INDOOR LOCATION ESTIMATION BASED ON TOA DATA AND BIAS ESTIMATION USING GAMMA REGRESSION

Atsushi Yoshida[*]**, Takenori Sakumura**[†] **and Toshinari Kamakura**[†]

ABSTRACT

We aim at improving the accuracy of indoor position estimation through a statistical approach. In this study, we propose a position estimation method based on Time-of-Arrival (ToA). ToA data are often useful. However, ToA data include a positive bias due to the reflection of radio waves. Therefore, it is difficult to estimate the TAG position from ToA data directly without an accurate bias correction. In this paper, we propose a maximum likelihood estimation method for the TAG position using gamma regression and a rotated distribution, and we show that the estimation with bias correction is more accurate than the estimation without bias correction. In addition, we show that our method also provides a confidence region for the TAG position.

1. Introduction

We aim at improving the accuracy of indoor position estimation through a statistical approach. Since there are prospects of utilization in fields of such as marketing science and location-based service, the study on indoor localization is important. In marketing science, it is considered for use to get an interesting product's shelf information for a customer and to analyze the customer's behavior in the store. In location-based service, it may become possible for a visitor to receive the navigation service anywhere in a shopping mall or a museum (Chong, Watanabe, and Inamura, 2006; Sahinoglu, Gezici and Guvenc, 2008).

The Global Positioning System (GPS) is well known as a typical localization method. However, its accuracy is at best within about 1 to 5 meters even if it is outside, and it not anticipated that further accuracy can be achieved in urban areas and the indoor environment. Hence, a method using a Wireless Sensor Network (WSN) is widely used for indoor localization. The localization method in WSN is mainly classified as either the *range-free* method or the *range-based* method. The *range-free* method estimates a position from a network-like view with many devices, and provides a much less accurate position than the *range-based* method that uses ranges between devices. Therefore, we use the latter in this study.

To estimate the location using the *range-based* method, we need to prepare several devices as anchor nodes (ANCHOR) and target nodes (TAG) and handle the distances between them. For measuring distance, there are various kinds of radio signal information to measure a distance, such as Received Signal Strength (RSS) (Okusa and Kamakura, 2015), Time-of-Arrival (ToA) (Kamakura and Okusa, 2013; Venkatraman, Caffery and You, 2004; Watabe and Kamakura, 2010), and Angle-of-Arrival (AoA). We have to select or

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[*]Graduate School of Science and Engineering, Chuo University, 1-13-27, Kasuga, Bunkyo-ku, Tokyo 112-8551, Japan   E-mail: `jundoll33@gmail.com`
[†]Faculty of Science and Engineering, Chuo University, 1-13-27, Kasuga, Bunkyo-ku, Tokyo 112-8551, Japan
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combine these sources of information to estimate the position of TAG. In this study, we propose a position estimation method based on ToA.

ToA is the radio signal information to estimate the distance between ANCHOR and TAG based on time. In this paper, we call a measurement method using the ToA information a "ToA method." This method measures the radio signal's travel time between an ANCHOR and TAG that are completely synchronized, and calculates the distance. For instance, when a radio signal is transmitted from TAG at time t_0 and it is received by ANCHOR at time t_1, the distance R is calculated as follows:

$$R = C(t_1 - t_0),$$

where C is the speed of light. Thus, if the distance was measured correctly, we can get the position of TAG using trilateration. However, since the distance often includes positive systematic error, not random error in the actual environment, it is difficult to estimate the accurate position of TAG.

The cause of the systematic error is in Non Line-of-Sight conditions, multipath and clock off (Go and Chong, 2015; Sahinoglu, *et al.*, 2008). In this paper, we call a difference between a true distance and a measured distance a *bias*, and focus on the bias.

In this paper, we devise a new rotated distribution to define the likelihood of the position of TAG, and propose a method to obtain the location of TAG based on maximum likelihood estimation using the estimated bias by the gamma regression model. In addition, our proposed method can also calculate a confidence region for the estimated position.

