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Reliability analysis of a linear consecutive 2-out-of-3 system in the presence of supporting device and repairable service station

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Abstract **—** This paper studies the reliability characteristics of a linear consecutive 2-out-of-3 cold standby repairable system operating with the help of a repairable external supporting device with preventive maintenance. A repairable service station is set aside to repair any failed unit. The system is analyzed using first order linear differential equation to develop the explicit expression for steady-state availability, busy period, profit function and mean time to system failure (MTSF). Based on assumed numerical values given to system parameters, graphical illustrations are given to highlight important results. Comparisons are performed to highlight the impact of preventive maintenance and found that the 2-out-of-3 cold standby system with preventive maintenance, supporting device and a repairable service station is better.
Keywords — supporting device, preventive maintenance, service station.

1. INTRODUCTION

Systems are usually studied with intention to the evaluation of their reliability measures in terms of mean time to system failure (MTSF), busy period of repairman, availability and generated revenue. There exist systems that cannot work without the help of external supporting devices connect to such systems. These external supporting devices are systems themselves that are frowned to failure as such require preventive maintenance to improve their reliability. One of the forms of redundancy is the *k*-out-of-*n* system which has wide application in industrial setting. Moreover, the *k*-out-of-*n* works if and only if at least *k* of the *n* components work. The *k*-out-of-*n* reliability system is one of the most popular and widely used systems in practice. Example of the *k*-out-of-*n* system can be seen in a communication system with three transmitters and the average message load may be such that at least two transmitters must be operational at all times or critical messages may be lost. Thus, the transmission subsystem functions as a 2-out-of-3: G system. One of the form of *k*-out-of-*n* is 2-out-of-3 redundant systems under different assumptions. Examples of 2-out-of-3 redundancy can be seen in aircrafts, nuclear plants, satellites, electric generators, computer systems, power plants, manufacturing systems, and industrial systems. Improving the reliability of such systems with their supporting device is vital in ensuring quality of products. The study of reliability of repairable systems is an important topic in engineering and operation research. System reliability is a very meaningful measure, and achieving required level of reliability and availability is an essential requisite. System reliability and availability depends on the system structure.

2. LITERATURE REVIEW

Reliability is vital for proper utilization and maintenance of any system. It involves technique for increasing system effectiveness through reducing failure frequency and maintenance cost. For this reason, many researchers have studied reliability problem of redundant systems such as *k*-out-of-*N* under different operational situations and circumstances in assessing their reliability characteristics. For example, Bhardwj and Chander (2007) dealt with reliability and cost benefit analysis of 2-out-of-3 redundant system with general distribution of repair and waiting time. Chander and Bhardwaj (2009) present the reliability and economic analysis of 2-out-of-3 redundant system with priority to repair. Chander and Bhardwaj (2009) present reliability modelling of 2-out-of-3 system subject to conditional arrival of server. Bharwaj and Malik (2010) performed MTSF of 2-out-of-3 cold standby system with probability of repair and inspection. Yusuf and Hussaini (2012) present the evaluation of reliability and availability characteristics of 2-out-of-3 standby system under a perfect repair condition.

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Adequate preventive maintenance practice will enable manufacturers, maintenance managers, reliability engineers to maximize output, system availability, and generated revenue, minimize cost, and assure ongoing quality of the parts being produced. Many research results have been reported on system reliability in the presence of preventive maintenance. This preventive maintenance may include inspection, preventive repair or replacement of system. These include Adhikary et al. (2013) who examined cost-effective preventive maintenance scheduling of coal-fired power plants, Haggag (2009) who examined the cost analysis of two unit cold standby system involving preventive maintenance, Mujahid and Abdurrahim (2010) who investigated the optimal preventive maintenance warranty policy for repairable products with periodically increasing failure rate, Mahmoud and Moshref (2010) analyzed a two unit cold standby system with hardware, human error and preventive maintenance, Nourelfath *et al* (2010) proposed an integrated model for production and preventive maintenance planning in multi state systems, Wang (2013) proposed integrated model of production planning and imperfect preventive maintenance policy for single machine system, Wu and Zuo (2010) examined the linear and non linear preventive maintenance model and Uemura *et al* (2010) analyzed the availability of an intrusion tolerant distributed server system with preventive maintenance. Yusuf (2013) performed comparative analysis between two redundant systems requiring supporting units for their operation while recently Yusuf *et al.* (2014) dealt with analysis of MTSF of two unit cold standby system with a supporting device and repairable service station. Extensive research works exist in the reliability and availability modelling and analysis of 2-out-of-3 system, there is a lack of quality-based modelling of 2-out-of-3 systems with their corresponding supporting devices in the reliability analysis. Existing literatures either ignores the impact of preventive maintenance on both the system and its supporting device or exclude the supporting device when the need for preventive maintenance arise. Such works laid emphasis of preventive maintenance to the system alone without paying much attention to the external supporting device. More sophisticated models of systems connected to an external supporting device should be developed to assist in reducing operating costs and the risk of a catastrophic breakdown.

In many practice applications, preventive maintenance is usually adopted for extending the availability and working time, reducing the average cost and increasing the revenue generated for a system. The problem considered in this paper is different from the work of discussed authors above.The purpose of this paper is to analyze the reliability measures of a 2-out-of-3 repairable system with supporting device for its operation.

The rest of the paper is organized as follows. Section 2 presents literature review of the study. Section 3 presents notation used in the study. Section 4 gives the description of the system. Section 5 is the deals model formulation. The results of our numerical simulations are presented and discussed in Section 6. The paper is concluded in Section 7.

3. NOTATIONS

$A_{V1}(\infty) / A^{*}_{V1}(\infty)$: Steady-state availability of the system with/without preventive maintenance

PF_1 / PF^{*}_2 : Profit of the system with/without preventive maintenance.

$MTSF_1 / MTSF^{*}_2$: Mean time to system failure of the system with/without preventive maintenance

$B_{V1} / B_{V2} / B_{V3} / B_{V4}$: Steady-state busy period due preventive maintenance/ failure of unit/failure of service station/failure of supporting device of system with preventive maintenance.

$B^{*}_{V2} / B^{*}_{V3} / B^{*}_{V4}$: Steady-state busy period due failure of unit/failure of service station/failure of supporting device of system without preventive maintenance.

