

# Robustness multi-objective optimization for parallel robot based on subregional meta-heuristic iteration

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## Abstract

**Purpose** – This work aims to provide a rapid robust optimization design solution for parallel robots or mechanisms, thereby circumventing inefficiencies and wastage caused by empirical design, as well as numerous physical verifications, which can be employed for creating high-quality prototypes of parallel robots in a variety of applications.

**Design/methodology/approach** – A novel subregional meta-heuristic iteration (SMI) method is proposed for the optimization of parallel robots. Multiple subregional optimization objectives are established and optimization is achieved through the utilisation of an enhanced meta-heuristic optimization algorithm, which roughly employs chaotic mapping in the initialization strategy to augment the diversity of the initial solution. The non-dominated sorting method is utilised for updating strategies, thereby achieving multi-objective optimization.

**Findings** – The actuator error under the same trajectory is visibly reduced after SMI, with a maximum reduction of 6.81% and an average reduction of 1.46%. Meanwhile, the response speed, maximum bearing capacity and stiffness of the mechanism are enhanced by 63.83, 43.98 and 97.51%, respectively. The optimized mechanism is more robust and the optimization process is efficient.

**Originality/value** – The proposed robustness multi-objective optimization via SMI is more effective in improving the performance and precision of the parallel mechanisms in various applications. Furthermore, it provides a solution for the rapid and high-quality optimization design of parallel robots.

**Keywords** Robustness multi-objective optimization, Logistics parallel robot, Subregional meta-heuristic iteration (SMI)

**Paper type** Research paper

## 1. Introduction

Parallel robots work dependably in intelligent manufacturing, logistics industry, medical engineering and numerous other fields rely on their high precision and reliability. It also has the advantage of small size which makes them well-suited for space constrained applications. Nevertheless, these are related to the structural parameters of the mechanism, and the optimized structural design is a permanent challenge.

Several existing researches focus on designing innovative structures. Riabtsev *et al.* (2022) present a 2-DOF active lockable joint. Ye *et al.* (2020) developed a 1R1T parallel mechanism as a remote center of motion mechanism. Others focus on the optimization of classical structures.

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Dastjerdi *et al.* (2020) explored how to obtain the smallest robot structural design by using dimensionally integrated analysis for parallel robots, ensuring the specified workspace was met. Quintero-Riaza *et al.* (2019) studied the optimal size design method for planar parallel robots, which enables the best dexterity index, force transfer efficiency and stiffness of the robots.

Comprehensive mathematical modeling is fundamental for structural optimization. Altuzarra *et al.* (2023) analyzed the kinematics of a three-degree-of-freedom (3-DOF) planar parallel continuum mechanism and developed a procedure for solving the fully inverse and forward kinematics of a planar 3-DOF system. The application of enhanced optimisation algorithms can enhance the efficiency of optimisation processes. Laribi *et al.* (2007) developed an optimal dimensional synthesis method of the Delta parallel mechanism for a prescribed workspace using a genetic algorithm-based method.

Most of the structural optimization design methods are empirical and obsolete. These methods rely heavily on multiple experiments and trials, which can lead to inaccuracies or inefficiencies. Based on the previous work (Tao *et al.*, 2024; Xu *et al.*, 2023), a complete mathematical description of the classical structure is developed and the specific work situation is taken into account, which further completes the professional robust multi-objective optimization via an improved meta-heuristic intelligent optimization algorithm.

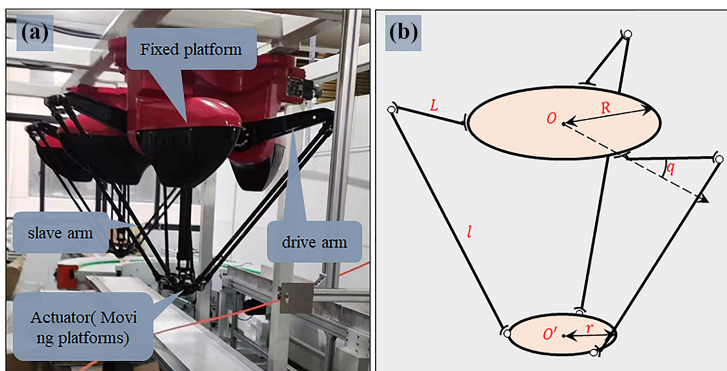
## 2. Parallel mechanism and its mathematical model