2. The proposed method - Indoor Location Estimation algorithm

We perform location estimation by a maximum likelihood method. The likelihood for a location of TAG is represented a density function defined over a 2-dimensional plane (x, y). Here, the ToA method provides only information for distances. We propose a new density function $q(x, y; \boldsymbol{\theta})$ rotated over the 2-dimensional plane (x, y) which is generated from the density function $p(t; \boldsymbol{\theta})$ $(t \geq 0)$ for the measured distances t;

$$q(x, y; \boldsymbol{\theta}) = \frac{1}{2\pi\mu(\boldsymbol{\theta})} p(\sqrt{x^2 + y^2}; \boldsymbol{\theta}),$$

where $\mu(\boldsymbol{\theta})$ represents the mean of the density function $p(t; \boldsymbol{\theta})$. This density function is given by calculating the volume of a solid of revolution generated from the underlying density function. The volume becomes $2\pi\mu(\boldsymbol{\theta})$ by Pappus's Centroid's theorem (see Goodman and Goodman (1969)). We call the above $q(x, y; \boldsymbol{\theta})$ the rotated distribution generated from the positive distribution $p(t; \boldsymbol{\theta})$. For instance, when $p(t)(= e^{-t})$ denotes an exponential distribution with rate parameter $\lambda = 1$, shown in Figure 1, the rotated distribution $q(x, y)$ will be obtained as Figure 2.

For the indoor location estimation our proposed method will be described as the following 4 steps.

Step 1: We consider the gamma regression model for the bias γ_{ijk} using the measured distance ℓ_{ijk}. Sahinoglu *et al.* (2008) mentioned the gamma distributions for the correction. We will use the gamma distribution regression model described as follows:

$$\log(E(\gamma_{ijk})) = \beta_0 + \beta_1 \ell_{ijk},$$
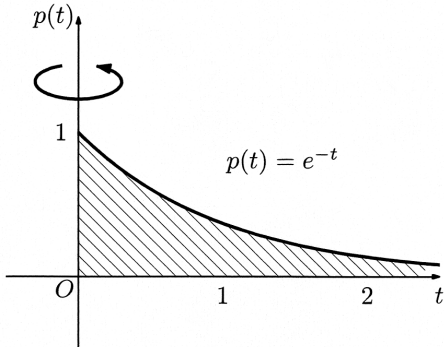
Fig. 1: Exponential distribution $p(t)$
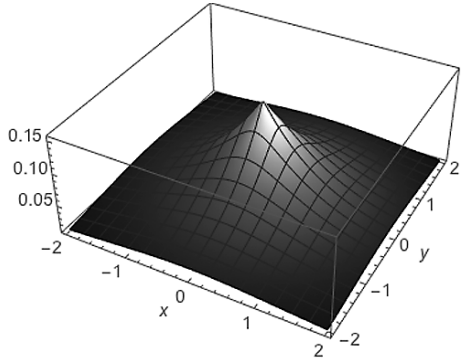
Fig. 2: A rotated distribution of $q(x, y)$

where β_0 and β_1 are unknown parameters, the bias γ_{ijk} is a positive value which subtracted the actual distance R_{jk} from the ith measured distance ℓ_{ijk} between the jth TAG and the kth ANCHOR ($i = 1, 2, \ldots, n_0$, $j = 1, 2, \ldots, J_0$, $k = 1, 2, \ldots, K_0$). We measure the distance under various conditions on the TAG position in advance, and estimate common parameters β_0, β_1 using the observed data for all ANCHORs by the maximum likelihood method. In that way, we construct the model for the relationship between measured distance and actual bias beforehand. Figure 3 shows the relationships between the measured distances and the actual biases, and the fitted gamma regression curve.
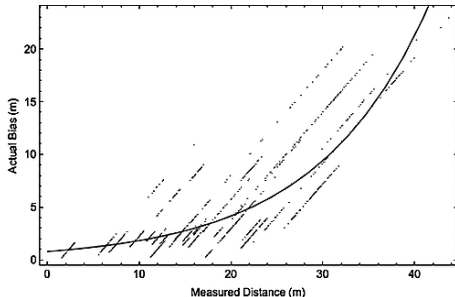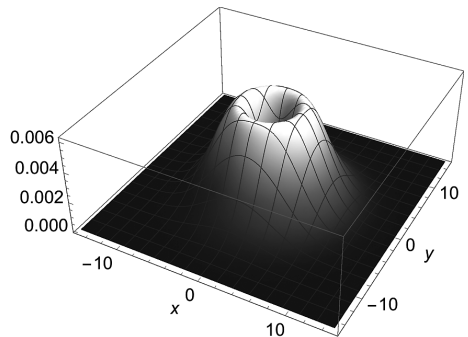