S_k : State of the system, $k = 0,1,2,\ldots,10$

$P_k(t)$: Probability that the system is in state k at time $t \geq 0$.

PF_1 : Profit incurred to the system.

$B(\infty)$: Accumulated busy period due to unit failure

$F(\infty)$: Frequency of preventive maintenance

4. DESCRIPTION AND STATES OF THE SYSTEM

In this paper, a 2-out-of-3 redundant system is considered. The system is connected to one external supporting device for its operation. The system undergoes preventive maintenance before the failure while the system is working with δ and η as rate of going to and preventive maintenance rate respectively. When a unit failed with rate β_1, it is sent to a repairable service station where it repaired with rate α_1 and the standby unit is switch into operation. It is assumed that the switch from standby to operation is perfect. System failure occurs when two units failed. At the failure of external supporting device with failure and repair rate β_3 and α_3 respectively, the system is in idle. The service station can break down at rest or on the course of serving the failed unit with exponentially distributed break down rate of β_2 and repair rate of α_2 .

5. MODEL FORMULATIONS

5.1. Mean time to system failure calculation

Let system initiation be the instant when the system operation begins for the first time. The system and all its units are assumed to be new working correctly. It is well known that reliability analysis and evaluation is becoming an increasingly important subject in designing, operating and managing systems. Thus, it is important to develop mathematical models for the evaluation of system performance. Attention on 2-out-of-3 system connected to a supporting device is paid here, and the explicit expressions for system availability, busy period of repairman, profit function and the mean time to system failure that are use in testing the system effectiveness will be derive in the subsequently. Following Trivedi (2002), Wang and Kuo (2000) and Wang *et al.*(2006), the state-transition-rate diagram of the system is shown in Figure 1 below. The probability vector $P(t)$ of system is defined as: $P(t)=\left[P_0(t),P_1(t),P_2(t),P_3(t),...,P_{10}(t)\right]$

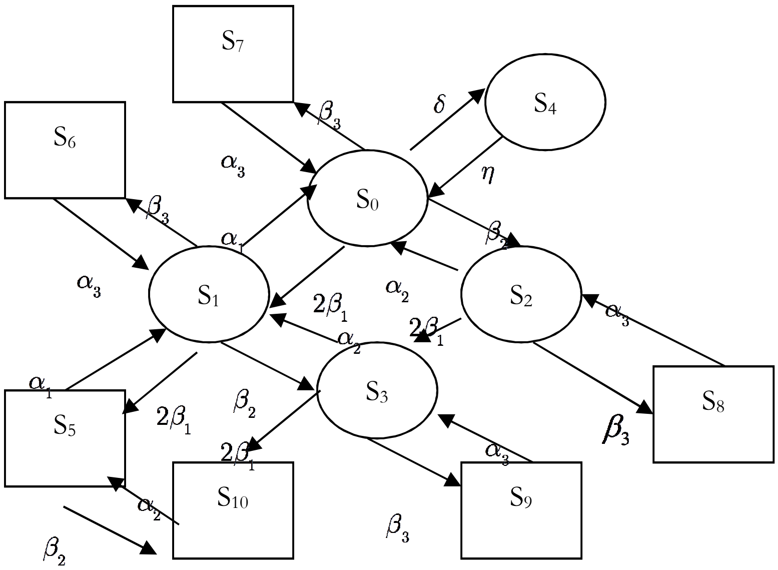

Figure 1: Schematic diagram of the System

State S_0: Unit I and II are working and the supporting device, unit III is in standby. The system is working.

State S_1: Unit I failed and is under attention of service station, unit II and III, and the supporting device are working. The system is working.

State S_2: Unit I and II are working and the supporting device, unit III is in standby and the service station breakdown. The system is working.

State S_3: Unit I failed and is waiting for repair, the service station breakdown and is under repair, unit II and III, and the supporting device are working. The system is working.

State S_4: The units and supporting device are under online preventive maintenance, service station is idle. The system is working

State S_5: Unit I failed and is waiting for repair, service station is busy repairing unit II, unit III and the supporting devices are idle. The system failed.

State S_6: Unit I failed and is under attention of service station, unit II and III are idle, the supporting device failed and is under repair. The system failed.

State S_7: Unit I and II and the service station are idle, unit III is in standby, the supporting device failed and is under repair. The system failed.

State S_8: Unit I and II are idle, unit III is in standby, the service station and supporting device have failed and are under repair. The system failed.

State S_9: Unit I failed and is waiting for repair, unit II and III are idle, and the supporting device and service station have failed and are under repair. The system failed.

State S_{10}: Unit I and II have failed and are waiting for repair, unit III and the supporting device are idle, the service station breakdown and is under repair. The system failed.

Relating the state of the system at time t and $t+dt$, the steady-state differential equations for the system obtained from Figure 1 above are as follow:

$$
\begin{aligned}
\frac{dP_0(t)}{dt} &= -t_1P_0(t)+\alpha_1P_1(t)+\alpha_2P_2(t)+\eta P_3(t)+\alpha_3P_7(t)\\
\frac{dP_1(t)}{dt} &= -t_2P_1(t)+2\beta_1P_0(t)+\alpha_2P_3(t)+\alpha_1P_5(t)+\alpha_3P_6(t)\\
\frac{dP_2(t)}{dt} &= -t_3P_2(t)+\beta_2P_0(t)+\alpha_3P_8(t)\\
\frac{dP_3(t)}{dt} &= -t_4P_3(t)+\beta_2P_1(t)+2\beta_1P_2(t)+\alpha_3P_4(t)\\
\frac{dP_4(t)}{dt} &= -\eta P_4(t)+\delta P_0(t)\\
\frac{dP_5(t)}{dt} & - t_5P_5(t)+2\beta_1P_1(t)+\alpha_2P_{10}(t)\\
\frac{dP_6(t)}{dt} &= -\alpha_3P_6(t)+\beta_3P_1(t)\\
\frac{dP_7(t)}{dt} &= -\alpha_3P_7+\beta_3P_0(t)\\
\frac{dP_8(t)}{dt} &= -\alpha_3P_8(t)+\beta_3P_2(t)\\
\frac{dP_9(t)}{dt} &= -\alpha_3P_9(t)+\beta_3P_3(t)\\
\frac{dP_{10}(t)}{dt} &= -\alpha_2P_{10}(t)+2\beta_1P_1(t)+\beta_2P_5(t)
\end{aligned}
\tag{1}
$$

