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Vahagn Galstyan  
*Trinity College Dublin*

Adnan Velic  
*Technological University Dublin, adnan.velic@tudublin.ie*

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## Taxation, Debt and Relative Prices in the Long Run: The Irish Experience

Vahagn Galstyan\*

*Trinity College Dublin*

Adnan Velic<sup>†</sup>

*Dublin Institute of Technology*

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**Abstract:** This paper investigates the effects of public debt and distortionary labour taxation on the long-run behaviour of Irish relative non-traded goods prices. We highlight that higher public debt, acting through higher taxes, has an equivocal impact on the relative supply of non-traded goods and, correspondingly, relative prices. Our empirical analysis for Ireland suggests that taxes and public debt play significant roles in the long run, comoving negatively with the relative price of non-tradables. Accordingly, shifts in public debt and taxation bear implications for the country's international price competitiveness.

### I INTRODUCTION

What are the implications of higher public debt for a country's international price competitiveness in the long run? This fundamental question in international macroeconomics has been receiving increasing attention in recent times. According to the IMF's Fiscal Monitor,<sup>1</sup> the average ratio of gross government debt to GDP remains above 100 per cent in advanced economies, with some countries facing upward projections in debt paths and postponed turning points.<sup>2</sup> Spending cuts and tax hikes are two instruments available to the government

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\*Email: v.galstyan@tcd.ie

<sup>†</sup>Email: adnan.velic@dit.ie

<sup>1</sup> See Bova *et al.* (2015) and Clark *et al.* (2016).

<sup>2</sup> Notably, the compound annual growth rate of government debt across the globe has risen from 5.8 per cent during 2000-2007 to 9.3 per cent over the period 2007-2014.

for reducing the debt level in the long run. Quite often, the latter channel is a significant source of adjustment. However, if taxation is distortionary, it provides a route through which higher public debt levels can influence relative prices. Our objective in this paper is to examine the interaction between relative prices and fiscal variables (public debt, public expenditures and distortionary taxation) in the long run.

A large number of studies have examined the effects of government expenditure on relative prices in the long run. Amongst others, Froot and Rogoff (1991), De Gregorio *et al.* (1994), Chinn (1999), Lee *et al.* (2008) and Ricci *et al.* (2013) find that increases in government consumption are associated with medium- to long-run relative non-traded goods price increases or real exchange rate appreciation, while Galstyan and Lane (2009a; 2009b) indicate that the long-run effects of government investment are more equivocal. However, Galstyan and Velic (2016) show that it is also important to focus on the role of distortionary taxation and public debt in the determination of long-run relative price movements.

Galstyan and Velic (2016) illustrate that, although the endogenous labour supply response to higher debt and taxes is proportionate across sectors, the relative non-tradables supply response is ambiguous and ultimately depends on factor intensities. A non-traded sector that is more labour intensive than the traded sector can yield configurations in which relative supply reacts negatively, thus raising the relative price of non-traded goods. However, it is also possible for a relatively labour intensive non-traded sector to lead to a relative price decline if the traded sector's private capital share is sufficiently higher than that of the non-traded sector.

Guided by Galstyan and Velic (2016), we focus on Irish data over the period 1980-2007. Ireland provides an important case study for the analysis of the impact of government finances.<sup>3</sup> As a volatile economy and a member of a currency union that does not have monetary or exchange rate policy autonomy, the macroeconomic aspects of its fiscal policy bear non-negligible implications for the economy. For instance, under EMU, determining the long-run path of relative prices is important for understanding inflation differentials. Furthermore, internal relative sectoral price changes form the theoretical underpinning of real exchange rate fluctuations which are highly relevant for the small open economy in the context of external competitiveness, not only in terms of trade but also the location of production. Consistent with the studies of Canzoneri *et al.* (1999), Lane and Milesi-Ferretti (2002a; 2002b; 2004), Galstyan (2015), Galstyan and Velic (2016; 2017) and others, we adopt a cointegration approach to analysing the long run. Our empirical analysis for Ireland suggests that taxes and public debt play significant roles in the long run, comoving negatively with the relative price of non-tradables.

<sup>3</sup> Bénétrix and Lane (2009) examine the short-run effects of government spending shocks on the output and real exchange rate of Ireland using a VAR framework, while Bénétrix and Lane (2012) analyse the cyclicity of Irish fiscal policy.

The remainder of the paper is organised as follows. Section II lays out the theoretical framework and derives steady-state relations. Section III describes the empirical methodology adopted for the long-run analysis, while Section IV provides an overview of the data. In Section V we discuss our empirical findings for Ireland. Lastly, Section VI concludes.

## II THE MODEL

In this section we describe the model developed by Galstyan and Velic (2016). Consider a small open economy that produces two composite goods, tradables ( $T$ ) and non-tradables ( $N$ ), and faces an exogenous terms of trade and world interest rate with the price of traded goods, equalling the corresponding world price, normalised to one. In what follows, we first present the various elements of the model. Afterwards, we provide the solution and obtain steady-state relations given our interest in the long run. The time subscript  $t$  on variables in this section is suppressed whenever possible for brevity.

### 2.1 Firms

Outputs are given by Cobb-Douglas production functions of the labour ( $L$ ) and private capital ( $K$ ) employed, with both sectors in addition depending on the exogenous public capital stock ( $Z$ ) available

$$Y_T = A_T^* F(L_T, K_T) = [A_T Z^{\alpha_Z}] L_T^{\alpha_L} K_T^{\alpha_K} \quad (1)$$

and

$$Y_N = A_N^* G(L_N, K_N) = [A_N Z^{\beta_Z}] L_N^{\beta_L} K_N^{\beta_K} \quad (2)$$

where  $\alpha_Z + \alpha_L + \alpha_K = 1$ ,  $\beta_Z + \beta_L + \beta_K = 1$  and the  $A_i$  are sector-specific productivity shifters.<sup>4</sup> Thus, total factor productivity  $A_i^*$  can be viewed as a product of a sector-specific productivity term and the public capital level. Furthermore, we allow the positive effect of the public capital stock on total productivity to be potentially different across sectors (if  $\alpha_Z \neq \beta_Z$ ). Both labour and private capital are mobile intersectorally. As is standard in the literature, we assume international mobility of private capital but immobility of labour. With decreasing returns to scale in private inputs, both sectors generate non-zero profits, which are subsequently distributed to consumers.

<sup>4</sup> See Barro (1990) on the inclusion of public capital in the production function.

