CHAPTER 5

The Structure of Partially Coherent Fields

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1. PREFACE

The general framework of optical coherence theory is now well established and has been described in numerous publications (see Beran & Parrent, 1964; Born & Wolf, 1999; Goodman, 1985; Mandel & Wolf, 1995; Marathay, 1982; Perina, 1985; Schouten & Visser, 2008; Troup, 1967; Wolf, 2007b). In this article, we provide an overview of recent advances, both theoretical and experimental, that have been made in a number of areas of classical optical coherence. These advances have been spurred on by the introduction of the space-frequency representation of partially coherent fields, to be discussed in Section 2, and an increased emphasis on the

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spatial coherence properties of wave fields. The fundamental experiment to measure spatial coherence is, of course, Young's double-slit experiment, which still provides many insights to this day; recent developments will be discussed in Section 3.

A number of important optical processes are influenced by the coherence properties of the wave field. Results relating to the propagation of partially coherent wavefields are too numerous to be comprehensively covered here, but Section 4 highlights some of the significant results relating to optical beams. In Section 5, the influence of coherence on focusing is summarized and reviewed. In Section 6, the scattering of partially coherent wave fields, and its relation to inverse scattering problems, is discussed.

In recent years, it has been shown that spatial correlation functions have interesting topological properties associated with their phase singularities; these properties and the relevant literature are discussed in Section 7. The coherent mode representation and its applications are described in Section 8. Several techniques for the numerical simulation of wave fields with a prescribed statistical behavior are explained in Section 9. Many novel applications of partially coherent fields are described in the concluding Section 10.

As noted, optical coherence is a mature field of study, and a single review article cannot comprehensively cover all of the important developments. This article is restricted to results from the classical theory of optical coherence, and excludes discussion of the quantum theory. A number of other developments are discussed only in the context of the specific topics mentioned earlier. Among these are *correlation-induced spectral changes* and the relatively recent *unified theory of coherence and polarization*. "Correlation-induced spectral changes" refers to the important observation that the spectrum of a partially coherent wave field can change on propagation or scattering; a thorough review of research on the phenomenon was undertaken by Wolf and James (1996). The "unified theory of coherence and polarization" refers to a new formulation of the electromagnetic theory of optical coherence that has been used, among other things, to characterize the changes in the state and degree of polarization of electromagnetic fields on propagation; the fundamental results are reviewed by Wolf (2007b).

The topics discussed in this review are unified, in part, by the realization that the ability to manipulate the spatial coherence of a wave field provides an additional degree of control over the properties of that wave field. Many of the advances in optical coherence have come from the design of fields with unusual structural properties that are optimized for different applications.

2. THE SPACE-FREQUENCY REPRESENTATION

Optical coherence theory is the study of the statistical properties of light and their influence on the observable characteristics of optical fields. The beginnings of coherence theory can be traced back to Verdet (1865), who estimated the spatial coherence of sunlight on the Earth's surface, and van Cittert (1934) and Zernike (1948), who calculated the evolution of the spatial coherence of light propagating from an incoherent source.[1]

The modern theory of optical coherence, as championed by Wolf and others, began with the study of the mutual coherence function $\Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau)$ of wide-sense statistically stationary optical fields, defined as

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau) = \langle U^*(\mathbf{r}_1, t_1) U(\mathbf{r}_2, t_2) \rangle, \tag{2.1}$$

where the time difference $\tau \equiv t_2 - t_1$ and the angle brackets represent time averaging or, equivalently for ergodic fields, ensemble averaging. The field $U(\mathbf{r}, t)$ is typically taken to be scalar, with polarization effects neglected, but the formalism can be readily extended to the fully electromagnetic case, as discussed in detail in Wolf (2007b). It was shown by Wolf (1955) that the mutual coherence function satisfies a pair of wave equations in free space, namely,

$$\left(\nabla_1^2 - \frac{1}{c^2} \frac{\partial^2}{\partial \tau^2} \right) \Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau) = 0, \tag{2.2}$$

$$\left(\nabla_2^2 - \frac{1}{c^2} \frac{\partial^2}{\partial \tau^2} \right) \Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau) = 0, \tag{2.3}$$

where ∇_i^2 is the Laplacian with respect to the Cartesian coordinates of position vector $\mathbf{r}_i$ and c is the speed of light. From these equations one can see that the statistical properties of light evolve in a well-defined way on propagation, and much of the research in optical coherence theory has involved the study of the consequences of these equations of evolution.

Just as it is possible to study the behavior of deterministic wave fields in the time domain or the frequency domain, it is also possible to study the behavior of partially coherent wave fields in either time or frequency. The *cross-spectral density function* $W(\mathbf{r}_1, \mathbf{r}_2, \omega)$ is defined as the temporal

[1] More details on the history of optical coherence theory can be found in Born and Wolf (1999), section 10.1, and Wolf (2001). Reprints of a number of classic articles can be found in Mandel and Wolf (1990).

Fourier transform of the mutual coherence function with respect to the time variable τ, i.e.,

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau) e^{-i\omega\tau} d\tau. \tag{2.4}$$

The cross-spectral density will then satisfy a pair of Helmholtz equations,

$$\left(\nabla_1^2 + k^2\right) W(\mathbf{r}_1, \mathbf{r}_2, \omega) = 0, \tag{2.5}$$

$$\left(\nabla_2^2 + k^2\right) W(\mathbf{r}_1, \mathbf{r}_2, \omega) = 0, \tag{2.6}$$

where $k = \omega/c$ is the wave number of light corresponding to frequency ω. This pair of elliptic partial differential equations for the cross-spectral density function is, in general, easier to solve than the pair of hyperbolic wave equations for the mutual coherence function; the mutual coherence function can, however, be readily determined by taking an inverse Fourier transform of the cross-spectral density.

The cross-spectral density is commonly written in terms of two other functions, the *spectral density* $S(\mathbf{r}, \omega)$ and the *spectral degree of coherence* $\mu(\mathbf{r}_1, \mathbf{r}_2, \omega)$, as

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \sqrt{S(\mathbf{r}_1, \omega)}\sqrt{S(\mathbf{r}_2, \omega)}\mu(\mathbf{r}_1, \mathbf{r}_2, \omega). \tag{2.7}$$

The spectral density $S(\mathbf{r}, \omega)$ represents the intensity of the wave field at position $\mathbf{r}$ and frequency ω, and it may be written in terms of the cross-spectral density function as

$$S(\mathbf{r}, \omega) \equiv W(\mathbf{r}, \mathbf{r}, \omega). \tag{2.8}$$

The spectral degree of coherence $\mu(\mathbf{r}_1, \mathbf{r}_2, \omega)$ is a measure of the degree of correlation of the field at the two positions $\mathbf{r}_1$ and $\mathbf{r}_2$ and at frequency ω, and may be written in terms of the cross-spectral density function and spectral density as

$$\mu(\mathbf{r}_1, \mathbf{r}_2, \omega) \equiv \frac{W(\mathbf{r}_1, \mathbf{r}_2, \omega)}{\sqrt{S(\mathbf{r}_1, \omega)S(\mathbf{r}_2, \omega)}}. \tag{2.9}$$

It can be shown that the absolute value of the spectral degree of coherence is restricted to the values

$$0 \leq |\mu(\mathbf{r}_1, \mathbf{r}_2, \omega)| \leq 1, \tag{2.10}$$

where 0 represents complete spatial incoherence, and 1 represents full spatial coherence. The physical significance of $\mu(\mathbf{r}_1, \mathbf{r}_2, \omega)$ will be discussed in more detail in Section 3.

An important milestone in the development of coherence theory in the space-frequency domain was the observation by Wolf (1982) that the cross-spectral density itself may be represented as a correlation function derived from an ensemble of monochromatic realizations of the field. This can be proven by first noting that the cross-spectral density is Hermitian, i.e.,

$$W(\mathbf{r}_2, \mathbf{r}_1, \omega) = W^*(\mathbf{r}_1, \mathbf{r}_2, \omega), \tag{2.11}$$

and that it is non-negative definite, such that

$$\int_D \int_D W(\mathbf{r}_1, \mathbf{r}_2, \omega) f^*(\mathbf{r}_1) f(\mathbf{r}_2)\, \mathrm{d}^2 r_1 \mathrm{d}^2 r_2 \geq 0, \tag{2.12}$$

where $f(\mathbf{r})$ is an arbitrary square-integrable function and, for a secondary source with a field propagating from $z = 0$, the domain of integration D is the source plane. Assuming that the cross-spectral density is also square-integrable over this domain, it represents a *Hilbert–Schmidt kernel*; by *Mercer's theorem*[2], it may be expanded in a series of orthogonal functions of the form

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \sum_n \lambda_n(\omega) \phi_n^*(\mathbf{r}_1, \omega) \phi_n(\mathbf{r}_2, \omega), \tag{2.13}$$

where the eigenvalues $\lambda_n(\omega)$ and the eigenfunctions $\phi_n(\mathbf{r}, \omega)$ satisfy the integral equation

$$\int_D W(\mathbf{r}_1, \mathbf{r}_2, \omega) \phi(\mathbf{r}_1, \omega)\, \mathrm{d}^2 r_1 = \lambda_n(\omega) \phi_n(\mathbf{r}_2, \omega). \tag{2.14}$$

The summation, in general, may be over multiple indices, and may be a finite or infinite sum. The eigenvalues are non-negative, and the eigenfunctions are orthogonal and typically taken to be orthonormal. Equation (2.13) represents what is now known as the *coherent mode representation* of the cross-spectral density, to be discussed further in Section 8.

The coherent mode representation may be used to construct an ensemble of monochromatic wave fields whose second-order average reproduces

[2]Mercer's theorem and Hilbert–Schmidt kernels are introduced in the theory of integral equations; see, for instance, Moisewitsch (1977).

a given cross-spectral density. To do so, we introduce an ensemble of fields defined by

$$U(\mathbf{r},\omega) = \sum_n a_n(\omega)\phi_n(\mathbf{r},\omega), \tag{2.15}$$

where the coefficients a_n are random variables. We choose these variables such that the average of them over the entire ensemble of fields (denoted by $\langle\cdots\rangle_\omega$) satisfies the condition

$$\langle a_n^*(\omega)a_m(\omega)\rangle_\omega = \lambda_n(\omega)\delta_{nm}. \tag{2.16}$$

It then follows that the cross-spectral density function may be written as

$$W(\mathbf{r}_1,\mathbf{r}_2,\omega) = \langle U^*(\mathbf{r}_1,\omega)U(\mathbf{r}_2,\omega)\rangle_\omega. \tag{2.17}$$

On substitution from Equation (2.15) into Equation (2.17), we readily find that Equation (2.13) is satisfied. Furthermore, on substitution from Equation (2.17) into Equations (2.5) and (2.6), it follows that the individual realizations $U(\mathbf{r},\omega)$ each satisfy the Helmholtz equation and represent valid monochromatic, and therefore coherent, wave fields.

