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Mathematical Methods in Continuum Mechanics of Solids

Springer

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Preface

> Mechanics is the paradise of mathematical sciences, because with that one comes to the mathematical fruit.
>
> LEONARDO DI SER PIERO DA VINCI (1452–1519)

> Pure mathematicians sometimes are satisfied with showing that the non-existence of a solution implies a logical contradiction, while engineers might consider a numerical result as the only reasonable goal. Such one sided views seem to reflect human limitations rather than objective values.[1]
>
> RICHARD COURANT (1888–1972)

> To those who do not know mathematics it is difficult to get across a real feeling as to the beauty, the deepest beauty, of nature... If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in. ... It's an appreciation of the mathematical beauty of nature, of how she works inside.
>
> RICHARD PHILLIPS FEYNMAN (1918–1988)

> It is certain that continuum mechanics, which emerged during the 1950th, would not have experienced such rapid growth without amazing efficiency of the mathematical methods that were so well adapted to the problems formulated in the context of theoretical mechanics. [217]
>
> PAUL GERMAIN (1920–2009)

Mechanics have accompanied the development of mankind since ancient times and ultimately called for rational thinking, primarily based on more or less rigorous mathematical tools. And conversely, in this way, mechanics have served historically as a constant and probably main inspiration for mathematics. Such *Interaction between Mechanics and Mathematics*, being reflected also by the name of this Springer series, accelerated the development of modern mathematical tools in twentieth century when mathematics becomes relatively well applicable also to various nonlinear and coupled problems in mechanics and thermomechanics.

A very particular interaction has been developed between mechanics of continuum media and the theory of partial differential equations. Important concepts have been created on both sides, and a lot of mechanicians have become fairly applied mathematicians and vice versa.

In spite of such occasional and well-connecting "bridges", there are barriers between continuum mechanics and applied mathematics, caused probably partly by a

[1] See [129].

traditional way of education on specialized departments with very diverse professional accents. In this way, mechanical engineers and computational physicists do not support their models even by a very basic qualitative analysis and very typically use models whose solutions may even not exist under some generic circumstances. Usually, they also use approximations whose numerical stability and convergence are not granted and even sometimes there is a numerical evidence that they do not converge to any solution of a continuous model (in some cases because such solutions even may not exist). And quite often, prefabricated software packages are used without a solid knowledge what they really calculate, and such unrealistic trust and overuse of digital technology sometimes result in the deterioration of cognitive abilities of young generations.[2] A large portion of computational simulations performed in engineering and in physics, nowadays under euphemistic labels like “computational modeling”, “numerical modeling”, or “computational analysis”, have unintentionally moved rather to a position that can be called, with possibly a little exaggeration, a “computer-assisted science fiction”. On the other hand, this is also partly due to mathematicians because they often slide into very academic models, which have a little or no relevance for real-world demands, and into very particular results which, when in addition expressed in complicated mathematical language, are not understandable even for mathematically oriented mechanicians and physicists.

Viewed from the optimistic perspective, these (both historically developed and newly arising) barriers yield even more challenges both to design more applicable mathematics and to apply it to even more interesting (often of a “multi-character”)[3] problems in continuum (thermo)mechanics. It is certainly not possible to smear out these barriers by only a single book. Anyhow, this book aims at contributing at least a bit to make these barriers smaller. It tries to present selected basic mechanical concepts in the context of their (at least to some extent rigorous) mathematical handling, typically focused on existence of solutions of particular models possibly together with some additional attributes as smoothness or uniqueness, outlining their approximations that would suggest computationally implementable algorithms. To make it readable for engineers or computational physicists, advanced analytical tools and results are suppressed to a minimal reasonable level, most of them being only briefly exposed in the four Appendices (A–D) without proofs, and we intentionally avoid really “exotic” concepts like non-metrizable topologies, measures which are only finitely additive, or convergence in terms of nets instead of conventional sequences. We also reduce advanced analytical tools handling set-valued and nonsmooth mappings, except those which arise from convex analysis.

The primal focus is on static boundary-value problems arising in mechanics of solids at large or at small strains (Part I) and their various evolution variants (Part II). As the title already suggests, we intentionally exclude fluids, although some spots

[2] A certain general parallel and a possible origin of this phenomenon has been articulated by M. Spitzer [491].

[3] To advertise complexity of models from various aspects, the popular adjective “multi” is used in literature in connection of multi-scale, multi-component, multi-continuum, multi-phase, multi-field, multi-physics, multi-disciplinary, multi-functional, multi-ferroic, and, of course, multi-dimensional.

as Sects. 3.6, 5.7, 6.6, 7.6, or 8.6 have a slight relevance to fluid mechanics, too. This still represents a very ambitious plot, henceforth some (otherwise important) areas are omitted: in particular contact mechanics, i.e. phenomena like friction, adhesion, or wear will not be addressed. Also, homogenization methods for composite materials and various dimensional reductions of three-dimensional continua to two-dimensional plates, shells, or membranes, or one-dimensional beams, trusses, or rods are omitted, too. Numerical approximation as a wide and important area of computational mechanics is presented only in its very minimal extent. At this point, specialized monographs at pp. 473–478 as a further reading are advisable.

The book can serve as an advanced textbook and introductory scientific monograph for graduate or Ph.D. students in programs such as mathematical modeling, applied mathematics, computational continuum physics, or mechanical engineering. Henceforth, also some exercises (sometimes with solutions outlined on pp. 557–574) are involved, too. Besides, we believe that experts actively working in theoretical or computational continuum mechanics and thermomechanics of solids will find useful material here.

The book reflects both our experience with graduate classes within the program "Mathematical modeling" at Charles University in Prague taught during 2005–2018 and some other occasional teaching activities in this area,[4] reflecting also our own research[5] during the past several decades. Particular spots of the book benefit from our own computational activity in this area of continuum mechanics during many decades and from our collaboration with experts in mechanical engineering. The presented computer simulations have been provided by Barbora Benešová, José Reinoso, Jan Valdman, and Roman Vodička to whom we thus express our truly deep thanks. We are also deeply indebted to Katharina Brazda and Riccarda Rossi for careful reading of some chapters.

Eventually, we truly appreciate a constructive attitude of the Springer publisher allowing for printing this book essentially from our own pdf-file (only with reflecting a partial language corrections made in India[6]).

Prague, July 2018 *Martin Kružík & Tomáš Roubíček*

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[4] In particular, it concerns regular courses taught by M.K. at Technical University München in 2006-8 and at University of Würzburg in 2016, and by T.R. at University Vienna in 2017. Also, it concerns short intensive courses by T.R. about damage and plasticity at SISSA, Trieste, in 2015 and at Humboldt University Berlin in 2016.

[5] At this occasion, we would like to acknowledge the support by the Czech Science Foundation under grants 16-03823S "Homogenization and multi-scale computational modeling of flow and nonlinear interactions in porous smart structures", 16-34894L "Variational structures in thermomechanics of solids", 17-04301S "Advanced mathematical methods for dissipative evolutionary systems", 18-03834S "Localization phenomena in shape memory alloys: experiments & modeling", "Large strain challenges in materials science", and 19-04956S "Dynamic and nonlinear behaviour of smart structures; modelling and optimization".

[6] After reducing the production procedure in India to minimum, the proofs and final corrections of our own files were thus accomplished in January 2019.

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Part II EVOLUTION PROBLEMS