Journal of Fundamental and Applied Sciences

ISSN 1112-9867

Available online at http://www.jfas.info

Research Article

ANALYTIC TECHNIQUE TO DETERMINE THE ENERGY OF CONFORMAL ANTENNAS

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Received: 04 February 2021 / Accepted: 18 August 2021/ Published online: 01 September 2021

ABSTRACT

The conformal mapping (CM) mode is engaged for conformal antenna strategy using the geometric representation of the CM. For some conformal antennas (CAs) have the same shape, actuality able to segment a shared rough system, which is an essential improvement of this method in these CA optimizations (CAOs). Different parameters of CAs are optimized. The function reactions in this sample are selected as starlike distribution. The assessments of efficiency, gain and accuracy with the commercial software Mathematica 11.2 are prepared. The suggested method is mainly appropriate for CAO. Moreover, we formulate the energy operator in a 2D geometry, when the discharged field is detected over a semi-circumference in the far zone.

Keywords: Conformal mapping; open unit disk; starlike conformal function; conformal antenna; energy; univalent function; subordination and superordination

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doi: http://dx.doi.org/10.4314/jfas.v13i3.19

1. INTRODUCTION

A conformal map (CM) is a function that locally preserves angles, but not essentially lengths. It preserves both angles and the shapes of infinitesimally small data, but not certainly their magnitude or curve. The conformal property can be indicated in expressions of the Jacobian derivative matrix of a organize transformation. The transformation is conformal when the Jacobian at each point is a positive scalar time an alternation matrix (orthogonal matrix satisfies that the determinant is equal to one). Some researchers define CM to contain orientation-reversing mappings whose Jacobians can be inscribed as some scalar times of an orthogonal matrix. In the complex plane, CM is analytic univalent on the open unit disk.

Conformal antennas (CAs) are increasing attention in numerous requests in sensor and moveable communication structures. Array antennas, whose components are placed on a curved surface, may display some compensations, not only from the aerodynamically point of view, since they can survey the surface of pitchers or airliners, but also from the electrical one. In information, they can emit matching concentrating designs within higher angular areas, as necessary in some investigation (see [1-5]). Specifically, the diagnostics of CAs are necessary to resolve an inverse source problem, which of the renovation of source flows from the information on the energy field. Its result totals to capsizing a linear integral operator. Actually, the energy source and the energy field are associated with an integral operator and in order to properly attitude the problem, not only the energy operator but also the source and field functional spaces need to be detailed. In applying antenna scheme, full power transmission will happen when the antenna's response, impedance is used to a conjugate-impedance equal to the individual impedance of the communication line. Consequently, CAO would contain not only the energy properties, but also the impedance appearances. Here, we shall organize a starlike CA by using the geometric function theory such that antenna indicated on the cylinder (proposal model). We shall deal with the frequency, accuracy and gain of the model with comparison with the recent works. To complete our investigation, we present the energy operator of the system with analytic presentations.

2. METHODOLOGY

The function φ is titled conformal (or angle-preserving) at a point x_0 if it preserves angles between directed curves through x_0, as well as preserving orientation. Conformal maps preserve both shapes, angles and the small figures, but without fail their size or curvature. In this effort, we present a conformal mapping based on the geometric function theory. Suppose that Λ is the set of all analytic functions $\Upsilon \in \cup, \cup := \{\xi : |\xi| < 1\}$ (the open unit disk) and normalized by the conditions $\Upsilon(0)=0$ and $\Upsilon'(0)=1$ expanded by

$$\Upsilon(\xi) = \xi + \sum_{n=2}^{\infty} \Upsilon_n \xi^n, \xi \in \cup$$

A sub-class of Λ is the univalent functions (one-to-one) indicating by ***S.*** Moreover, a function $\Upsilon \in \cup$ is known as starlike in $\cup$, which represents by $\mathfrak{S}^*$, if and only if

$$\Re\left(\frac{\xi \Upsilon'(\xi)}{\Upsilon(\xi)}\right) > 0, \xi \in \cup$$

And a function $\Upsilon \in \cup$ is convex, which represents by ***C***, if and only if

$$1 + \Re\left(\frac{\xi \Upsilon''(\xi)}{\Upsilon'(\xi)}\right) > 0, \xi \in \cup$$

The category of univalent function is classified as a set of special conformal mappings in the open unit disk (for detail see [6]).

