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On the (M,D) number of a graph

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Abstract

For a connected graph $G = (V, E)$, a monophonic set of G is a set $M \subseteq V(G)$ such that every vertex of G is contained in a monophonic path joining some pair of vertices in M. A subset D of vertices in G is called dominating set if every vertex not in D has at least one neighbour in D. A monophonic dominating set M is both a monophonic and a dominating set. The monophonic,dominating,monophonic domination number $m(G), \gamma(G), \gamma_m(G)$ respectively are the minimum cardinality of the respective sets in G. Monophonic domination number of certain classes of graphs are determined. Connected graph of order p with monophonic domination number $p-1$ or p is characterised. It is shown that for every two intigers $a, b \geq 2$ with $2 \leq a \leq b$, there is a connected graph G such that $\gamma_m(G) = a$ and $\gamma_g(G) = b$, where $\gamma_g(G)$ is the geodetic domination number of a graph.

Keywords: *monophonic number, domination number, monophonic domination number, geodetic domination number.*

AMS Subject classification: *05C05,05C69*

1. Introduction

By a graph $G = (V, E)$,we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. For basic graph theoretic terminology we refer to Harary [1]. The vertices u and v in a connected graph G,the distance $d(u, v)$ is the length of a shortest $u - v$ path in G. The *eccentricity* $e(v)$ of a vertex v in G is the maximum distance from v and a vertex of G. The minimum eccentricity among the vertices of G is the *radius*, rad G or $r(G)$ and the maximum eccentricity is its *diameter*, *diamG* of G. An $u-v$ path of length $d(u, v)$ is called an $u-v$ *geodesic*. A vertex x is said to lie on a $u-v$ geodesic P if x is a vertex of P including the vertices u and v. A geodetic set of G is a set $S \subseteq V(G)$ such that every vertex of G contained in a geodesic joining some pair of vertices in S. The geodetic number $g(G)$ of G is the minimum order of its geodetic sets and any geodetic set of order $g(G)$ is a geodetic basis of G . The geodetic number was introduced in[7] and further studied studied in [4,8]. A chord of a path P is an edge joining two non adjacent vertices of P. A path P is called monophonic if it is a chordless path. A monophonic set of G is set $M \subseteq V$ such that every vertex of G is contained in a monophonic path joining some pair of vertices in M. The monophonic number $m(G)$ of G is the minimum order of its monophonic sets and any monophonic set of order $m(G)$ is a minimum monophonic set or simply a $m-$ set of G. The monophonic number of a graph G is studied in [5,6,9]. If $e = uv$ is an edge of a graph G with $d(u) = 1$ and $d(v) > 1$, then we call e a *pendent edge*, u a leaf and v a support vertex. Let $L(G)$ be the set of all leaves of a graph G. We denote by P_p, C_p and $K_{r,s}$, the path on p vertices, the cycle on p vertices and complete bipartite graph in which one partite set has r vertices and the other partite set has s vertices respectively. For any set M of vertices of G, induced subgraph $< M >$ is the maximal subgraph of G with vertex set M.For any connected graph G, a vertex $v \in V(G)$ is called a *cut vertex* of G if $< V - \{v\} >$ is no longer connected. A maximum connected induced subgraph without a cut vertex is called a block of G. A graph G is a block graph if every block in G is complete. Sum of two graphs G_1 and G_2 is the union of G_1 and G_2 together with all the lines joining vertices of G_1 to vertices of G_2 . Let $N(v) = \{u \in V(G) : uv \in E(G)\}$ is called the neighborhood of the vertex v in G. A vertex v is a simplicial vertex of a graph G if $< N(v) >$ is complete. A simplex of a graph G is a subgraph of G which is a complete graph. A vertex v in a graph G dominates itself and its neighbours. A set of vertices D in a graph G is a dominating set if each vertex of G is dominated by some vertices of D. The dominating number $\gamma(G)$ of G is the minimum cardinality of a dominating set of G. For references on domination parameters in graphs see [2,3]. A set of vertices M in G is called a geodetic dominating set if M is both a geodetic set and a dominating set. The minimum cardinality of a geodetic dominating set of G is its geodetic domination number and is denoted by $\gamma_g(G)$. A geodetic dominating set of size $\gamma_g(G)$ is said to be a γ_g- set. The geodetic domination number of a graph was introduced and studied in [8]. It is easily seen that a dominating set is not in general a monophonic set in a graph G. Also the converse is not valid in general. This has motivated us to study the new domination conception of monophonic domination. We investigate subsets of vertices of a graph that are both a monophonic set and a dominating set. We call these sets as a monophonic dominating sets. We call the minimum cardinality of the monophonic dominating set of G,the monophonic domination number of G. Throughout this paper G denotes simple connected graph with at least two vertices

The following theorems are used in sequel.

