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Window Analysis and MPI for Efficiency and Productivity Assessment Under Fuzzy Data:

Window Analysis and MPI

Abbas Al-Refaie, University of Jordan, Jordan*
https://orcid.org/0000-0002-3291-0805

ABSTRACT

This research develops a procedure for DEA window analysis and MPI evaluation of a manufacturing process with fuzzy inputs and outputs. A real case study was provided to illustrate relative efficiency and MPI assessment of a blowing machine over a period of one a year. The proposed approach was implemented to measure the technical, pure technical, and scale efficiency scores for decision making unit. The results showed that the blowing process was technically inefficient due to scale inefficiency. Therefore, management should optimize the size of operations and better utilize resources. Then, the lower and upper MPI values and their corresponding technology change and efficiency change were calculated. The MPI results revealed the reasons behind MPI progress or regress in the current period measured with respect to the next period. This procedure provides great assistance to process engineering in obtaining reliable feedback on process performance and guide them to take proper actions.

KEYWORDS

Efficiency, Fuzzy Data, Malmquist Productivity Index, Window Analysis

1. INTRODUCTION

In practice, production engineers regularly assess efficiency and productivity of manufacturing processes to achieve business goals (Park et al., 2018. Typically, measurement of a production unit-performance is crucial in determining whether it has achieved its objectives or not, and it generates a phase of management process that consists of feedback motivation phases (Kumar and Gulati, 2008; Al-Refaie et al., 2015). An effective technique for measuring processes' relative efficiency is the Data envelopment analysis (DEA) method, in which a production frontier is constructed from a set of comparable Decision making Units (DMUs) and data on their inputs and outputs. The efficiency of each DMU is deðned by its relative distance from the production frontier (Al-Refaie et al., 2016a; Al-Refaie et al., 2016b; Ennen and Batool, 2018). Two common DEA models can be used for this purpose Charnes, Cooper, and Rhodes (CCR) and Banker, Chang, and Cooper (*BCC*) by Charnes *et al*. (1978) and Banker *et al*. (1984), respectively.

However, when using the *CCR* and *BCC* models, an important rule of thumb is that the number of DMUs is at least twice the sum of the number of inputs and outputs (Arcos-Vargaset al., 2017).

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Otherwise, the model may produce numerous relatively efficient units and decrease discriminating power. To resolve this difficulty, DEA window analysis was introduced in which the performance of a *DMU* in any period can be compared with its own performance in other periods as well as to the performance of other *DMU*s (Al-Refaie et al., 2014). DEA window analysis is based on a dynamic perspective, regarding the same *DMU* in different period of time as entirely different *DMU*s (Jia and Yuan, 2017). The window analysis technique relies on the traditional *CCR* and *BCC* models for estimating technical efficiency (*TE*) and pure technical efficiency (*PTE*) scores for each *DMU*. DEA window analysis is usually followed by the evaluation of the Malmquist productivity index (*MPI*), which is a formal time-series analysis method for conducting performance comparisons of DMUs over time by solving traditional DEA type models. The *MPI* measures the productivity change of DMU over time. The productivity of DMU from period p and $p+1$ is improved when *MPI* is larger than one, remained unchanged when *MPI* equals one, and deteriorated when *MPI* is less than one. The productivity change can be decomposed into two parts, namely technological change (*TC*) and efficiency change (*TEC*) component, which measures the change in relative efficiency over time (Balcerzak et al., 2017).

In the traditional DEA window analysis and *MPI*, the main assumption is that the inputs and outputs are measured by exact values on a ratio scale (Kao and Liu, 2000). In some real applications, however, the inputs and outputs may be expressed by fuzzy values, and hence the use of the traditional window analysis makes the analysis unreliable, which may lead to obtain erroneous decisions. Consequently, this research contributes to ongoing research by proposing a procedure for DEA window analysis and *MPI* evaluation that is used to assess the relative efficiency and productivity of a production process under fuzzy input and output data. This procedure can more realistically represents real problems than the traditional DEA models and provides reliable analysis and enables taking accurate decisions with proper actions on production inputs and outputs that lead to enhance production performance and quality, and better utilize input resources. The assessment of the efficiency and productivity of a blowing machine, which is used in manufacturing plastic products, under fuzzy input and output data over a period of one year will be presented to illustrate the proposed procedure. The remaining of this paper including the introduction is organized as follows. Section 2 presents background and literature review. Section 3 develops research methodology and application. Section 4 discusses research results. Section 5 summarizes research conclusions.

2. DEA BACKGROUND AND LITERATURE REVIEW

2.1 DEA Relevant Background

Generally, in DEA the *CCR* efficiency score measures technical efficiency (*TE*), which reflects the firm's ability to maximize output from a given set of inputs assuming that the size of operation of *DMU* is optimal. Consider a set of n DMUs. For a specific DMU_j (j =1,…, n), DMU_k, let y_{rk} denote the level of rth (r = 1, …, s) output and x_{ik} the level of the ith (i = 1,…, m) input. The efficiency score, θ_k, of DMU_k is then calculated as by solving the dual input-oriented *CCR* model as follows (Charnes *et al*., 1978):.

$$Min\ \theta_k \tag{1a}$$

Subject to:

$$\sum_{j=1}^{n} \lambda_{kj} x_{ij} - \theta_k x_{ik} \le 0, \forall i \tag{1b}$$

$$\sum_{j=1}^{n} \lambda_{kj} y_{rj} \geq y_{rk}, \forall r \tag{1c}$$

$$\lambda_{kj} \geq 0, \forall j \tag{1d}$$

where θ_k unrestricted in sign. The optimal θ_k is denoted by θ_k^* satisfies 0 £ θ_k^* £ 1. If θ_k^* equals to one, the *DMU* under measurement is then technically efficient. The *CCR* model assumes constant return to scale (*CRS*) where an increase in the input results an increase in the output result. Whereas, the *BCC* model assumes that the *DMU* operates under variable returns to scale (VRS) if it is suspected that an increase in inputs does not result in a proportional change in the outputs. The BCC model measures the Pure Technical Efficiency (*PTE*), which ignores the impact of the scale size by only comparing a *DMU* to a unit of similar scale. The *PTE* measures how a *DMU* utilizes its sources under exogenous environments; a low value of *PTE* implies that the *DMU* inefficiently manages its resources. To calculate the *PTE* score, the following dual input-oriented *BCC* model is used (Banker *et al.*, 1984):

$$Min\ \theta_k \tag{2a}$$

Subject to:

$$\sum_{j+1}^{n} \lambda_{kj} x_{ij} - \theta_k x_{ik} \leq 0\ ,\ \ \forall\ j \tag{2b}$$

$$\sum_{j=1}^{n} \lambda_{kj} y_{rj} \geq y_{rk}, \forall\ r \tag{2c}$$

$$\sum_{j=1}^{n} \lambda_{kj} = 1$$
$$\lambda_{kj} \geq 0,\ \forall k, j \tag{2d}$$

Typically, the BCC model divides the *TE* into two parts: *(i) PTE* which ignores the impact of scale size by only comparing a *DMU* to a unit of similar scale and measures how a DMU utilizes its sources under exogenous environment and *(ii)* Scale Efficiency (*SE*) which measures how the scale size affects efficiency. The *SE* measures how the scale size affects efficiency. The *SE* provides the ability of the management to choose the optimal size of resources and is calculated using Eq. (3).

$$SE = \frac{TE}{PTE} \tag{3}$$

Given the input and output data for a manufacturing process over a period of time, the traditional window analysis in DEA divides the time period into time windows. Each window is treated as a *DMU*. Then, the *DMU* 's input and output data are utilized to measure the *TE* and *PTE* scores using the *CCR* (model 1) and *BCC* (Model 2), respectively, at each time unit in this *DMU*. Finally, the *DMU* 's *TE* and *PTE* averages are calculated and then used to estimate the corresponding *SE* averages using Eq. (3). Based on the window's *TE*, *PTE* and *SE* scores, proper improvement actions are suggested to enhance production performance. The *MPI* follows to determine both the efficiency change (catch-up) and technological change (frontier-shift). Review of relevant previous studies is presented in the following section.