This result, which seems very formal and almost trivial at first glance, is perhaps one of the most useful results in modern coherence theory, because it implies that a valid cross-spectral density can be found by any suitable averaging process over a set of monochromatic realizations. This is used, for instance, in the "beam wander" model discussed in Section 7.

It is to be noted that it is possible to extend the space-frequency theory to higher-order correlation functions, as done by Wolf (1986b) and Agarwal and Wolf (1993); the formalism becomes significantly more complicated, however.

The theory of optical coherence has developed rapidly with the introduction of the space-frequency representation. Perhaps the most significant result to arise as yet is the theory of correlation-induced spectral changes, in which the degree of spatial coherence of a source can affect the properties of the radiated spectral density. The results arising from this theory are too numerous to be included here; a comprehensive review was provided some time ago by Wolf and James (1996).

At its heart, the theory of optical coherence may be said to be the *optics of observable quantities*. Although traditional optics focuses on the behavior of wave fields $U(\mathbf{r},t)$ that are not directly observable, coherence theory describes the behavior of second-order and higher moments of the wave field such as the mutual coherence function and the cross-spectral density function, which can be measured through interference experiments. An early discussion of this point of view was given by Wolf (1954).

3. PARTIALLY COHERENT FIELDS IN YOUNG'S EXPERIMENT

The state of coherence of a wave field is intimately related to its ability to form an interference pattern. The relation between the visibility of the fringes that are produced in Young's celebrated experiment (see Young, 1804 and Young, 1807) and the state of coherence of the field at the two pinholes was first studied by Zernike (1938).[3] To see this relation in the space-frequency domain, let us first consider the case of a partially coherent, scalar wave field that impinges on an opaque screen $\mathcal{A}$ with two identical small apertures at positions $Q(\mathbf{r}'_1)$ and $Q(\mathbf{r}'_2)$. (See Figure 1.) The field at a point $P(\mathbf{r})$ on the observation screen $\mathcal{B}$ is given by the formula

$$U(\mathbf{r},\omega) = K_1 U(\mathbf{r}'_1,\omega) + K_2 U(\mathbf{r}'_2,\omega), \tag{3.1}$$

where

$$K_i = -\frac{\mathrm{i}kA}{2\pi}\frac{\exp(\mathrm{i}kR_i)}{R_i} \quad (i = 1, 2) \tag{3.2}$$

is the propagator that relates the field at $Q(\mathbf{r}'_i)$ to the field at $P(\mathbf{r})$. Here A is the area of each pinhole, R_i denotes the distance from $Q(\mathbf{r}'_i)$ to P, and k is the wave number associated with the angular frequency ω. It follows from Equation (3.1) that the spectral density of the field at P equals

$$\begin{aligned} S(\mathbf{r},\omega) &= |K_1|^2\, S(\mathbf{r}'_1,\omega) + |K_2|^2\, S(\mathbf{r}'_2,\omega) \\ &\quad + 2\sqrt{S(\mathbf{r}'_1,\omega)S(\mathbf{r}'_2,\omega)}\mathrm{Re}\left\{K_1^* K_2 \mu_{12}(\omega)\right\}, \end{aligned} \tag{3.3}$$

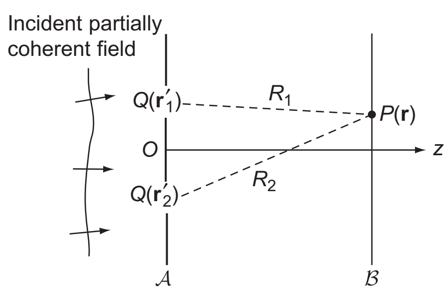

FIGURE 1 Young's interference experiment with partially coherent light. The perforated screen is situated in the plane $z = 0$, and the origin O of the coordinate system is taken in between the two pinholes.

[3] A historical overview of the role of Young's experiment in the development of coherence theory was given by Wolf (2007a).

where

$$\mu_{12}(\omega) = \frac{\langle U^*(\mathbf{r}_1', \omega) U(\mathbf{r}_2', \omega) \rangle}{\sqrt{S(\mathbf{r}_1', \omega) S(\mathbf{r}_2', \omega)}} \tag{3.4}$$

is the spectral degree of coherence of the field at the two pinholes. Equation (3.3) is the so-called *spectral interference law* for partially coherent fields.

In the often occurring case that $|K_1| \approx |K_2| = K$ and $S(\mathbf{r}_1', \omega) \approx S(\mathbf{r}_2', \omega) = S(\omega)$, and on writing

$$\mu_{12}(\omega) = |\mu_{12}(\omega)| \, \mathrm{e}^{\mathrm{i}\phi}, \tag{3.5}$$

and

$$K_1^* K_2 = K^2 \mathrm{e}^{\mathrm{i}k(R_2 - R_1)}, \tag{3.6}$$

Equation (3.3) reduces to

$$S(\mathbf{r}, \omega) = 2K^2 S(\omega) \left\{ 1 + |\mu_{12}(\omega)| \cos[\phi + k(R_2 - R_1)] \right\}. \tag{3.7}$$

In the immediate neighborhood of the observation point P on the plane $\mathcal{B}$, the phase factor $k(R_2 - R_1)$ will take on all values between 0 and 2π, whereas K remains approximately unchanged. Hence, in the vicinity of P, the maximum spectral density equals

$$S_{\max}(\omega) = 2K^2 S(\omega) \left(1 + |\mu_{12}(\omega)|\right), \tag{3.8}$$

and the minimum spectral density equals

$$S_{\min}(\omega) = 2K^2 S(\omega) \left(1 - |\mu_{12}(\omega)|\right). \tag{3.9}$$

Suppose now that the two pinholes are covered by narrow-band filters, centered around the frequency ω. If we define the spectral visibility (or "sharpness") of the interference fringe that is formed near P as

$$\mathcal{V}(P, \omega) = \frac{S_{\max}(\omega) - S_{\min}(\omega)}{S_{\max}(\omega) + S_{\min}(\omega)}, \tag{3.10}$$

it immediately follows that

$$\mathcal{V}(P, \omega) = |\mu_{12}(\omega)|. \tag{3.11}$$

Hence, as derived by Mandel & Wolf (1976), *the visibility of the fringes that are produced in Young's interference experiment is a direct measure of the modulus of the spectral degree of coherence of the field at the two pinholes.*

It is readily seen from Equation (3.7) that, in general, the spectral density of the field that is observed on the screen $\mathcal{B}$ differs from that of the field at the pinholes. This is due to (a) the appearance of the wave number k in the propagators K_i and (b) the modulus and phase of the spectral degree of coherence $\mu_{12}(\omega)$. Such spectral changes were analyzed in James and Wolf (1991a,b) for both filtered and broadband thermal light. In the latter case, significant spectral changes may occur. Experimental observations of spectral changes in a double-slit setup were presented by Santarsiero and Gori (1992). It is clear that Equation (3.7) can also be used to determine the spectral degree of coherence of the field at the apertures by comparing the spectral density in the far zone with that at the perforated screen. Such a study was carried out by Kandpal, Vaishya, Chander, Saxena, and Joshi (1992), and Kandpal and Vaishya (2000).

Another consequence of the spectral interference law, as expressed by Equation (3.3), is that the state of coherence of the field in the region of superposition may be different from that at the two pinholes. One might expect that when the incident field is partially coherent, i.e., $0 < |\mu_{12}(\omega)| < 1$, the same holds true for the field on the observation screen $\mathcal{B}$. This turns out not always to be the case. Extending the work of Ponomarenko and Wolf (1999), it was predicted by Schouten, Visser, and Wolf (2003) that at certain pairs of points the light is fully coherent, i.e., $|\mu(\mathbf{r}_1, \mathbf{r}_2, \omega)| = 1$, regardless of the state of coherence of the light at the two pinholes. The other extreme, namely the complete absence of coherence can also occur. This was demonstrated by Schouten, Gbur, Visser, and Wolf (2003) who found that at certain pairs of observation points $\mu(\mathbf{r}_1, \mathbf{r}_2, \omega) = 0$. This means that at these points, the spectral degree of coherence is singular. It is important to realize that such a *coherence singularity* occurs in a six-dimensional $(\mathbf{r}_1, \mathbf{r}_2)$-space, in contrast to the classical phase singularities that are found in two or three dimensions. A fuller discussion of coherence singularities is presented in Section 7.

Next we turn our attention to Young's experiment with stochastic, electromagnetic beams. A study by Wang and Lü (2002) described how covering the apertures with linear polarizers can lead to changes in the spectral density and the state of polarization. Their work was based on the "beam coherence-polarization matrix" approach as developed by Gori (1998). Here we concentrate on the recently developed "unified theory of coherence and polarization," described in Wolf (2003b) and Wolf (2003a). In that theory, the state of coherence and polarization of a random beam is characterized by the *electric cross-spectral density matrix*, which is defined as

$$\mathbf{W}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \begin{bmatrix} W_{xx}(\mathbf{r}_1, \mathbf{r}_2, \omega) & W_{xy}(\mathbf{r}_1, \mathbf{r}_2, \omega) \\ W_{yx}(\mathbf{r}_1, \mathbf{r}_{2,\omega}) & W_{yy}(\mathbf{r}_1, \mathbf{r}_2, \omega) \end{bmatrix}, \tag{3.12}$$

where

$$W_{ij}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \langle E_i^*(\mathbf{r}_1, \omega) E_j(\mathbf{r}_2, \omega) \rangle, \qquad (i, j = x, y). \tag{3.13}$$

Here $E_i(\mathbf{r}, \omega)$ is a Cartesian component of the electric field at a point specified by a position vector $\mathbf{r}$ at frequency ω, of a typical realization of the statistical ensemble representing the beam. A method to determine the elements of the matrix $\mathbf{W}(\mathbf{r}_1, \mathbf{r}_2, \omega)$ is described by Roychowdhury and Wolf (2004). Several observables can be derived from knowledge of the cross-spectral density matrix. The spectral density is given by the expression

$$S(\mathbf{r}, \omega) = \mathrm{Tr}\, \mathbf{W}(\mathbf{r}, \mathbf{r}, \omega), \tag{3.14}$$

where Tr denotes the trace. The degree of coherence $\eta(\mathbf{r}_1, \mathbf{r}_2, \omega)$ of the field is defined as

$$\eta(\mathbf{r}_1, \mathbf{r}_2, \omega) = \frac{\mathrm{Tr}\, \mathbf{W}(\mathbf{r}_1, \mathbf{r}_2, \omega)}{[\mathrm{Tr}\, \mathbf{W}(\mathbf{r}_1, \mathbf{r}_1, \omega)\, \mathrm{Tr}\, \mathbf{W}(\mathbf{r}_2, \mathbf{r}_2, \omega)]^{1/2}}. \tag{3.15}$$

In a manner quite similar to that for the spectral degree of coherence of scalar wave fields (see Section 2), one can show that

$$0 \leq |\eta(\mathbf{r}_1, \mathbf{r}_2, \omega)| \leq 0, \tag{3.16}$$

with the extreme values 0 and 1 corresponding to complete incoherence and complete coherence, respectively. The sharpness of the interference fringes that are produced in Young's experiment are related to the modulus of $\eta(\mathbf{r}_1, \mathbf{r}_2, \omega)$ in complete analogy to the scalar case described in the first part of this section. The absence of off-diagonal matrix elements in the definition of the spectral degree of coherence reflects a generalization of the classical Fresnel-Arago interference laws, according to which mutually orthogonal components of the electric field do not give rise to interference. A third observable is the (spectral) degree of polarization $\mathcal{P}(\mathbf{r}, \omega)$. This is defined as the ratio of the spectral density of the polarized part of the beam and its total spectral density (see Born & Wolf, 1999). One can show that

$$\mathcal{P}(\mathbf{r}, \omega) = \sqrt{1 - \frac{4\mathrm{Det}\, \mathbf{W}(\mathbf{r}, \mathbf{r}, \omega)}{[\mathrm{Tr}\, \mathbf{W}(\mathbf{r}, \mathbf{r}, \omega)]^2}}, \tag{3.17}$$

where Det denotes the determinant.