3. RESULT AND DISCUSSION

3.1. **Optimize Method**

A Our approach is based on the class of Briot-Bouquet differential equation of complex variable taking the formula (see [7])

$$\frac{\xi \Upsilon'(\xi)}{\alpha \Upsilon(\xi) + \beta} + \Upsilon(\xi) = E(\xi), \alpha \neq 0$$

where E represents to the error function, which is convex in the open unit disk and satisfying $E(0)=0$ and $\Upsilon(\xi)$ indicates the optimal coarse model design. By letting $\alpha = -1$ and $\beta=0$ with $0<Re(E)<1$ we have

$$\Upsilon(\xi)-\frac{\xi\Upsilon'(\xi)}{\Upsilon(\xi)}=E(\xi),\xi\in\cup.$$

The iteration formula becomes

$$E_{i+1}(\xi)=\left(\Upsilon(\xi)-\frac{\xi\Upsilon'(\xi)}{\Upsilon(\xi)}\right)_i,i=1,2,3,....$$

The process dismisses if |E| converts adequately slight or the scheme qualifications is fulfilled.

Example 1. Consider

$$\Upsilon(\xi)=\xi-\gamma\xi^2,$$

then we have

$$(\xi-\gamma\xi^2)-\left(\frac{\xi-2\gamma\xi^2}{\xi-\gamma\xi^2}\right)=E(\xi).$$

Solving the above equation for ξ, we arrive at $|\xi|=0.48$ is the value that implies $E(\xi)=0$

Now consider the function

$$\Upsilon(\xi)=\xi^2\Gamma(\xi),$$

where

$$\Upsilon(\xi)\approx\xi-\gamma\xi^2+0.989\xi^3.$$

To determine the error function, we have the following computation

$$(\xi-\gamma\xi^2+0.989\xi^3)-\left(\frac{\xi-2\gamma\xi^2+2.967\xi^3}{\xi-\gamma\xi^2+0.989\xi^3}\right)=E(\xi).$$

To minimize the function E, we have that $|\xi|$ is minimizing E.

3.2. **CA-Optimization**

This section deals with the conformal antenna shaped by the stralike conformal mapping graph. We consider the Cylindrical Conformal antenna and quasi-Cylindrical Conformal antenna (see Fig.1).

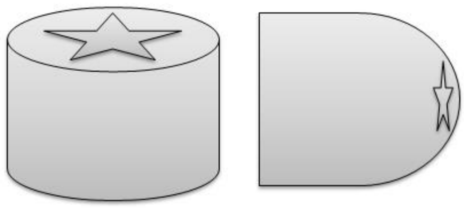

Fig.1. Starlike antenna placed on the cylinder and quasi-cylinder respectively

3.3. **Cylindrical Conformal antenna**

The design of starlike-shaped antenna (see Fig.1) is produced on a reproduced circuit board with r1,h=1.5mm. The restriction request to be optimized is the half-length of the reverberation edges (each one). We express the formal construction as $f=\Upsilon(\xi)$. The surface of a cylinder platform has the data: radius is 10-mm, and cylinder high is 20-mm. The CA is published on the same substrate, and the optimal construction is given by the starlike combination $g=(\xi\Upsilon'(\xi))/(\Upsilon(\xi))$ accounting that $R(g)>0$. Comparing the optimal evaluation bode of f and g where $|\xi|$ with E=0 when $|\xi|=0.48<1$, we confirm that the edges are minimized. In this place, we indicate that, one can reduce or increase the high h and get the same result. We note that, the iteration in this case is only two to converge to the optimal value (see Fig.2 and Fig.3 for the functions f and g and Fig.4 and Fig.5 for the energy corresponding to f and g).

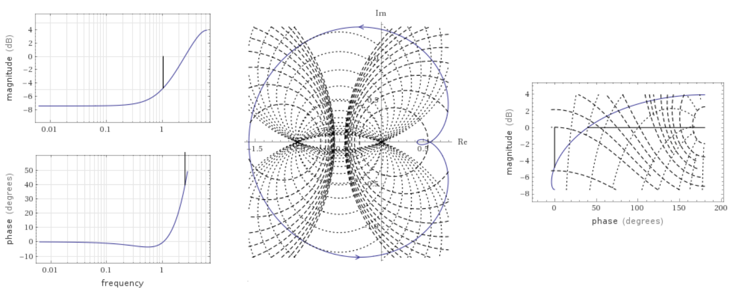

Fig.2. Frequency, phase and magnitude of f. The stability of the magnitude and the phase at the boundary of ***U***

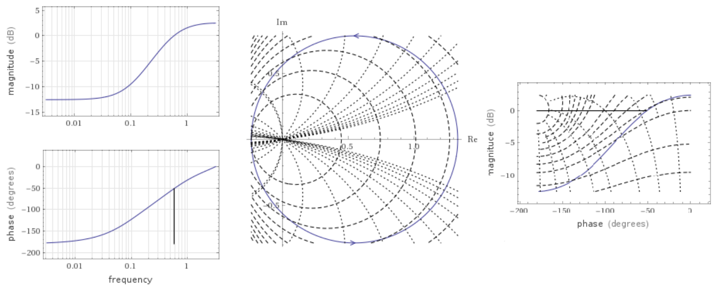

Fig.3. Frequency, phase and magnitude of g. The stability of the magnitude and the phase at the boundary of ***U***