2.2 Literature Review

Window analysis in DEA has been applied in several studies. For example, Kumar and Gulati (2008) measured the extent of technical, pure technical, and scale efficiencies in 27 public sector banks operating in India in the year 2004/2005 using DEA. Mahajan *et al*. (2012) measured technical efficiencies, slacks and input/output targets for 50 large Indian pharmaceutical firms. This study uses DEA approach. Mugera (2013) measured technical efficiency of dairy farms with imprecise data using a fuzzy data envelopment analysis approach. Azadeh *et al*. (2014) developed an integrated fuzzy simulation fuzzy data envelopment analysis approach for optimum maintenance planning. Al-Refaie, *et al*. (2015) analyzed the growth potentials of five production machines in a plastic industry by employing window analysis and Malmquist productivity index. Campos *et al*. (2016) used DEA to evaluate the efficiency of public resource usage in health systems of autonomous communities in Spain. Jia and Yuan (2017) evaluated and compared operational efficiencies of different hospitals before and after their establishment of branched hospitals using DEA. Balcerzak, et al. (2017) proposed a methodology for a comprehensive evaluation of operational efficiency of the banking sectors in EU countries using DEA. Aye et al. (2018) used a two-stage fuzzy approach efficiency in South African agriculture. Ennen and Batool (2018) investigated 12 major airports in Pakistan for potential cost inefficiencies using DEA. Barak and Dahooei (2018) proposed fuzzy DEA for airlines safety evaluation. Park et al. (2018) suggested a new DEA-based efficiency evaluation model and conducted efficiency evaluations and benchmarking for 13 Korean national university hospitals. Anouze and Bou-Hamad (2019) employed DEA and data mining to efficiency estimation and evaluation. Zhou and Xu (2020) overviewed fuzzy DEA models in the presence of undesirable outputs. The input/output data were represented by triangular fuzzy numbers. Then, two virtual fuzzy DMUs called fuzzy ideal *DMU* and fuzzy anti-ideal *DMU* were introduced into proposed fuzzy DEA framework. Nandy and Singh (2021) employed a combination of fuzzy data envelopment analysis approach to yield crisp DEA efficiency values by converting the fuzzy DEA model into a linear programming problem and machine learning algorithms for better evaluation and prediction of the variables affecting the farm efficiency. Sánchez-Ortiz et al. (2021) employed DEA window analysis and Malmquist index to assess efficiency and productivity in the Spanish electricity sector The study defined a model that showed how the efficiency problems associated with electricity distribution companies such as productive overcapacity or tariff deficit can be measured based on the theory of constraints and theory of economic regulation.

In most previous studies, the window analysis and *MPI* were performed without consideration of the impreciseness in the input and output data which gives wrong efficient frontier (Wanke et al., 2016). Hence, research contributions are necessary to develop and efficient procedure for assessing efficiency and productivity of a manufacturing process under fuzzy input and output data. Nevertheless, few studies were reported on the use window analysis in DEA and MPI for assessing the efficiency of a manufacturing process under fuzzy input and output data.

3. RESEARCH METHODOLOGY AND APPLICATION

3.1 Research Methodology

The methodology for conducting window analysis in DEA and Malmquist analysis with fuzzy inputs and outputs is outlined as follows:
Step 1: Select the production process for which the efficiency scores is to be evaluated and then measure the process's low, middle, and high values of the inputs and outputs over a time horizon of *T*. Let *p* denotes any period in time horizon *T*. Collect the data for each period $t = 1, ..., T$. Then, determine the fuzzy inputs and the outputs that will be used for data envelopment window analysis. Assume each period *t;* where $t=1, \ldots, T$, has fuzzy inputs $\tilde{x}_i^t = (x_i^{t,L}, x_i^{t,M}, x_i^{t,H})$; where $x_i^{t,L}, x_i^{t,M}$, and $x_i^{t,H}$ are the low, middle, and high values of the i^{th} input; $i = 1, \ldots, m$, and fuzzy outputs $\tilde{y}_r^t = (y_r^{t,L}, y_r^{t,M}, y_r^{t,H})$; where $y_r^{t,L}, y_r^{t,M}$, and $y_r^{t,H}$ are the low, middle, and high values of the r^{th} output; $r = 1, \ldots, s$. Divide the time horizon, *T*, into *n* time windows w_js; where w_1: $t_1 = p_1 \rightarrow p_w$, w_2: $t_2 = p_2 \rightarrow p_{w+1}$, ..., w_k: $t_k = p_k \rightarrow p_{k+w-1}$, and so on. Treat each time window as a decision making unit (DMU_j); $j = 1, ..., n$. Consequently, the matrices of fuzzy inputs and outputs, $\tilde{X}_k$ and $\tilde{Y}_k$, of a specific *DMU*, DMU_k, can be expressed respectively as:

$$\tilde{X}_k = \begin{bmatrix} \tilde{x}_1^{t=p_k} & \tilde{x}_2^{p_k} & \cdots & \tilde{x}_m^{p_k} \\ \tilde{x}_1^{t=p_k+1} & \tilde{x}_2^{p_k+1} & \cdots & \tilde{x}_m^{p_k+1} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_1^{t=p_k+w-1} & \tilde{x}_2^{p_k+w-1} & \cdots & \tilde{x}_m^{p_k+w-1} \end{bmatrix} \tag{4}$$

and

$$\tilde{Y}_k = \begin{bmatrix} \tilde{y}_1^{t=p_k} & \tilde{y}_2^{p_k} & \cdots & \tilde{y}_s^{p_k} \\ \tilde{y}_1^{t=p_k+1} & \tilde{y}_2^{p_k+1} & \cdots & \tilde{y}_s^{p_k+1} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{y}_1^{t=p_k+w-1} & \tilde{y}_2^{p_k+w-1} & \cdots & \tilde{y}_s^{p_k+w-1} \end{bmatrix} \tag{5}$$

Calculate the technical efficiency, pure technical efficiency, and scale efficiency scores for each DMU_j; $j=1, ..., J$, as follows. That is, for a specific *DMU*, DMU_k, which covers the period p_k till p_{k+w-1}, the optimal technical efficiency score, θ^*_{k,p_k} , for DMU_k at time p_k is calculated from the inputs and outputs of p_k till p_{k+w-1} including the inputs and outputs of p_k by solving the following dual model:

$$Min\ \theta_{k,p_k} \tag{6a}$$

Subject to:

$$\sum_{t=p_k}^{p_k+w-1} \lambda_t x_{i,t}^L - \theta_{k,p_k} x_{i,p_k}^L \le 0 \quad , \quad \forall i \tag{6b}$$

$$\sum_{t=p_k}^{p_k+w-1} \lambda_t x_{i,t}^{M} - \theta_{k,p_k} x_{i,p_k}^{M} \leq 0 \quad , \quad \forall i \tag{6c}$$

$$\sum_{t=p_k}^{p_k+w-1} \lambda_t x_{i,t}^{H} - \theta_{k,p_k} x_{i,p_k}^{H} \leq 0 \quad , \quad \forall i \tag{6d}$$

$$\sum_{t=p_k}^{p_k+w-1} \lambda_t y_{r,t}^{L} - y_{r,p_k}^{L} \geq 0, \quad \forall r \tag{6e}$$

$$\sum_{t=p_k}^{p_k+w-1} \lambda_t y_{r,t}^{M} - y_{r,p_k}^{M} \geq 0, \quad \forall r \tag{6f}$$

$$\sum_{t=p_k}^{p_k+w-1} \lambda_t y_{r,t}^{H} - y_{r,p_k}^{H} \geq 0, \quad \forall r \tag{6g}$$

$$\lambda_t \geq 0 \ , \ \ \forall t \in \left[p_k,\ p_k + w\text{-}1\right] \tag{6h}$$