This can be written in the matrix form as

$$P' = T_1P(t) \tag{2}$$

where

$$
T_1 = \begin{bmatrix}
-t_1 & \alpha_1 & \alpha_2 & 0 & \eta & 0 & 0 & \alpha_3 & 0 & 0 & 0\\
\beta_1 & -t_2 & 0 & \alpha_2 & 0 & \alpha_1 & \alpha_3 & 0 & 0 & 0 & 0\\
\beta_2 & 0 & -t_3 & 0 & 0 & 0 & 0 & 0 & \alpha_3 & 0 & 0\\
0 & \beta_2 & \beta_1 & -t_4 & 0 & 0 & 0 & 0 & 0 & \alpha_3 & 0\\
\delta & 0 & 0 & 0 & -\eta & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \beta_1 & 0 & 0 & 0 & -t_5 & 0 & 0 & 0 & 0 & \alpha_2\\
0 & \beta_3 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 & 0\\
\beta_3 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0\\
0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0\\
0 & 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & 0 & -\alpha_3 & 0\\
0 & 0 & 0 & \beta_1 & 0 & \beta_2 & 0 & 0 & 0 & 0 & -\alpha_2
\end{bmatrix},
$$

$$t_1=(\delta+2\beta_1+\beta_2+\beta_3),\ t_2=(\alpha_1+2\beta_1+\beta_2+\beta_3),\ t_3=t_4=(\alpha_2+2\beta_1+\beta_3),t_5=(\alpha_1+\beta_2)$$

It is difficult to evaluate the transient solutions, the procedure to develop the explicit expression for $MTSF_1$ is to delete the rows and fifth, sixth and column of absorbing state of matrix T_1 and take the transpose to produce a new matrix, say M . The expected time to reach an absorbing state is obtained from

$$E\left[T_{P(0)\to P(absorbing)}\right] = MTSF_1 = P(0)(-M^{-1})\begin{bmatrix}1 & 1 & 1 & 1 & 1\end{bmatrix}^T \tag{3}$$

where

$$M = \begin{bmatrix} -(\delta+2\beta_1+\beta_2+\beta_3) & \beta_1 & \beta_2 & 0 & \delta \\ \alpha_1 & -(\alpha_1+2\beta_1+\beta_2+\beta_3) & 0 & \beta_2 & 0 \\ \alpha_2 & 0 & -(\alpha_2+2\beta_1+\beta_3) & \beta_1 & 0 \\ 0 & \alpha_2 & 0 & -(\alpha_2+2\beta_1+\beta_3) & 0 \\ \eta & 0 & 0 & 0 & -\eta \end{bmatrix}$$

This method is successful because of the following:

$$E\left[T_{P(0)\to P(absorbing)}\right] = P(0)\int_0^\infty e^{Mt}\,dt \tag{4}$$

and

$$\int_0^\infty e^{Mt}\,dt = -M^{-1} \tag{5}$$

$$E\left[T_{P(0)\to P(absorbing)}\right] = MTSF_1 = \frac{N_6}{D_2} \tag{6}$$

$$\begin{aligned} N_6 = {} & \eta(\alpha_2^2\beta_1 + \alpha_1\alpha_2^2 + \alpha_2^2\beta_3 + \alpha_2\beta_1\beta_2 + \alpha_2\beta_2\beta_3 + 2\alpha_2\beta_3^2 + 2\alpha_1\alpha_2\beta_1 + 2\alpha_2\beta_1^2 + 4\alpha_2\beta_1\beta_3 + 2\alpha_1\alpha_2\beta_3 \\ & + 2\beta_1\beta_2\beta_3 + \beta_2\beta_3^2 + \beta_1^2\beta_2 + 3\beta_1\beta_3^2 + 2\alpha_1\beta_1\beta_3 + \alpha_1\beta_1^2 + 3\beta_1^2\beta_3 + \beta_3^3 + \alpha_1\beta_3^2) + \beta_1\eta(\alpha_2^2 + 2\alpha_2\beta_3 + \alpha_2\beta_2 \\ & + 2\alpha_2\beta_1 + \beta_1^2 + 2\beta_1\beta_3 + \beta_3^3) + \beta_2\eta(\alpha_2\beta_1 + \alpha_1\alpha_2 + \alpha_2\beta_3 + \beta_1\beta_2 + \beta_2\beta_3 + \alpha_1\beta_1 + \alpha_1\beta_3 + 2\beta_1\beta_3 + \beta_1^2 + \beta_3^2) \\ & + \beta_1\beta_2\eta(\alpha_2 + \beta_2 + 2\beta_3 + \alpha_1 + 2\beta_1) + \delta(\alpha_2^2\beta_1 + \alpha_1\alpha_2^2 + \alpha_2^2\beta_3 + \alpha_2\beta_1\beta_2 + \alpha_2\beta_2\beta_3 + 2\alpha_2\beta_3^2 + 2\alpha_1\alpha_2\beta_1 + 2\alpha_2\beta_1^2 \\ & + 4\alpha_2\beta_1\beta_3 + 2\alpha_1\alpha_2\beta_3 + 2\beta_1\beta_2\beta_3 + \beta_2\beta_3^2 + \beta_1^2\beta_2 + 3\beta_1\beta_3^2 + 2\alpha_1\beta_1\beta_3 + \beta_1^3 + \alpha_1\beta_1^2 + 3\beta_1^2\beta_3 + \beta_3^3 + \alpha_1\beta_3^2) \end{aligned}$$

$$\begin{aligned} D_2 = {} & 2\alpha_2\beta_1^2\beta_2 + 2\beta_1^3\beta_2 + 2\alpha_2\beta_2\beta_3^2 + \alpha_1\beta_1^2\beta_2 + \alpha_1\alpha_2^2\beta_3 + 2\alpha_1\alpha_2\beta_3^2 + 6\alpha_2\beta_1\beta_3^2 + 6\alpha_2\beta_1^2\beta_3 + 2\alpha_2^2\beta_1\beta_3 + \alpha_2^2\beta_1^2 \\ & + 2\alpha_2\beta_1^3 + 6\beta_1\beta_2\beta_3^2 + 6\beta_1^2\beta_2\beta_3 + 2\alpha_2\beta_3^3 + \alpha_1\beta_2\beta_3^2 + 4\beta_1^3\beta_3 + 6\beta_1^2\beta_3^2 + \alpha_1\beta_3^3 + 4\beta_1\beta_3^3 + 2\beta_1\beta_2^2\beta_3 + 2\alpha_1\beta_1\beta_2\beta_3 \\ & + \alpha_1\beta_1^2\beta_3 + 2\alpha_1\beta_1\beta_3^2 + \alpha_1\alpha_2\beta_2\beta_3 + \beta_1^2\beta_2^2 + \beta_2^2\beta_3^2 + 2\beta_2\beta_3^3 + 4\alpha_2\beta_1\beta_2\beta_3 + \alpha_2^2\beta_3^2 + \beta_1^4 + \beta_3^4 + 2\alpha_1\alpha_2\beta_1\beta_3 \end{aligned}$$