## 2.2 Consumers

Consumers maximise the present discounted value of lifetime utility in aggregate consumption  $C$  and labour  $L$

$$U_t = \sum_{j=0}^{\infty} \beta^j \left[ \ln C_{t+j} - \frac{L_{t+j}^{1+\psi}}{1+\psi} \right] \quad (3)$$

subject to the flow budget constraint present in each period

$$\Delta B = rB + r(K_T + K_N) + w(1 - \tau)(L_T + L_N) - (I_T^K + I_N^K) - PC + \Pi_N + \Pi_T \quad (4)$$

where  $\beta \in (0, 1)$  is the discount factor,  $\psi$  is the inverse of the Frisch elasticity of labour supply,  $B$  is an internationally traded bond that pays the fixed rate  $r$ ,  $w$  is the wage rate,  $\tau$  is the rate of distortionary labour taxation,  $I_i^K$  is private capital investment which is assumed to require only the tradable good as an input,  $P$  is the aggregate price, and  $\Pi_T = (1 - \alpha_L - \alpha_K)Y_T$ ,  $\Pi_N = (1 - \beta_L - \beta_K)P_N Y_N$  capture aggregate profits in the traded and non-traded sectors respectively. Notably, the rate of capital depreciation is set equal to zero.<sup>5</sup>

The intratemporal labour-leisure optimality condition sets the ratio between marginal utilities of work effort and consumption equal to the net real wage

$$L^\psi = \frac{1}{C} \frac{w}{P} (1 - \tau) \quad (5)$$

where  $P = P_N^\gamma$  is the aggregate welfare-based price index and total consumption is defined as<sup>6</sup>

$$C = \frac{C_T^{1-\gamma} C_N^\gamma}{(1-\gamma)^{1-\gamma} \gamma^\gamma}. \quad (6)$$

This definition implies that the optimal consumer expenditure shares on traded and non-traded goods are fixed at  $1 - \gamma$  and  $\gamma$  respectively, with the intratemporal elasticity of substitution between the two types of good standing at 1. More specifically, optimality requires the allocation

$$C_N = \gamma P_N^{-1} PC \text{ and } C_T = (1 - \gamma) PC. \quad (7)$$

<sup>5</sup> Since our interest is in steady-state relations, a zero depreciation rate of the capital stock allows us to disregard the investment process altogether.

<sup>6</sup> Assuming that the price of non-traded goods in the rest of the world is fixed and normalised to 1, changes in  $P$  correspond to changes in the real exchange rate.

### 2.3 Government

The government consumes both traded and non-traded goods. To finance spending, it can borrow or tax labour income. Thus, the flow budget constraint facing the public sector takes the following form

$$\Delta D + \tau wL = rD + G_T + P_N(G_N + I^Z) \quad (8)$$

where  $D$  is the level of public debt,  $G_T$  and  $G_N$  are the levels of public consumption of the traded and non-traded goods respectively, and  $I^Z$  is the level of public investment which is assumed to require only the non-traded good.

### 2.4 Equilibrium

Equations characterising equilibrium in the labour market and non-traded goods market are given by

$$L = L_N + L_T \quad (9)$$

and

$$Y_N = C_N + G_N + I^Z \quad (10)$$

while equilibrium in the traded goods market is described by

$$\Delta N = rN + Y_T - C_T - G_T - (I_T^K + I_N^K) \quad (11)$$

where  $N = B - D$  is the net foreign asset position and  $Y_T - C_T - G_T - (I_T^K + I_N^K)$  is the trade balance.<sup>7</sup>

### 2.5 Solution

Appendix A summarises the general system in steady state, where  $\Delta B = \Delta D = I = 0$ . Our primary interest is in the long-run relation between relative prices and fundamentals, including productivities, net foreign assets, public debt, taxation and fiscal spending. Accordingly, we first solve the system for the benchmark steady state in which the net foreign asset position, public debt, fiscal spending and taxes are set equal to zero, while sector-specific productivity levels and the exogenous public capital stock are normalised to one. It is possible to show that equilibrium labour in this benchmark steady state is given by

$$\bar{L} = \left( (1 - \gamma)\alpha_L + \gamma\beta_L \right)^{\frac{1}{1+\psi}} \quad (12)$$

<sup>7</sup> To be consistent with Galstyan and Lane (2009b), in the empirical section we rely on the steady-state negative link between the trade balance and net foreign asset position, and instead use the former variable as a regressor with an expected negative sign. For further discussion see Galstyan and Velic (2016; 2017).

with traded and non-traded sector allocations of  $\bar{L}_N = \theta \bar{L}$  and  $\bar{L}_T = (1 - \theta) \bar{L}$ , where  $\theta = \gamma \beta_L ((1 - \gamma) \alpha_L + \gamma \beta_L)^{-1}$ .

Defining  $\hat{X} = \ln X - \ln \bar{X} \simeq (X - \bar{X}) \bar{X}^{-1}$  as percentage deviations from corresponding steady-state values  $\bar{X}$ ,  $\tilde{N} = (B - D) \bar{Y}_T^{-1}$ ,  $\tilde{D} = D \bar{Y}_T^{-1}$  and  $\tilde{G}_i = G_i \bar{Y}_i^{-1}$  for  $i = \{T, N\}$ , we next log-linearise the general system around the aforementioned benchmark. The first equation of interest is the linearised version of the long-run government constraint

$$\tau = \frac{1 - \theta}{\alpha_L} r \tilde{D} + \frac{1 - \theta}{\alpha_L} \tilde{G}_T + \frac{\theta}{\beta_L} \tilde{G}_N \tag{13}$$

which states that in the long run, for a given level of public spending, higher public debt is associated with a higher tax rate.