Then, estimate the corresponding optimal pure technical efficiency score, γ^{*}_{k,p_k}, of DMU_k at time p_k is estimated as follows:

$$Min \ \gamma_{k,p_k} \tag{7a}$$

Subject to:

$$\sum_{t=p_k}^{p_k+w-1} \lambda_t x_{i,t}^{L} - \gamma_{k,p_k} x_{i,p_k}^{L} \leq 0 \quad , \quad \forall i \tag{7b}$$

$$\sum_{t=p_k}^{p_k+w-1} \lambda_t x_{i,t}^{M} - \gamma_{k,p_k} x_{i,p_k}^{M} \leq 0 \quad , \quad \forall i \tag{7c}$$

$$\sum_{t=p_k}^{p_k+w-1} \lambda_t x_{i,t}^{H} - \gamma_{k,p_k} x_{i,p_k}^{H} \le 0 \quad , \quad \forall i \tag{7d}$$

$$\sum_{t=p_k}^{p_k+w-1} \lambda_t y_{r,t}^{L} - y_{r,p_k}^{L} \ge 0, \quad \forall r \tag{7e}$$

$$\sum_{t=p_k}^{p_k+w-1} \lambda_t y_{r,t}^{M} - y_{r,p_k}^{M} \ge 0, \quad \forall r \tag{7f}$$

$$\sum_{t=p_k}^{p_k+w-1} \lambda_t y_{r,t}^{H} - y_{r,p_k}^{H} \ge 0, \quad \forall r \tag{7g}$$

$$\sum_{t=p_k}^{p_k+w-1} \lambda_t = 1, \tag{7h}$$

$$\lambda_t \ge 0 \ , \ \ \forall t \in \left[p_k, \, p_k + w\text{-}1\right] \tag{7i}$$

Finally, the optimal scale efficiency score, η_{k,p_k}^{*}, is calculated by dividing the optimal *TE* score, θ_{k,p_k}^{*}, by its corresponding optimal *PTE* score, γ_{k,p_k}^{*}. Mathematically,

$$\eta_{k,p_k}^{*} = \frac{\omega_{k,p_k}^{*}}{\gamma_{k,p_k}^{*}} \tag{8}$$

In a similar manner, the other values of the $\theta_{k,t}^{*}$, $\gamma_{k,t}^{*}$ and $\eta_{k,t}^{*}$; $\forall t \in \left[p_k, \, p_k + w\text{-}1\right]$ and $t^{1} p_k$, of DMU_k are estimated. Similarly, calculate the optimal efficiency scores of $\theta_{j,t}^{*}$, $\gamma_{j,t}^{*}$ and $\eta_{j,t}^{*}$; $j \ne k$. Calculate the averages of the optimal technical, pure technical, and scale efficiency scores, θ_j^{*}, γ_j^{*} and η_j^{*}, respectively, of DMU_j using Eqs. (9) to (11), respectively.

$$\theta_j^{*} = \frac{1}{w} \sum_{t=p_j}^{p_j+w-1} \theta_{j,t}^{*} \tag{9}$$

$$\gamma_j^* = \frac{1}{w}\sum_{t=p_j}^{p_j+w-1} \gamma_{j,\ t}^* \tag{10}$$

$$\eta_j^* = \frac{1}{w}\sum_{t=p_j}^{p_j+w-1} \eta_{j,p_j}^* \tag{11}$$

where

$$\eta_{j,p_j}^* = \frac{\theta_{j,p_j}^*}{\gamma_{j,p_j}^*} \tag{12}$$

Estimate the Malmquist Productivity Index (*MPI*). In practice, the production line is evaluated at three-month periods. This results in four evaluation periods each of three months. Let *e* denotes the category; $e = 1,..., E$. Let $\theta_{e'}^{p,L}(E^p)$ and $\theta_{e'}^{p,U}(E^p)$ denote the optimal lower and upper efficiency of month *p* in a specific evaluation period *e'*, which are calculated from the inputs and outputs of the month *p* in all *E* evaluation periods, respectively by solving the formulas (10) and (11), respectively (Ebrahimnejad and Amani, 2021; Jahanshahloo *et al*., 2006).

$$\theta_{e'}^{p,L}(E^p) = Min\ \delta_1 \tag{13a}$$

Subject to:

$$\delta_1 x_{ie'}^{p,U} - \sum_{e \neq e', e=1}^{E} \lambda_e x_{ie}^{p,L} - \lambda_{e'} x_{ie'}^{p,U} \geq 0, \quad \forall i \tag{13b}$$

$$y_{re'}^{p,L} - \sum_{e \neq e', e=1}^{E} \lambda_e y_{re}^{p,U} - \lambda_{e'} y_{re'}^{p,L} \geq 0, \quad \forall r \tag{13c}$$

$$\lambda_{e'} \geq 0, \ \forall e \tag{13d}$$

and

$$\theta_{e'}^{p,U}(E^p) = Min\ \delta_2 \tag{14a}$$

Subject to:

$$\delta_2 x_{ie'}^{o,L} - \sum_{e \neq e', e=1}^{E} \lambda_e x_{ie}^{p,U} - \lambda_{e'} x_{ie'}^{p,L} \geq 0, \quad \forall i \tag{14b}$$

$$y_{re'}^{p,U} - \sum_{e \neq e', e=1}^{E} \lambda_e y_{re}^{p,L} - \lambda_{e'} y_{re'}^{p,U} \geq 0, \quad \forall r \tag{14c}$$

$$\lambda_e \geq 0, \quad \forall e \tag{14d}$$

Using $p+1$ instead of p, the $\theta_{e'}^{p+1,L}(E^{p+1})$ and $\theta_{e'}^{p+1,U}(E^{p+1})$ are calculated for the lower and upper bounds, respectively. Furthermore, let $\theta_{e'}^{p+1,L}(E^{p})$ and $\theta_{e'}^{p+1,U}(E^{p})$ denote the optimal lower and upper efficiency of month $p+1$ in evaluation period *e'*, which are estimated from the inputs and outputs of month p in all E evaluation periods by solving formulas (15) and (16), respectively.