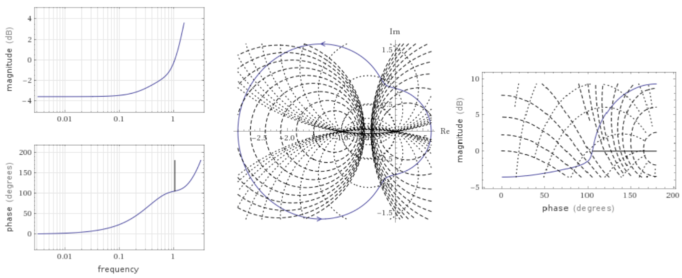

Fig.4. Frequency, phase and magnitude of E(f). The stability of the magnitude and the phase at the boundary of ***U***

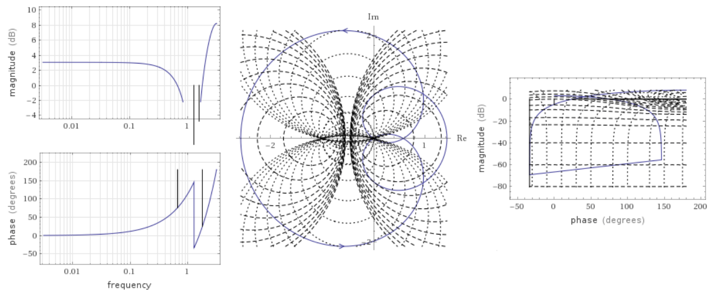

Fig.5. Frequency, phase and magnitude of E(g). The stability of the magnitude and the phase at the boundary of ***U***

4. APPLICATIONS

In this section, we define our suggested technique for the open unit disk conformal parameterizations. The disk conformal parameterizations are satisfied the support of an effective iterative algorithm. The central notion of the iterative system is to progress the conformality distortions adjacent the edges step by step. Note that the conformality distortion near the edges is small. Based on the above section and by using the definition of the starlike structure g together with the normalized function, we define the discrete harmonic energy as follows:

$$\mathcal{E}(\Upsilon)=\sum_{\theta}K_{\theta}\,\|\Upsilon-g(\Upsilon)\|^{2},$$

where θ is the angle between two edges and K_θ is a constant depends on the angle θ. One can assume that K_θ =cos(θ)+sin(θ) or any other formula depending on the application. The Laplace operator is formalized by

$$\mathcal{L}(\Upsilon)=\sum_{i}^{n}K_{\theta_i}\left(\Upsilon-g(\Upsilon)\right)\left(\xi^{i}\right).$$

Thus, the Laplace equation formulates a polynomial of order $\mathcal{L}(\Upsilon)$ =0 subjects to the arc-length parameterized boundary constriction. The linear system can be competently resolved utilizing the conjugate gradient process. Consequently, the initialization of our suggested technique can be calculated resourcefully.

Example 2. Consider the following function

$$\Upsilon(\xi)=\left(\xi-\gamma\xi^{2}\right)$$

then we obtain

$$\left(\xi-\gamma\xi^{2}\right)-\left(\frac{\xi-2\gamma\xi^{2}}{\xi-\gamma\xi^{2}}\right)=1+(1-2\gamma)\xi-\left(2\gamma^{2}+\gamma\right)\xi^{2}.$$

Now consider the function K_θ =cos(θ)+sin(θ) for all i then we have

$$\mathcal{L}(\Upsilon)=K_{\theta}+(1-2\gamma)K_{\theta}\xi-\left(2\gamma^{2}+\gamma\right)K_{\theta}\xi^{2}.$$

Solve the above equation for ξ, we obtain a set of solutions. Minimize this set; we obtain the value of ξ that minimizes the energy and consequently the edges. For the recent example the interval of solution is (-0.9,0.8) for all θ. Hence, the value |ξ|<0.88 minimizes the energy.