The second equation of interest is that of relative non-tradable prices given by

$$\begin{aligned} \hat{P}_N = & \frac{1 - \beta_K}{1 - \alpha_K} \hat{A}_T^* - \hat{A}_N^* + \\ & + \underbrace{\left[ \alpha_Z \frac{1 - \beta_K}{1 - \alpha_K} \frac{1 + \psi \theta}{1 + \psi} + \beta_Z \frac{\psi(1 - \theta)}{1 + \psi} \right]}_{> 0} r \tilde{N} + \underbrace{\left[ \frac{1 - \beta_K}{1 - \alpha_K} \frac{\alpha_Z}{1 + \psi} - \frac{\beta_Z}{1 + \psi} \right]}_{\Leftrightarrow 0} \tau + \\ & - \underbrace{\left[ \alpha_Z \frac{1 - \beta_K}{1 - \alpha_K} \frac{1 + \psi \theta}{1 + \psi} + \beta_Z \frac{\psi(1 - \theta)}{1 + \psi} \right]}_{> 0} \tilde{G}_T + \underbrace{\left[ \alpha_Z \frac{1 - \beta_K}{1 - \alpha_K} \frac{\psi \theta}{1 + \psi} + \beta_Z \frac{1 + \psi(1 - \theta)}{1 + \psi} \right]}_{> 0} \tilde{G}_N \end{aligned} \tag{14}$$

where

$$\frac{1 - \beta_K}{1 - \alpha_K} \hat{A}_T^* - \hat{A}_N^* = \frac{1 - \beta_K}{1 - \alpha_K} \hat{A}_T - \hat{A}_N + \underbrace{\left[ \frac{(1 - \beta_K) \alpha_Z - (1 - \alpha_K) \beta_Z}{1 - \alpha_K} \right]}_{\Leftrightarrow 0} \hat{Z}. \tag{15}$$

Equations (13) and (14) together show the link between public debt, taxation and relative prices. In particular, they demonstrate that the relation between public debt and relative non-tradable prices can be either positive or negative. To see why this happens, note that the relative price of non-traded goods is determined by the intersection of relative demand and supply curves. More specifically, relative demand and relative supply are given by

$$RD = -\hat{P}_N + \tilde{G}_N - \tilde{G}_T + r \tilde{N} \tag{16}$$

and

$$RS = \hat{Y}_N - \hat{Y}_T = \hat{A}_N^* - \frac{1 - \beta_K}{1 - \alpha_K} \hat{A}_T^* + (\beta_L + \beta_K) \hat{L}_N - (\alpha_L + \alpha_K) \hat{L}_T + \frac{(1 - \alpha_L - \alpha_K)(\beta_K - \alpha_K)}{1 - \alpha_K} \hat{L}_T \quad (17)$$

where equilibrium labour supply is governed by the following equations

$$\hat{L} = \frac{\theta}{1 + \psi} \tilde{G}_N + \frac{1 - \theta}{1 + \psi} \tilde{G}_T - \frac{1 - \theta}{1 + \psi} r\tilde{N} - \frac{1}{1 + \psi} \tau \quad (18)$$

$$\hat{L}_T = -\frac{\psi\theta}{1 + \psi} \tilde{G}_N + \frac{1 + \psi\theta}{1 + \psi} \tilde{G}_T - \frac{1 + \psi\theta}{1 + \psi} r\tilde{N} - \frac{1}{1 + \psi} \tau \quad (19)$$

$$\hat{L}_N = \frac{1 + \psi(1 - \theta)}{1 + \psi} \tilde{G}_N - \frac{\psi(1 - \theta)}{1 + \psi} \tilde{G}_T + \frac{\psi(1 - \theta)}{1 + \psi} r\tilde{N} - \frac{1}{1 + \psi} \tau. \quad (20)$$

In the long run, for a given level of public spending, high public debt is associated with a higher tax rate. Higher taxation, in turn, reduces aggregate labour supply proportionally in both sectors. However, the response of the relative supply of non-traded goods is ambiguous due to the presence of the exogenous public capital stock in both sectors and depends on relative labour and capital factor intensities

$$\frac{\partial RS}{\partial \tau} = \frac{\alpha_L(1 - \beta_K) - \beta_L(1 - \alpha_K)}{(1 - \alpha_K)(1 + \psi)} \begin{cases} > 0 & \text{if } \alpha_L\beta_Z > \beta_L\alpha_Z \\ = 0 & \text{if } \alpha_L\beta_Z = \beta_L\alpha_Z \\ < 0 & \text{if } \alpha_L\beta_Z < \beta_L\alpha_Z \end{cases} \quad (21)$$

A non-traded sector that exhibits a higher labour share than the traded sector ( $\alpha_L < \beta_L$ ) can yield configurations in which relative supply declines, therefore increasing the relative price of non-traded goods. Nevertheless, it is also possible for  $\alpha_L < \beta_L$  to lead to a deterioration of the relative price if the traded sector is sufficiently more sensitive to the private capital stock than the non-traded sector ( $\alpha_K > \beta_K + c$ ).<sup>8</sup> Symmetrically,  $\alpha_L > \beta_L$  can result in scenarios in which the relative supply rises, thus reducing the relative price. On the other hand, if  $\alpha_L > \beta_L$  and the non-traded sector is sufficiently more private capital intensive than the traded sector ( $\alpha_K + c < \beta_K$ ), relative prices can increase. With similar labour shares,  $\alpha_L \approx \beta_L$ , a more productive non-traded sector private capital stock ( $\beta_K > \alpha_K$ ) generates a relative price increase while  $\beta_K < \alpha_K$  engenders the opposite outcome. Moreover, note that the response is zero when  $\alpha_j = \beta_j \forall j \in \{Z, L, K\}$  or when  $\alpha_Z = \beta_Z = 0$ .

<sup>8</sup> Where  $c$  is a function of the discrepancy between  $\alpha_L$  and  $\beta_L$ .



Consequently, the sign of the relation between public debt and relative prices is an empirical matter.

Regarding the remaining variables in Equation (14), observe that a positive net foreign asset position, reflecting a positive wealth transfer from the rest of the world, is associated with a higher relative non-tradables price. Higher total productivity in the traded sector or lower total productivity in the non-traded sector causes a rise in the relative price of non-traded goods via the Balassa-Samuelson mechanism. Decomposing total productivities, Equation (15) shows that the relation between the public capital stock and relative prices is equivocal and depends on relative factor intensities across sectors.<sup>9</sup> Meanwhile, for a given level of the tax rate, greater public consumption of non-traded goods is associated with higher relative prices. Finally, observe that when the share of the public capital stock is zero across sectors, relative prices are solely driven by the sector-specific productivity differential.