$$\theta_{e'}^{p+1,L}(E^{p}) = Min\ \omega_1 \tag{15a}$$

Subject to:

$$\omega_1 x_{ie'}^{p,U} - \sum_{e=1}^{E} \lambda_e x_{ie}^{p+1,L} \geq 0, \quad \forall i \tag{15b}$$

$$y_{re'}^{p,L} - \sum_{e=1}^{E} \lambda_e y_{re}^{p+1,U} \geq 0, \quad \forall r \tag{15c}$$

$$\lambda_e \geq 0, \quad \forall e \tag{15d}$$

and

$$\theta_{e'}^{p+1,U}(E^{p}) = Min\ \omega_2 \tag{16a}$$

Subject to:

$$\omega_2 x_{ie'}^{p,L} - \sum_{e=1}^{E} \lambda_e x_{ie}^{p+1,U} \geq 0, \quad \forall i \tag{16b}$$

$$y_{re'}^{p,U} - \sum_{e=1}^{E} \lambda_e y_{re}^{p+1,L} \geq 0, \qquad \forall r \tag{16c}$$

$$\lambda_e \geq 0, \qquad \forall e \tag{16d}$$

Using the $p+1$ instead of p and vice versa, for the above models, the $\theta_{e'}^{p,L}(E^{p+1})$ and $\theta_{e'}^{p,U}(E^{p+1})$ values can be obtained, respectively, for the lower and upper bounds. Consequently, the lower and upper *MPI*; $MPI_{e',p}^{L}$ and $MPI_{e',p}^{U}$, respectively, are calculated using Eqs. (17) and (18), respectively.

$$MPI_{e',p}^{L} = \left[\frac{\theta_{e'}^{p,U}(p)}{\theta_{e'}^{p+1,U}(p)} \times \frac{\theta_{e'}^{p,L}(p+1)}{\theta_{e'}^{p+1,L}(p+1)}\right]^{1/2} \times \left[\frac{\theta_{e'}^{p+1,L}(p+1)}{\theta_{e'}^{p,U}(p)}\right] = \left[\frac{\theta_{e'}^{p,L}(p+1)}{\theta_{e'}^{p,U}(p)} \times \frac{\theta_{e'}^{p+1,L}(p+1)}{\theta_{e'}^{p+1,U}(p)}\right]^{1/2} \tag{17}$$

$$MPI_{e',p}^{U} = \left[\frac{\theta_{e'}^{p,L}(p)}{\theta_{e'}^{p+1,L}(p)} \times \frac{\theta_{e'}^{p,U}(p+1)}{\theta_{e'}^{p+1,U}(p+1)}\right]^{1/2} \times \left[\frac{\theta_{e'}^{p+1,U}(p+1)}{\theta_{e'}^{p,L}(p)}\right] = \left[\frac{\theta_{e'}^{p,U}(p+1)}{\theta_{e'}^{p,L}(p)} \times \frac{\theta_{e'}^{p+1,U}(p+1)}{\theta_{e'}^{p+1,L}(p)}\right]^{1/2} \tag{18}$$

where $MPI_{e',p}^{L}$ larger than one indicates a progress; $MPI_{e',p}^{U}$ smaller than one indicates a regress in productivity from time p to $p+1$. Otherwise, nothing can be said.
Step 6. Analyze and discuss the obtained optimal technical, pure technical, and scale efficiency scores. Then, identify the reasons for inefficiency and suggest proper actions. Further, analyze and discuss the results of Malmquist analysis.

3.2 Application

The data is obtained from production reports for a blowing machine over year 2019 as displayed in Table 1, where three inputs; planned production quantity (*PP*, $\tilde{x}_1^t$), defect quantity (*DQ*, $\tilde{x}_2^t$), and idle time (*IT*,, $\tilde{x}_3^t$), and a single output actual production quantity (*PQ*, $\tilde{y}^t$) were identified for each month, *t*.

Table 1. The collected data for blowing machine.

Month (*t*)	Input			Output
	$\tilde{x}_1^t$ (PP, unit)	$\tilde{x}_2^t$ (DQ, unit)	$\tilde{x}_3^t$ (IT, unit)	$\tilde{y}^t$ (PQ, unit)
1	(24150, 24192, 25100)	(179, 185, 192)	(1383,1426, 1483)	(21099, 22300, 23103)
2	(24100, 24192, 24300)	(91, 94, 97)	(7756, 7996, 8315)	(15550, 15731, 17832)
3	(24000, 24192, 25159)	(66, 69, 71)	(3054, 3149, 3274)	(20549, 21419, 22318)
4	(24000, 24192, 25000)	(94, 97, 100)	(6221, 6414, 6670)	(18231, 18359, 23721)
5	(20113, 20736,21565)	(170, 176, 183)	(1876, 1935, 2012)	(17128, 17221, 19762)
6	(26818, 27648, 28753)	(137, 142, 147)	(49, 51, 53)	(26810, 27620, 27640)
7	(23466, 24192, 25159)	(116, 120, 124)	(3317, 3420, 3556)	(18089, 19456, 22941)

Table 1 continued on next page

Table 1 continued

Month (t)	Input			Output
	$\tilde{x}_1^t$ (PP, unit)	$\tilde{x}_2^t$ (DQ, unit)	$\tilde{x}_3^t$ (IT, unit)	$\tilde{y}^t$ (PQ, unit)
8	(20113, 20736, 21565)	(51, 53, 55)	(2018, 2081, 2164)	(19012, 19616, 20000)
9	(13409, 13824, 14376)	(31, 32, 33)	(657, 678, 705)	(12984, 13500, 13561)
10	(10056, 10368, 10782)	(70, 73, 75)	(2769, 2855, 2969)	(8195, 8318, 9939)
11	(15085, 15552, 16174)	(111, 115, 119)	(2873, 2962, 3080)	(13294, 13350, 15230)
12	(21790, 22464, 23362)	(237, 245, 254)	(2268, 2339, 2432)	(20137, 20750, 22252)

(1) Window analysis

The window length is set a period of six months. This results in seven windows or *DMUs*; t_1-t_6, t_2-t_7, ..., and t_7-t_{12} treated as DMU_1 to DMU_7, respectively. Table 2 displays the optimal technical for all periods and DMU_j s. That is, the optimal technical efficiency, θ^*_{j,p_j} , scores for each period of DMU_j are obtained by solving Eq. (6). Similarly, the optimal pure technical efficiency, $\gamma^*_{j,\, p_j}$, scores are estimated for all periods of DMU_j by solving Eq. (7). Then, the optimal technical efficiency, θ^*_j , and pure technical efficiency, γ^*_j , of DMU_j are using Eqs. (9) and (10), respectively. Finally, the optimal scale efficiency, η^*_j , of DMU_j is determined using Eqs. (11). For example, the optimal *TE* score, $\theta^*_{1,1}$, of DMU_1 at period p_1 is estimated by solving Eq. (6) and found to be 0.9575. Similarly the $\theta^*_{1,2}$ to $\theta^*_{1,6}$ are obtained. The optimal *TE* score, $\theta^*_1 = 0.9493$, of DMU_1 is calculated as the average of $\theta^*_{1,1}$ to $\theta^*_{1,6}$ values. In a similar manner, the θ^*_j values are estimated. Furthermore, the coefficient of variation (CV_j%) of DMU_j is calculated as the standard deviation of the θ^*_{j,p_j} values divided by θ^*_j . Finally, the $\theta^*_{j,1}$ for the first period and the coefficient of variation (CV_t %) are calculated. In Table 2, the estimated *TE* scores listed in each period t (column) show stable performance, because the differences between the efficiency values are (CV_t %) values in column are smaller than 0.05). However, the coefficient of variance (CV_j%) listed in rows are relatively large for DMU_1 and DMU_2, which means that the dispersion is significant and hence there exists a trend in their corresponding θ^*_{j,p_j} scores. Moreover, it is found that all the θ^*_j scores are less than one, while the smallest and largest θ^*_j values are 0.9488 and 0.9853, which correspond to θ^*_2 and θ^*_7 , respectively. Theoretically, all DMU_js are, therefore, concluded inefficient. For each period t, the average *TE*, θ^*_t , it found that the θ^*_t is equal to one at periods 6, 9, and 12, whereas the θ^*_t scores are less than one at each of the remaining periods. In other words, the blowing machine was technically- efficient in two out of twelve months.