Theorem 1.1. *[9]Each simplicial vertex of a connected graph G belongs to every monophonic set of G. In particular every end vertex of a connected graph G belongs to every monophonic set of G.*

Theorem 1.2. *[8]Each simplicial vertex of a connected graph G belongs to every geodetic dominating set of G. In particular every end vertex of a connected graph G belongs to every geodetic dominating set of G.*

2. The Monophonic Domination Number Of a Graph

Definition 2.1. *Let G be a connected graph. A set of vertices M in G is called a monophonic dominating set or simply (M, D)-set if M is both a monophonic set and a dominating set. The minimum cardinality of a (M, D)- set of G is its monophonic domination number or simply (M, D)-number and is denoted by $\gamma_m(G)$. A (M, D)-set of size $\gamma_m(G)$ is said to be a γ_m-set.*

Example 2.2. *For the graph G is given in Figure 2.1, $M = \{v_1, v_4\}$ is a (M, D)-set of G so that $\gamma_m(G) = 2$.*

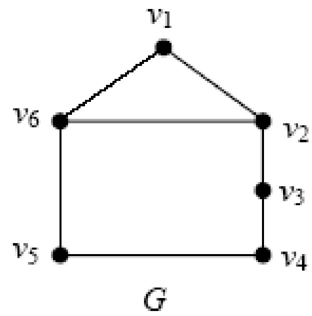


Figure 2.1

Remark 2.3. *Each simplicial vertex of a connected graph G belongs to every (M, D)- set of G.*

Remark 2.4. *Let G be a connected graph and v be a cut-vertex of G. Then every (M, D)- set contains at least one element from each component of $G - v$.*

Remark 2.5. *If G is a connected graph of order p,then*
$2 \leq \max\{m(G), \gamma(G)\} \leq \gamma_m(G) \leq p$.

Remark 2.6. *For any cycle $C_P, (p \geq 4)$, $\gamma_m(C_P) = \gamma(C_P) = \lceil p/3 \rceil$.*

In the following, we determine the (M,D)- number of some standard graphs.

Theorem 2.7. *For the complete graph $K_p(p \geq 2)$, $\gamma_m(K_p) = p$.*

Proof. Since every vertex of the complete graph $K_p(p \geq 2)$ is a simplical vertex, the vertex set of K_p is the unique (M, D)-set of K_p. Thus $\gamma_m(K_P) = p$. □

Theorem 2.8. *For the wheel $G = W_p(p \geq 4)$,*

$\gamma_m(w_p) = 4$, if $p = 4$;
2, if $p = 5, 6$;
3, if $p \geq 7$.

Proof. Let $\{x, v_1, v_2...v_{p-1}\}$ be the vertices of $G = W_p (p \geq 4)$, with $deg(x) = p-1$.

Case(i) Let $p = 4$. Then $G = K_4$ and by Theorem 2.7,$\gamma_{m(W_p)} = 4$.

Case(ii) Let $p = 5$ or 6. Then $M = \{v_1, v_3\}$ is a (M, D)-set of G so that $\gamma_{m(W_p)} = 2$.

Case(iii) Let $p \geq 7$. Let $M = \{x, v_i, v_j\}$ $(1 \leq i \neq j \leq p-1)$, where v_i and v_j are any two non adjacent vertices of G. Then M is a (M, D)-set of G so that $\gamma_m(G) \leq 3$. Suppose that $\gamma_m(G) = 2$. Then there exists a (M, D)-set M' such that $|M'| = 2$. If $M' = \{x, v_i\}$,$(1 \leq i \leq p-1)$, then xv_i, $(1 \leq i \leq p-1)$ is a chord of path $x - v_i$ and so M' is not a (M, D)-set of G, which is a contradiction. If $M' = \{v_i, v_j\}(1 \leq i \neq j \leq p-1)$ then M' is a monophonic set of G which is not a dominating set of G,which is a contradiction. Therefore $\gamma_m(W_p) = 3$. □

Theorem 2.9. *For the complete bipartite graph* $G = K_{m,n}$,$\gamma_{m(K_{m,n})} =$
2, *if* $m = n = 1$
n *if* $n \geq 2, m = 1$
$\min\{m, n, 4\}$ *if* $m, n \geq 2$.