As pointed out in Wolf (2003a), the elements of the electric cross-spectral density matrix change as the beam propagates. It is therefore to be expected that the observable quantities $S(\mathbf{r}, \omega)$, $\eta(\mathbf{r}_1, \mathbf{r}_2, \omega)$, and $\mathcal{P}(\mathbf{r}, \omega)$, which are derived from the matrix, will also change on propagation. Examples of such correlation-induced changes are discussed in Section 4.

Within the present context, that of Young's experiment, we need to know the electric field at two points $P(\mathbf{r}_1)$ and $P(\mathbf{r}_2)$, both located on the observation screen depicted in Figure 1. It is given by the expressions

$$E_x(\mathbf{r}_1, \omega) = K_{11} E_x(\mathbf{r}_1', \omega) + K_{21} E_x(\mathbf{r}_2', \omega), \tag{3.18}$$

$$E_y(\mathbf{r}_1, \omega) = K_{11} E_y(\mathbf{r}_1', \omega) + K_{21} E_y(\mathbf{r}_2', \omega), \tag{3.19}$$

$$E_x(\mathbf{r}_2, \omega) = K_{12} E_x(\mathbf{r}_1', \omega) + K_{22} E_x(\mathbf{r}_2', \omega), \tag{3.20}$$

$$E_y(\mathbf{r}_2, \omega) = K_{12} E_y(\mathbf{r}_1', \omega) + K_{22} E_y(\mathbf{r}_2', \omega), \tag{3.21}$$

with the propagators K_{ij} defined as

$$K_{ij} = -\frac{\mathrm{i}kA}{2\pi} \frac{\mathrm{e}^{\mathrm{i}kR_{ij}}}{R_{ij}}, \qquad (i, j = 1, 2), \tag{3.22}$$

and where R_{ij} denotes the distance from the pinhole $Q(\mathbf{r}_i')$ to the point $P(\mathbf{r}_j)$. As was shown by Roychowdhury and Wolf (2005b), substitution from Equations (3.18)–(3.21) into definition (3.12) yields the cross-spectral density matrix in the region of superposition, expressed entirely in terms of that matrix at the two pinholes. They applied this formalism to the case of incident Gaussian Schell-model beams (see Section 4) and found that the degree of polarization of the field on the observation screen depends on (1) the position of observation, (2) the degree of polarization of the incident light, and (3) the degree of coherence of the field at the pinholes. The degree of coherence of the field that is observed, however, only depends on the degree of coherence of the incident field. Another striking prediction made by Roychowdhury and Wolf is that light that is completely unpolarized at the pinholes may become partially polarized across the fringe pattern. Experimental confirmation of this prediction was presented in Gori, Santarsiero, Borghi, and Wolf (2006). A further study of the observable quantities in the region of superposition was presented by Li, Lee, and Wolf (2006).

Generalizing the work concerning scalar fields of Schouten et al. (2003), it was shown by Agarwal, Dogariu, Visser, and Wolf (2005) that there exist special pairs of points at which the field is spatially fully coherent, irrespective of the state of coherence and polarization of the field that is incident at the two pinholes.

4. THE EVOLUTION OF PARTIALLY COHERENT BEAMS

In was noted in Section 2 that coherence functions obey certain propagation equations: the mutual coherence function $\Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau)$ satisfies a pair of wave equations and the cross-spectral density function $W(\mathbf{r}_1, \mathbf{r}_2, \omega)$ satisfies a pair of Helmholtz equations. The coherence functions therefore have a well-defined behavior as they propagate; however, other properties derived from those coherence functions are not solutions of a differential equation and can evolve in nontrivial and unexpected ways on propagation. Among the properties that can change on propagation are the spectrum and the spectral degree of coherence of a wave field, defined by Equations (2.8) and (2.9), respectively, as well as the state of polarization and degree of polarization of an electromagnetic wave field.

Of particular interest is the propagation and evolution of partially coherent beams, i.e., wave fields that are highly directional. Partially coherent beams can be generated, for instance, by the distortion of a fully coherent laser beam, using a rotating ground-glass plate or a liquid crystal spatial light modulator. Numerous articles have been published on the behavior of partially coherent beams, in fact, more than can reasonably be covered here. In this section, we highlight some of the most significant results and discuss their theoretical foundations.

We consider first a partial coherent scalar wave field propagating from the plane $z = 0$ into the half-space $z > 0$; in the plane $z = 0$, the cross-spectral density has the form $W_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega)$, where $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ represent transverse coordinates in the plane. Using the space-frequency representation of partially coherent wave fields, it can be readily shown that the cross-spectral density at points $\mathbf{r}_1$ and $\mathbf{r}_2$ in the half-space may be written in integral form as

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \iint W_0(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2, \omega) G^*(\boldsymbol{\rho}'_1, \mathbf{r}_1, \omega) G(\boldsymbol{\rho}'_2, \mathbf{r}_2, \omega)\, \mathrm{d}^2\rho'_1 \mathrm{d}^2\rho'_2, \quad (4.1)$$

where $G(\boldsymbol{\rho}', \mathbf{r}, \omega)$ is the free-space propagator for the Helmholtz equation. The propagator may be written as

$$G(\boldsymbol{\rho}', \mathbf{r}, \omega) = \frac{1}{2\pi} \frac{\partial}{\partial z} \left(\frac{\mathrm{e}^{\mathrm{i}ks}}{s} \right), \quad (4.2)$$

with $s \equiv |\mathbf{r} - \boldsymbol{\rho}'|$ and $\mathbf{r} = (\boldsymbol{\rho}, z)$.

If we assume for the moment that the field is highly directional, it follows that the cross-spectral density must be negligible outside of a narrow cone centered on the z-axis. The propagator may then be approximated by its paraxial form,

$$G(\boldsymbol{\rho}', \mathbf{r}, \omega) = -\frac{ik}{2\pi z} \exp[ik[(x - x')^2 + (y - y')^2]/2z]. \tag{4.3}$$

It is to be noted that there is no precise criterion for what constitutes a "beamlike" wave field. For a monochromatic wave field, a beam condition is typically formulated using the angular spectrum representation of the wave field. With this representation, the field in the half-space $z > 0$ can be written as

$$U(\mathbf{r}, \omega) = \int a(\mathbf{k}_\perp) \exp[i\mathbf{k} \cdot \mathbf{r}] \, d^2k, \tag{4.4}$$

where $\mathbf{k} = (\mathbf{k}_\perp, k_z)$, $|\mathbf{k}| = k$, and

$$k_z = \sqrt{k^2 - k_\perp^2}. \tag{4.5}$$

The quantity $a(\mathbf{k}_\perp)$ is the *angular spectrum* of the wave field, defined as

$$a(\mathbf{k}_\perp) = \frac{1}{(2\pi)^2} \int U_0(\boldsymbol{\rho}', \omega) \exp[-i\mathbf{k}_\perp \cdot \boldsymbol{\rho}'] \, d^2\rho', \tag{4.6}$$

where $U_0(\boldsymbol{\rho}, \omega)$ is the wave field in the plane $z = 0$. Equation (4.4) expresses the field as a coherent superposition of plane waves propagating into the positive half-space. For $|\mathbf{k}_\perp| \leq k$, the quantity k_z is real-valued, and the plane wave has a constant amplitude on propagation. For $|\mathbf{k}_\perp| > k$, however, k_z is imaginary and the plane wave decays exponentially in the z-direction; it is an evanescent wave. The total wave of Equation (4.4) is said to be beamlike if

$$|a(\mathbf{k}_\perp)| \approx 0 \text{ unless } |\mathbf{k}_\perp| \ll k. \tag{4.7}$$

A similar definition exists, almost by analogy, for a partially coherent field. We introduce the angular spectrum of the cross-spectral density function as

$$\mathcal{A}(\mathbf{k}_{1\perp}, \mathbf{k}_{2\perp}) = \frac{1}{(2\pi)^2} \iint W_0(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2', \omega) \exp[i\mathbf{k}_{1\perp} \cdot \boldsymbol{\rho}_1'] \times \exp[-i\mathbf{k}_{2\perp} \cdot \boldsymbol{\rho}_2'] \, d^2\rho_1' d^2\rho_2'. \tag{4.8}$$

The cross-spectral density may be written in terms of plane waves as

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \iint \mathcal{A}(\mathbf{k}_{1\perp}, \mathbf{k}_{2\perp}) \exp[-i\mathbf{k}_1 \cdot \mathbf{r}_1] \exp[i\mathbf{k}_2 \cdot \mathbf{r}_2]\, d^2k_{1\perp} d^2k_{2\perp}. \tag{4.9}$$

It can be shown (for details see, for instance, section 5.6.3 of Mandel & Wolf, 1995) that the field will be beamlike if

$$|\mathcal{A}(\mathbf{k}_\perp, \mathbf{k}_\perp)| \approx 0 \text{ unless } |\mathbf{k}_\perp| \ll k. \tag{4.10}$$

A number of special classes of fields have been used because of their analytic simplicity and their relevance to physically realizable optical fields. The first of these is produced by a so-called Schell-model source (Schell, 1961), for which the spectral degree of coherence is a function of the spatial difference variable alone, i.e.,

$$\mu_0(\mathbf{r}_1, \mathbf{r}_2, \omega) = \mu_0(\mathbf{r}_2 - \mathbf{r}_1, \omega). \tag{4.11}$$

The most analytically tractable class of partially coherent sources are known as *Gaussian Schell-model sources*, for which the spectral density and spectral degree of coherence in the plane $z = 0$ are both of Gaussian shape, namely

$$S_0(\boldsymbol{\rho}, \omega) = A^2 e^{-\rho^2/2\sigma_S^2}, \tag{4.12}$$

$$\mu_0(\boldsymbol{\rho}', \omega) = e^{-\rho'^2/2\sigma_\mu^2}. \tag{4.13}$$

Here A represents the amplitude of the wave, σ_S represents the width of the source and σ_μ represents the transverse correlation length of the source; all quantities are in general frequency dependent. It can be readily shown that the spectral density and spectral degree of coherence retain a Gaussian form on propagation. An early study of the directionality of beams produced by such sources was done by Foley and Zubairy (1978).