5.2. Availability, busy period and profit

For the availability, busy period and profit cases of Figure 1 using the initial condition in section 4 for this system, the differential equations in (1) can be expressed as

$$\begin{bmatrix} P_0' \\ P_1' \\ P_2' \\ P_3' \\ P_4' \\ P_5' \\ P_6' \\ P_7' \\ P_8' \\ P_9' \\ P_{10}' \end{bmatrix} = \begin{bmatrix} -t_1 & \alpha_1 & \alpha_2 & 0 & \eta & 0 & 0 & \alpha_3 & 0 & 0 & 0 \\ \beta_1 & -t_2 & 0 & \alpha_2 & 0 & \alpha_1 & \alpha_3 & 0 & 0 & 0 & 0 \\ \beta_2 & 0 & -t_3 & 0 & 0 & 0 & 0 & 0 & \alpha_3 & 0 & 0 \\ 0 & \beta_2 & \beta_1 & -t_4 & 0 & 0 & 0 & 0 & 0 & \alpha_3 & 0 \\ \delta & 0 & 0 & 0 & -\eta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & 0 & -t_5 & 0 & 0 & 0 & 0 & \alpha_2 \\ 0 & \beta_3 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 & 0 \\ \beta_3 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 \\ 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 \\ 0 & 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & \beta_2 & 0 & 0 & 0 & 0 & -\alpha_2 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \\ P_{10} \end{bmatrix} \tag{7}$$

The steady-state availability and busy periods can be obtained using the method below. In the steady-state, the derivatives of the state probabilities become zero which enable us to compute the steady-state probabilities with

$$A_{V1}(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) \tag{8}$$

$$B_{V1}(\infty) = P_4(\infty) \tag{9}$$

$$B_{V2}(\infty) = P_1(\infty) + P_3(\infty) + P_5(\infty) + P_{10}(\infty) \tag{10}$$

$$B_{V3}(\infty) = P_2(\infty) + P_3(\infty) \tag{11}$$

$$B_{V4}(\infty) = P_6(\infty) + P_7(\infty) + P_8(\infty) + P_9(\infty) \tag{12}$$

and (2) become

$$T_1 P(\infty) = 0 \tag{13}$$

which in matrix form as

$$\begin{bmatrix} -t_1 & \alpha_1 & \alpha_2 & 0 & \eta & 0 & 0 & \alpha_3 & 0 & 0 & 0 \\ \beta_1 & -t_2 & 0 & \alpha_2 & 0 & \alpha_1 & \alpha_3 & 0 & 0 & 0 & 0 \\ \beta_2 & 0 & -t_3 & 0 & 0 & 0 & 0 & 0 & \alpha_3 & 0 & 0 \\ 0 & \beta_2 & \beta_1 & -t_4 & 0 & 0 & 0 & 0 & 0 & \alpha_3 & 0 \\ \delta & 0 & 0 & 0 & -\eta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & 0 & -t_5 & 0 & 0 & 0 & 0 & \alpha_2 \\ 0 & \beta_3 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 & 0 \\ \beta_3 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 \\ 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 \\ 0 & 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & \beta_2 & 0 & 0 & 0 & 0 & -\alpha_2 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \\ P_9(\infty) \\ P_{10}(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{14}$$

Using the normalizing condition below:

$$\sum_{k=0}^{10} P_k(\infty) = 1 \tag{15}$$

Substituting (11) in any one of the redundant rows of (10) to give

$$\begin{bmatrix} -t_1 & \alpha_1 & \alpha_2 & 0 & \eta & 0 & 0 & \alpha_3 & 0 & 0 & 0 \\ \beta_1 & -t_2 & 0 & \alpha_2 & 0 & \alpha_1 & \alpha_3 & 0 & 0 & 0 & 0 \\ \beta_2 & 0 & -t_3 & 0 & 0 & 0 & 0 & 0 & \alpha_3 & 0 & 0 \\ 0 & \beta_2 & \beta_1 & -t_4 & 0 & 0 & 0 & 0 & 0 & \alpha_3 & 0 \\ \delta & 0 & 0 & 0 & -\eta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & 0 & -t_5 & 0 & 0 & 0 & 0 & \alpha_2 \\ 0 & \beta_3 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 & 0 \\ \beta_3 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 \\ 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 \\ 0 & 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \\ P_9(\infty) \\ P_{10}(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{16}$$

The solution of (12) gives the steady-state probabilities. The explicit expressions for availability and busy periods are given by

$$A_{V1}(\infty) = \frac{N_1}{D_1}$$

$$B_{V1}(\infty) = \frac{N_2}{D_1}$$

$$B_{V2}(\infty) = \frac{N_3}{D_1}$$

$$B_{V3}(\infty) = \frac{N_4}{D_1}$$

$$B_{V4}(\infty) = \frac{N_5}{D_1}$$

where

$$\begin{aligned} N_1 = {} & \alpha_1^2\alpha_2\alpha_3\eta(\alpha_2^2 + 2\alpha_2\beta_1 + \beta_1^2) + \alpha_1\alpha_2\alpha_3\beta_1\eta(\alpha_2^2 + \alpha_2\beta_2 + 2\alpha_2\beta_1 + \beta_1\beta_2 + \beta_1^2) + \alpha_1^2\alpha_2\alpha_3\beta_2\eta(\alpha_2 + \beta_1) \\ & + \alpha_1\alpha_2\alpha_3\beta_1\beta_2\eta(\alpha_1 + \alpha_2 + \beta_1 + \beta_2) + \alpha_1^2\alpha_2\alpha_3\delta(\alpha_1^2 + 2\alpha_2\beta_1 + \beta_1^2) \end{aligned}$$