### III EMPIRICAL APPROACH

#### 3.1 ARDL Bounds Testing

Our interest lies in estimating the long run relation shown in steady-state Equation (14). We first test for the existence of a level or cointegrating relation between the relevant variables by applying the methodology of Pesaran *et al.* (2001). Importantly, the proposed bounds testing procedure is applicable irrespective of whether the underlying time series are purely  $I(0)$ , purely  $I(1)$  or mutually cointegrated. More precisely, the test is based on a modified version of the autoregressive distributed lag (ARDL) model

$$\Delta y_t = a + \omega_y y_{t-1} + \omega_x' \mathbf{x}_{t-1} + \sum_{i=1}^{p-1} \mathfrak{D}'_i \Delta \mathbf{z}_{t-i} + \kappa' \Delta \mathbf{x}_t + u_t \quad (22)$$

with null hypothesis  $\omega_y = 0$  and  $\omega_x = \mathbf{0}$ , where  $\mathbf{z}'_t = [y_t \ \mathbf{x}'_t]$ .<sup>10</sup> If the computed F-statistic falls outside the critical value bounds, a conclusive inference can be drawn. A calculated F-statistic found below the lower bound, corresponding to the polar case of  $I(0)$  variables, implies that the variables are stationary so that cointegration is not possible by definition. Conversely, obtaining an F-statistic that exceeds the upper bound, corresponding to the polar assumption of  $I(1)$  variables, indicates the presence of a cointegrating long-run relation. Meanwhile, the test is inconclusive if the realised F-statistic lies between the bounds.

<sup>9</sup> We employ public investment instead of the public capital stock in regressions as the measurement error effects associated with the latter variable would be exacerbated in small samples. This choice also ensures consistency with Galstyan and Lane (2009b).

<sup>10</sup> Equation (22) is often referred to as the “unrestricted” or “unconstrained” error-correction model. Pesaran *et al.* (2001) refer to it as the “conditional” error-correction model.

### 3.2 Cointegration and Error-Correction

Accordingly, once the existence of a cointegrating long-run relation amongst variables is established, we estimate the following ARDL model<sup>11</sup>

$$y_t = \alpha + \rho y_{t-1} + \sum_{j=1}^n \sum_{i=0}^1 \phi_i^j x_{t-i}^j + \varepsilon_t \quad (23)$$

and subsequently back out the long-run equation

$$\bar{y} = \underbrace{\frac{\alpha}{1-\rho}}_{\lambda^0} + \sum_{j=1}^n \underbrace{\frac{\sum_{i=0}^1 \phi_i^j}{1-\rho}}_{\lambda^j} \bar{x}^j \quad (24)$$

where standard errors corresponding to long-run coefficients,  $\boldsymbol{\lambda}' = [\lambda^0 \lambda^1 \lambda^2 \dots \lambda^n]$ , are estimated via the delta method.  $\lambda^j$  represents the impact of a shift in the long-run value of  $x^j$  ( $\bar{x}^j$ ) on the long-run value of  $y$  ( $\bar{y}$ ).<sup>12</sup> The ARDL order adopted is selected by both the Schwarz-Bayesian and Akaike information criteria with the maximum lag length set equal to 1.<sup>13</sup>

Lastly, in order to gauge short-run dynamics, we also estimate the error-correction model

$$\Delta y_t = \mu + \delta GAP_{t-1} + \sum_{j=1}^n \pi^j \Delta x_t^j + \eta_t \quad (25)$$

where  $GAP_{t-1}$  denotes the deviation  $y_{t-1} - \bar{y}_{t-1} = y_{t-1} - \boldsymbol{\lambda}' \mathbf{x}_{t-1}$ .<sup>14,15</sup> That is, the *GAP* term represents the discrepancy between the actual relative price and fundamentals-based long-run equilibrium relative price.

## IV DATA

Our time series analysis is conducted at the annual frequency level over the period 1980-2007. We note that the sample size is governed by data availability. The external trade balance on goods and services is defined as exports minus imports and is expressed as a share of GDP. The terms of trade series is generated by taking

<sup>11</sup> Note that Pesaran (1997) associates cointegration with the empirical analysis of steady-state relations.

<sup>12</sup> In Equation (22), long-run coefficients on regressors would be given by  $-\left(\frac{\omega_{x^1}}{\omega_y}\right), -\left(\frac{\omega_{x^2}}{\omega_y}\right), \dots, -\left(\frac{\omega_{x^n}}{\omega_y}\right)$ .

<sup>13</sup> Pesaran and Shin (1999) and Panopoulou and Pittis (2004) demonstrate that the ARDL estimator is superior to alternative long-run estimators.

<sup>14</sup> Note that the  $\mathbf{x}_t$  vector has been augmented to account for the intercept term.

<sup>15</sup> Equation (25) implicitly includes the same lagged level variables as in Equation (22). However, the corresponding coefficients in the latter are unrestricted.

the ratio of the export price deflator to the import price deflator. Data for both variables are obtained from the World Bank's World Development Indicators repository.

Public debt, government consumption, government investment and labour taxes comprise the fiscal variables. General government gross debt as a share of GDP is an end of year stock variable that is sourced from the International Monetary Fund's World Economic Outlook. Government final consumption expenditure and gross fixed capital formation, as fractions of GDP, are adopted from the OECD Economic Outlook database.<sup>16</sup> Labour taxation is gauged by the implicit tax rate on labour. Following Eurostat methodology, this is calculated as the sum of all direct and indirect taxes, and employees' and employers' social contributions, divided by the total economic remuneration of employees working in the economic territory.<sup>17</sup> The data used to generate these variables are retrieved from the OECD and Eurostat databases.<sup>18</sup>

All data required for the construction of relative non-tradable prices and labour productivities are gathered from the EU KLEMS dataset (see O'Mahony and Timmer, 2009).<sup>19</sup> Manufacturing and services are used to proxy for the traded and non-traded sectors respectively. In particular, manufacturing is measured by "*total manufacturing*" as reported in KLEMS, while services is measured by the sum of "*wholesale and retail trade*", "*hotels and restaurants*", "*transport and storage and communication*", "*finance, insurance, real estate, and business services*" and "*community, social and personal services*".<sup>20,21</sup> We aggregate prices and quantities

<sup>16</sup> Government spending on transfer programs such as social security or welfare is not included in the analysis since such transfers only redistribute resources across private-sector entities.

<sup>17</sup> See Galstyan and Velic (2016) for more details.

<sup>18</sup> Our computed labour tax rates are significantly correlated with the average personal income tax rates from the Andrew Young School World Tax Indicators dataset. In robustness checks, we applied the latter series and found similar empirical results.

<sup>19</sup> Our price and productivity data are based on the ISIC revision 3 industry classification (NACE 1) and are not available after the year 2007. While EU KLEMS is rolling out new data based on the ISIC revision 4 industry classification (NACE 2) that cover the post-2007 period, at the time of writing, these data are not yet available for Ireland.

