE-only models show better performance compared to the EGM08 at the spectral band from approximately degree 110 to 200. However, a significant difference in behavior can be observed

Table 2. Descriptive statistics of differences between GPS-leveling and GGMs as well as EGM08/RTM solutions.

Model name	Omission error	Statistics (meters)					Improvement rate
		Max	Mean	Min	SD	RMSE	
EGM08 (2190)	-	0.719	0.014	−0.730	0.224	0.224	-
EGM08 (720)	-	0.676	−0.000	−0.765	0.223	0.222	0.7%
EGM08 (2190)	RTM (2160)	0.739	0.016	−0.733	0.225	0.225	−0.3%
DGM-1S (250)	-	1.142	−0.028	−1.086	0.358	0.359	−37.5%
DGM-1S (250)	EGM08 (251-2190)	0.772	0.025	−0.956	0.257	0.257	−12.9%
DGM-1S (250)	EGM08 (251-2190) + RTM (2160)	0.763	0.027	−0.950	0.257	0.258	−13.2%
SPW-R1 (210)	-	1.416	−0.038	−1.384	0.426	0.427	−47.5%
SPW-R1 (210)	EGM08 (211-2190)	0.996	0.015	−0.715	0.270	0.270	−16.9%
SPW-R1 (210)	EGM08 (211-2190) + RTM (2160)	0.987	0.017	−0.709	0.271	0.271	−17.2%
SPW-R2 (240)	-	1.290	−0.040	−1.165	0.396	0.397	−43.5%
SPW-R2 (240)	EGM08 (241-2190)	1.110	0.020	−0.998	0.288	0.288	−22.3%
SPW-R2 (240)	EGM08 (241-2190) + RTM (2160)	1.100	0.022	−0.993	0.288	0.288	−22.3%
DIR-R1 (240)	-	1.094	−0.042	−0.885	0.343	0.345	−35.0%
DIR-R1 (240)	EGM08 (241-2190)	0.579	0.018	−0.797	0.201	0.202	11.1%
DIR-R1 (240)	EGM08 (241-2190) + RTM (2160)	0.580	0.020	−0.792	0.203	0.203	10.3%
DIR-R2 (240)	-	1.187	−0.033	−1.105	0.380	0.380	−41.1%
DIR-R2 (240)	EGM08 (241-2190)	0.902	0.028	−1.017	0.290	0.291	−22.9%
DIR-R2 (240)	EGM08 (241-2190) + RTM (2160)	0.892	0.029	−1.012	0.290	0.291	−22.9%
DIR-R3 (240)	-	1.145	−0.033	−1.066	0.356	0.356	−37.1%
DIR-R3 (240)	EGM08 (241-2190)	0.777	0.028	−0.978	0.249	0.250	−10.3%
DIR-R3 (240)	EGM08 (241-2190) + RTM (2160)	0.782	0.029	−0.973	0.250	0.251	−10.7%
DIR-R4 (260)	-	1.052	−0.039	−0.861	0.350	0.352	−36.3%
DIR-R4 (260)	EGM08 (261-2190)	0.653	0.013	−0.891	0.248	0.248	−9.5%
DIR-R4 (260)	EGM08 (261-2190) + RTM (2160)	0.653	0.015	−0.895	0.249	0.249	−9.8%
TIM-R1 (224)	-	1.414	−0.051	−1.340	0.400	0.403	−44.4%
TIM-R1 (224)	EGM08 (225-2190)	0.959	0.008	−0.901	0.262	0.262	−14.4%
TIM-R1 (224)	EGM08 (225-2190) + RTM (2160)	0.950	0.009	−0.895	0.263	0.263	−14.7%
TIM-R2 (250)	-	1.167	−0.031	−1.087	0.352	0.353	−36.5%
TIM-R2 (250)	EGM08 (251-2190)	0.814	0.023	−1.036	0.263	0.263	−14.9%
TIM-R2 (250)	EGM08 (251-2190) + RTM (2160)	0.805	0.024	−1.030	0.263	0.264	−15.1%
TIM-R3 (250)	-	1.067	−0.036	−1.101	0.355	0.356	−37.1%
TIM-R3 (250)	EGM08 (251-2190)	0.707	0.017	−1.050	0.254	0.254	−11.9%
TIM-R3 (250)	EGM08 (251-2190) + RTM (2160)	0.718	0.019	−1.045	0.255	0.256	−12.3%
TIM-R4 (250)	-	0.986	−0.035	−0.863	0.342	0.343	−34.8%
TIM-R4 (250)	EGM08 (251-2190)	0.600	0.018	−0.846	0.244	0.244	−8.2%
TIM-R4 (250)	EGM08 (251-2190) + RTM (2160)	0.600	0.020	−0.849	0.245	0.245	−8.6%

Note: Improvement rates are given in terms of the RMSE related to EGM08 d/o 2190/2158.

for the DIR-R1 (combined model), it performs better than all models from degree 30 up to 240 and the lowest RMSE (~0.201 m) occurs at the d/o 230. It is remarkable that by adding GOCE data to the combined *a-priori* model a significant improvement in height anomalies can be observed (Gruber et al. 2011). This plays an important role in areas where the accuracy of the geoid model, as for example in Brazil, is approximately 0.28 m (de Matos et al. 2012).