Fig. 3: Measured Distance vs. Actual Bias

Fig. 4: A rotated Weibull distribution
($m=2$, $\eta=5$, $x_0=0$, $y_0 = 0$, $\gamma = 0$)

Step 2: The observed distance data are assumed to follow a rotated Weibull distribution. The probability density function $f(x, y; x_0, y_0, m, \eta, \gamma)$ is given by the following equation

$$f(x, y; x_0, y_0, m, \eta, \gamma) = \frac{1}{2\pi\eta\Gamma(1+\frac{1}{m})} \left(\frac{m}{\eta}\right) \left(\frac{d(x,y,x_0,y_0)+\gamma}{\eta}\right)^{m-1} \exp\left\{-\left(\frac{d(x,y,x_0,y_0)+\gamma}{\eta}\right)^m\right\},$$

where x, y are latent location x-y coordinates, x_0, y_0 are location coordinates of an ANCHOR, $d(x, y, x_0, y_0)$ is the euclidean distance between (x, y) and (x_0, y_0), m is a shape parameter, η is a scale parameter, γ is the bias stated as a location parameter and $\Gamma(\cdot)$

is the gamma function. Figure 4 shows the probability density function about the rotated Weibull distribution with $m = 2$, $\eta = 5$ and $x_0 = y_0 = \gamma = 0$. The likelihood for the ith measured distance ℓ_{ik} between an unknown TAG and the kth ANCHOR $(i = 1, 2, \ldots, n, \ k = 1, 2, \ldots, K)$ will become

$$
\begin{aligned}
&L_1(m_k, \eta_k) \\
&= \prod_{i=1}^{n} f\left(x_{ik}, y_{ik}; x_k, y_k, m_k, \eta_k, \gamma_k\right) \\
&= \prod_{i=1}^{n} \frac{1}{2\pi\eta_k \Gamma(1 + \frac{1}{m_k})} \left(\frac{m_k}{\eta_k}\right) \left(\frac{d(x_{ik}, y_{ik}, x_k, y_k) + \gamma_k}{\eta_k}\right)^{m_k - 1} \exp\left\{-\left(\frac{d(x_{ik}, y_{ik}, x_k, y_k) + \gamma_k}{\eta_k}\right)^{m_k}\right\},
\end{aligned}
$$

where x_{ik}, y_{ik} are latent variables on ith measurement. In actual calculation, we can obtain maximum likelihood estimators $\hat{m}_k$ and $\hat{\eta}_k$ by maximizing L_1 in which $d(x_{ik}, y_{ik}, x_k, y_k) + \gamma_k$ is replaced with ℓ_{ik}.

Step 3: With the estimated parameters $\hat{m}_k$, $\hat{\eta}_k$ in Step 2, the TAG position $(x_{\text{TAG}}, y_{\text{TAG}})$ is defined as the parameters giving the maximum values of the following joint probability density function,

$$
L_2(x_{\text{TAG}}, y_{\text{TAG}}) = \prod_{k=1}^{K} f\left(x_{\text{TAG}}, y_{\text{TAG}}; x_k, y_k, \hat{m}_k, \hat{\eta}_k, \hat{\gamma}_k\right),
$$

where $(x_k, \ y_k)$ is the kth ANCHOR position and γ_k is estimated by the equation $\hat{\gamma}_k = \sum_{i=1}^{n} \exp\left(\hat{\beta}_0 + \hat{\beta}_1 \ell_{ik}\right)/n$. In this step, we regard $\hat{\gamma}_k$ as an estimator of the bias. The maximum likelihood estimator of the TAG position $(\hat{x}_{\text{TAG}}, \hat{y}_{\text{TAG}})$ is obtained by solving numerically the following simultaneous equations.