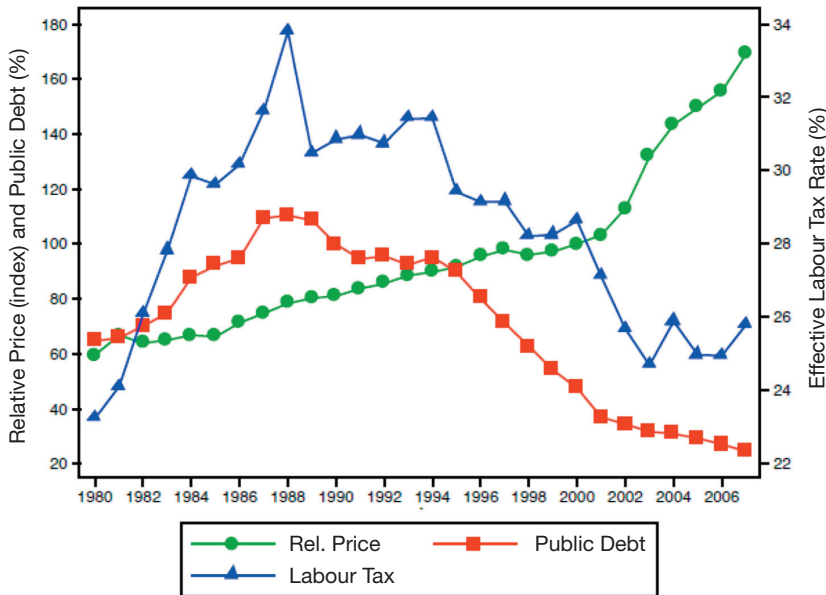
<sup>20</sup> The sectoral allocation should be viewed as reflecting degrees of tradability, with more trade occurring in the "traded" sector.

<sup>21</sup> In robustness checks, we employed broader definitions of the traded and non-traded sectors. In one check, we took the current set of definitions of this paper and augmented the non-traded side with the "*Construction*" sub-sector given its relative importance to the Irish economy. In another check, following Canzoneri *et al.* (1999), Obstfeld (2009) and Galstyan and Lane (2009a), we defined the traded sector as the sum of "*Agriculture, Hunting, Forestry and Fishing*", "*Mining and Quarrying*", "*Total Manufacturing*" and "*Electricity, Gas and Water Supply*", and the non-traded sector as the sum of "*Construction*", "*Wholesale and Retail Trade*", "*Hotels and Restaurants*", "*Transport, Storage and Communication*", "*Finance, Insurance, Real Estate and Business Services*", "*Public Administration and Defence; Compulsory Social Security*", "*Education*", "*Health and Social Work*" and "*Other Community, Social and Personal Services*". On both occasions, we found very similar results.

across the non-traded sub-sectors using the Fisher index.<sup>22</sup> The relative price of non-traded goods is constructed by taking the ratio of the aggregate services price index to the manufacturing price index. Labour productivity in each sector is given by the ratio of value added in constant terms to the total number of employees in the sector. Analogously, the labour productivity differential across sectors is defined as the ratio of services labour productivity to manufacturing labour productivity.

Figure 1 provides some cursory evidence on the raw link between gross public debt, labour taxes and relative non-tradable prices by plotting the three series over the sample period. Visual inspection of the graph suggests that public debt and the labour tax rate comove positively, while both variables tend to predominantly covary negatively with the relative price of non-traded goods. Table 1 displays the corresponding bilateral correlation coefficients. Both the Pearson and Spearman correlation statistics in the first two panels of the table corroborate the initial observations. At conventional significance levels, they indicate a strong positive

**Figure 1: Relative Non-Tradable Prices, Public Debt and Labour Taxes, 1980-2007**



<sup>22</sup> The Fisher index can be defined as a geometric average of the Paasche and Laspeyres indices. For this reason it is known as the “ideal” index. More precisely, for a given price or quantity index ( $P_{0,t}$  or  $Q_{0,t}$ ) and the value index ( $V_{0,t}$ ), the residual component can be backed out from the factor reversal test

$$P_{0,t} \times Q_{0,t} = \sqrt{\frac{\sum p_{j,t} q_{j,0}}{\sum p_{j,0} q_{j,t}} \times \frac{\sum p_{j,t} q_{j,t}}{\sum p_{j,0} q_{j,t}}} \times \sqrt{\frac{\sum q_{j,t} p_{j,0}}{\sum q_{j,0} p_{j,0}} \times \frac{\sum q_{j,t} p_{j,t}}{\sum q_{j,0} p_{j,t}}} = \frac{\sum p_{j,t} q_{j,t}}{\sum p_{j,0} q_{j,0}} = V_{0,t}$$

gross relation between government debt and labour taxes, with each variable exhibiting an inverse link with relative prices that is more moderate in magnitude. The bottom panel of the table also shows the correlations between relative prices and each of public debt and labour taxes after netting out the potential effects of other covariates present in our model. The partial correlations reveal a more pronounced negative association that is this time statistically significant even at the 1 per cent level.

**Table 1: Pairwise Gross and Partial Correlations**

	<i>Rel. Price</i>	<i>Public Debt</i>	<i>Labour Tax</i>
<i>Level Correlation</i>			
Rel. Price	1.00		
	–		
Public Debt	–0.77 [0.00]	1.00	
		–	
Labour Tax	–0.43 [0.02]	0.83 [0.00]	1.00
			–
<i>Rank Correlation</i>			
Rel. Price	1.00		
	–		
Public Debt	–0.65 [0.00]	1.00	
		–	
Labour Tax	–0.32 [0.09]	0.88 [0.00]	1.00
			–
<i>Partial Correlation</i>			
Public Debt	–0.81 [0.00]		
Labour Tax	–0.66 [0.00]		

*Notes:* The first two panels of the table provide gross correlations. The top panel gives the Pearson correlation coefficients while the middle panel shows the Spearman correlation coefficients. Note that the latter is less sensitive to outlier observations and will indicate a perfect correlation when the two variables in question are perfectly monotonically related, even if their relation is not linear. The partial correlations in the bottom panel are obtained by fitting a linear regression of the relative non-tradable price on relevant covariates. The partial correlation coefficient is then calculated as  $\frac{t}{\sqrt{t^2 + n - k}}$  where  $t$  is the corresponding

$t$ -statistic,  $n$  is the number of observations, and  $k$  is the number of independent variables, including the constant. P-values are given in square brackets.