When the width of the spectral degree of coherence function is much narrower than the width of the spectral density function, one may further approximate a Schell-model source by using the *quasi-homogeneous* approximation, such that

$$W_0(\mathbf{r}_1, \mathbf{r}_2, \omega) \approx S_0\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}, \omega\right) \mu_0(\mathbf{r}_2 - \mathbf{r}_1, \omega). \tag{4.14}$$

A study of the propagation characteristics of Gaussian quasi-homogeneous beams was undertaken by Collett and Wolf (1980); the concept of quasi-homogeneity is also used to describe scatterers, and it will be further discussed in Section 6.

Directionality is often assumed to require a high degree of spatial coherence, as in a laser, and it would seem to exclude the possibility of beamlike fields from quasi-homogeneous sources. It was shown in Collett and Wolf (1978) and Wolf and Collett (1978) that one can create partially coherent sources of nearly any degree of spatial coherence that produce the same directionality as a laser. The far-field spatial coherence properties of such laser-equivalent sources was studied in Shirai and Wolf (2002).

The earliest studies of the evolution of partially coherent fields investigated the change in spatial coherence on propagation; most notable are those of van Cittert (1934) and Zernike (1948), who investigated the coherence of light emanating from a spatially incoherent planar source. More generally, it has been shown by Friberg and Wolf (1983) that there exist reciprocity relations between the intensity and spatial coherence of the source and the spatial coherence and intensity of the radiation in the far zone, respectively. Another investigation of the reciprocal relationship between source and far zone has been undertaken by Friberg, Visser, and Wolf (2000).

Although typically the spatial coherence of a field increases on propagation, it was shown by Devaney, Friberg, Kumar, and Wolf (1997) that it is possible to produce fields whose spatial coherence decreases on propagation through the mechanism of phase conjugation. Furthermore, Pedersen and Stamnes (2000) used a radiometric approach to show that if on propagation an increase of the intensity occurs, i.e., when the light is being concentrated, the spatial degree of coherence decreases.

An unusual class of spatially coherent beams are the so-called non-diffracting or Bessel beams; a review of the subject was presented by Bouchal (2003). It was shown by Turunen, Vasara, and Friberg (1991) that fields that have Bessel *correlations* can also possess a degree of propagation-invariance, or even revivals of spatial coherence on propagation.

We have already noted that the spectrum of light of a partially coherent field can change on propagation, even in free space, a phenomenon known as a correlation-induced spectral change. An early study of the spectral changes of beams on propagation was done by Dačić and Wolf (1988).

One can readily extend the formalism of partially coherent scalar beams to partially coherent electromagnetic beams. Within the paraxial limit, the electric and magnetic fields will be completely transverse to the direction of propagation, formally chosen as the z-axis. The second-order coherence properties of the electromagnetic beam can then be characterized by the 2×2 cross-spectral density matrix,

$$\mathbf{W}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \begin{bmatrix} \langle E_x^*(\mathbf{r}_1, \omega) E_x(\mathbf{r}_2, \omega) \rangle & \langle E_x^*(\mathbf{r}_1, \omega) E_y(\mathbf{r}_2, \omega) \rangle \\ \langle E_y^*(\mathbf{r}_1, \omega) E_x(\mathbf{r}_2, \omega) \rangle & \langle E_y^*(\mathbf{r}_1, \omega) E_y(\mathbf{r}_2, \omega) \rangle \end{bmatrix}, \tag{4.15}$$

where $E_x(\mathbf{r}, \omega)$ and $E_y(\mathbf{r}, \omega)$ are monochromatic realizations of the x and y-components of the electric field, respectively. Each component of this matrix can be propagated using Equation (4.1) for a scalar partially coherent wave field.

At any point in the wave field, the degree of polarization is defined by the following expression, previously noted in Section 3,

$$\mathcal{P}(\mathbf{r}, \omega) = \sqrt{1 - \frac{4\mathrm{Det}\{\mathbf{W}(\mathbf{r}, \mathbf{r}, \omega)\}}{[\mathrm{Tr}\{\mathbf{W}(\mathbf{r}, \mathbf{r}, \omega)\}]^2}}, \tag{4.16}$$

where Det indicates the determinant and Tr the trace of the cross-spectral density matrix. The value $\mathcal{P} = 0$ indicates a completely unpolarized field, while the value $\mathcal{P} = 1$ indicates a completely polarized field.

When the correlation and polarization properties of the source are spatially-varying, it is possible for the degree of polarization of the light field to change on propagation; this seems to have first been observed by James (1994), and was later discussed by Agrawal and Wolf (2000), well before a unified theory of coherence and polarization was introduced by Wolf (2003b) (and discussed in detail in Wolf, 2007b). This unified theory was used by Wolf (2003a) to study the correlation-induced changes in coherence, polarization, and spectrum of a partially coherent electromagnetic beam. The far zone behavior of these properties in quasi-homogeneous electromagnetic beams was investigated by Korotkova, Hoover, Gamiz, and Wolf (2005). It has also been shown by Korotkova and Wolf (2005) that the state of polarization (ellipticity, orientation and handedness of the polarization ellipse) of an electromagnetic beam may change on propagation in free space. A further study of the changes in the degree of polarization was done by Salem and Wolf (2008).

There is a well-known theorem due to Stokes (1852) regarding the decomposition of an arbitrary beam into polarized and unpolarized components, in which he states,

> *...it is always possible to represent the given group by a stream of common light combined with a stream of elliptically polarized light independent of the former.*

It has recently been shown that this assertion is incorrect (Wolf, 2008): the decomposition of a beam into a polarized part and an unpolarized part was further shown by Korotkova, Visser, and Wolf (2008) to be a local, rather than global, property of the field (i.e., the decomposition may be different at different points).

From the definition (4.16) of the degree of polarization, it is to be noted that it depends only on the diagonal elements ($\mathbf{r}_1 = \mathbf{r}_2 = \mathbf{r}$) of the cross-spectral density matrix. This implies that there are many different coherence matrices that can produce the same degree of polarization; in particular, Visser, Kuebel, Lahiri, Shirai, and Wolf (2009) demonstrated that completely unpolarized beams may have a variety of coherence properties. Gbur and James (2000) used a similar observation to theoretically construct three-dimensional unpolarized primary radiation sources that produce nearly fully polarized fields.

Additional effects can arise when a partially coherent electromagnetic beam propagates through a homogeneous and isotropic weakly scattering medium such as the turbulent atmosphere. The effect of atmospheric turbulence on beam propagation can be made analytically tractable by treating the scattered wave field by a perturbative approximation such as the Born or Rytov series. A more direct connection to the earlier equations of this section may be made by using the so-called extended Huygens–Fresnel principle, in which the effect of turbulence is treated as a perturbation of the free-space Green's function. Equation (4.1) can then be written in the form,

$$W(\mathbf{r}_1,\mathbf{r}_2,\omega) = \iint W_0(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\omega)C_\psi(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\mathbf{r}_1,\mathbf{r}_2) \times G^*(\boldsymbol{\rho}_1',\mathbf{r}_1,\omega)G(\boldsymbol{\rho}_2',\mathbf{r}_2,\omega)\,\mathrm{d}^2\rho_1'\mathrm{d}^2\rho_2', \tag{4.17}$$

where $\psi(\boldsymbol{\rho},\mathbf{r})$ is the phase distortion induced by the turbulence on the field on propagation from $\boldsymbol{\rho}$ to $\mathbf{r}$, and

$$C_\psi(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2',\mathbf{r}_1,\mathbf{r}_2) = \langle \exp[\psi^*(\boldsymbol{\rho}_1',\mathbf{r}_1)]\exp[\psi(\boldsymbol{\rho}_2',\mathbf{r}_2)]\rangle_T \tag{4.18}$$

is the correlation function of that phase distortion. The average $\langle\cdots\rangle_T$ is an ensemble average over realizations of the atmospheric turbulence, and this average is, in general, independent of the ensemble average of the wave field. The calculation of C_ψ is nontrivial and requires a number of simplifying, sometimes dubious, assumptions; details can be found in Lutomirski and Yura (1971) and the book by Andrews and Phillips (2005). Equation (4.17) can be used to propagate each component of the cross-spectral density matrix to study the effects of turbulence on polarization.

Polarization changes of beams in turbulence were first studied by Roychowdhury, Ponomarenko, and Wolf (2005) and Salem, Korotkova, Dogariu, and Wolf (2004); it was surprisingly shown that the degree of polarization tends to its initial value after propagation over a sufficiently long distance. The far zone behavior of the degree of polarization was investigated by Korotkova, Salem, and Wolf (2004). Changes in the state of polarization in turbulence, and its return to the initial state, were discussed by Korotkova, Salem, Dogariu, and Wolf (2005). Spectral changes of electromagnetic beams in turbulence were considered by Korotkova, Pu, and Wolf (2008).

Propagation through other types of random media have also been considered; Gao and Korotkova (2007) considered the changes of polarization on propagation through tissue.

Discussions of the behavior of the scintillation of partially coherent beams in turbulence will be considered in Section 10.

5. FOCUSING OF PARTIALLY COHERENT WAVE FIELDS

The classical theory of focusing deals with monochromatic wave fields that can be scalar or vectorial in nature; an excellent overview is given by Stamnes (1986). In the present section, we examine the focusing of partially coherent scalar fields. In particular, the effect of the state of coherence of the field in the exit pupil on the distribution of the spectral density and the coherence properties of the field in the focal region will be discussed. In addition, the focal shift phenomenon will also be addressed.