$$
F = \frac{\partial \log L_2}{\partial x_{\text{TAG}}} = 0,
$$
$$
G = \frac{\partial \log L_2}{\partial y_{\text{TAG}}} = 0.
$$

Step 4: Since the position $(\hat{x}_{\text{TAG}}, \hat{y}_{\text{TAG}})$ in Step 3 is a maximum likelihood estimator, a confidence region for the estimated position is calculated by Kamakura and Okusa (2013) as follows:

$$
(x - \hat{x}_{\text{TAG}}, \ y - \hat{y}_{\text{TAG}}) \left[H^T \hat{\Sigma} H\right]^{-1} \begin{pmatrix} x - \hat{x}_{\text{TAG}} \\ y - \hat{y}_{\text{TAG}} \end{pmatrix} \leq \chi_2^2(p),
$$

where $H^T \hat{\Sigma} H$ represents the asymptotic covariance matrix (ACov). The asymptotic variance (AVar) of the x-location is given by

$$
\text{AVar}[x_{\text{TAG}}(\hat{m}_1, \hat{\eta}_1, \ldots, \hat{m}_K, \hat{\eta}_K)] = \boldsymbol{h}_1^T \hat{\Sigma} \boldsymbol{h}_1,
$$
$$
\text{where } \boldsymbol{h}_1 = \left(\frac{\partial \hat{x}_{\text{TAG}}}{\partial \hat{m}_1} \ \frac{\partial \hat{x}_{\text{TAG}}}{\partial \hat{\eta}_1} \ \cdots \ \frac{\partial \hat{x}_{\text{TAG}}}{\partial \hat{m}_K} \ \frac{\partial \hat{x}_{\text{TAG}}}{\partial \hat{\eta}_K}\right)^T,
$$

and we substitute the estimators for the position and distribution parameters into these after solving for simplicity. Similarly, the AVar of the y-location is given by

$$\mathrm{AVar}[y_{\mathrm{TAG}}(\hat{m}_1, \hat{\eta}_1, \ldots, \hat{m}_K, \hat{\eta}_K)] = \boldsymbol{h}_2^T \hat{\Sigma} \boldsymbol{h}_2,$$
$$\text{where } \boldsymbol{h}_2 = \left(\frac{\partial \hat{y}_{\mathrm{TAG}}}{\partial \hat{m}_1} \ \frac{\partial \hat{y}_{\mathrm{TAG}}}{\partial \hat{\eta}_1} \ \cdots \ \frac{\partial \hat{y}_{\mathrm{TAG}}}{\partial \hat{m}_K} \ \frac{\partial \hat{y}_{\mathrm{TAG}}}{\partial \hat{\eta}_K} \right)^T.$$

Combining the above variances, the ACov is given by

$$\mathrm{ACov}[(\hat{x}_{\mathrm{TAG}}, \hat{y}_{\mathrm{TAG}})] = H^T \hat{\Sigma} H,$$
$$\text{where } H = \begin{pmatrix} \boldsymbol{h}_1 \\ \boldsymbol{h}_2 \end{pmatrix}^T = \begin{pmatrix} \frac{\partial \hat{x}_{\mathrm{TAG}}}{\partial \hat{m}_1} & \frac{\partial \hat{x}_{\mathrm{TAG}}}{\partial \hat{\eta}_1} & \cdots & \frac{\partial \hat{x}_{\mathrm{TAG}}}{\partial \hat{m}_K} & \frac{\partial \hat{x}_{\mathrm{TAG}}}{\partial \hat{\eta}_K} \\ \frac{\partial \hat{y}_{\mathrm{TAG}}}{\partial \hat{m}_1} & \frac{\partial \hat{y}_{\mathrm{TAG}}}{\partial \hat{\eta}_1} & \cdots & \frac{\partial \hat{y}_{\mathrm{TAG}}}{\partial \hat{m}_K} & \frac{\partial \hat{y}_{\mathrm{TAG}}}{\partial \hat{\eta}_K} \end{pmatrix}^T.$$