**Table 2: Bounds Test Results**

	(1)	(2)	(3)	(4)
Labour Tax	✓		✓	
Govt. Debt		✓		✓
Govt. Consumption	✓	✓	✓	✓
Govt. Investment	✓	✓	✓	✓
Prod. Differential	✓	✓	✓	✓
Trade Balance	✓	✓	✓	✓
Terms of Trade			✓	✓
F-statistic	5.34	10.98	4.38	7.80
I(0)a	3.41	3.41	3.15	3.15
I(1)a	4.68	4.68	4.43	4.43
I(0)b	2.62	2.62	2.45	2.45
I(1)b	3.79	3.79	3.61	3.61
I(0)c	2.26	2.26	2.12	2.12
I(1)c	3.35	3.35	3.23	3.23

*Notes:* The letters a, b and c correspond to the 1, 5 and 10 per cent critical value bounds respectively. The null hypothesis is that no levels relation exists. The bounds cover all possible classifications of regressors into  $I(0)$ ,  $I(1)$  and mutually cointegrated processes, with lower and upper bounds matching the polar cases of purely  $I(0)$  and purely  $I(1)$  forcing variables. Lag length of bounds test is restricted to 1. Asymptotic critical values in the case of an unrestricted intercept and no trend are adopted from Pesaran *et al.* (2001).

## V RESULTS

For different groups of variables, Table 2 displays the results of the bounds test for the existence of a cointegrating relation. In particular, the computed F-statistics and critical value bounds covering all possible classifications of regressors are provided.<sup>23</sup> Regarding the latter, the lower bound values assume that the forcing variables are purely  $I(0)$  while the upper bound values treat the regressors as purely  $I(1)$ , with letters *a*, *b* and *c* corresponding to 1, 5 and 10 per cent significance levels respectively. As can be observed from the table, all four F-statistics lie outside the 0.05 critical value bounds. Moreover, three of the four are found to lie above the 0.01 upper bound. Thus, we are able to reject the null hypothesis that there is no levels relative price equation. Therefore, we can infer that the variables are  $I(1)$  amongst which a common stochastic trend is shared.<sup>24</sup>

<sup>23</sup> Asymptotic critical values are adopted from Pesaran *et al.* (2001) and correspond to the case of an unrestricted intercept and no trend.

<sup>24</sup> Applying standard unit root tests to individual variables also revealed non-stationarity across series.

Table 3 provides the empirical equivalents of Equation (14), showing the long-run coefficient estimates for the relative price of non-traded goods. In all four columns, the dependent variable is the natural logarithm of the relative price. In relation to covariates, logarithms of relative non-traded productivity and the terms

**Table 3: Long-Run Equation Estimates, Ireland 1980-2007**

	(1)	(2)	(3)	(4)
Labour Tax	-2.90 (1.03)**		-2.12 (1.05)*	
Govt. Debt		-0.48 (0.13)***		-0.40 (0.16)**
Govt. Consumption	7.90 (1.69)***	8.21 (1.42)***	7.64 (1.45)***	7.98 (1.37)***
Govt. Investment	-14.02 (2.87)***	-18.31 (3.20)***	-13.11 (2.53)***	-16.91 (3.50)***
Prod. Differential	-1.10 (0.10)***	-0.97 (0.09)***	-1.06 (0.09)***	-0.98 (0.09)***
Trade Balance	-1.39 (0.53)**	-1.47 (0.41)***	-1.42 (0.45)***	-1.49 (0.39)***
Terms of Trade			-0.58 (0.46)	-0.36 (0.46)
<i>ECM</i>				
Gap(-1)	-0.44 (0.09)***	-0.52 (0.09)***	-0.51 (0.10)***	-0.55 (0.10)***
Half-Life	1.20	0.94	0.97	0.87

*Notes:* The dependent variable is the logarithm of relative non-traded goods prices. The productivity differential and terms of trade appear in logs, while all other regressors are in original levels. Long-run estimates are backed out of the estimated ARDL specification given in Equation (23). The ARDL order employed is selected by both the Schwarz-Bayesian and Akaike Information criteria using a maxlag of 1. Standard errors are given in parentheses. Long-run standard errors are constructed using the delta method. Half-life is measured in years. Asterisks \*\*\*, \*\*, \* indicate significance at 1, 5 and 10 per cent levels respectively.

of trade are employed, with all other variables remaining untransformed. Note that, in contrast to the theoretical equivalent, the estimated specifications incorporate the sector-specific productivity differential  $\ln\left(\frac{A_N}{A_T}\right)$  as opposed to separate productivities  $\ln(A_N)$  and  $\ln(A_T)$  in light of the limited number of degrees of freedom. Columns (1) and (2) alternate between the use of labour taxation and public debt in the long-run equation, while columns (3) and (4) expand these specifications with the terms of trade.

Noting the estimates, government debt enters significantly into the formulation and is accompanied by the expected sign. Column (2) suggests that a 1 per cent of GDP rise in start of period public debt is associated with a long-run decrease of 0.48 per cent in relative non-traded goods prices. Augmenting the specification with the terms of trade reduces the estimate to  $-0.40$ , with the coefficient still statistically significant at the 5 per cent level. Replacing public debt with the implicit labour tax rate provides evidence in favour of the core hypothesis: for a given level of government spending, public debt is relevant if taxation matters for the determination of relative prices. Column (1) indicates that a 1 percentage point rise in the effective labour tax rate is related to a  $-2.90$  per cent change in relative prices. Inclusion of the terms of trade in column (3) yields a marginally weaker economically and statistically significant result. Thus, according to our model, this is consistent with relative factor intensities that satisfy  $\alpha_L\beta_Z > \beta_L\alpha_Z$ , such that the relative supply of non-traded goods responds positively.