Proof. Case(i). Let *m=n=1*. Then $G = K_2$.By Theorem 2.7 $\gamma_m(G) = 2$.

Case(ii). Let $m = 1, n \geq 2$. Then $G = K_{1,n}$. Let M be the set of n end vertices of G. Then by Remark 2.3,$\gamma_m(G) \geq n$. It is clear that M is a (M,D)-set of G so that $\gamma_m(G) = n$.

Case(iii) Let $2 \leq m \leq n$. Let $U = \{u_1, u_2...u_m\}$ and $V = \{v_1, v_2...v_n\}$ be the bipartite sets of G

Subcase iiia. Let $m = 2, n \geq 2$. Then $U = \{u_1, u_2\}$ is a (M, D)-set of G so that $\gamma_m(G) = 2$.

Subcase iiib. Let $m = 3$ and $n \geq 3$. Then $M = \{u_1, u_2, u_3\}$ is a (M, D)-set of G and so $\gamma_m(G) \leq 3$. Let M' be a $(M, D)-$ set of G with $|M'| = 2$. If $M' \subset U$, then there exists $x \in U$ such that $x \notin M'$. Then the vertex x doesnot lie on a monophonic path joining a pair of vertices of M', which is a contradiction. If $M' \subset W$, then there exists at least one $y \in W$ such that $y \notin M'$. Then the the vertex y doesnot lie on monophonic path joining a pair of vertices of M', which is contradiction. If $M' \subset U \cup W$, then $M' = \{u_i, w_j\}(1 \leq i \leq 3), (1 \leq j \leq n)$. Since u_iw_j is a chord of the path $u_i - w_j$, M' is not a (M, D)-set of G,which is a contradiction. Therefore $\gamma_m(G) = 3$.

Subcase iiic. Let $m \geq 4$ and $n \geq 4$. Then $M = \{u_1, u_2, v_1, v_2\}$ is a (M, D) set of G and so that $\gamma_m(G) \leq 4$. By the similar argument given in Subcase iiib, there is no (M, D)-set M' such that $\mid M' \mid= 2$ or $\mid M' \mid= 3$. Hence $\gamma_m(G) = 4$. □

Theorem 2.10. *If G is a non complete connected graph such that it has a minimum cut set, then $\gamma_m(G) \leq p - k(G)$.*

Proof. Since G is non complete, it is clear that $1 \leq k(G) \leq p - 2$. Let $U = \{u_1, u_2, ..., u_k\}$ be a minimum cut set of G. Let $G_1, G_2, ..., G_r (r \geq 2)$ be the components of $G - U$ and let $M = V(G) - U$. Then every vertex $u_i (1 \leq i \leq k)$ is adjacent to at least one vertex of G_j for every $j (1 \leq j \leq r)$. It is clear that M is a (M, D)-set of G so that $\gamma_m(G) \leq p - k(G)$. □

Theorem 2.11. *Let G be a connected graph of order $p \geq 2$. Then $\gamma_m(G) = 2$ if and only if there exist a (M, D)-set $M = \{u, v\}$ of G such that $d(u, v) \leq 3$.*

Proof. Suppose $\gamma_m(G) = 2$. Let $M = \{u, v\}$ be a (M, D)-set of G. Suppose that $d(u, v) \geq 4$. Then the diametrical path contains at least three internal vertices. Therefore $\gamma_m(G) \geq 3$, which is a contradiction. Therefore $d(u, v) \geq 3$. The converse is clear. □

Theorem 2.12. *Let G be a connected graph of order $p \geq 2$. Then $\gamma_m(G) = p$ if and only if G is the complete graph on p vertices.*