Furthermore, we write down the notations to obtain the ACov. These are calculated by a theorem on implicit functions. Because the simultaneous equations

$$\begin{cases} F = F(\hat{x}_{\mathrm{TAG}}, \hat{y}_{\mathrm{TAG}}, \hat{m}_1, \ldots, \hat{m}_K, \hat{\eta}_1, \ldots, \hat{\eta}_K) = 0 \\ G = G(\hat{x}_{\mathrm{TAG}}, \hat{y}_{\mathrm{TAG}}, \hat{m}_1, \ldots, \hat{m}_K, \hat{\eta}_1, \ldots, \hat{\eta}_K) = 0 \end{cases}$$

and

$$\Delta = \left| \frac{\partial(F, G)}{\partial(\hat{x}_{\mathrm{TAG}}, \hat{y}_{\mathrm{TAG}})} \right| \neq 0$$

exist, we can obtain the relation

$$\begin{cases} \hat{x}_{\mathrm{TAG}} = x(\hat{m}_1, \ldots, \hat{m}_K, \hat{\eta}_1, \ldots, \hat{\eta}_K) \\ \hat{y}_{\mathrm{TAG}} = y(\hat{m}_1, \ldots, \hat{m}_K, \hat{\eta}_1, \ldots, \hat{\eta}_K). \end{cases}$$

And then, with partial differentiation for F, G with respect to the distribution parameters m_k, η_k for $k = 1, \ldots, K$, one obtains the equations (1) and (2),

$$\begin{cases} \dfrac{\partial F}{\partial \hat{x}_{\mathrm{TAG}}} \dfrac{\partial \hat{x}_{\mathrm{TAG}}}{\partial \hat{m}_k} + \dfrac{\partial F}{\partial \hat{y}_{\mathrm{TAG}}} \dfrac{\partial \hat{y}_{\mathrm{TAG}}}{\partial \hat{m}_k} + \dfrac{\partial F}{\partial \hat{m}_k} = 0 \\ \dfrac{\partial G}{\partial \hat{x}_{\mathrm{TAG}}} \dfrac{\partial \hat{x}_{\mathrm{TAG}}}{\partial \hat{m}_k} + \dfrac{\partial G}{\partial \hat{y}_{\mathrm{TAG}}} \dfrac{\partial \hat{y}_{\mathrm{TAG}}}{\partial \hat{m}_k} + \dfrac{\partial G}{\partial \hat{m}_k} = 0, \end{cases} \tag{1}$$

$$\begin{cases} \dfrac{\partial F}{\partial \hat{x}_{\mathrm{TAG}}} \dfrac{\partial \hat{x}_{\mathrm{TAG}}}{\partial \hat{\eta}_k} + \dfrac{\partial F}{\partial \hat{y}_{\mathrm{TAG}}} \dfrac{\partial \hat{y}_{\mathrm{TAG}}}{\partial \hat{\eta}_k} + \dfrac{\partial F}{\partial \hat{\eta}_k} = 0 \\ \dfrac{\partial G}{\partial \hat{x}_{\mathrm{TAG}}} \dfrac{\partial \hat{x}_{\mathrm{TAG}}}{\partial \hat{\eta}_k} + \dfrac{\partial G}{\partial \hat{y}_{\mathrm{TAG}}} \dfrac{\partial \hat{y}_{\mathrm{TAG}}}{\partial \hat{\eta}_k} + \dfrac{\partial G}{\partial \hat{\eta}_k} = 0, \end{cases} \tag{2}$$