Table 2. The estimated optimal TE scores.

Time period t														
DMU_j	1	2	3	4	5	6	7	8	9	10	11	12	θ_j^*	$CV_j\%$
DMU_1	0.9575	0.7851	1.0000	1.0000	0.9533	1.0000							0.9493	8.78%
DMU_2		0.7851	1.0000	1.0000	0.9533	1.0000	0.9544						0.9488	8.78%
DMU_3			0.9528	1.0000	0.9533	1.0000	0.9510	1.0000					0.9762	2.67%
DMU_4				0.9944	0.9533	1.0000	0.9497	0.9817	1.0000				0.9799	2.34%
DMU_5					0.9533	1.0000	0.9497	0.9817	1.0000	0.9590			0.9740	2.37%
DMU_6						1.0000	0.9497	0.9817	1.0000	0.9590	0.9796		0.9783	2.12%
DMU_7							0.9638	0.9830	1.0000	0.9721	0.9926	1.0000	0.9853	1.52%
θ_t^*	0.9575	0.7851	0.9843	0.9986	0.9533	1.0000	0.9531	0.9856	1.0000	0.9634	0.9861	1.0000		
$CV_t\%$	0.00%	0.00%	2.77%	0.28%	0.00%	0.00%	0.58%	0.82%	0.00%	0.79%	0.93%	0.00%		

The pure technical efficiency reflects the managerial performance to organize inputs of the blowing machine. For each DMU_j , the estimated optimal *PTE* scores; γ_{j,p_j}^* and γ_j^* , are calculated using Eq. (7) and (10), respectively. The CV_j % and CV_t % values reveal the lack of existence of less dispersion and trend in the *PTE* scores for all DMU_js and periods. In Table 3, it is noted that two (DMU_2 and DMU_3) out of the seven DMU_js are found pure technically-efficient. On the other hand, it is found that eight out of the twelve periods are concluded pure technically-efficient. Compared with the technical efficiency values in Table 2, it is found that the γ_j^* and γ_t^* are larger than their corresponding θ_j^* and θ_t^* , respectively. Further, the γ_1^* $(= 0.9998)$ for DMU_1 indicates the same level of output could be produced by 99.98% of the recourses taking into consideration that the scale size is ignored, in addition 0.02% of all recourses could be saved by raising the performance of the machine to the highest level.

Table 3. The estimated optimal PTE scores.

Time period t														
DMU_j	1	2	3	4	5	6	7	8	9	10	11	12	γ_j^*	$CV_j\%$
DMU_1	0.9990	1.0000	1.0000	1.0000	1.0000	1.0000							0.9998	0.04%
DMU_2		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000						1.0000	0.00%
DMU_3			1.0000	1.0000	1.0000	1.0000	1.0000	1.0000					1.0000	0.00%
DMU_4				1.0000	0.9603	1.0000	0.9521	1.0000	1.0000				0.9854	2.31%
DMU_5					0.9603	1.0000	0.9521	1.0000	1.0000	1.0000			0.9854	2.31%
DMU_6						1.0000	0.9521	1.0000	1.0000	1.0000	0.9943		0.9911	1.94%
DMU_7							1.0000	1.0000	1.0000	1.0000	0.9956	1.0000	0.9993	0.18%
γ_t^*	0.9990	1.0000	1.0000	1.0000	0.9841	1.0000	0.9761	1.0000	1.0000	1.0000	0.9949	1.0000		
$CV_t\%$	0.00%	0.00%	0.00%	0.00%	2.21%	0.00%	2.69%	0.00%	0.00%	0.00%	0.09%	0.00%		

Table 4 lists the optimal *SE* efficiency scores; η^*_{j,p_j} . The scare efficiency score, η^*_j, and the corresponding CV_t % are then calculated. It is found that all η^*_j are less than one for all DMU_js. However, the η^*_t scores are equal to one for eight periods, and hence the size is optimal at these periods.

Table 4. The estimated optimal SE scores.

Time period *t*													
DMU_j	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	η^*_j	$CV_j\%$
DMU_1	0.9584	0.7851	1.0000	1.0000	0.9533	1.0000							0.9495	8.78%
DMU_2		0.7851	1.0000	1.0000	0.9533	1.0000	0.9544						0.9488	8.78%
DMU_3			0.9528	1.0000	0.9533	1.0000	0.9510	1.0000					0.9762	2.67%
DMU_4				0.9944	0.9927	1.0000	0.9975	0.9817	1.0000				0.9944	0.69%
DMU_5					0.9927	1.0000	0.9975	0.9817	1.0000	0.9590			0.9885	1.62%
DMU_6						1.0000	0.9975	0.9817	1.0000	0.9590	0.9852		0.9872	1.61%
DMU_7							0.9638	0.9830	1.0000	0.9721	0.9970	1.0000	0.9860	1.58%
η^*_t	0.9584	0.7851	0.9843	0.9986	0.9691	1.0000	0.9769	0.9856	1.0000	0.9634	0.9911	1.0000		
$CV_t\%$	0.00%	0.00%	2.77%	0.28%	2.23%	0.00%	2.34%	0.82%	0.00%	0.79%	0.84%	0.00%		

Generally, the *PTE* and *SE* scores provide an indication for the reason behind the inefficiency in the *TE*, $\overline{\theta}^*_j$, values for each *DMU*. For DMU_j, if the *PTE* inefficiency, $\overline{\gamma}^*_j$, value is larger than its corresponding *SE* inefficiency score, $\overline{\eta}^*_j$, then the reason behind the $\overline{\theta}^*_j$ is managerial. However, if the $\overline{\gamma}^*_j$ value is smaller than its corresponding $\overline{\eta}^*_j$, then the reason of the $\overline{\theta}^*_j$ is the size of operation. Finally, the size of operation is optimal when $\overline{\gamma}^*_j$ and $\overline{\eta}^*_j$ scores are equal.

(2) MPI Analysis for Blowing Machine

The Malmquist productivity index (*MPI*) is calculated and displayed in Table 5, in which four evaluation periods each of a planning horizon of three months are considered. For illustration, the $\theta_1^{1,L}(E^1)$ value of 0.5557 is calculated as follows Eq. (13):

$$\theta_1^{1,L}(E^1) = Min\ \delta_1$$

Subject to:

$$\delta_1 x_{1,1}^{1,U} - \sum_{e\neq 1',e=2}^{4} \lambda_e x_{1,e}^{1,L} - \lambda_1 x_{1,1}^{1,U} \geq 0,$$

$$\delta_1 x_{2,1}^{1,U} - \sum_{e\neq 1',e=2}^{4} \lambda_e x_{2e}^{1,L} - \lambda_1 x_{2,1}^{1,U} \geq 0,$$

$$\delta_1 x_{3,1}^{1,U} - \sum_{e \neq 1', e=2}^{4} \lambda_e x_{3e}^{1,L} - \lambda_1 x_{3,1}^{1,U} \geq 0,$$

$$y_{1,1}^{1,L} - \sum_{e \neq 1, e=2}^{4} \lambda_e y_{1e}^{1,U} - \lambda_1 y_{1,1}^{1,L} \geq 0,$$