Proof. Suppose $G = K_p$. Then by Theorem 2.7, $\gamma_m(G) = p$. Conversely, let $\gamma_m(G) = p$. Suppose that G is non complete. Then by Theorem 2.10, $\gamma_m(G) \leq p - 1$, which is a contradiction. It follows that G is complete. □

Theorem 2.13. *Let G be a connected graph of order $p \geq 2$. Then $\gamma_m(G) = p - 1$ if and only if $G = K_1 + \bigcup m_j K_j$, where $\sum m_j \geq 2, j \geq 1$.*

Proof. Suppose $\gamma_m(G) = p - 1$. Then by Theorem 2.10, $k(G) = 1$. Therefore G contains only one cut vertex, say v. We show that each component of $G - \{v\}$ is complete. Suppose that there exist a component G_1 of $G - \{v\}$ such that G_1 is non complete. Then $|G_1| \geq 2$. Let u be the non simplicial vertex of G_1. Then $M = V(G) - \{u, v\}$ is a (M, D)-set of G so that $\gamma_m(G) \leq p - 2$, which is a contradiction. Hence each component of $G - \{v\}$ is complete. Therefore $G = K_1 + \bigcup m_j K_j$, where $\sum m_j \geq 2$. Conversely suppose $G = K_1 + \bigcup m_j K_j$ where $\sum m_j \geq 2$. Then it is clear that $\gamma_m(G) = p - 1$. □

Remark 2.14. *If G is a graph of order p, then $\gamma_m(G) + \gamma_m(\bar{G}\ ) \leq 2p$ and $\gamma_m(G) + \gamma_m(\bar{G}) = 2p$ if and only if $G = K_p$ or $\bar{G} = K_p$.*

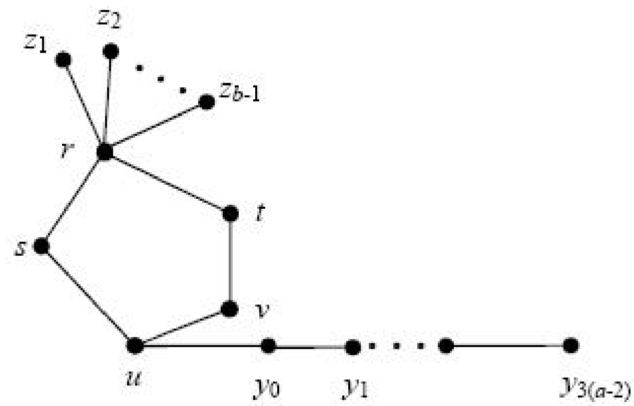
Theorem 2.15. *If G is a connected graph of order p, then $\gamma_m(G) + \gamma_m(\bar{G}) = 2p - 1$ if and only if $p \geq 3$ and $G = K_{1,p-1}$ or $\bar{G} = K_{1,p-1}$.*

Proof. Suppose $p \geq 3$ and $G = K_{1,p-1}$ or $\overline{G} = K_{1,p-1}$. Then by Theorem 2.9(ii), $\gamma_m(G) + \gamma_m(\overline{G}) = 2p - 1$. Conversely suppose $\gamma_m(G) + \gamma_m(\overline{G}) = 2p - 1$. Then $\gamma_m(G) = p$ or $\gamma_m(\overline{G}) = p$. Without loss of generality, we assume that $\gamma_m(\overline{G}) = p$. Then $\gamma_m(G) = p - 1$. We prove that the components of $\overline{G}$ are complete graphs. If not, then $\overline{G}$ contains a component H with two non adjacent vertices u and v. Let P be a path in $u - v$ geodesic in H and x be a vertex of P adjacent to u. Let $S = V(\overline{G}) - \{x\}$. Then S is a monophonic dominating set of G so that $\gamma_m(\overline{G}) \leq p - 1$,which is a contradiction to $\gamma_m(\overline{G}) = p$. If $\overline{G}$ is not connected, then $p \geq 2$ and G is connected. By Theorem 2.13, we find that there exists a vertex v in G such that v is adjacent to every other vertex of G and $G - v$ is the union of at least two complete graphs. Therefore $p \geq 3$. Since $\gamma_m(\overline{G}) = p$, the components of $G - \{v\}$ are isolated vertices. This shows that $G = K_{1,p-1}$. □