Figure 1(a) demonstrates a logistics sorting parallel robot working on a conveyor line. It is constructed as a delta-type parallel mechanism, which has 3 degrees of freedom (3-DOF) for movement in the XYZ direction, and the schematic is shown in Figure 1(b), where  $R$  is the radius of the fixed platform with center  $O$ ,  $r$  is the radius of the actuator with center  $O'$ ,  $L$  is the length of the drive arm,  $l$  is the length of the slave arm, and  $q$  is the drive angle.

There are already numerous contributions from scholars for mathematical modeling of the delta parallel mechanism (Altuzarra *et al.*, 2023), and some of our previous works have advanced the theoretical study of it (Tao *et al.*, 2024; Xu *et al.*, 2023). To summarize, we will employ kinematic models, performance parameter models and error models.

The kinematic model includes forward, inverse and velocity models. The forward model  $f_{fwd}$  calculates the actuator position  $O'$  of the mechanism based on the drive angle  $q$ , and the inverse model  $f_{inv}$  solves for the drive angle  $q$  based on the end position  $X$ .

$$O' = f_{fwd}(q) \quad (1)$$



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Figure 1.  
Logistics sorting  
parallel robot and  
structure schematic

$$q = f_{ivs}(O') \quad (2)$$

The velocity model calculates the mechanism actuator velocity  $\dot{X}$  based on the drive angular velocity  $\dot{q}$  with a transfer matrix called the Jacobian matrix  $J$ .

$$\dot{O}' = J(q)\dot{q} \quad (3)$$

Error models may be classified as absolute error models or probability error models.

$$e_{out} = J_e e_{in} \quad (4)$$

$$\sigma_{out} = J_\sigma \sigma_{in} \quad (5)$$

where  $e_{in}$  is the input absolute error,  $e_{out}$  is the output absolute error,  $\sigma_{in}$  is the standard deviation of the input error,  $\sigma_{out}$  is the standard deviation of the output error,  $J_e$  is the absolute error transfer matrix, and  $J_\sigma$  is the probability error transfer matrix.

The performance parameters models encompass the response speed, bearing capacity and stiffness of the mechanism, which are all related to the Jacobi matrix.

$$k = \frac{\sigma_{max}(J)}{\sigma_{min}(J)} k \in R^+ \quad (6)$$

$$F_{max} = \sigma_{max}(J^{-1} \cdot (J^T)^{-1}) F_{max} \in R^+ \quad (7)$$

$$D_{max} = \sigma_{max}(J \cdot J^T) D_{max} \in R^+ \quad (8)$$

where  $\sigma_{max}()$  represents computing the maximum singular value,  $\sigma_{min}()$  represents computing the minimum singular value, and  $T$  is the matrix transpose symbol.

### 3. The principle of meta-heuristic iteration

Various optimization algorithms have been proposed with the objective of facilitating the rapid and optimal design of prototypes. This is essential for the diffusion and application of novel mechanical structures, by which the design parameters of mechanical structures are iterated in an accelerated manner. It is also possible to reduce the cost of testing and production, the former being achieved by numerical simulation techniques, and the latter using cost as one of the considerations for the optimization objective.

Currently, meta-heuristic optimization algorithms are the subject of extensive study due to their problem-free dependency and their ability to solve non-convex problems effectively. The whole optimization search process can be generally divided into three steps: population initialization, meta-heuristic optimization strategy and population update strategy.

Population initialization provides a set of initial random solutions within the specified boundaries. In contrast to the use of simple random number generators, chaotic mapping techniques are capable of generating initial solutions that exhibit a more extensive distribution. We have employed the Cubic chaotic mapping with good chaotic properties with the expression:

$$x_{n+1} = \alpha x_n (1 - x_n^2) \quad (9)$$

where  $x_n$  is a random value ranging from (0, 1),  $x_{n+1}$  is a chaotic value, and  $\alpha$  is a control parameter.