We consider first a monochromatic, spherical wave of frequency ω that emerges from a circular aperture with radius a, and which converges toward a geometrical focus O (see Figure 2). The field in the focal region is, according to the Huygens–Fresnel principle (see Born & Wolf, 1999, chapter 8), given by the expression

$$U(\mathbf{r},\omega) = -\frac{\mathrm{i}}{\lambda}\int_S U^{(0)}(\mathbf{r}',\omega)\frac{\mathrm{e}^{iks}}{s}\,\mathrm{d}S. \tag{5.1}$$

Here $U^{(0)}(\mathbf{r}',\omega)$ is the field on the wavefront S that fills the aperture, $k=\omega/c=2\pi/\lambda$ is the wave number associated with frequency ω, with c the speed of light and λ the wavelength. Furthermore, $s=|\mathbf{r}'-\mathbf{r}|$ denotes the distance from a point of integration $Q(\mathbf{r}')$ to the observation point $P(\mathbf{r})$. From Equation (5.1), one can derive the so-called Debye

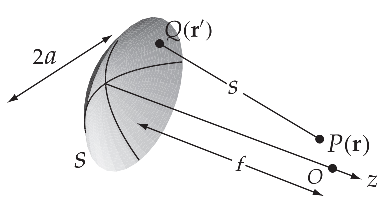

FIGURE 2 Illustrating the focusing configuration. The origin of a right-handed cartesian coordinate system is taken at the geometrical focus O.

integral (see Born & Wolf, 1999, chapter 8)

$$U(\mathbf{r},\omega) = -\frac{\mathrm{i}}{\lambda}\int\limits_{\Omega} a(\mathbf{s},\omega)\mathrm{e}^{ik\mathbf{s}\cdot\mathbf{r}}\,\mathrm{d}\Omega, \tag{5.2}$$

where Ω is the solid angle subtended by the aperture at the geometrical focus, which is spanned by the real unit vectors $\mathbf{s} = (s_x, s_y, s_z > 0)$. The amplitude function $a(\mathbf{s},\omega)$ is assumed to be real, apart from a possible constant phase factor. The Debye integral expresses the field in the focal region as a superposition of plane waves, with direction-dependent amplitudes. Equations (5.1) and (5.2) are valid provided that $\lambda \ll a \ll f$, where f is the radius of curvature of the wavefront. In addition, the Fresnel number $N = a^2/\lambda f$ must be large compared with unity.

The above two expressions can both be generalized to deal with partially coherent fields. Recall the definition (2.17) of the cross-spectral density function of the field at a pair of points $\mathbf{r}_1$ and $\mathbf{r}_2$,

$$W(\mathbf{r}_1,\mathbf{r}_2,\omega) = \langle U^*(\mathbf{r}_1,\omega)U(\mathbf{r}_2,\omega)\rangle_\omega. \tag{5.3}$$

On substituting from Equation (5.1) into Equation (5.3) and interchanging the order of integration and ensemble averaging, we obtain the following formula for the cross-spectral density in the focal region,

$$W(\mathbf{r}_1,\mathbf{r}_2,\omega) = \frac{1}{\lambda^2}\iint\limits_{S} W^{(0)}(\mathbf{r}_1',\mathbf{r}_2',\omega)\frac{\mathrm{e}^{ik(s_2-s_1)}}{s_1 s_2}\,\mathrm{d}S_1\,\mathrm{d}S_2, \tag{5.4}$$

where

$$W^{(0)}(\mathbf{r}_1',\mathbf{r}_2',\omega) = \langle U^{(0)^*}(\mathbf{r}_1',\omega)U^{(0)}(\mathbf{r}_2',\omega)\rangle_\omega \tag{5.5}$$

denotes the cross-spectral density of the field in the exit pupil, and

$$s_1 = |\mathbf{r}_1' - \mathbf{r}_1|, \tag{5.6}$$

$$s_2 = |\mathbf{r}_2' - \mathbf{r}_2|. \tag{5.7}$$

The spectral density $S(\mathbf{r},\omega) = W(\mathbf{r},\mathbf{r},\omega)$ in the focal region is then given by the expression

$$S(\mathbf{r},\omega) = \frac{1}{\lambda^2}\iint\limits_{S} W^{(0)}(\mathbf{r}_1',\mathbf{r}_2',\omega)\frac{\mathrm{e}^{ik(s_2'-s_1')}}{s_1' s_2'}\,\mathrm{d}S_1\mathrm{d}S_2, \tag{5.8}$$

with

$$s'_1 = |\mathbf{r}'_1 - \mathbf{r}|, \tag{5.9}$$

$$s'_2 = |\mathbf{r}'_2 - \mathbf{r}|. \tag{5.10}$$

Alternatively, one can use the *angular correlation function* defined as

$$\mathcal{A}(\mathbf{s}_1, \mathbf{s}_2, \omega) = \langle a^*(\mathbf{s}_1, \omega) a(\mathbf{s}_2, \omega) \rangle_\omega. \tag{5.11}$$

On substituting from Equations (5.2) and (5.11) into Equation (5.3), we obtain the expression

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \frac{1}{\lambda^2} \iint_\Omega \mathcal{A}(\mathbf{s}_1, \mathbf{s}_2, \omega) \mathrm{e}^{\mathrm{i}k(\mathbf{s}_2 \cdot \mathbf{r}_2 - \mathbf{s}_1 \cdot \mathbf{r}_2)} \, \mathrm{d}\Omega_1 \mathrm{d}\Omega_2. \tag{5.12}$$

Expression (5.12) is known as the *generalized Debye integral*. The spectral density $S(\mathbf{r}, \omega) = W(\mathbf{r}, \mathbf{r}, \omega)$ in the focal region is now given by the formula

$$S(\mathbf{r}, \omega) = \frac{1}{\lambda^2} \iint_\Omega \mathcal{A}(\mathbf{s}_1, \mathbf{s}_2, \omega) \mathrm{e}^{\mathrm{i}k\mathbf{r} \cdot (\mathbf{s}_2 - \mathbf{s}_1)} \, \mathrm{d}\Omega_1 \mathrm{d}\Omega_2. \tag{5.13}$$

Equations (5.4) and (5.8) allow one to study the state of coherence and the spectral density of the field in the focal region for a given cross-spectral density $W^{(0)}(\mathbf{r}'_1, \mathbf{r}'_2, \omega)$ of the field in the aperture. Alternatively, Equations (5.12) and (5.13) can be used for the same purpose when the angular correlation function $\mathcal{A}(\mathbf{s}_1, \mathbf{s}_2, \omega)$ of the field in the aperture is known.

To specify the position of an observation point near the geometrical focus we use the dimensionless Lommel variables, which are defined as

$$u = k \left(\frac{a}{f} \right)^2 z, \tag{5.14}$$

$$v = k \left(\frac{a}{f} \right) \sqrt{x^2 + y^2}. \tag{5.15}$$

Friberg and Turunen (1988) studied the imaging of Gaussian Schell-model sources by generalizing the familiar *ABCD* ray-transfer formalism. They derived expressions for the size and position of the image waist. Wang, Friberg, and Wolf (1997) used the generalized Debye integral, Equation (5.12), to calculate the axial spectral density distribution of focused, partially coherent cylindrical waves with a Gaussian angular correlation function. They noticed that the peak intensity decreases as the coherence of the field in the aperture is reduced. Also, the focal spot size was seen to increase in that case. The effect of the state of coherence on the full three-dimensional spectral density near focus was examined by Visser, Gbur, and Wolf (2002). Starting from Equation (5.4), the field in the exit pupil was assumed to be of the Gaussian Schell-model type, i.e.,

$$W^{(0)}(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2, \omega) = S^{(0)}(\omega)\mathrm{e}^{-(\boldsymbol{\rho}'_2-\boldsymbol{\rho}'_1)^2/2\sigma_{\mathrm{g}}^2}. \tag{5.16}$$

Here $S^{(0)}(\omega)$ denotes the spectral density and σ_{g} the effective coherence length of the field in the aperture. Furthermore, $\boldsymbol{\rho}' = (x', y')$ is the two-dimensional projection, considered as a two-dimensional vector, of the position vector $\mathbf{r}'$ of the point Q onto the xy-plane (see Figure 2). It was shown for such fields that the spectral density distribution is symmetric about the geometrical focus. Also, the maximum spectral density, which occurs at the geometrical focal point, decreases when σ_{g} decreases. The intricate focal field structure that is typical of coherent fields gradually disappears when σ_{g} becomes smaller than the aperture radius a. In addition, the maximum spectral density decreases with decreasing coherence length. These trends can be seen from Figures 3 and 4. The focusing of twisted anisotropic Gaussian-Schell model beams was examined by Cai and Lin (2003). The coupling of partially coherent light into a planar waveguide was discussed by Saastamoinen, Kuittinen, Vahimaa, and Turunen (2004).

It was noted by Visser et al. (2002) that cross-spectral density functions that are not positive for all values of their spatial arguments may

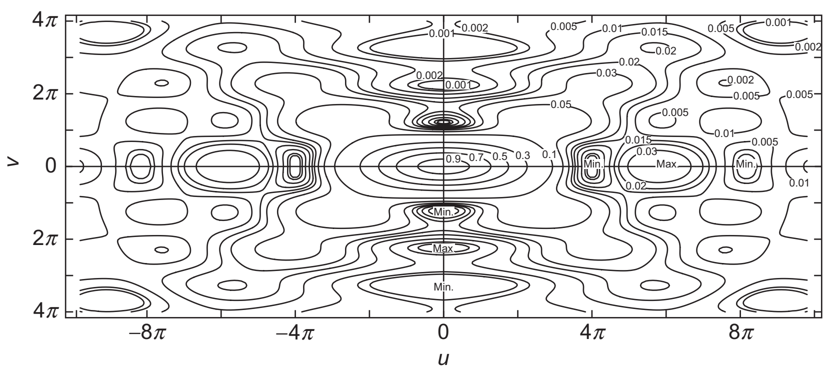

FIGURE 3 Contours of constant spectral density of a fully coherent field in the neighborhood of the geometrical focus. Positions are indicated using Lommel variables as defined in Equations (5.14) and (5.15) (Adapted from Visser, Gbur, & Wolf, 2002).

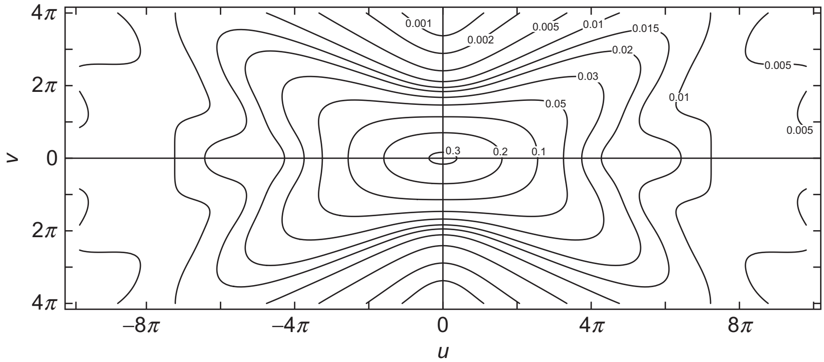

FIGURE 4 Contours of constant spectral density of a converging partially coherent Gaussian Schell-model field with $\sigma_g/a = 0.25$, in the neighborhood of the geometrical focus. The normalization is equal to that used in Figure 3. (Adapted from Visser, Gbur and Wolf, 2002.)

produce a spectral density distribution that is still symmetric about the focal plane, but with a maximum that does not coincide with the geometrical focus. Following up on this observation, cross-spectral densities of the type

$$W^{(0)}(\mathbf{r}_1', \mathbf{r}_2', \omega) = S^{(0)}(\omega) J_0[\beta(r_2 - r_1)], \tag{5.17}$$

were studied by Gbur and Visser (2003a), Pu, Nemoto, and Liu (2004), and van Dijk, Gbur, and Visser (2008). In Equation (5.17), $S^{(0)}(\omega)$ denotes the (uniform) spectral density of the aperture field, J_0 is the Bessel function of zeroth order, and β is, roughly speaking, the inverse effective coherence length. Using a coherent mode decomposition (see Section 8), it was calculated how the spectral density distribution in the focal region changes as the parameter $a\beta$, with a the aperture radius, is varied. It was indeed found that the spectral density at focus can be a local minimum, rather than a maximum, when $a\beta < 1$. This is illustrated in Figure 5. An experimental observation of such a partially coherent "bottle beam" was made by Pu, Dong, and Wang (2006).