3. Realization results

Theorem 3.1. *For any two integers $a, b \geq 2$, there is a connected graph G such that $\gamma(G) = a$,$m(G) = b$ and $\gamma_m(G) = a + b$.*

Proof. Let $F : r, s, u, v, t, r$ be a copy of C_5. Let H be a graph obtained from F by adding the new vertices $z_1, z_2, ..., z_{b-1}$ and join each to the vertex r. Let G be the graph obtained from H by taking a copy of the path on $3(a-2)+1$ vertices $y_0, y_1, ..., y_{3(a-2)}$ and joining y_0 to the vertex u as shown in the Figure 3.1.Let $Z = \{r, u, y_2, y_5, ..., y_{3(a-2)-1}\}$.Then it is clear that Z is a minimum dominating set of G so that $\gamma(G) = a$. Let $Z' = \{z_1, z_2, ..., z_{b-1}, y_{3(a-2)}\}$. Then by Theorem 1.1, Z' is a subset of every monophonic set of G and so $m(G) \geq b$. Now Z' is a monophonic set of G so that $m(G) = b$. By Remark 2.3, Z' is subset of every (M, D)-set of G. Now, let $M = Z \cup Z'$. It is clear that M is a minimum (M, D)-set of G so that $\gamma_m(G) = a + b$. □



G
Figure 3.1

Theorem 3.2. *For any two integers $a, b \geq 2$ with $2 \leq a \leq b$, there is a connected graph G such that $\gamma_m(G) = a$ and $\gamma_g(G) = b$.*

Proof. Let $P : x, y, z$ be a path on three vertices.Let $P_i : u_i, v_i(1 \leq i \leq (b - a + 2)$ be a path on two vertices. Let H be a graph obtained from P and P_i by joining each $u_i(1 \leq i \leq b - a + 2)$ with x and each $v_i(1 \leq i \leq b_{a+2})$ with z. Let G be a graph obtained from H by adding the new vertices $z_1, z_2, ..., z_{a-2}$ and joining each $z_i(1 \leq i \leq a - 2)$ with x and y as shown in Figure 3.2. First we show that $\gamma_m(G) = a$. Let $Z' = \{z_1, z_2, ..., z_{a-2}\}$ be the set of all of simplicial vertices of G. By Remark 2.3, Z is subset of every (M, D)-set of G. It is clear that Z is not a (M, D)-set of G. It is easily verified that $Z \cup \{v\}$, where $v \notin Z$ is not a (M, D)-set of G and so $\gamma_m(G) \geq a$.However $M = Z \cup \{x, z\}$ is a (M, D)-set of G so that $\gamma_m(G) = a$. Next, we show that $\gamma_g(G) = b$. By Theorem 1.2 Z is subset of every geodetic dominating