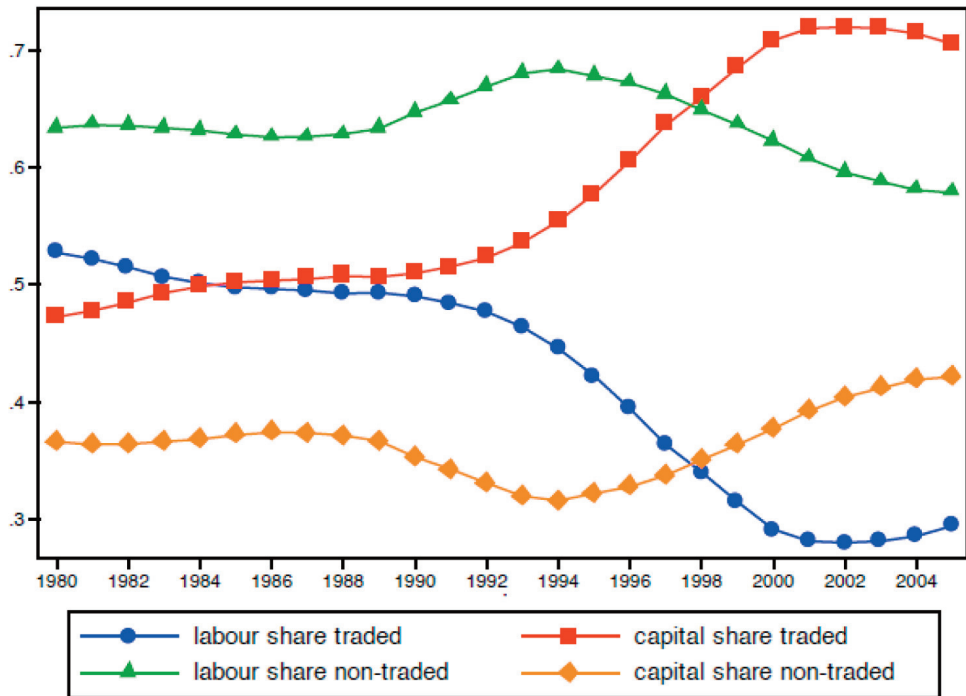
Using EU KLEMS data on factor compensations which assume constant returns to scale in conventional inputs, Figure 2 plots the labour and capital shares across sectors for Ireland. The figure illustrates that the manufacturing sector has been more capital intensive while the services sector has been more labour intensive, a trend suggesting that our result of a fall in relative prices is more likely to coincide with the scenario of  $\alpha_L < \beta_L$  and  $\alpha_K > \beta_K + c$ .<sup>25</sup> More generally, Obstfeld and Rogoff (1996), Piketty (2014), Alvarez-Cuadrado *et al.* (2015) and Lawrence (2015), among other studies in the literature, emphasise that employment and labour's income share in manufacturing have been declining across advanced economies. For the US, Lawrence (2015) notes that, although relatively stable up until 1987, labour's share in manufacturing has fallen below that of non-manufacturing. On the other hand, despite a similar decline, Alvarez-Cuadrado *et al.* (2015) report that the labour share in manufacturing has on average been higher than that in services over the period 1970–2007 for a group of developed countries excluding Ireland, with the reverse holding for capital's share. Thus, it is worth noting that an extended analysis across OECD countries may very well reveal heterogeneous effects.

Turning attention to the remaining regressors, all coefficients apart from those pertaining to the terms of trade are statistically significant at the 5 per cent level, exhibiting good congruence with the results of Galstyan and Lane (2009b). The average coefficients on government consumption and investment across the four columns are quite sizable and stand at 7.93 and  $-15.59$  respectively. Therefore, both the level and composition of government spending matter for the evolution of the relative price of non-tradables. Government consumption expenditure typically falls more heavily on the non-traded sector, thus engendering a rise in relative prices.

<sup>25</sup> In other words, it is a trend that broadly coincides with the empirical result if maintained once the public capital stock is introduced with  $\alpha_Z, \beta_Z \neq 0$ .



**Figure 2: Labour and Capital Shares, 5-year moving averages, 1980-2005**

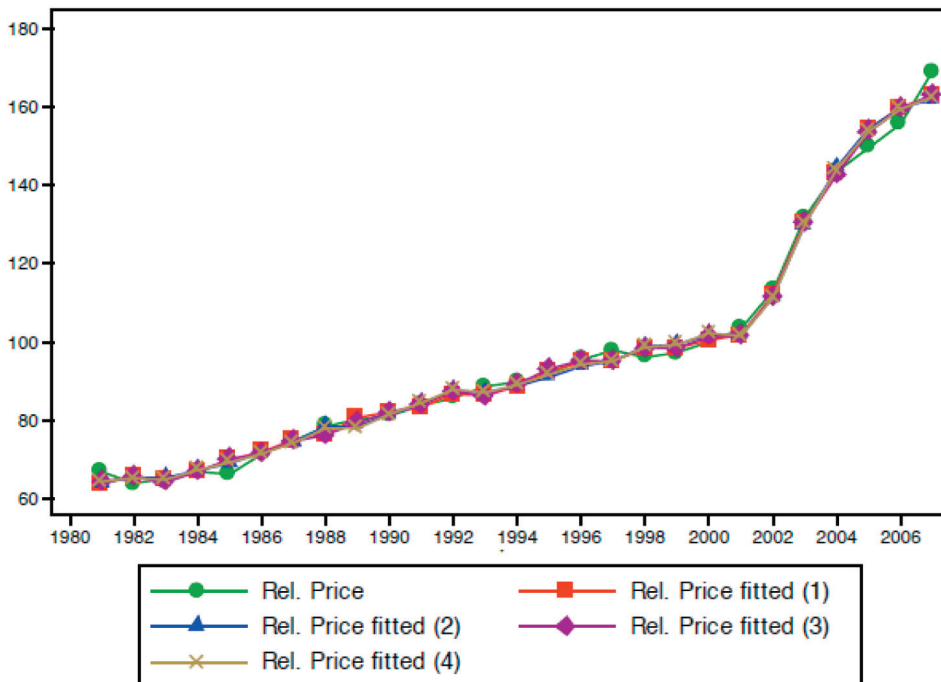


Notes: The source for the factor compensations and value added is EU KLEMS. The sectoral labour and capital shares can be defined as  $\frac{w_\nu L_\nu}{P_\nu Y_\nu} = \frac{\sum_{i \in I_\nu} w_{\nu,i} L_{\nu,i} P_{\nu,i} Y_{\nu,i}}{P_\nu Y_\nu}$  and  $\frac{r_\nu K_\nu}{P_\nu Y_\nu} = \frac{\sum_{i \in I_\nu} r_{\nu,i} K_{\nu,i} P_{\nu,i} Y_{\nu,i}}{P_\nu Y_\nu}$  where  $P_\nu Y_\nu = \sum_{i \in I_\nu} P_{\nu,i} Y_{\nu,i}$ ,  $\nu \in [N, T]$  and  $I_\nu = \{1, 2, \dots, m_\nu\}$  represents the subsectors of sector  $\nu$ .