$$N_2 = \alpha_1^2\alpha_2\alpha_3\delta(\alpha_2^2 + 2\alpha_2\beta_1 + \beta_1^2)$$

$$\begin{aligned} N_3 = {} & \alpha_1\alpha_2\alpha_3\beta_1\eta(\alpha_2^2 + \alpha_2\beta_2 + 2\alpha_2\beta_1 + \beta_1\beta_2 + \beta_1^2) + \alpha_1\alpha_2\alpha_3\beta_1\beta_2\eta(\alpha_1 + \alpha_2 + \beta_1 + \beta_2) + \alpha_2\alpha_3\beta_1^2\eta(\alpha_1\beta_2 \\ & + \alpha_2^2 + 2\alpha_2\beta_2 + 2\alpha_2\beta_1 + 2\beta_1\beta_2 + \beta_1^2 + \beta_2^2) + \alpha_3\beta_1^2\beta_2\eta(\alpha_1^2 + \alpha_1\beta_1 + 2\alpha_1\beta_2 + \alpha_1\alpha_2 + \beta_1^2 + 2\beta_1\beta_2 + \alpha_2^2 + \beta_2^2 \\ & + 2\alpha_2\beta_1 + 2\alpha_2\beta_2) \end{aligned}$$

$$N_4 = \alpha_1^2\alpha_2\alpha_3\beta_2\eta(\alpha_2 + \beta_1) + \alpha_1\alpha_2\alpha_3\beta_1\beta_2\eta(\alpha_1 + \alpha_2 + \beta_1 + \beta_2)$$

$$\begin{aligned} N_5 = {} & \alpha_2\alpha_3\beta_1^2\eta(\alpha_1\beta_2 + \alpha_2^2 + 2\alpha_2\beta_2 + 2\alpha_2\beta_1 + 2\beta_1\beta_2 + \beta_1^2 + \beta_2^2) + \alpha_1^2\alpha_2\beta_3\eta(\alpha_2^2 + 2\alpha_2\beta_1 + \beta_1^2) \\ & + \alpha_1^2\alpha_2\beta_2\beta_3\eta(\alpha_2 + \beta_1) + \alpha_1\alpha_2\beta_1\beta_3\eta(\alpha_2^2 + \alpha_2\beta_2 + 2\alpha_2\beta_1 + \beta_1\beta_2 + \beta_1^2) \end{aligned}$$

$$\begin{aligned} D_1 = {} & 2\alpha_1\alpha_2^2\beta_1\beta_2\beta_3\eta + \alpha_1\alpha_2\beta_1\beta_2^2\beta_3\eta + 2\alpha_1\alpha_2\beta_1^2\beta_2\beta_3\eta + 2\alpha_1^2\alpha_2\beta_1\beta_2\beta_3\eta + 2\alpha_1^2\alpha_2^2\beta_1\beta_3\eta + \alpha_1^2\alpha_2\beta_1^2\beta_3\eta \\ & + 2\alpha_1^2\alpha_2\alpha_3\beta_1\beta_2\eta + 4\alpha_2\alpha_3\beta_1^3\beta_2\eta + 2\alpha_1^2\alpha_2^2\alpha_3\beta_1\delta + \alpha_1\alpha_2\beta_1^3\beta_3\eta + 2\alpha_1\alpha_2^2\alpha_3\beta_1\beta_2\eta + \alpha_1\alpha_2\alpha_3\beta_1\beta_2^2\eta \\ & + 4\alpha_1\alpha_2\alpha_3\beta_1^2\beta_2\eta + \alpha_1^2\alpha_2^2\beta_2\beta_3\eta + \alpha_1^2\alpha_2\alpha_3\beta_1^2\delta + 3\alpha_2^2\alpha_3\beta_1^2\beta_2\eta + 3\alpha_2\alpha_3\beta_1^2\beta_2^2\eta + \alpha_1\alpha_2\alpha_3\beta_1^3\eta + 2\alpha_3\beta_1^3\beta_2^2\eta \\ & + \alpha_2\alpha_3\beta_1^4\eta + \alpha_3\beta_1^4\beta_2\eta + \alpha_1^2\alpha_2^3\alpha_3\delta + \alpha_1\alpha_3\beta_1^3\beta_2\eta + \alpha_1^2\alpha_2^3\beta_3\eta + \alpha_1^2\alpha_2^2\alpha_3\beta_2\eta + 2\alpha_1^2\alpha_2^2\alpha_3\beta_1\eta + \alpha_3\beta_1^2\beta_2^3\eta \\ & + \alpha_1^2\alpha_2\alpha_3\beta_1^2\eta + \alpha_1^2\alpha_3\beta_1^2\beta_2\eta + \alpha_1\alpha_2^3\beta_1\beta_3\eta + 2\alpha_1\alpha_2^2\beta_1^2\beta_3\eta + \alpha_1\alpha_2^3\alpha_3\beta_1\eta + 2\alpha_1\alpha_2^2\alpha_3\beta_1^2\eta + \alpha_2^3\alpha_3\beta_1^2\eta \\ & + 2\alpha_1\alpha_3\beta_1^2\beta_2^2\eta + 2\alpha_2^2\alpha_3\beta_1^3\eta + \alpha_1^2\alpha_2^3\alpha_3\eta \end{aligned}$$

The units, service station and supporting device are subjected to corrective maintenance and preventive maintenance as can be observed in states 2, 3,4,…,10 of Figure 1. Let C_0 and C_1 be the revenue generated when the system is in working state and no income when in failed state and cost of each repair (corrective maintenance), and overhaul (preventive maintenance) respectively. The expected total profit per unit time incurred to the system in the steady-state is

Profit=total revenue generated – cost incurred by the repair man due to preventive maintenance – cost incurred when repairing the failed units.

$$PF_1 = C_0 A_{v1}(\infty) - C_1(B(\infty) + F(\infty)) \tag{17}$$

where

PF_1 is the profit incurred to the system.
$B(\infty)$ is the accumulated busy period due to unit failure
$F(\infty)$ is frequency of preventive maintenance