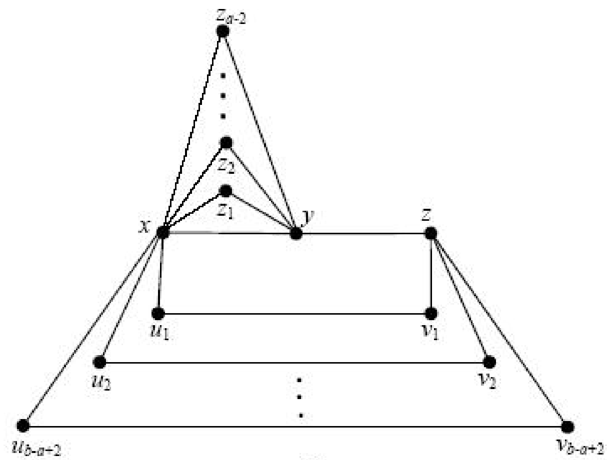


Figure 3.2

set of G. It can be easily verified that Z is not a geodetic dominating set of G. Now M=$Z \cup \{v_1, v_2, ..., v_{b-a+2}\}$ is a geodetic dominating set of G so that $\gamma_g(G) \leq b$. Let $H_i = \{u_i, v_i\}(1 \leq i \leq b - a + 2)$. Let S be a geodetic dominating set of G. Suppose that $z \in S$. Then S contains at least one element of each $H_i(1 \leq i \leq b - a + 2)$. If not suppose that $u_1, v_1 \notin S$. Then u_1, v_1 do not lie on a geodesic joining a pair of vertices of S, which is a contradiction. Therefore S contains at least one element of each $H_i(1 \leq i \leq b - a + 2)$. Hence it follows that $\gamma_g(G) \geq a - 2 + 1 + b - a + 2 = b + 1$, which is a contradiction. Therefore $z \notin S$. Let $G_i = \{u_i, v_{i+1}\}(1 \leq i \leq b - a + 1)$, $Q_i = \{v_i, u_{i+1}\}(1 \leq i \leq b - a + 1)$ and $S' = \{v_1, v_2, ..., v_{b-a+2}\}$. It is easily observed that S contain at least one element from each $G_i(1 \leq i \leq b - a + 2)$ or least one element from each $Q_i(1 \leq i \leq b-a+2)$ or $S' \subseteq S$. Hence it follows that $\gamma_g(G) = a - 2 + b - a + 2 = b$. □

4. Block Graphs

Theorem 4.1. *Let G be a connected block graph of order $p \geq 2$, and let M be the set of simplicial vertices of G. Then M is the unique minimum monophonic set of G.*

Proof. The theorem is obvious when $M = V(G)$. Hence assume that $M \subseteq V(G)$. Let $v \in V(G) - M$ be an arbitrary vertex. It follows that v is a cut-vertex of G. Let H and H' be two components of $G - \{v\}$ and H and H' are also block graphs. Let $x \in V(H)$ and $x' \in V(H')$ be two simplicial vertices of G. Let P be a monophonic path from x to x' in G. Since v is a cut-vertex of G containing v, the monophonic path contains v. Hence P is a $x - x'$ monophonic path of G containing v. Then $J[M] = V(G)$. Thus M is a monophonic set of G. As every monophonic set M' of G must contain M, the set M is the unique monophonic set of G. □

Theorem 4.2. *If G is a connected block graph of order $p \geq 2$, then the following conditions are equivalent.*

(a) $\gamma_m(G) = m(G) = \gamma(G)$.

(b) The set M of simplicial vertices of G is a minimum dominating set of G.

(c) Every block of G contains at most one simplicial vertex, and every vertex of G belongs to exactly one simplex of G.

Proof. $(a) \Rightarrow (b)$. Suppose $\gamma_m(G) = m(G) = \gamma(G)$. Then by Theorem 4.1, the set M of simplicial vertices of G is a minimum dominating set of G.
$(b) \Rightarrow (a)$. Suppose the set M of simplicial vertices of G is a minimum dominating set of G. It follows that $\gamma_m(G) = m(G) = \gamma(G)$.

$(c) \Rightarrow (b)$. Let $G_1, G_2, ...G_k$ be the simplexes of G with simplicial vertices $v_i \in V(G_i)$ for $i = \{1, 2, ..., k\}$. Clearly each simplex G_i is also a block of G. Since every block of G contains at most one simplicial vertex, v_i is the only simplicial vertex of G_i,(i=1,2,...,k). The hypothesis that every vertex of G belongs to exactly one simplex of G shows that $V(G) = V(G_1) \cup V(G_2) \cup ... \cup V(G_k)$. Therefore $M = \{v_1, v_2, ..., v_k\}$ is a dominating set of G. On the contrary, suppose that G contains a dominating set M' with $|M'| < |M|$. This implies that there exists a vertex $y \in M'$such that y dominates at least two simplicial vertices, say v_1 and v_2 which is a contradiction. This contradiction shows that y belongs to the simplexes $G_1$and G_2. Hence M is a minimum dominating set of G.

$(b) \Rightarrow (c)$. Suppose that the set M of simplicial vertices of G is a minimum dominating set of G. If there is a block containing two simplicial vertices u and v, then $M - \{u\}$ is also a dominating set of G, which is a contradiction. This shows that every block of G contains at most one simplicial vertex. If there exists a vertex which does not belong to any simplex of G, then M is not a dominating set of G, which is a contradiction. Finally, on the contrary, suppose that there is a vertex u belonging to at least two simplexes of G_1 and G_2. If v_1 and v_2 are simplicial vertices of G_1 and G_2 then $(M - \{v_1, v_2\}) \cup \{u\}$ is a dominating set of G, which is a contradiction. Hence, every vertex of G belong to exactly one simplex of G. □

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