Now, we suppose that

$$\left(\xi-\gamma\xi^{2}\right)-\left(\frac{\xi-2\gamma\xi^{2}}{\xi-\gamma\xi^{2}}\right)=1+\left(1-2\gamma\right)\xi-\left(2\gamma^{2}+\gamma\right)\xi^{2}-2\gamma^{3}\xi^{3},$$

then, the Laplace operator is given by

$$\mathcal{L}\left(\Upsilon\right)=K_{\theta}+\left(1-2\gamma\right)K_{\theta}\xi-\left(2\gamma^{2}+\gamma\right)K_{\theta}\xi^{2}-2\gamma^{3}K_{\theta}\xi^{3}.$$

The solution interval for all θ is (-1,0.7) thus we get the value of minimization |ξ|<0.756. While

$$\left(\xi-\gamma\xi^{2}\right)-\left(\frac{\xi-2\gamma\xi^{2}}{\xi-\gamma\xi^{2}}\right)=1+\left(1-2\gamma\right)\xi-\left(2\gamma^{2}+\gamma\right)\xi^{2}-2\gamma^{3}\xi^{3}-2\gamma^{4}\xi^{4},$$

implies that |ξ|<0.73. We continue the iteration until n=10 we get |ξ|<0.72. We conclude that |ξ|<0.72 is the minimal value to minimize the energy. The application of energy for the conformal antenna can be seen in Fig.6 for different iterations. It is clear that the frequency is maximized for advance iteration.

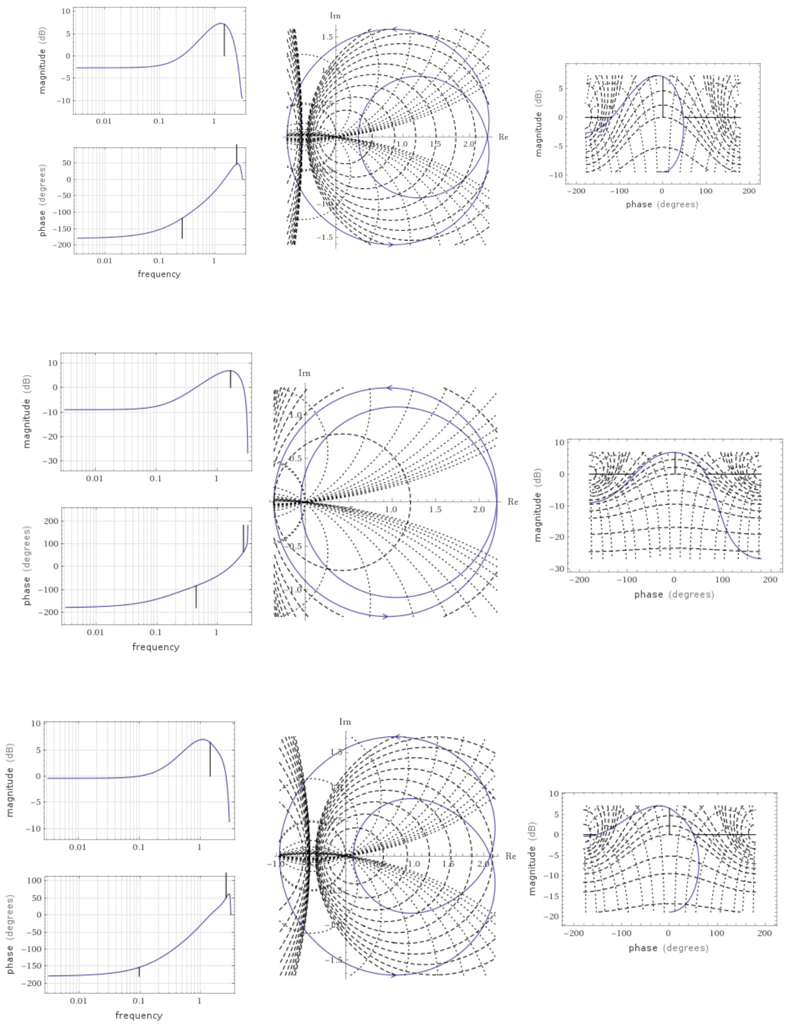

Fig.6. Frequency, phase and magnitude. The stability of the magnitude and the phase at the boundary for n = 2,3,4 respectively

5. CONCLUSION

From above, we conclude that the conformal mapping has a great activation on the design of antennas. The size and accuracy are established in the design. It has been seen that the small size of CA implies high frequency and accuracy. We established out CA by using the starlike conformal mapping. This map is defined in the open unit disk. Moreover, we formulated the integral energy equation utilizing starlike conformal mapping. This presentation is a theoretical study in CA. In addition, another point of view in such a research, we have used a simple computation, which is very clear and affected for the reader. In the future, one can apply the suggested CA to develop a class of antennas such as the fractal in the open unit disk.

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How to cite this article:
Ibrahim RW, Hadid SB, Momani S. Analytic technique to determine the energy of conformal antennas. J. Fundam. Appl. Sci., 2021, *13(3)*, *1447-1456.*