After completing the initialization, it is necessary to optimize these solutions by executing the selected meta-heuristic optimization strategy on the generated populations. Ultimately, the fitness of the original and updated solutions must be evaluated in order to determine which solutions

should be retained. When faced with multi-objective optimization problems, we focus on finding the Pareto frontier. The non-dominated ordering update strategy is capable of computing the non-dominated level of hierarchical relationships among the individuals of the population, thereby obtaining the Pareto-optimal solution set under the multi-objective condition. When there are an excessive number of population individuals in the highest dominance level, the crowding distance is further computed in order to remove some of the overly concentrated solutions, thus broadening the Pareto front.

$$D_{crowd}(X_i) = \sum_{i=1}^m \left( \frac{Obj_i(X_{i+1}) - Obj_i(X_{i-1})}{\max(Obj_i) - \min(Obj_i)} \right) \quad (10)$$

where  $D_{crowd}$  is the crowding distance,  $X_{i+1}$  and  $X_{i-1}$  are the two solutions adjacent to  $X_i$ ,  $Obj_i$  denotes the  $i$ -th optimization objective, and  $m$  is the total number of optimization objectives.

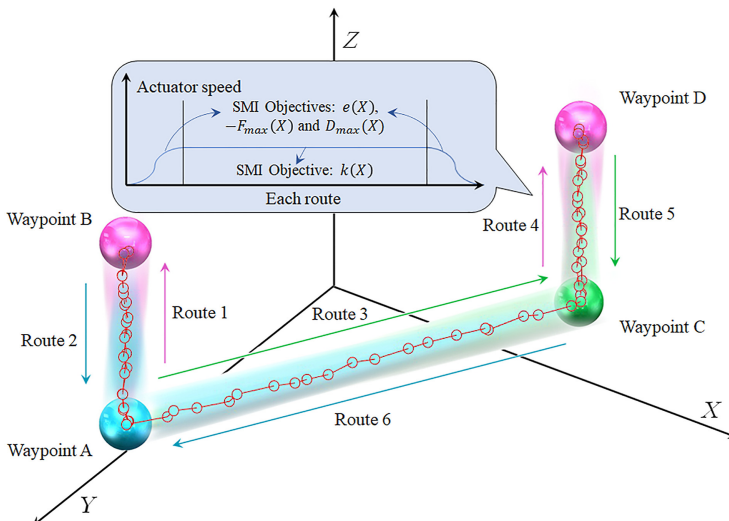
#### 4. Results of optimization for a specific case

For the parallel robot used in logistics sorting, its optimization problem can be described as:

$$\begin{cases} \text{Find } X = [R, r, L, l] \in R^4 \\ \min f(X) = \min [e(X), k(X), -F_{max}(X), D_{max}(X)] \\ \text{s.t. } X_{lb} \leq X \leq X_{ub} \end{cases} \quad (11)$$

where  $X$  represents the design variable to be optimized.  $R^4$  indicates that the viable domain of the design variable is a 4-dimensional solution space.  $e(X)$  means to optimize the error.  $k(X)$ ,  $-F_{max}(X)$  and  $D_{max}(X)$  mean to optimize the performance parameters.  $X_{lb}$  and  $X_{ub}$  are the lower and upper bounds of the design variable, respectively.

In particular, depending on our application, it is preferable to limit the optimized objectives in accordance with the respective trajectory segments. We restrict the optimization of  $e(X)$  and  $k(X)$  to smoothly moving trajectory segments, and the optimization of  $-F_{max}(X)$  and  $D_{max}(X)$  to trajectory segments with large acceleration, as shown in Figure 2.