It is worth to be mentioned that the combination of GOCE-only model TIM-R4 up to degree 190 and EGM08 from degree 191 up to 2190 provides an improvement over approximately 58% of the points with an average of 6.5 cm. However, DIR-R3 reduces the RMSE by approximately 1.4% compared to TIM-R4 at degree 190. It is noted that TIM-R4 seems to perform better than DIR-R3 in terms of maximum, minimum and mean values over the GPS-leveling stations. Furthermore, TIM-R4 and DIR-R4 seem to perform better

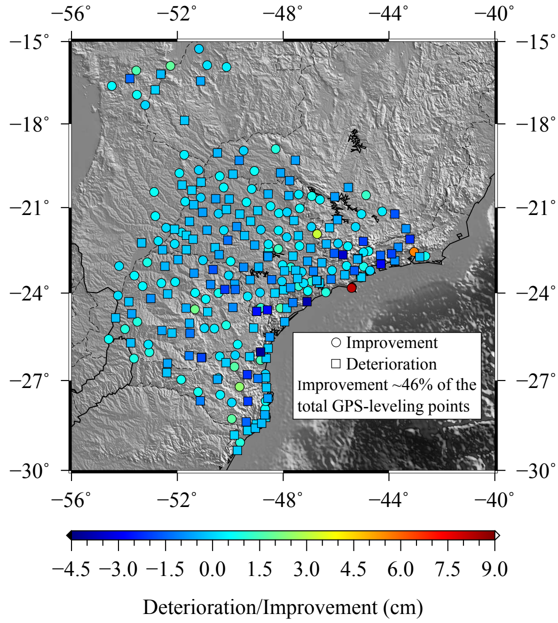

Figure 3. Improvement/deterioration (see Eq. 14 of the residual between EGM08 d/o 2190 and the omission errors estimated by RTM-based height anomalies.

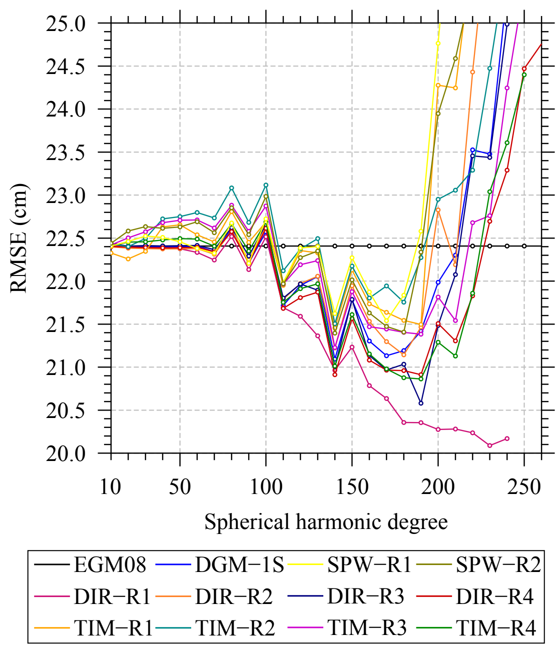

Figure 4. RMSE of the "height anomaly" differences for the selected truncation degrees for GOCE-based models with respect to EGM08.

than DIR-R3 in the spectral band from 10 to 250 (d/o 190 is an exception). Related to the omission error estimated from EGM08, we did not perform analyses for the spectral bands from 180 to 2190 (the spectral band 180-720 contains a large portion of regional gravity anomalies information) to verify for which d/o EGM08 delivers the lowest RMSE. This is one limitation of our study since we used EGM08 to estimate the GOCE's omission error; we therefore plan to repeat the validation in future work by using regional gravity anomalies and topography information to improve and extend the GOCE-based GGMs.