Thus far, we have discussed focusing configurations for which the Fresnel number $N = a^2/\lambda f \gg 1$. For systems with $N \approx 1$, the location of maximum spectral density is no longer at the geometrical focus, but it occurs closer toward the aperture. This so-called focal shift phenomenon is described in detail in Stamnes (1986). Lü, Zhang, and Cai (1995) and later Friberg, Visser, Wang, and Wolf (2001) analyzed this effect for converging partially coherent fields. It was found that the focal shift depends not just on the Fresnel number, as it does for fully coherent fields, but also

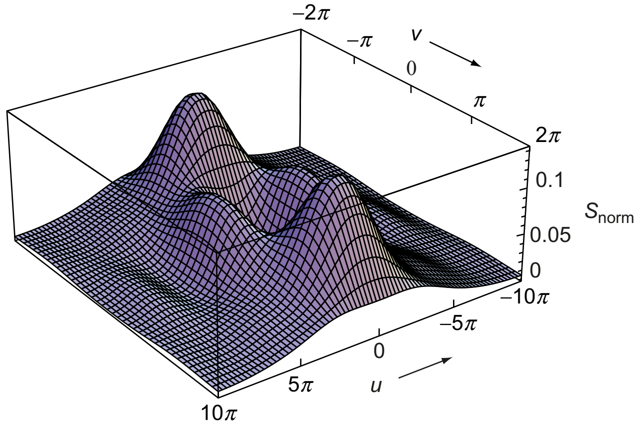

FIGURE 5 Example of the three-dimensional normalized spectral density distribution near focus, produced by a converging, Bessel-correlated field. The minimum at the geometrical focus is clearly visible. (Adapted from van Dijk, Gbur, & Visser, 2008.)

on the state of coherence. Experimental confirmation of these predictions was reported by Wang, Cai, and Korotkova (2009).

It has long been known that the focusing and diffraction of polychromatic light generally results in spectral changes. In studying the diffraction of partially coherent light, Pu, Zhang, and Nemoto (1999) noted that the spectrum can, in fact, change very rapidly under a gradual change of system parameters. Such "spectral switches" on diffraction were studied in more detail by Pu and Nemoto (2000) and Pu and Nemoto (2002). Similar spectral switches that arise on focusing were investigated by Gbur, Visser, and Wolf (2002a), Gbur, Visser, and Wolf (2002b), and Visser and Wolf (2003); it was pointed out that the switches can be associated with the phase singularities of coherent waves (to be discussed in Section 7). These spectral anomalies at focus were experimentally measured by Popescu and Dogariu (2002), and the connection between phase singularities and spectral changes was described further by Foley and Wolf (2002). The spectral changes associated with the focusing of high-order Bessel beams was discussed by Hu and Pu (2006).

The above-mentioned studies dealt exclusively with spectral densities (intensities) of focused, partially coherent fields. The coherence properties of such fields were studied by Fischer and Visser (2004). It was found for Gaussian-correlated fields that, depending on the effective coherence length of the field in the aperture, the effective width of the spectral degree of coherence can be either larger or smaller than that of the spectral density distribution. Moreover, the spectral degree of coherence was shown to possess phase singularities, even though such singularities are not present in the aperture field (see also Section 7). The coherence properties of focused, Bessel-correlated fields were analyzed by van Dijk et al. (2008).

In this section, we have restricted ourselves to scalar waves, but we do mention that a few studies have been devoted to the focusing of partially coherent electromagnetic fields, e.g., Zhang, Pu, and Wang (2008), Foreman and Török (2009), Salem and Agrawal (2009a), and Salem and Agrawal (2009b).

6. SCATTERING OF PARTIALLY COHERENT WAVE FIELDS BY RANDOM AND DETERMINISTIC MEDIA

The scattering of wave fields by a particulate medium such as, for example, a colloidal suspension, is a problem of fundamental importance. The seminal work by Mie (1908) laid the groundwork for an entire field of study. Here we review the scattering of partially coherent fields from both deterministic and random media. Also, the role of coherence in inverse problems is explained.

We consider first a fully coherent field, propagating in free space, that is incident on a deterministic scatterer. The total field $U(\mathbf{r}, \omega)$ can be written as the sum of the incident field, $U^{(i)}(\mathbf{r}, \omega)$, and the scattered field, $U^{(s)}(\mathbf{r}, \omega)$, i.e.,

$$U(\mathbf{r}, \omega) = U^{(i)}(\mathbf{r}, \omega) + U^{(s)}(\mathbf{r}, \omega). \tag{6.1}$$

Here $\mathbf{r}$ denotes the position and ω the frequency. The total field satisfies the following integral equation (see Born & Wolf, 1999, section 13.1):

$$U(\mathbf{r}, \omega) = U^{(i)}(\mathbf{r}, \omega) + \int_V F(\mathbf{r}', \omega) U(\mathbf{r}', \omega) G(\mathbf{r} - \mathbf{r}', \omega)\, \mathrm{d}^3 r', \tag{6.2}$$

where V is the volume occupied by the scatterer,

$$F(\mathbf{r}, \omega) = \frac{k^2}{4\pi} \left[n^2(\mathbf{r}, \omega) - 1 \right] \tag{6.3}$$

is the *scattering potential* and $n(\mathbf{r}, \omega)$ the refractive index. Furthermore,

$$G(\mathbf{r} - \mathbf{r}', \omega) = \frac{\mathrm{e}^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r} - \mathbf{r}'|} \tag{6.4}$$

is the outgoing free-space Green's function associated with the Helmholtz operator. For weak scatterers, i.e., scatterers whose refractive index differs only slightly from unity, we may approximate the total field $U(\mathbf{r}', \omega)$ in the integral of Equation (6.2) by the incident field $U^{(i)}(\mathbf{r}', \omega)$. This so-called first-order Born approximation yields the expression

$$U(\mathbf{r}, \omega) = U^{(i)}(\mathbf{r}, \omega) + \int_V F(\mathbf{r}', \omega) U^{(i)}(\mathbf{r}', \omega) G(\mathbf{r} - \mathbf{r}', \omega)\, \mathrm{d}^3 r'. \tag{6.5}$$

We notice that Equation (6.5) involves an ordinary integral, and is therefore typically easier to solve than an integral equation of the form of Equation (6.2).

Let us now consider the case where the incident field is not fully coherent, but partially coherent. Such a field is characterized by a cross-spectral density function (see Section 2)

$$W^{(i)}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \langle U^{(i)*}(\mathbf{r}_1, \omega) U^{(i)}(\mathbf{r}_2, \omega) \rangle_\omega. \tag{6.6}$$

Because the incident field is random, the scattered field, represented by the last term of Equation (6.5), will also be random. Its cross-spectral density function

$$W^{(s)}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \langle U^{(s)*}(\mathbf{r}_1, \omega) U^{(s)}(\mathbf{r}_2, \omega) \rangle_\omega \tag{6.7}$$

can readily be found by substituting from Equation (6.5) into Equation (6.7) and interchanging the order of averaging and integration. The resulting expression is

$$W^{(s)}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \iint_V F^*(\mathbf{r}_1', \omega) F(\mathbf{r}_2', \omega) W^{(i)}(\mathbf{r}_1', \mathbf{r}_2', \omega) \times G^*(\mathbf{r}_1 - \mathbf{r}_1', \omega) G(\mathbf{r}_2 - \mathbf{r}_2', \omega)\, \mathrm{d}^3 r_1' \mathrm{d}^3 r_2'. \tag{6.8}$$

Expression (6.8) pertains to the scattering of a partially coherent field by a deterministic scatterer. It can easily be generalized to scatterers whose refractive index, and hence also their scattering potential $F(\mathbf{r}, \omega)$, is a random function of position. This is achieved by replacing the product $F^*(\mathbf{r}_1', \omega) F(\mathbf{r}_2', \omega)$ by the correlation function

$$C_F(\mathbf{r}_1', \mathbf{r}_2', \omega) = \langle F^*(\mathbf{r}_1', \omega) F(\mathbf{r}_2', \omega) \rangle_F, \tag{6.9}$$

where the symbol $\langle \ldots \rangle_F$ denotes an average taken over an ensemble of scatterers. On making use of Equation (6.9) and again interchanging the order of averaging and integration, we obtain the formula

$$W^{(s)}(\mathbf{r}_1,\mathbf{r}_2,\omega) = \iint_V C_F(\mathbf{r}'_1,\mathbf{r}'_2,\omega)W^{(i)}(\mathbf{r}'_1,\mathbf{r}'_2,\omega) \times G^*(\mathbf{r}_1-\mathbf{r}'_1,\omega)G(\mathbf{r}_2-\mathbf{r}'_2,\omega)\,\mathrm{d}^3r'_1\mathrm{d}^3r'_2. \tag{6.10}$$

The radiant intensity (the rate at which the field radiates energy at frequency ω in direction $\mathbf{s}$ per unit solid angle) of the scattered field in a direction specified by the unit vector $\mathbf{s}$ equals

$$J(r\mathbf{s},\omega) = r^2W^{(s)}(r\mathbf{s},r\mathbf{s},\omega), \tag{6.11}$$

$$= r^2\iint_V C_\mathrm{F}(\mathbf{r}'_1,\mathbf{r}'_2,\omega)W^{(i)}(\mathbf{r}'_1,\mathbf{r}'_2,\omega) \times G^*(r\mathbf{s}-\mathbf{r}'_1,\omega)G(r\mathbf{s}-\mathbf{r}'_2,\omega)\,\mathrm{d}^3r'_1\mathrm{d}^3r'_2. \tag{6.12}$$

Equation (6.12) brings into evidence that the radiant intensity depends on both the statistical properties of the scatterer [represented by the function $C_\mathrm{F}(\mathbf{r}'_1,\mathbf{r}'_2,\omega)$] and those of the incident field [represented by the function $W^{(i)}(\mathbf{r}'_1,\mathbf{r}'_2,\omega)$]. Using the above formalism, Jannson, Jannson, and Wolf (1988) studied how the degree of coherence of the incident field affects the directionality of the field scattered by a Gaussian-correlated medium. They found that the effective angular width of the radiant intensity increases as the correlation length of the scatterer decreases. Furthermore, the radiant intensity was found to be more peaked in the forward direction for a fully coherent beam than for a partially coherent field. Gori, Palma, and Santarsiero (1990) confirmed experimentally that for a planar random medium (in their case, a rotating ground-glass plate), the width of the radiant intensity distribution of the scattered field indeed increases when the spectral degree of coherence of the incident field decreases.