$$\lambda_e, \geq 0, \quad e = 1, \ldots, 4$$

The $\theta_1^{1,U}(E^1)$ of 0.5557 is estimated by solving Eq. (14). The other $\theta_{e'}^{p,L}(E^p)$ and $\theta_{e'}^{p,U}(E^p)$ are estimated similarly. Next, the $\theta_{e'}^{p+1,L}(E^{p+1})$ and $\theta_{e'}^{p+1,U}(E^{p+1})$ are estimated by replacing p by $p+1$. On the other hand, the efficiency of period $p+1$ (=2) in evaluation period e'=1, $\theta_1^{2,L}(E^1) = 0.6474$, is calculated aby solving Eq. (15):

$$\theta_1^{2,L}(E^1) = Min\ \omega_1$$

Subject to:

$$\omega_1 x_{1,1}^{1,U} - \sum_{e=1}^{4} \lambda_e x_{1,e}^{2,L} \geq 0,$$
$$\omega_1 x_{2,1}^{1,U} - \sum_{e=1}^{4} \lambda_e x_{2e}^{2,L} \geq 0,$$
$$\omega_1 x_{3,1}^{1,U} - \sum_{e=1}^{4} \lambda_e x_{3e}^{2,L} \geq 0,$$

$$y_{1,1}^{1,L} - \sum_{e=1}^{E} \lambda_e y_{1e}^{2,U} \geq 0,$$

$$\lambda_e \geq 0, \quad e = 1, \ldots, 4$$

The $\theta_1^{2,U}(E^1) = 1.0749$ are calculated by solving Eq. (16). The other $\theta_{e'}^{p+1,L}(E^p)$ and $\theta_{e'}^{p+1,U}(E^p)$ are calculated in a similar manner. Next, the $\theta_{e'}^{p,L}(E^{p+1})$ and $\theta_{e'}^{p,U}(E^{p+1})$ are estimated similarly. Finally, the $MPI_{e',p}^{L}$ and $MPI_{e',p}^{U}$ are calculated using Eqs. (17) and (18), respectively. Table 6 displays the estimated values of $MPI_{e',p}^{L}$ and $MPI_{e',p}^{U}$ with the corresponding technological and efficiency changes.

Table 5. The results of the MPI components.

Evaluation period	p	$\theta_{e'}^{p,L}(E^p)$	$\theta_{e'}^{p+1,L}(E^{p+1})$	$\theta_{e'}^{p+1,L}(E^p)$	$\theta_{e'}^{p,L}(E^{p+1})$	$\theta_{e'}^{p,U}(E^p)$	$\theta_{e'}^{p+1,U}(E^{p+1})$	$\theta_{e'}^{p+1,U}(E^p)$	$\theta_{e'}^{p,U}(E^{p+1})$
e_1	1	0.5557	0.4088	0.6474	0.2684	1.0000	0.8393	1.0749	1.9014
	2	0.4088	0.6616	0.2109	0.3901	0.8393	1.0000	1.0548	0.8055
	3	0.6616	0.9218	0.7141	1.2967	1.0000	1.0000	1.0743	2.0779

Table 5 continued on next page

Table 5 continued

Evaluation period	p	$\theta_{e'}^{p,L}(E^p)$	$\theta_{e'}^{p+1,L}(E^{p+1})$	$\theta_{e'}^{p+1,L}(E^p)$	$\theta_{e'}^{p,L}(E^{p+1})$	$\theta_{e'}^{p,U}(E^p)$	$\theta_{e'}^{p+1,U}(E^{p+1})$	$\theta_{e'}^{p+1,U}(E^p)$	$\theta_{e'}^{p,U}(E^{p+1})$
e_2	1	0.9218	0.2387	0.8225	0.1329	1.0000	1.0000	1.1689	1.1211
	2	0.2387	1.0000	9.5424	0.7706	1.0000	1.0000	64.2054	1.0538
	3	1.0000	0.5781	0.7000	30.2813	1.0000	1.0000	1.2656	39.64800
e_3	1	0.5781	1.0000	1.6200	0.1063	1.0000	1.0000	2.5088	1.1089
	2	1.0000	1.0000	0.3474	0.8712	1.0000	1.0000	2.3494	1.0980
	3	1.0000	0.7690	0.7375	2.2617	1.0000	1.0000	1.0600	3.2885
e_4	1	0.7690	0.8266	0.8370	0.0796	1.0000	1.0000	1.2139	1.1211
	2	0.8266	0.8363	0.1562	0.8363	1.0000	1.0000	1.1583	1.0828
	3	0.8363				1.0000			

In order to determine the sources of *MPI* regress, the components of the *MPI*; efficiency change and technology change, were calculated as also shown in Table 6.

Table 6. The results of the lower and upper MPI values.

No.	e’,p	Technology Change $\left[\frac{\theta_{e'}^{p,U}(p)}{\theta_{e'}^{p+1,U}(p)} \times \frac{\theta_{e'}^{p,L}(p+1)}{\theta_{e'}^{p+1,L}(p+1)}\right]^{1/2}$	Efficiency Change $\left[\frac{\theta_{e'}^{p+1,L}(p+1)}{\theta_{e'}^{p,U}(p)}\right]$	$MPI_{e',p}^{L}$	Technology Change $\left[\frac{\theta_{e'}^{p,L}(p)}{\theta_{e'}^{p+1,L}(p)} \times \frac{\theta_{e'}^{p,U}(p+1)}{\theta_{e'}^{p+1,U}(p+1)}\right]^{1/2}$	Efficiency Change $\left[\frac{\theta_{e'}^{p+1,U}(p+1)}{\theta_{e'}^{p,L}(p)}\right]$	$MPI_{e',p}^{U}$
1	1, 1	0.7815	0.4088	0.3195	1.3945	1.5103	2.1061
2	1, 2	0.6850	0.7883	0.5399	1.2495	2.4462	3.0566
3	1, 3	1.1443	0.9218	**1.0548**	1.3875	1.5115	2.0972
4	2, 1	0.6902	0.2387	0.1647	1.1209	1.0848	1.2160
5	2, 2	0.1096	1.0000	0.1096	0.1624	4.1894	**0.6802**
6	2, 3	6.4334	0.5781	**3.7191**	7.5260	1.0000	7.5260
7	3, 1	0.2058	1.0000	0.2058	0.6291	1.7298	1.0881
8	3, 2	0.6089	1.0000	0.6089	1.7778	1.0000	1.7778
9	3, 3	1.6657	0.7690	**1.2809**	2.1116	1.0000	2.1116
10	4, 1	0.2817	0.8266	0.2328	1.0149	1.3004	1.3198
11	4, 2	0.9292	0.8363	0.7771	2.3938	1.2098	2.8959

In Table 6, it is noted that $MPI_{e',p}^{L}$ (worst) is only larger than one in three periods; 3, 6, and 9, which indicates a progress. However, the $MPI_{e',p}^{U}$ (best) is only smaller than one in period 5, which indicates a regress in productivity from time p to $p+1$.

4. RESEARCH RESULTS

For window analysis, the inefficiency scores for all seven *DMUs* were calculated and then displayed in Table 7 and Fig. 1. It is clear in Fig. 1 that the main reason behind the $\overline{\theta}_j^*$ for five *DMUs* is contributed by the size of operations; scale inefficiency. However, the source of $\overline{\theta}_j^*$ for two *DMUs* is contributed by management. Further, the inefficiency scores for all months were estimated and then shown in Table 8 and Fig. 2, where it is found that the scale is optimal in only three months; t =6, 9, and 12, out of the twelve months. Moreover, the technical inefficiency, $\overline{\theta}_t^*$, is attributed by scale inefficiency, $\overline{\eta}_t^*$, in eight months. Finally, the pure technical efficiency, $\overline{\gamma}_j^*$, is the reason behind the $\overline{\theta}_j^*$ in one month, $t = 7$.