5.3. Special case: 2-out-of-3 with supporting device and repairable service station without preventive maintenance

To highlight the importance of preventive maintenance, the system is treated here without the application of preventive maintenance.

$$A^*_{V2}(\infty) = \frac{N_1^*}{D_1^*} \tag{18}$$

$$B^*_{V2} = \frac{N_2^*}{D_1^*} \tag{19}$$

$$B^*_{V3} = \frac{N_3^*}{D_1^*} \tag{20}$$

$$B_{V4}^{*} = \frac{N_4^*}{D_1^*} \tag{21}$$

$$N_1^* = \alpha_1^2\alpha_2\alpha_3(\alpha_2^2 + 2\alpha_2\beta_1 + \beta_1^2) + \alpha_1\alpha_2\alpha_3\beta_1(\alpha_2^2 + 2\alpha_2\beta_1 + \alpha_2\beta_2 + \beta_1\beta_2 + \beta_1^2) + \alpha_1^2\alpha_2\alpha_3\beta_2(\alpha_2 + \beta_1) + \alpha_1\alpha_2\alpha_3\beta_1\beta_2(\alpha_1 + \alpha_2 + \beta_1 + \beta_2)$$

$$N_2^* = \alpha_1\alpha_2\alpha_3\beta_1(\alpha_2^2 + 2\alpha_2\beta_1 + \alpha_2\beta_2 + \beta_1\beta_2 + \beta_1^2) + \alpha_1\alpha_2\alpha_3\beta_1\beta_2(\alpha_1 + \alpha_2 + \beta_1 + \beta_2) + \alpha_2\alpha_3\beta_1^2(\alpha_1\beta_2 + \alpha_2^2 + 2\alpha_2\beta_1 + 2\alpha_2\beta_2 + \beta_2^2 + 2\beta_1\beta_2 + \beta_1^2) + \alpha_3\beta_1^2\beta_2(\alpha_1^2 + \alpha_1\alpha_2 + 2\alpha_1\beta_2 + \alpha_1\beta_1 + \alpha_2^2 + 2\alpha_2\beta_1 + 2\alpha_2\beta_2 + \beta_1^2 + \beta_2^2 + 2\beta_1\beta_2)$$

$$N_3^* = \alpha_1^2\alpha_2\alpha_3\beta_2(\alpha_2 + \beta_1) + \alpha_1\alpha_2\alpha_3\beta_1\beta_2(\alpha_1 + \alpha_2 + \beta_1 + \beta_2) + \alpha_3\beta_1^2\beta_2(\alpha_1^2 + \alpha_1\alpha_2 + 2\alpha_1\beta_2 + \alpha_1\beta_1 + \alpha_2^2 + \beta_1^2 + 2\alpha_2\beta_1 + 2\alpha_2\beta_2 + \beta_2^2 + 2\beta_1\beta_2)$$

$$N_4^* = \alpha_1^2\alpha_2\beta_3(\alpha_2^2 + 2\alpha_2\beta_1 + \beta_1^2) + \alpha_1\alpha_2\beta_1\beta_3(\alpha_2^2 + \alpha_2\beta_2 + 2\alpha_2\beta_1 + \beta_1\beta_2 + \beta_1^2) + \alpha_1\alpha_2\beta_1\beta_2\beta_3(\alpha_1 + \alpha_2 + \beta_1 + \beta_2) + \alpha_1^2\alpha_2\beta_2\beta_3(\alpha_2 + \beta_1)$$

$$D_1^* = \alpha\beta\beta_2 + 2\alpha_3\beta_1^3\beta_2^2 + \alpha_3\beta_1^2\beta_2^3 + \alpha_2^3\alpha_3\beta_1^2 + 2\alpha_2^2\alpha_3\beta_1^3 + \alpha_2\alpha_3\beta_1^4 + \alpha_1^2\alpha_2^3\alpha_3 + \alpha_1^2\alpha_2^3\beta_3 + 2\alpha_1^2\alpha_2^2\alpha_3\beta_1 + \alpha_1^2\alpha_2^2\alpha_3\beta_2 + \alpha_1^2\alpha_2\alpha_3\beta_1^2 + \alpha_1\alpha_2^3\alpha_3\beta_1 + 2\alpha_1\alpha_2^2\alpha_3\beta_1^2 + \alpha_1\alpha_2\alpha_3\beta_1^3 + 2\alpha_1^2\alpha_2\beta_1\beta_2\beta_3 + 2\alpha_1^2\alpha_2^2\beta_1\beta_3 + \alpha_1\alpha_2\beta_1^3\beta_3 + 2\alpha_1\alpha_2\beta_1^2\beta_2\beta_3 + \alpha_1\alpha_2\beta_1\beta_2^2\beta_3 + \alpha_1^2\alpha_2^2\beta_2\beta_3 + 2\alpha_1^2\alpha_2\alpha_3\beta_1\beta_2 + \alpha_1^2\alpha_2\beta_1^2\beta_3 + \alpha_1\alpha_2^3\beta_1\beta_3 + 2\alpha_1\alpha_2^2\alpha_3\beta_1\beta_2 + 2\alpha_1\alpha_2^2\beta_1^2\beta_3 + 2\alpha_1\alpha_2^2\beta_1\beta_2\beta_3 + \alpha_1\alpha_2\alpha_3\beta_1\beta_2^2 + 4\alpha_1\alpha_2\alpha_3\beta_1^2\beta_2 + 4\alpha_2\alpha_3\beta_1^3\beta_2 + \alpha_1\alpha_3\beta_1^3\beta_2 + 3\alpha_2^2\alpha_3\beta_1^2\beta_2 + \alpha_1^2\alpha_3\beta_1^2\beta_2 + 2\alpha_1\alpha_3\beta_1^2\beta_2^2 + 3\alpha_2\alpha_3\beta_1^2\beta_2^2$$