An important class of scatterers is formed by so-called *quasi-homogeneous media*. These were introduced by Carter and Wolf (1988), who defined the degree of spatial correlation of the scattering potential [given by Equation (6.9)] as

$$\mu_\mathrm{F}(\mathbf{r}_1,\mathbf{r}_2,\omega) \equiv \frac{C_\mathrm{F}(\mathbf{r}_1,\mathbf{r}_2,\omega)}{\sqrt{I_\mathrm{F}(\mathbf{r}_1,\omega)I_\mathrm{F}(\mathbf{r}_2,\omega)}}. \tag{6.13}$$

Here the quantity $I_\mathrm{F}(\mathbf{r},\omega) \equiv C_\mathrm{F}(\mathbf{r},\mathbf{r},\omega)$ represents the average value of the second moment of the scattering potential. Just as for the spectral degree of coherence of a wave field, one can show that

$$0 \leq |\mu_\mathrm{F}(\mathbf{r}_1,\mathbf{r}_2,\omega)| \leq 1. \tag{6.14}$$

The extreme values $|\mu_F(\mathbf{r}_1, \mathbf{r}_2, \omega)| = 1$ and $\mu_F(\mathbf{r}_1, \mathbf{r}_2, \omega) = 0$ correspond to complete correlation and the total absence of correlation, respectively. As discussed by Silverman (1958), for many scatterers such as fluids, plasmas, or the atmosphere, the degree of spatial correlation of the scattering potential will depend only on the two positions $\mathbf{r}_1$ and $\mathbf{r}_2$ through the difference $\mathbf{r}_2 - \mathbf{r}_1$. We can then write

$$\mu_F(\mathbf{r}_1, \mathbf{r}_2, \omega) = g_F(\mathbf{r}_2 - \mathbf{r}_1, \omega) \tag{6.15}$$

A scatterer is said to be quasi-homogeneous if the function $g_F(\mathbf{r}, \omega)$ varies much faster with $\mathbf{r}$ than the function $I_F(\mathbf{r}', \omega)$ varies with $\mathbf{r}'$. Carter and Wolf (1988) studied the scattering of a monochromatic plane wave by a quasi-homogeneous medium. They showed that, within the validity of the first-order Born approximation, the far zone field satisfies two reciprocity relations, namely

$$J(r\mathbf{s}, \omega) = \tilde{I}_F(\mathbf{0}, \omega)\tilde{g}_F[k(\mathbf{s} - \mathbf{s}_0), \omega], \tag{6.16}$$

$$\mu(r\mathbf{s}_1, r\mathbf{s}_2, \omega) = \tilde{I}_F[k(\mathbf{s}_2 - \mathbf{s}_1), \omega]/\tilde{I}_F(\mathbf{0}, \omega). \tag{6.17}$$

Here $\mathbf{s}_0$ is the direction of propagation of the incident field, and $\tilde{I}_F(\mathbf{K}, \omega)$ and $\tilde{g}_F(\mathbf{K}, \omega)$ are the three-dimensional spatial Fourier transforms of $I_F(\mathbf{r}, \omega)$ and $g_F(\mathbf{r}, \omega)$, respectively. Equation (6.16) states that the radiant intensity is proportional to the Fourier transform of the degree of spatial correlation of the scattering potential. Equation (6.17) states that degree of coherence is proportional to the Fourier transform of the second moment of the scattering potential. It is to be noted that these relations are similar to the reciprocity relations satisfied by the fields of quasi-homogeneous beams, which are mentioned in Section 4.

Fischer and Wolf (1994) described how these two reciprocity relations can be used to reconstruct both the correlation function $g_F(\mathbf{r}, \omega)$, and the second moment $I_F(\mathbf{r}, \omega)$ of the scattering potential of a quasi-homogeneous medium from far-zone field data. Fischer and Cairns (1995) showed that by using pulses, rather than an incident monochromatic plane wave, the second moment of the dielectric susceptibility of a quasi-homogeneous medium can be reconstructed from far-zone intensity measurements alone. A theory of diffraction tomography for quasi-homogeneous media was later developed by Fischer and Wolf (1997), and generalized by Fischer (1998). Visser, Fischer, and Wolf (2006) examined how radiation generated by a quasi-homogeneous source (discussed in Section 4) is scattered by a quasi-homogeneous medium. They derived reciprocity relations for both the spectral degree of coherence and the spectral density of the field in the far zone. Recently, an "Ewald-sphere construction" for determining the structure of random media was proposed by Lahiri, Wolf, Fischer, and Shirai (2009).

The scattering of a two-dimensional electromagnetic field by a slit or a groove was studied by Huttunen, Friberg, and Turunen (1995). By using a coherent mode decomposition (see Section 8), the scattered field can be considered as an incoherent superposition of the field scattered by each individual mode. A strong decrease in the directionality of the scattered radiant intensity with decreasing spatial coherence was found.

As discussed in Section 4 (see also Wolf & James, 1996), the spectral density of the field that is generated by a partially coherent source can change on propagation, even when this propagation takes place in free space. Because of the well-known analogy that exists between radiation and scattering, one might expect that a similar effect will arise when a polychromatic wave is scattered by a medium whose dielectric susceptibility is a random function of position. Wolf, Foley, and Gori (1989) showed that, within the validity of the first-order Born approximation, this is indeed the case. In particular, if the two-point correlation function $C_F(\mathbf{r}'_1, \mathbf{r}'_2, \omega)$ of Equation (6.9) is a Gaussian, and if the spectrum of the incident light has a Gaussian profile, then the spectral density of the scattered field may be effectively blue-shifted or red-shifted, depending on the scattering angle. The analysis was extended to multiple scattering using the Rytov approximation by Shirai and Asakura (1995). Zhao, Korotkova, and Wolf (2007) discussed how the observation of these spectral changes in the far zone of the scatterer may be used to determine the correlation function of its scattering potential.

The spectral changes described above involve what may be referred to as a redistribution of the spectral density, in that light of different frequencies is scattered in different directions; no new frequencies are generated by the scattering. When the scattering medium is itself explicitly varying in time, however, it is possible to get true changes of the spectrum on scattering, including Doppler-like frequency shifts. This latter possibility was first introduced by Wolf (1989), and suggested by James, Savedoff, and Wolf (1990) as a possible explanation for some observed anomalies in quasar spectra. These Doppler-like shifts were further investigated by James and Wolf (1990).

Inverse scattering techniques have been developed that exploit the relationship between coherence and scattering. Baleine and Dogariu (2004a) developed the "variable coherence tomography" method. In this approach, the spatial coherence properties of the incident beam are tuned such that two separate volumes are created in which the field is strongly correlated. The correlation function of the scattering potential can then be determined by recording the spectral density of the scattered field in a single direction. An experimental demonstration of this method was presented by Baleine and Dogariu (2004b).

A general expression for the rate at which energy is removed by scattering and absorption from a partially coherent beam that is incident on a deterministic scatterer was derived by Carney, Wolf, and Agarwal (1997). Their result can be considered as a generalization of the optical cross-section theorem. This generalization was applied to derive an energy theorem for partially coherent beams by Carney and Wolf (1998), and it was used as the basis for a new diffraction tomography technique by Carney and Wolf (2001).

The scattering of a partially coherent wave field by a sphere, i.e., a generalization of the so-called Mie scattering, has been considered by several authors. Greffet, Cruz-Gutierrez, Ignatovich, and Radunsky (2003) applied the results from Carney et al. (1997) to study rotationally invariant scatterers. Using the Wigner transform, they showed that the extinction cross section does not depend on the coherence of the incident field. van Dijk, Fischer, Visser, and Wolf (2010) generalized the method of partial waves to the case of partially coherent fields. They predicted that when the correlation length of the incident field is comparable with or is smaller than the radius of the sphere, the angular distribution of the radiant intensity depends strongly on the degree of coherence. The occurrence of coherence vortices (see Section 7) in Mie scattering was studied by Marasinghe, Premaratne, and Paganin (2010). By varying the so-called "pointing stability" of the incident field, such singularities were found to be created or annihilated.

An inverse problem of fundamental importance is the determination of crystalline structures from diffraction experiments. Because of the small distances involved, one uses X-rays rather than radiation in the visible spectrum. The basis of this method, for which knowledge of both the phases and amplitudes of the diffracted beams is necessary, was provided by Laue (1912). This approach, however, suffers from a serious limitation because of the inability to measure these phases. It was explained in Wolf (2009) and Wolf (2010) that the phase of any physically realizable wave field is a meaningless concept due to the inherent fluctuations that the field undergoes. He pointed out that for a spatially coherent beam (which is not necessarily the same as a monochromatic beam, as was noted earlier by Roychowdhury and Wolf (2005a)), there exists an "equivalent" monochromatic beam whose phase can be determined from correlation measurements as in Young's two-slit experiment (see Section 3). Because spatially coherent X-ray beams can indeed be generated, as was shown by Liu et al. (2001), such an experiment can indeed be carried out. Knowledge of this phase then, together with the amplitude of the diffracted beams, allows one to reconstruct the crystal's structure unambiguously.

Interesting effects can also arise from the scattering of partially coherent wave fields from crystalline structures. Such effects were considered by Dušek (1995) and Sinha, Tolan, and Gibaud (1998).

7. PHASE SINGULARITIES OF COHERENCE FUNCTIONS

Researchers have long noticed that the phase of a wave field has an unusual behavior in the neighborhood of its zeros of amplitude; an early instance of this was described by Sommerfeld (1964) in his textbook on optics. Looking at the structure of a wave field consisting of plane waves of different frequency and direction, he noted that in most regions the field behaves locally like a plane wave, the exception being the behavior in the neighborhood of zeros. He concluded,

> *However, just because the amplitude vanishes there, they do not produce any stronger effect than other points of varying intensity.*

This view changed in the mid-1970s with the publication of an article by Nye and Berry (1974), in which it was noted that the zeros of wave fields and their phase have a well-defined structure that is analogous to dislocations in crystal structures. These zeros are generally referred to as *phase singularities,* and the study of these and comparable phenomena is now its own subfield, referred to as *singular optics*. Several review articles have been published on the subject, such as Soskin and Vasnetsov (2001) and Dennis, O'Holleran, and Padgett (2009), and singular optics is also discussed in the book by Nye (1999).

Philosophically, singular optics is distinct from other fields of optics in that it emphasizes the study of "common" or "generic" features of wave fields over the study of "possible" features, the latter of which require special circumstances, such as rotational symmetry, to occur. For instance, the most commonly-described interference experiment is Young's two-pinhole experiment, illustrated in Figure 6, and previously discussed in Section 3. The (approximate) zeros of the interference pattern observed on the screen are lines, which correspond to zero surfaces in three-dimensional space. If an interference experiment is done with three or more pinholes, however, the interference pattern has a fundamentally different behavior, also illustrated in Figure 6: the zeros on the measurement screen are points, which correspond to lines of zeros in three-dimensional space. These zero lines are said to be the typical, or *generic*, features of interference patterns; for instance, laser speckle patterns contain a large number of these zero lines.