Table 7. The estimated inefficiency scores for DMUs.

a	θ_j^*	γ_j^*	η_j^*	$\overline{\theta}_j^*$	$\overline{\gamma}_j^*$	$\overline{\eta}_j^*$	**Reason**
DMU_1	0.9493	0.9998	0.9495	0.0507	0.0002	0.0505	Scale
DMU_2	0.9488	1.0000	0.9488	0.0512	0.0000	0.0512	Scale
DMU_3	0.9762	1.0000	0.9762	0.0238	0.0000	0.0238	Scale
DMU_4	0.9799	0.9854	0.9944	0.0201	0.0146	0.0056	Management
DMU_5	0.9740	0.9854	0.9885	0.0260	0.0146	0.0115	Management
DMU_6	0.9783	0.9911	0.9872	0.0217	0.0089	0.0128	Scale
DMU_7	0.9853	0.9993	0.9860	0.0147	0.0007	0.0140	Scale

Figure 1. The inefficiency values for all DMUs.

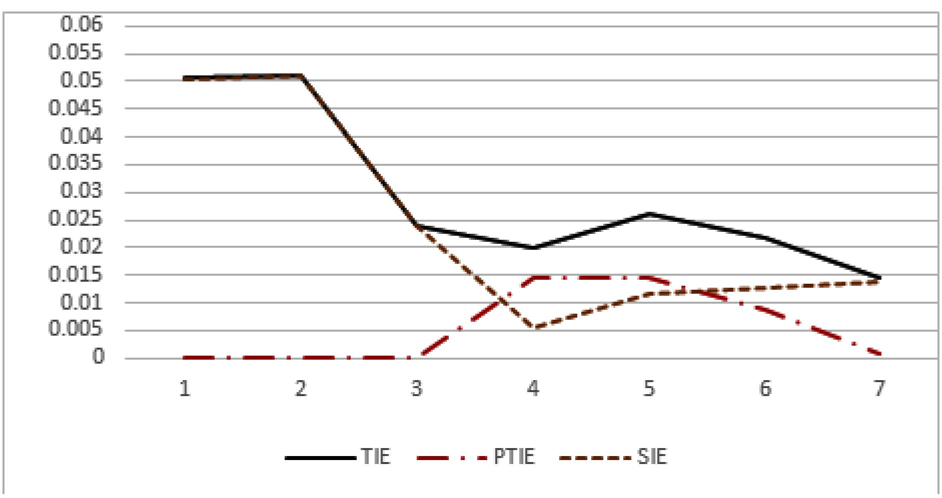
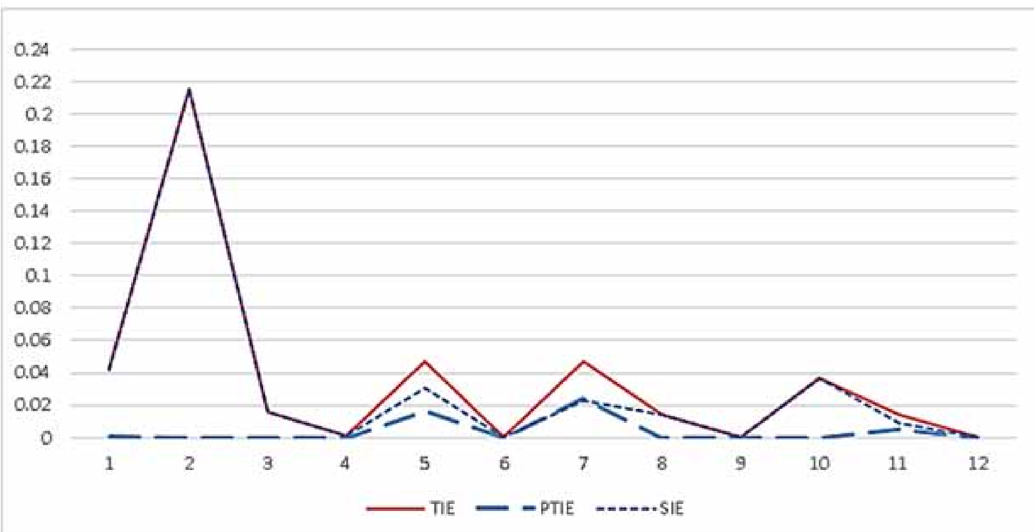


Table 8. The estimated inefficiency scores for months.

Period	θ_t^*	γ_t^*	η_t^*	$\overline{\theta}_t^*$	$\overline{\gamma}_t^*$	$\overline{\eta}_t^*$	Reason
1	0.9575	0.9990	0.9584	0.0425	0.0010	0.0416	Scale
2	0.7851	1.0000	0.7851	0.2149	0.0000	0.2149	Scale
3	0.9843	1.0000	0.9843	0.0157	0.0000	0.0157	Scale
4	0.9986	1.0000	0.9986	0.0014	0.0000	0.0014	Scale
5	0.9533	0.9841	0.9691	0.0467	0.0159	0.0309	Scale
6	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000	Optimal size
7	0.9531	0.9761	0.9769	0.0469	0.0239	0.0231	Management
8	0.9856	1.0000	0.9856	0.0144	0.0000	0.0144	Scale
9	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000	Optimal size
10	0.9634	1.0000	0.9634	0.0366	0.0000	0.0366	Scale
11	0.9861	0.9949	0.9911	0.0139	0.0051	0.0089	Scale
12	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000	Optimal size

Figure 2. The inefficiency values for all months.



In practice, to resolve the scale inefficiency due to size of operations, planning engineers may reschedule the allocation of the customer orders, size of inventory, or purchase new production machines. Further, pure technical inefficiency was caused because some *DMUs* resources were managed inefficiently. This problem can be addressed by increasing the actual production quantity by utilizing the available resources, adopting an effective quality control system to reduce defects quantity, and improving planning and machine reliability o reduce idle time.

For *MPI* analysis, on the other hand, the change (= $MPI^{U}_{e',p}$ - $MPI^{L}_{e',p}$) in *MPI* values are depicted in Fig. 3. Clearly, there is large change (say; greater than 1.5) in *MPI* values for periods 1, 2, 6, and

11 when each is compared to its following period. Fig. 4 displays the comparison between the lower and upper values for each of technology change and efficiency change. It is noted that differences between the upper and lower values does not exceed one in most at most time periods in both figures of technology change and efficiency change. This figure can provide useful information about the worst to best changes in technology and efficiency change. Further, Fig. 5 depicts the *MPI* and its components at the upper and lower bound, where it is found that whether the regress or progress in *MPI* was due to technological change or efficiency change. For example, at the lower bound of *MPI* at period 4, the reason behind *MPI* progress was the technological change. Similarly, the reason behind the *MPI* regress at the upper bound of period 5 was due to technological change. Such information provides valuable feedback about worst to best efficiency changes and technology changes from period *p* to *p*+1, and support decision makers in identifying the proper actions to enhance efficiency and/or introduce new technology to enhance *MPI*.

Figure 3. The lower and upper MPI values.