and the following notations are obtained by solving the simultaneous equations,

$$\frac{\partial \hat{x}_{\mathrm{TAG}}}{\partial \hat{m}_k} = \left| \frac{\partial(F, G)}{\partial(\hat{y}_{\mathrm{TAG}}, \hat{m}_k)} \right| \Big/ \Delta, \quad \frac{\partial \hat{y}_{\mathrm{TAG}}}{\partial \hat{m}_k} = - \left| \frac{\partial(F, G)}{\partial(\hat{x}_{\mathrm{TAG}}, \hat{m}_k)} \right| \Big/ \Delta,$$
$$\frac{\partial \hat{x}_{\mathrm{TAG}}}{\partial \hat{\eta}_k} = \left| \frac{\partial(F, G)}{\partial(\hat{y}_{\mathrm{TAG}}, \hat{\eta}_k)} \right| \Big/ \Delta, \quad \frac{\partial \hat{y}_{\mathrm{TAG}}}{\partial \hat{\eta}_k} = - \left| \frac{\partial(F, G)}{\partial(\hat{x}_{\mathrm{TAG}}, \hat{\eta}_k)} \right| \Big/ \Delta.$$

In addition, the following matrix is required to calculate the asymptotic covariance matrix,

$$\Sigma = \begin{pmatrix} I_1^{-1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & I_K^{-1} \end{pmatrix}.$$

Here, I_k is a Fisher information matrix of $\hat{m}_k$ and $\hat{\eta}_k$ as follows:

$$I_k = -E\left[\begin{pmatrix} \frac{\partial^2 \log L_1}{\partial^2 m_k} & \frac{\partial^2 \log L_1}{\partial m_k \partial \eta_k} \\ \frac{\partial^2 \log L_1}{\partial \eta_k \partial m_k} & \frac{\partial^2 \log L_1}{\partial^2 \eta_k} \end{pmatrix}\right].$$

3.   Numerical Experiments

3.1.   The Detail of Experiments

Figure 5 shows the environment of experiments. Gray boxes and white boxes show 4 ANCHORs and 18 TAGs respectively. We assume experiments in 4 ANCHORs located on 4 corners of the room and a specified TAG. We generated the independent sample ℓ_{ik} of TAG $(x_{\text{TAG}}, y_{\text{TAG}})$ for the kth ANCHOR (x_0, y_0) given by,

$$\ell_{ik} \sim Weib\left(shape = m_{\text{sim}},\ scale = \frac{d(x_{\text{TAG}}, y_{\text{TAG}}, x_0, y_0)}{\Gamma(1 + 1/m_{\text{sim}})}\right) + \gamma_{sim},$$

where, $m_{\text{sim}} = 5$, $\gamma_{\text{sim}} = 2$. In the parameter estimation of the gamma regression model, we used 100 pieces of the distance data between TAG and ANCHORs (i.e. $n_0 = 100$), and the information for the true locations of TAG.
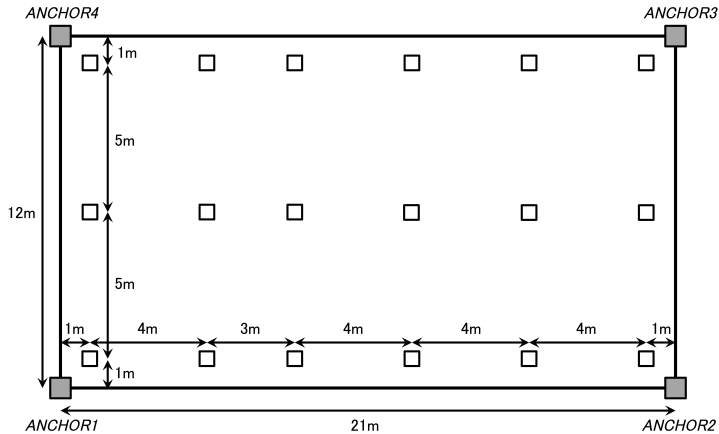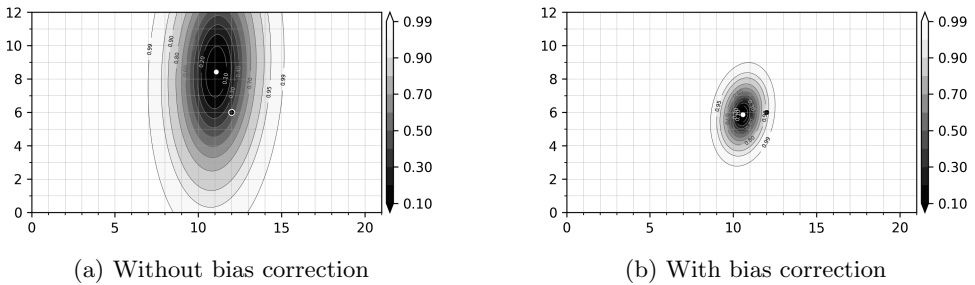