$$MTSF_2 = \frac{N_5^*}{D_2^*} \tag{22}$$

$$N_5^* = \alpha_1\alpha_2^2 + 2\alpha_1\alpha_2\beta_1 + 2\alpha_1\alpha_2\beta_3 + \alpha_1\beta_1^2 + 2\alpha_1\beta_1\beta_3 + \alpha_1\beta_3^2 + \alpha_2^2\beta_1 + 2\alpha_2\beta_1^2 + 4\alpha_2\beta_1\beta_3 + \beta_1^3 + 3\beta_1^2\beta_3 + \beta_3^3 + 3\beta_1\beta_3^2 + \alpha_2\beta_1\beta_2 + \alpha_2\beta_2\beta_3 + \beta_1^2\beta_2 + 2\beta_1\beta_2\beta_3 + \beta_2\beta_3^2 + \alpha_2^2\beta_3 + 2\alpha_2\beta_3^2 + \beta_1(\alpha_2^2 + 2\alpha_2\beta_1 + 2\alpha_2\beta_3 + \beta_1^2 + \beta_3^2 + 2\beta_1\beta_3 + \alpha_2\beta_2) + \beta_2(\alpha_1\alpha_2 + \alpha_1\beta_1 + \alpha_1\beta_3 + \alpha_2\beta_1 + \beta_1^2 + 2\beta_1\beta_3 + \beta_1\beta_2 + \beta_2\beta_3 + \alpha_2\beta_3 + \beta_3^2) + \beta_1\beta_2(\alpha_2 + 2\beta_1 + 2\beta_3 + \alpha_1 + \beta_2)$$

$$D_2^* = 6\beta_1\beta_2\beta_3^2 + 6\beta_1^2\beta_2\beta_3 + 2\beta_1\beta_2^2\beta_3 + \beta_2^2\beta_3^2 + \beta_1^2\beta_2^2 + \alpha_1\beta_2\beta_3^2 + 6\alpha_2\beta_1\beta_3^2 + 2\alpha_2\beta_1^2\beta_2 + 2\beta_1^3\beta_2 + 4\beta_1^3\beta_3 + \beta_1^4 + 2\alpha_1\beta_1\beta_2\beta_3 + \alpha_2^2\beta_1^2 + 2\alpha_2\beta_1^3 + 2\alpha_2^2\beta_1\beta_3 + \alpha_1\beta_1^2\beta_2 + 4\alpha_2\beta_1\beta_2\beta_3 + \alpha_2^2\beta_3^2 + 4\beta_1\beta_3^3 + 2\beta_2\beta_3^3 + \alpha_1\beta_3^3 + \beta_3^4 + 2\alpha_2\beta_3^3 + 2\alpha_2\beta_2\beta_3^2 + 2\alpha_1\alpha_2\beta_1\beta_3 + \alpha_1\alpha_2\beta_2\beta_3 + 6\beta_1^2\beta_3^2 + \alpha_1\beta_1^2\beta_3 + \alpha_1\alpha_2^2\beta_3 + 6\alpha_2\beta_1^2\beta_3 + 2\alpha_1\alpha_2\beta_3^2 + 2\alpha_1\beta_1\beta_3^2$$

6. RESULTS AND DISCUSSION

In this section, we numerically obtained the results for mean time to system failure, system availability, busy period and profit function for all the developed models. For the models analysis, the following set of parameters values are fixed throughout the simulations for consistency:

$\alpha_1 = 0.7$, $\alpha_2 = 0.8$, $\alpha_3 = 0.4$, $\beta_1 = 0.3$, $\beta_2 = 0.02$, $\beta_3 = 0.03$, $\delta = 0.8$, $\eta = 1$, $C_1 = 40{,}000$, $C_2 = 1{,}000$