Source(s): Authors' own work

**Figure 2.**  
Subregional  
optimization for an  
example motion  
trajectory

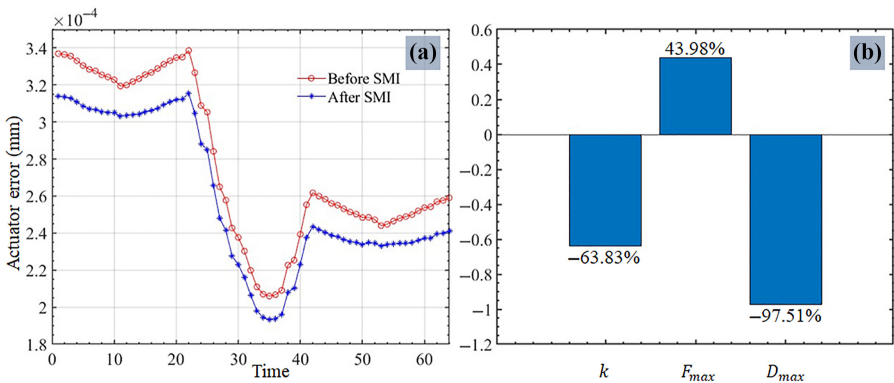
Settings  $X_{lb} = [80, 60, 110, 140]$ ,  $X_{ub} = [100, 70, 130, 160]$ . Table 1 illustrates the candidate design parameters of the mechanism after SMI optimization and their corresponding fitness. After verifying with simulation analysis and considering the structural rationality, the third set of candidate parameters was selected as the final optimization outcome. Figure 3(a) displays the error performance of the mechanism after SMI optimization over the full route of operation. It is evident that the error of the actuator after optimization is suppressed over the full path. The reduction is more significant in the acceleration route segment, which is one of the optimization objectives. The maximum reduction ratio in the path achieves 6.81% and the average error over the full path is reduced by 1.46%. Figure 3(b) illustrates the average change rate of performance parameters after SMI optimization. It can be observed that  $k$  of the mechanism becomes smaller,  $F_{max}$  increases and  $D_{max}$  decreases. This is reflected in practice resulting in a more flexible mechanism with greater load and stiffness. The optimized mechanism is more robust which broadens the range of applications.

**Table 1.**  
Candidate solutions  
obtained after  
proposed SMI and their  
corresponding fitness

Candidate solution	Design variable X (mm)				$e(X)$ (mm)	$k(X)$	$F_{max}(X)$	$D_{max}(X)$
	$R$	$r$	$L$	$l$				
1	88.8	65.0	122.2	149.1	2.71e-4	155.78	0.7010	2.88e5
2	88.7	63.2	120.3	148.3	2.66e-4	481.58	1.2406	1.45e6
3(chosen)	92.3	64.8	120.9	141.7	2.58e-4	114.40	0.7352	4.23e7
4	91.5	65.6	127.6	152.7	2.82e-4	42.53	0.0815	1.20e6
5	84.7	66.7	126.1	147.8	2.81e-4	29.62	0.0270	1.17e6

Source(s): Author's own work

**Figure 3.**  
Optimization outcomes  
where (a) is the  
actuator error  
comparison and (b) is  
the performance  
parameter  
improvement rate



Source(s): Authors' own work

## 5. Conclusion

- (1) A rapid and robust design method for the optimization of parallel mechanisms

A rapid and robust optimal design method for parallel mechanisms based on subregional meta-heuristic iteration (SMI) is intended to address the issues associated with the highly intricate structural design of parallel robots, which is heavily empirically dependent and inefficient due to a large number of physical experiments.

- (2) Improved meta-heuristics iterative strategy to accelerate the search for the global optimum

An improved meta-heuristic iterative strategy is proposed for the optimization of design parameters of parallel mechanisms. The initialization strategy is enhanced through the incorporation of chaotic mapping, thereby augmenting the diversity of initial solutions, which facilitates the rapid arrival at the global optimum and avoids local optimums. Meta-heuristics is employed to generate a novel generation of solutions, and through the implementation of a tailored policy, the algorithm is endowed with the capacity to evade local optimum. Non-dominated sorting is utilized as the update strategy, enabling the algorithm to adeptly address multi-objective optimization problems.

- (3) Optimization by constructing a subregional multiple optimization objective function

For the case of a logistics sorting parallel robot, a subregional multiple optimization objective function is constructed by analyzing its motion trajectory, which makes the optimization objective more directional and targeted. The actuator error is reduced by 6.81% at most and 1.46% on average across the entire path after proposed SMI optimization. Furthermore, the flexibility, bearing capacity, and stiffness performance of the mechanism are improved by 63.83%, 43.98%, and 97.51%, respectively, in comparison with the pre-optimisation stage.

In the future, more work will be carried out, including the development of more efficient meta-heuristics, the adaptive segmentation of the region for the optimization and the incorporation of additional mathematical models, such as the energy consumption and the dynamic vibration, among others.

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