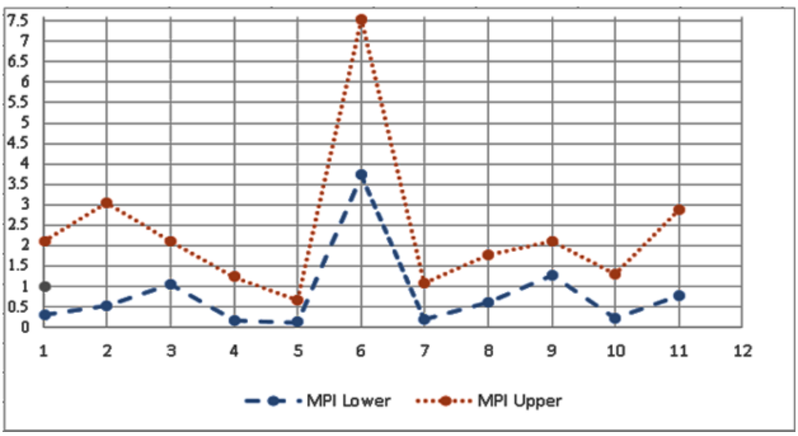


Figure 4. The comparison between lower and upper values for each of technology and efficiency changes.

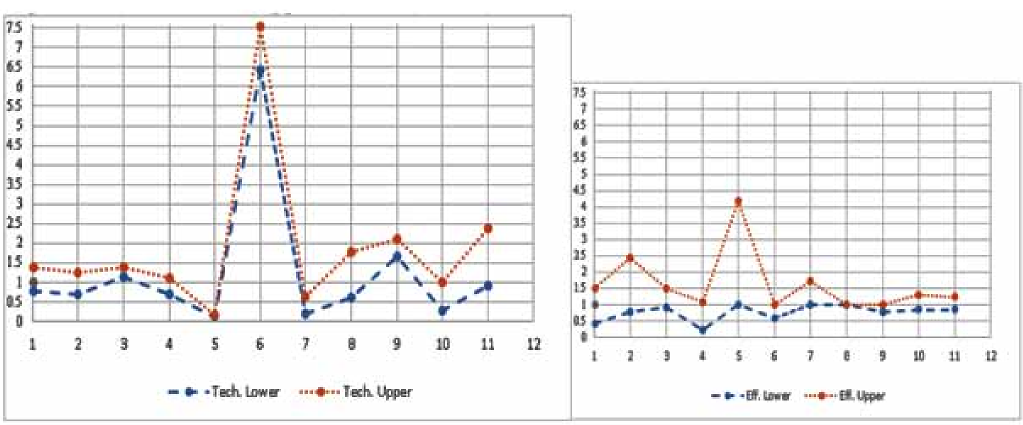
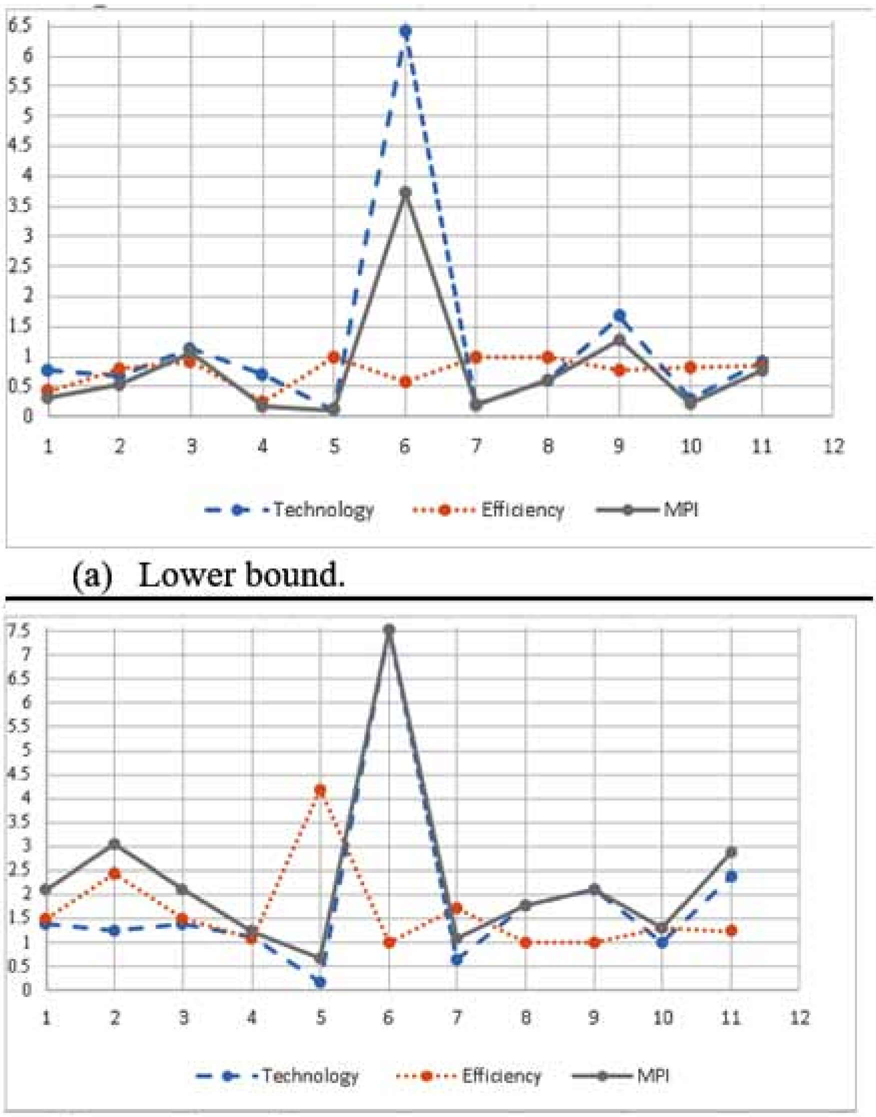


Figure 5. The technology change, efficiency change, and MPI.



(a) Lower bound.

(b) Upper bound.

5. CONCLUSION

This study proposed a procedure for window analysis followed by the Malmquist productivity index in DEA to assess the efficiency and productivity of manufacturing processes under fuzzy inputs and outputs. The proposed procedure was illustrated to measure efficiency and productivity of a blowing machine. For this process, the production quantity was the output, whereas the planned production quantity, defect quantity, and idle time in units were the inputs for all windows. Then, the technical, pure technical, and scale efficiencies were calculated using the proposed optimization models in window analysis. The sources for technical inefficiency were identified for each decision making unit and each period. Next, the lower and upper *MPI* values with the corresponding technology change and efficiency change were calculated to identify the reason behind productivity progress or regress in each period. The results of DEA window analysis showed that the main cause of technical inefficiency in blowing machine was the scale inefficiency. Hence, there is a need to optimize the size of operations. From productivity analysis, the *MPI* values indicated progress in productivity at three periods; 3, 6, and 9, whereas a regress in productivity was indicated at period 5. Such analysis provides valuable guidance on which *MPI* component to be enhanced. In conclusion, the proposed procedure can provide great assistance to decision maker when evaluating the efficiency and productivity of the manufacturing process and guide them to the proper actions to enhance its performance.

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Abbas Al-Refaie received his Ph.D. degree in Industrial Engineering and Systems Management in June,2008. He started is academic career as assistant professor of industrial engineering in Sep. 2008. Thereafter, he was promoted to the rank of Associate Professor in Sept. 2012 and to the rank of Full Professor of Industrial Engineering in Sept. 2016. In 2014, he received the UJ's Distinguished Researcher Award in Science for years 2012 and 2013. He also received the 2018- Ali Mango award for distinguished researcher in science in Jordan. His students won the first rank in the competition of applied projects in Jordan industry for years 2014, 2016, 2017, and 2018. His research focuses on experimental design, quality engineering, optimization, and engineering management.