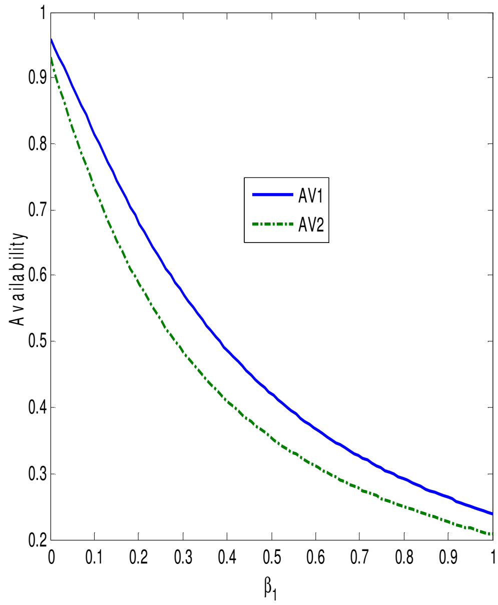

Figure 2: Availability against β_1

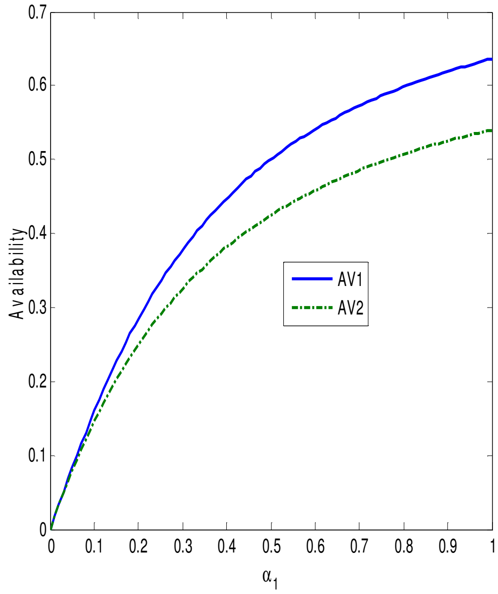

Figure 3: Availability against α_1

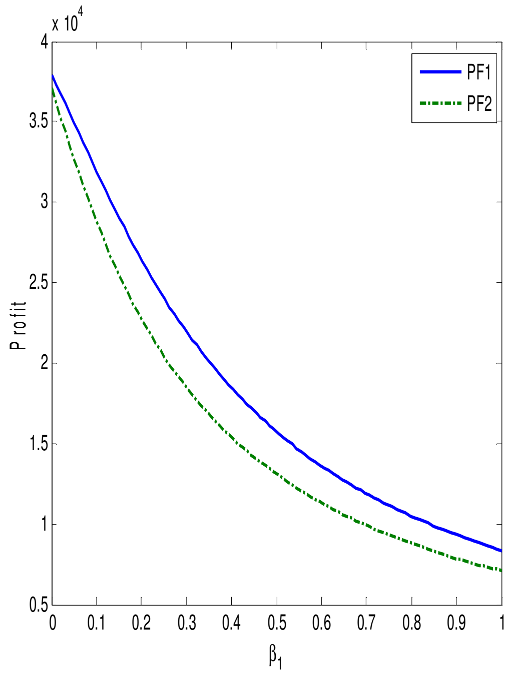

Figure 4: Profit against β_1

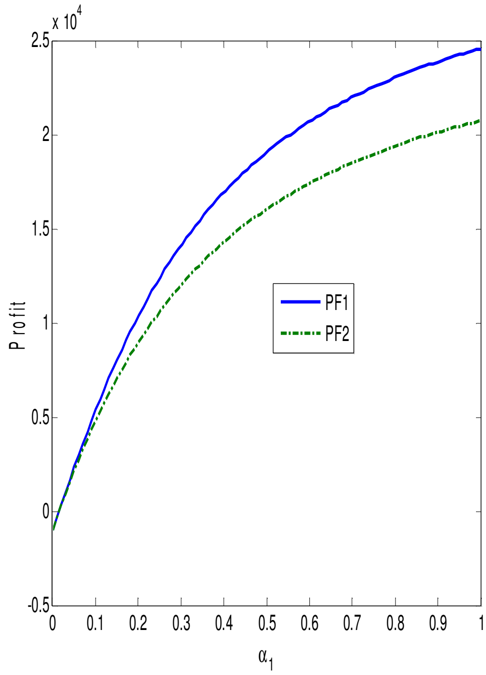

Figure 5: Profit against α_1

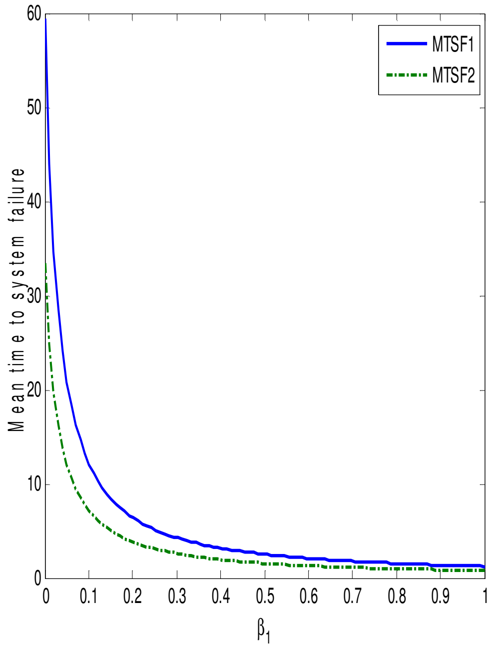

Figure6: MTSF against β_1

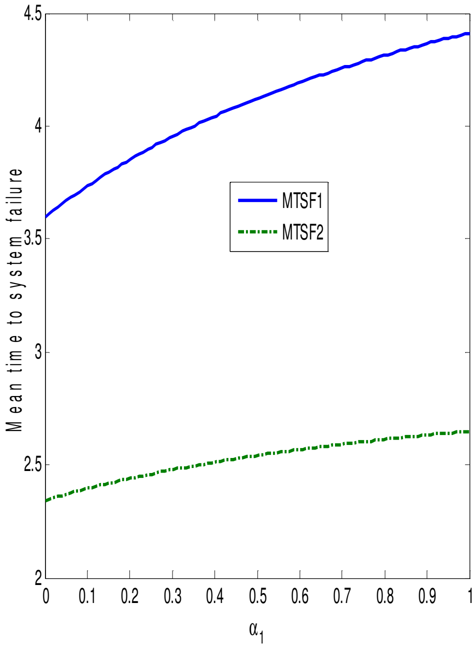

Figure 7: MTSF against α_1

Figures 2, 4 and 6 show the availability, profit and mean time to system failure results for the two systems being studied against the failure rate β_1 respectively. It is clear from the Figures that the 2-out-of-3 system with preventive maintenance has higher availability, profit and mean time to system failure as compared with the other system. The differences between availability, profit and mean time to system failure (MTSF) of the two systems widen as β_1, β_2 and β_3 increases. These Figures tend to suggest that the 2-out-of-3 system with preventive maintenance is better. Figures 3,5 and 7 show the availability, profit and mean time to system failure results for the two systems being studied against the repair rate α_1. It is evident from the Figures that the 2-out-of-3 system with preventive maintenance has higher availability, profit and mean time to system failure as compared with the other system. It is clear from the above Figures that the linear consecutive 2-out-of-3 system without preventive maintenance is decreasing faster than other with preventive maintenance. We can conclude as before that the linear consecutive 2-out-of-3 system with preventive maintenance is better than the other system without preventive maintenance. Thus,

$$A_{V1}(\infty) > A^{*}_{V2}(\infty)$$
$$PF_1 > PF^{*}_2$$
$$MTSF_1 > MTSF^{*}_2$$

7. CONCLUSION

In this paper, we analyzed reliability characteristics of linear consecutive 2-out-of-3 system with preventive maintenance in the presence of external supporting device and a repairable service station. Explicit expressions steady-state availability, busy period, profit function and mean time to system failure the system was derived and comparative analysis was also performed numerically to highlight the importance of preventive maintenance. It is evident from Figures 2-7 that the linear consecutive 2-out-of-3 system with preventive maintenance is better. Maintaining high or required level of reliability is often an essential requisite for improving system reliability. From the analysis, we conclude that system with preventive maintenance is better than the other systems without preventive maintenance. Maintenance managers, reliability engineers and system designers are faced with the challenges of competition and market globalisation on maintenance system to improve efficiency and reduce operational costs. Models developed in this paper are found to be highly beneficial to engineers, maintenance managers, system designers and plant management for proper maintenance analysis, optimal maintenance policy, decision, and evaluation of performance and for safety of the system as a whole. The results derived in this paper would be applied in practical fields by making some suitable modification and extensions. Further studies for such subject would be expected.The present study is important to engineers, maintenance managers, and plant management for proper maintenance analysis, decision and for safety of the system as a whole. The study will also assist engineers, decision makers and plant management to avoid an incorrect reliability assessment and consequent erroneous decision making which may lead to unnecessary expenditures, incorrect maintenance scheduling and reduction of safety standards.

There are several further research topics which will be studied in the future as follows. First, further work should be done to determine the impact of online and offline preventive maintenance. Second, 2-out-of-3 systems are more common in practice which components with the same failure rate. So modeling of load sharing 2-out-of-3 system where the failure rate differs will be address.

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