4. Conclusions

Various combinations for GOCE-based GGMs and EGM08 at different spectral bands were evaluated using GPS-leveling. Validation tests showed that the GOCE-derived geopotential model DIR-R1 (combined model) at d/o 230, spectrally enhanced with EGM08 from d/o 231 up to 2190, delivers the lowest RMSE (~0.201 m). For the satellite-only models, DIR-R3 at its d/o 190 combined with EGM08 from d/o 191 up to 2190 delivers the lowest RMSE (0.206 m) in comparison with other GOCE-based models. In fact, it was found that the long wavelength (>100 km) information derived from DIR-R3 improved the height anomalies from EGM08 by approximately 8% in terms of RMSE while TIM-R4 by approximately 7%. However, it is important be mentioned that TIM-R4 and DIR-R4 perform better than DIR-R3 in the spectral band from 10 to 250 (except for 190). The augmented EGM08 height anomalies, by using the RTM gravity forward modeling, improved the RMSE agreement over the EGM08's resolution for 46% of the total GPS-leveling stations. On average, augmentation with RTM omission error estimates deteriorated the RMSE by approximately 0.3%. Furthermore, the EGM08 from degree 721 up to 2190 does not deliver a significant contribution to the height anomalies determination over the study area. Future work will entail refining our validations by using a combination of the GOCE-based GGMs and EGM08 in the spectral domain, for example, by using the least-squares method. It is also important to identify the "optimal" combination between high resolution GGMs and RTM-based quantities over areas such as in this case study.

Acknowledgments

We gratefully acknowledge Prof. Nelsi Côgo de Sá for providing the GPS-leveling database. We thank Christian Hirt for making RTM-based height anomalies available. We thank the ICGEM for making the GGMs available on its home page. We also thank the Generic Mapping Tools (GMT) team. The comments provided by the two Reviewers and the Managing Editor are appreciated.

References

Barthelmes F., 2013, Definition of functionals of the geopotential and their calculation from spherical harmonic models.

Scientific Technical Report, GeoForschungsZentrum, Potsdam, Germany.

Becker J.J., Sandwell D.T., Smith W.H.F., Braud J., Binder B., Depner J., Fabre D., Factor J., Ingalls S., Kim S.-H., Ladner R., Marks K., Nelson S., Pharaoh A., Trimmer R., von Rosenberg J., Wallace G. and Weatherall P., 2009, Global bathymetry and elevation data at 30 arc seconds resolution: SRTM30_ PLUS. Mar. Geod., 32, 4, 355–371.

Bruinsma S.L., J. Marty C., Balmino G., Biancale R., Foerste C., Abrikosov O. and Neumayer H., 2010, GOCE gravity field recovery by means of the direct numerical method. ESA Living Planet Symposium 2010, June 27 – July 2 2010, Bergen, Norway.

Bruinsma S.L., Foerste C., Abrikosov O., Marty J.C., Rio M.-H., Mulet S. and Bonvalot S., 2013, The new ESA satellite-only gravity field model via the direct approach. Geophys. Res. Lett., doi:10.1002/grl.50716.

Burša M., Kenyon S., Kouba J., Šíma Z., Vatrt V. and Vojtíšková M., 2004, A global vertical reference frame based on four regional vertical datums. Stud. Geophys. Geod., 48, 3, 493–502.

Burša M., Kenyon S., Kouba J., Šíma Z., Vatrt V., Vítek V. and Vojtíšková M., 2006, The geopotential value W_0 for specifying the relativistic atomic time scale and a global vertical reference system. J. Geod., 81, 2, 103–110.

de Matos A.C.O.C., Blitzkow D., Guimarães G.N., Lobianco M.C.B. and Costa S.M.A., 2012, Validation of the MAPGEO2010 and comparison with recent geopotential models (in Portuguese). Bol. Cienc. Geod., 18, 1, 101–122.

Ferreira V.G., de Freitas S.R.C. and Heck B., 2011, The separation between geoid and quasigeoid: an analysis in the Brazilian context (in Portuguese). Rev. Bras. Cartogr., 63, 39–50.

Ferreira V.G. and de Freitas S.R.C., 2012, An attempt to link the Brazilian Height System to a World Height System. Bol. Cienc. Geod., 18, 3, 363–377.

Forsberg R., 1984, A study of terrain reductions, density anomalies and geophysical inversion methods in gravity field modeling. Rep 355, Department of Geodetic Science, The Ohio State University, Columbus, Ohio.

Gruber T., Visser P.N.A.M., Ackermann C. and Hosse M., 2011, Validation of GOCE gravity field models by means of orbit residuals and geoid comparisons. J. Geod., 85, 11, 845–860.

Gruber T., Gerlach C. and Haagmans R., 2012, Intercontinental height datum connection with GOCE and GPS-levelling data. J. Geod. Sci., 2, 4, 270–280.

Guimarães G.N., Matos A.C.O.C. and Blitzkow D., 2012, An evaluation of recent GOCE geopotential models in Brazil. J. Geod. Sci., 2, 2, 144–155.

Hashemi-Farahani H., Ditmar P., Klees R., da Encarnação J.T., Liu X., Zhao Q. and Guo J., 2013, Validation of static gravity field models using GRACE K-band ranging and GOCE gradiometry data. Geophys. J. Int. 194, 2, 751–771.

Heck B., 2003, Rechenverfahren un auswertemodelle der landesvermessung. 3rd edition, Wichman, Karlsruhe.