Conversely, as originally documented in Galstyan and Lane (2009a), the effects of a long-run increase in public investment are more equivocal. While an increase in government investment that propagates a productivity gain in the traded sector may result in a relative price increase, public investment that disproportionately raises productivity in the non-traded sector may ultimately manufacture a relative price decline. Our estimates are in line with the latter scenario. Regarding the sectoral productivity differential, we obtain an average long-run estimate of  $-1.03$ , implying that a 1 per cent increase in non-traded sector productivity relative to traded sector productivity is associated with approximately a 1 per cent fall in the relative price of non-traded goods. Finally, the typical trade balance coefficient lies around  $-1.44$ , suggesting that trade balance surpluses have been associated with lower relative non-tradable prices.

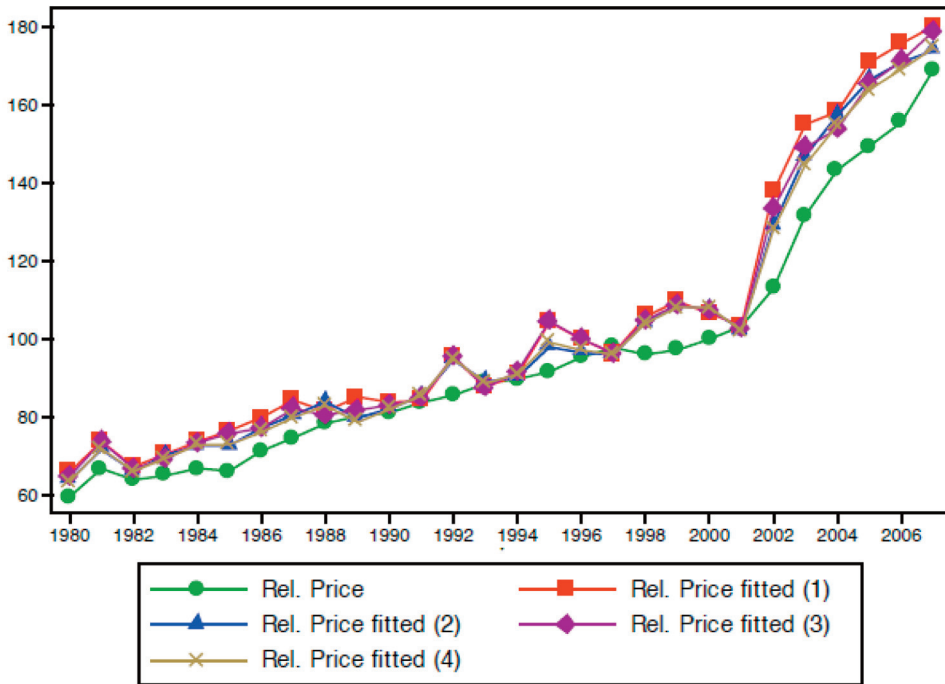
Figures 3 and 4 illustrate the overall performance of each of the estimated ARDL and corresponding long-run relative price equations across the four columns by plotting the actual and fitted series. The figures demonstrate that the estimated relative prices are able to track quite closely the positive trend in the actual relative price, thus implying that the models yield good fits. One can interpret the graph in Figure 4 as indicating that the actual relative price moves in accordance with the fundamentals-based equilibrium relative price path. Lastly, we provide some insight on the short-run dynamics of relative prices by reporting speeds of adjustment from the error-correction model. Examining the estimates, one can see that the gap coefficient stands at around  $-0.51$  and is statistically significant at the 1 per cent level, thus indicating that short-run relative price movements are inversely linked to deviations from the long-run relation. The magnitude of the coefficient implies that these misalignments diminish quite fast, with a half-life of less than one year assuming a monotonic rate of decay.

**Figure 3: Relative Price of Non-Tradables: Actual vs. ARDL Fitted Values**



Notes: Numbers in parentheses correspond to column numbers in Table 3.

**Figure 4: Relative Price of Non-Tradables: Actual vs. Long-Run Fitted Values**



Notes: Numbers in parentheses correspond to column numbers in Table 3.

## VI CONCLUSIONS

Our paper aims to empirically re-examine the determinants of relative prices in the long run in Irish data. The evidence provides support for our hypothesis that public debt and labour taxation play significant roles in the long run. Specifically, the findings indicate that higher public debt and higher effective labour taxation are associated with lower relative prices, and correspondingly a more depreciated real exchange rate.

In the context of a currency union, these results are quite pertinent given that fiscal policy is the primary macroeconomic tool over which control is retained by the member state. Moreover, the influence of public debt and taxes on the relative price structure in the economy can be invoked to explore the real exchange rate, and thus international competitiveness, implications. In particular, the assessment of real exchange rate misalignments is fundamental to the study of large external adjustment episodes.

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## APPENDIX A: THE SYSTEM

$$Y_T = A_T^* L_T^{\alpha_L + \alpha_K} \left( \frac{K_T}{L_T} \right)^{\alpha_K}$$

$$\alpha_L A_T^* L_T^{\alpha_L + \alpha_K - 1} \left( \frac{K_T}{L_T} \right)^{\alpha_K} = w$$

$$\beta_L P_N A_N^* L_N^{\beta_L + \beta_K - 1} \left( \frac{K_N}{L_N} \right)^{\beta_K} = w$$

$$L^\psi = \frac{1}{C} \frac{w}{P} (1 - \tau)$$

$$P = P_N^\gamma$$

$$Y_T = C_T + G_T - rN$$

$$L = L_N + L_T$$

$$Y_N = A_N^* L_N^{\beta_L + \beta_K} \left( \frac{K_N}{L_N} \right)^{\beta_K}$$

$$\alpha_K A_T^* L_T^{\alpha_L + \alpha_K - 1} \left( \frac{K_T}{L_T} \right)^{\alpha_K - 1} = r$$

$$\beta_K P_N A_N^* L_N^{\beta_L + \beta_K - 1} \left( \frac{K_N}{L_N} \right)^{\beta_K - 1} = r$$

$$C_T = \frac{1 - \gamma}{\gamma} P_N C_N$$

$$C = \frac{C_T^{1 - \gamma} C_N^\gamma}{(1 - \gamma)^{1 - \gamma} \gamma^\gamma}$$

$$Y_N = C_N + G_N$$

$$\tau w L = rD + G_T + P_N G_N$$