Fig. 5: The environment of experiments

3.2.   Results

The following Figures 6, 7 and 8 show the results of location estimation with the bias correction ($\hat{\gamma}_k = \sum_{i=1}^{n} \exp{(\hat{\beta}_0 + \hat{\beta}_1 \ell_{ik})}/n$) and without the bias correction ($\hat{\gamma}_k = 0$) in NLoS environments. In these figures, the true TAG position $(x_{\text{TAG}}, y_{\text{TAG}})$ is shown by a black dot, the estimated position $(\hat{x}_{\text{TAG}}, \hat{y}_{\text{TAG}})$ is shown by a white dot, and the contours of the confidence region are shaded using a gray scale. These are the estimation results when the sample size generated from each ANCHOR is 5 (i.e. $n = 5$). As seen, the estimated location with our proposed bias correction is closer to the true location than without the bias correction. Similarly in regard to the confidence region as well, we get good estimates. Table 1 shows the numerical results of location estimation for 18 patterns of the true TAG position with

(a) Without bias correction

(b) With bias correction
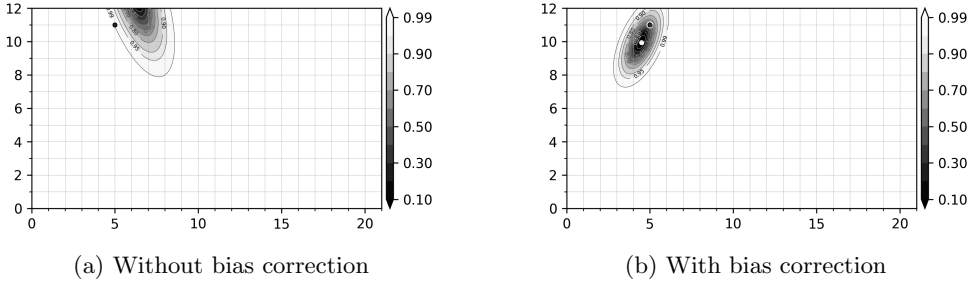
Fig. 6: The true TAG position $(x_{\text{TAG}}, y_{\text{TAG}}) = (5, 11)$

(a) Without bias correction

(b) With bias correction

Fig. 7: The true TAG position $(x_{\text{TAG}}, y_{\text{TAG}}) = (8, 6)$
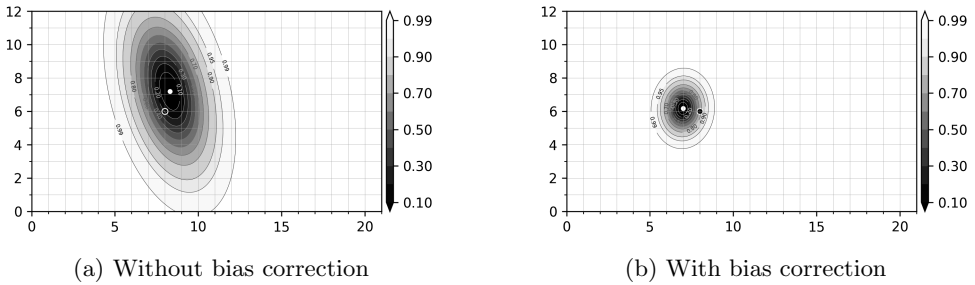
(a) Without bias correction

(b) With bias correction

Fig. 8: The true TAG position $(x_{\text{TAG}}, y_{\text{TAG}}) = (12, 6)$

the bias correction or without the bias correction. Table 2 shows the mean of the error distance (MED) and the standard deviation (SD) for two methods. From Table 1 and 2, it can be seen that our proposed method is more accurate for all results.