Hirt C., Featherstone W.E. and Marti U., 2010, Combining EGM2008 and SRTM/DTM2006.0 residual terrain model data to improve quasigeoid computations in mountainous areas devoid of gravity data. J. Geod., 84, 9, 557–567.

Hirt C., Kuhn M., Featherstone W.E. and Göttl F., 2012, Topographic/isostatic evaluation of new-generation GOCE gravity field models. J. Geophys. Res., 117, B5, B05407.

Hirt C., 2013, RTM gravity forward-modeling using topography/bathymetry data to improve high-degree global geopotential models in the coastal zone. Mar. Geod., 36, 2, 183-202.

Hofmann-Wellenhof B. and Moritz H., 2006, Physical Geodesy. 2nd corrected edition, SpringerWienNewYork, Austria.

Jarvis A., Reuter H.I., Nelson A. and Guevara E., 2008, Hole-filled SRTM for the globe Version 4. Available from the CGIAR-CSI SRTM 90m Database (http://srtm.csi.cgiar.org). Retrieved May 13, 2013, from http://www.studygs.net/citation.htm.

Jekeli C., Yang H.J. and Kwon J.H., 2009, Using gravity and topography-implied anomalies to assess data requirements for precise geoid computation. J. Geod., 83, 12, 1193–1202.

Migliaccio F., Reguzzoni M., Sansó F., Tscherning C.C. and Veicherts M., 2010, GOCE data analysis: the space-wise approach and the first space-wise gravity field model. ESA Living Planet Symposium 2010, June 27 – July 2 2010, Bergen, Norway.

Migliaccio F., Reguzzoni M., Gatti A., Sanso F. and Herceg M., 2011, A GOCE-only global gravity field model by the space-wise approach. Proceedings of the 4th International GOCE User Workshop, 31 March-1 April 2011, Munich.

Moritz H., 1980, Geodetic reference system 1980. Bull. Géod., 54, 3, 395–405.

Nagy D., Papp G. and Benedek J., 2000, The gravitational potential and its derivatives for the prism. J. Geod., 74, 7/8, 552–560.

Pail R., Goiginger H., Mayrhofer R., Schuh W., Brockmann J. M., Krasbutter I., Hoeck E. and Fecher T., 2010, GOCE gravity field model derived from orbit and gradiometry data applying the time-wise method. ESA Living Planet Symposium 2010, June 27 – July 2 2010, Bergen, Norway.

Pail R., Bruinsma S., Migliaccio F., Förste C., Goiginger H., Schuh W.-D., Höck E., Reguzzoni M., Brockmann J. M., Abrikosov O., Veicherts M., Fecher T., Mayrhofer R., Krasbutter I., Sansó F. and Tscherning C.C., 2011, First GOCE gravity field models derived by three different approaches. J. Geod., 85, 11, 819–843.

Pavlis N.K., Holmes S.A., Kenyon S.C. and Factor J.K., 2008, An earth gravitational model to degree 2160: EGM2008. Proceedings of European Geosciences Union, 13-14 April 2008, Vienna, Austria.

Pavlis N.K., Holmes S.A., Kenyon S.C. and Factor J.K., 2012, The development and evaluation of the Earth Gravitational Model 2008 (EGM2008). J. Geophys. Res., 117, B4, B04406.

Poutanen M., Vermeer M. and Mäkinen J., 1996, The permanent tide in GPS positioning. J. Geod., 70, 8, 499–504.

Rummel R., 2013, Height unification using GOCE. J. Geod. Sci., 2, 4, 355–362.

Sanchez L., 2007, Definition and realisation of the SIRGAS vertical reference system within a globally unified height system. In P. Tregoning and C. Rizos (Eds.), Dynamic Planet, International Association of Geodesy Symposia, 130, Springer Berlin Heidelberg, Berlin, 638–645.

Shen W., Li J., Tian B. and Han J., 2011, Determination of the quasi-geoid of Xinjiang and Tibet areas and the normal height of Mt. Everest based on EGM2008. Terr. Atmos. Ocean. Sci., 22, 2, 121–131.

Tenzer R., Vatrt V., Abdalla A. and Dayoub N., 2010, Assessment of the LVD offsets for the normal-orthometric heights and different permanent tide systems—a case study of New Zealand. Appl. Geom., 3, 1, 1–8.

Torge W. and Müller J., 2012, Geodesy. 4th Edition, Walter de Gruyter, Berlin.

Tziavos I.N., Vergos G.S. and Grigoriadis V.N., 2009, Investigation of topographic reductions and aliasing effects on gravity and the geoid over Greece based on various digital terrain models. Surv. Geophys., 31, 1, 23–67.

Ustun A. and Abbak R.A., 2010, On global and regional spectral evaluation of global geopotential models. J. Geophys. Eng., 7, 4, 369–379.