4. Case Study

We note results of the application for actual data in the same environment as the simulation case. The distance data are extracted from the data for three minutes excluding unexpected values. Figure 9 is one of the results, and shows that our method also gives good estimation using actual data. We can see that the estimated location with the bias correction is closer to the true location than without the bias correction, and the range of the confidence region with the bias correction is much smaller than without the bias correction.

Table 1: The results of estimated positions

	True	Without Correction		With Correction	
	position	position	error[m]	position	error[m]
1	(1, 1)	(3.35, 0.38)	2.433	(0.74, 1.80)	0.840
2	(1, 6)	(4.09, 6.27)	3.099	(0.92, 6.11)	0.136
3	(1, 11)	(3.23, 12.75)	2.829	(0.86, 10.32)	0.693
4	(5, 11)	(6.20, 12.87)	2.223	(4.50, 9.95)	1.164
5	(5, 6)	(6.25, 6.51)	1.351	(4.30, 6.17)	0.717
6	(5, 1)	(6.52, 0.63)	1.567	(4.56, 2.44)	1.510
7	(8, 1)	(8.30, -1.02)	2.040	(7.26, 2.57)	1.734
8	(8, 6)	(8.29, 7.20)	1.235	(6.99, 6.17)	1.024
9	(8, 11)	(7.92, 13.99)	2.994	(7.04, 9.88)	1.481
10	(12, 11)	(10.75, 14.86)	4.057	(10.46, 9.94)	1.868
11	(12, 6)	(11.07, 8.43)	2.599	(10.57, 5.88)	1.435
12	(12, 1)	(10.54, -3.27)	4.509	(10.93, 2.21)	1.610
13	(16, 1)	(14.37, -3.45)	4.739	(14.94, 1.30)	1.107
14	(16, 6)	(13.67, 6.81)	2.463	(14.13, 5.59)	1.917
15	(16, 11)	(14.12, 14.70)	4.153	(14.40, 10.53)	1.664
16	(20, 11)	(18.39, 14.68)	4.018	(18.74, 11.34)	1.301
17	(20, 6)	(15.67, 6.26)	4.334	(16.97, 5.69)	3.048
18	(20, 1)	(19.06, -3.04)	4.151	(18.89, 0.51)	1.216

Table 2: Mean error distance of the results

Method	MED[m]	SD[m]
without the bias correction	3.044	1.140
with the bias correction	1.359	0.624
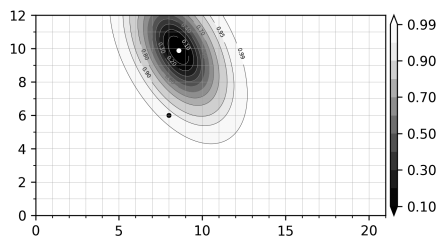
(a) Without bias correction
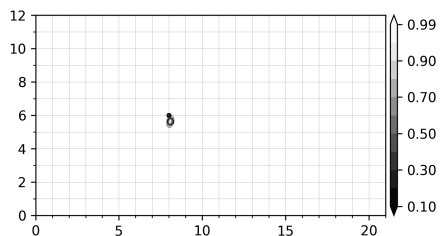
(b) With bias correction

Fig. 9: The results with the true TAG position $(x_{\mathrm{TAG}}, y_{\mathrm{TAG}}) = (8, 6)$ using actual data

Our method is able to obtain the location of TAG with higher accuracy. In addition, this result supports that the actual data contain a positive bias.

5. Conclusion

In order to estimate the spatial accurate location, ToA data are often useful. However, ToA data include a positive bias due to the reflection of radio waves. Therefore, it is difficult to estimate the TAG position from ToA data directly without an accurate bias correction. In this paper, we proposed a maximum likelihood estimation method for estimation of the TAG position using gamma regression and a rotated distribution. The result of a simulation that mimicked reality showed that the estimation with bias correction was more accurate than the estimation without bias correction. The case study also showed similar results to the simulation and supported the validity of our proposed method. In addition, we showed that our method also provides a confidence region for the TAG position.

Acknowledgements

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