A typical example of a phase singularity is present in a Laguerre–Gauss laser mode of azimuthal order $m = 1$ and radial order $n = 0$ propagating

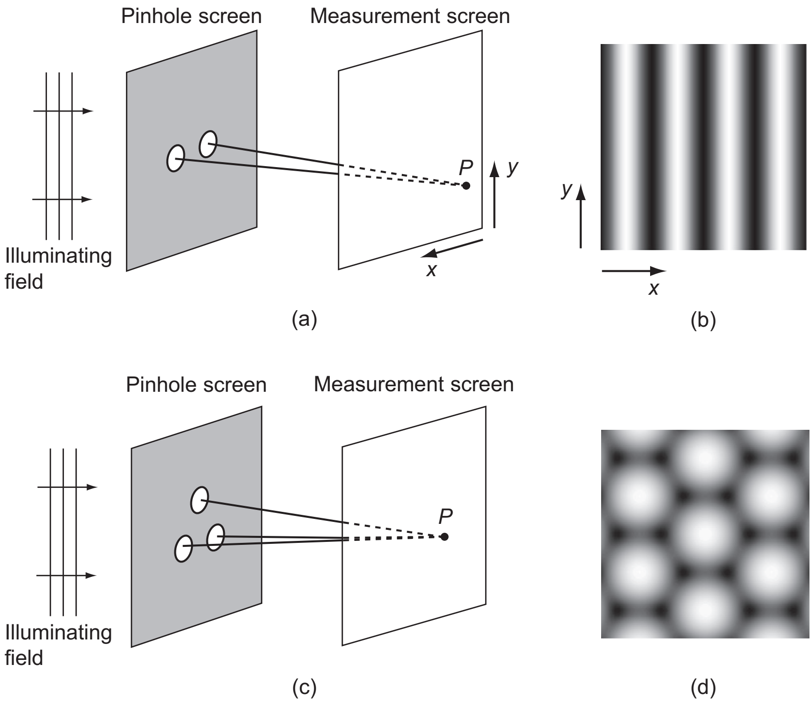

FIGURE 6 (a) Young's two-pinhole experiment. (b) The intensity of light observed on the measurement screen. (c) Young's three-pinhole experiment. (d) The intensity of light observed on the measurement screen. The zeros form a hexagonal pattern.

in the z-direction, which in the waist plane $z = 0$ has the form

$$U_0^1(\rho, \phi, 0) = \sqrt{2}U^{(0)}\frac{\rho}{w_0}e^{i\phi}\exp[-\rho^2/w_0^2], \tag{7.1}$$

where (ρ, ϕ) are polar coordinates in the transverse (x, y)-plane, w_0 is the width of the beam in the waist plane and $U^{(0)}$ is the field amplitude. The phase and intensity of this mode are illustrated in Figure 7. It can be seen that there is a zero of intensity on the central axis $\rho = 0$, and that the phase increases continuously by 2π as one traverses a closed path counterclockwise around the axis; for this reason, such phase singularities are commonly referred to as *optical vortices*.

More generally, it has become clear that nearly every property of a wave field that can be characterized by a spatially dependent amplitude and phase can also have singularities of that phase and associated generic features. For instance, vortices of the Poynting vector have been observed (see, for instance, Braunbek & Laukien, 1952; Boivin, Dow, & Wolf, 1967;

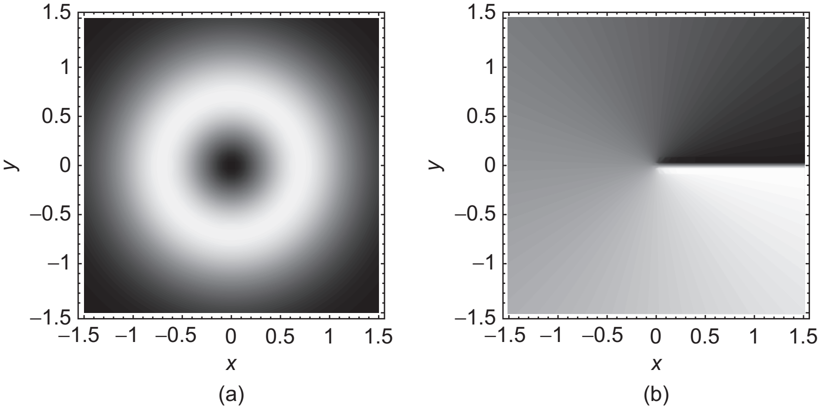

FIGURE 7 The (a) intensity and (b) phase of a Laguerre–Gauss beam of order $m = 1$, $n = 0$ in the waist plane $z = 0$.

Schouten, Visser, Gbur, Lenstra, & Blok, 2003), where the "phase" is, in this case, the direction of power flow. For inhomogeneously polarized light, singularities of the ellipticity and orientation axis of the polarization ellipse can arise; see, for instance, Nye (1983b), Nye (1983a), and Schoonover and Visser (2006).

Coherence functions of wave fields also possess spatially varying amplitude and phase structures, albeit more complicated ones, and it was natural for researchers to investigate the properties of such *correlation vortices* (also referred to as *coherence vortices*). Most studies have centered on singularities of the cross-spectral density function, i.e., regions in $(\mathbf{r}_1, \mathbf{r}_2)$-space such that $W(\mathbf{r}_1, \mathbf{r}_2, \omega) = 0$, and the phase structure of the cross-spectral density in the neighborhood of such regions. A complete description of the behavior of zeros requires the analysis of the cross-spectral density in six variables, $(\mathbf{r}_1, \mathbf{r}_2)$, a difficult prospect, and most research to date has involved studying lower-dimensional projections of the correlation function.

The earliest article on this subject seems to be that of Schouten et al. (2003), who studied singularities of the cross-spectral density of Young's interference experiment. It is to be noted, however, that these singularities are not generic features of the cross-spectral density.

Soon after, a number of authors approached the idea of correlation singularities from different viewpoints. Freund (2003), in the context of studying singularities of bichromatic optical fields, noted that correlation singularities could arise in correlations between orthogonal polarizations of the field. Gbur and Visser (2003b) studied the structure of correlation singularities in partially coherent beams in the special case in which one of the observation points is fixed, i.e., the behavior of $W(\mathbf{r}_1, \mathbf{r}_2)$ as a function of $\mathbf{r}_2$. Bogatyryova et al. (2003) performed a theoretical and experimental investigation of a class of partially coherent vortex beams, and observed singularities in the spectral degree of coherence. Palacios, Maleev, Marathay, and Swartzlander (2004) experimentally studied correlation singularities of the so-called cross-correlation function, the case such that $\mathbf{r}_2 = -\mathbf{r}_1$; this work was soon followed up with a theoretical analysis by Maleev, Palacios, Marathay, and Swartzlander (2004). An experimental observation of the fixed point correlation singularities, which possess a vortex structure and have been dubbed *correlation vortices*, was undertaken by Wang, Duan, Hanson, Miyamoto, and Takeda (2006).

A characteristic example of a correlation singularity can be derived from the Laguerre–Gauss beam of Equation (7.1) using the so-called beam wander model, introduced by Gbur, Visser, and Wolf (2004b). A partially coherent vortex beam is modeled by a Laguerre–Gauss beam with a central axis that is a random function of position; the cross-spectral density in the plane $z = 0$ is defined as

$$W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = \int U_0^{1*}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}', 0) U_0^1(\boldsymbol{\rho}_2 - \boldsymbol{\rho}', 0) f(\boldsymbol{\rho}')\, \mathrm{d}^2\rho', \tag{7.2}$$

where

$$f(\boldsymbol{\rho}') = \frac{1}{\delta\sqrt{\pi}} \mathrm{e}^{-\rho'^2/\delta^2} \tag{7.3}$$

is the probability density of the position of the central axis, and δ characterizes the average variance of the beam wander. This integral can be evaluated analytically, and takes on the form

$$\begin{aligned} W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = {} & \frac{2\sqrt{\pi}|U^{(0)}|^2}{w_0^6 A^3 \delta} \exp[-(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)^2 / w_0^4 A] \exp[-(\rho_1^2 + \rho_2^2)/\delta^2 w_0^2 A] \\ & \times \Big\{ \Big[\gamma^2 (x_1 + \mathrm{i}y_1) + (x_1 - x_2) + \mathrm{i}(y_1 - y_2) \Big] \\ & \times \Big[\gamma^2 (x_2 - \mathrm{i}y_2) - (x_1 - x_2) + \mathrm{i}(y_1 - y_2) \Big] + w_0^4 A \Big\}, \end{aligned} \tag{7.4}$$

where $\gamma \equiv w_0/\delta$, $\boldsymbol{\rho} \equiv (x, y)$ and

$$A \equiv \left(\frac{2}{w_0^2} + \frac{1}{\delta^2} \right). \tag{7.5}$$

The typical projections of correlation singularities are exhibited by this simple model. For instance, a plot of the cross-spectral density when $\mathbf{r}_1$ is fixed and $\mathbf{r}_2$ is varied is shown in Figure 8. It can be seen that a pair of correlation vortices are present, one associated with the original optical vortex of the Laguerre–Gauss beam and one that travels from the point at infinity. Figure 9 shows the cross-spectral density for the case when $\mathbf{r}_2 = -\mathbf{r}_1$. It can be seen that the correlation singularity is a zero ring centered on the origin, referred to as a *ring dislocation*; the phase jumps discontinuously by π across the boundary of the singularity. It is to be noted that these two very different behaviors – correlation vortex and ring dislocation – are different manifestations of the general correlation singularity in all variables $\mathbf{r}_1$, $\mathbf{r}_2$.

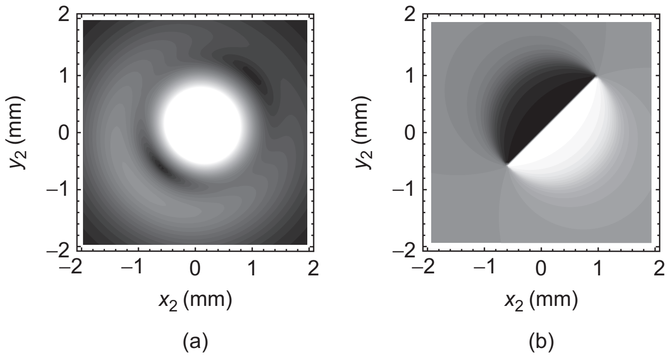

FIGURE 8 The (a) amplitude and (b) phase of the cross-spectral density $W(\mathbf{r}_1, \mathbf{r}_2, \omega)$, with $\mathbf{r}_1$ fixed. Here $w_0 = 1.0\,\text{mm}$, $\delta = 0.6\,\text{mm}$, $x_1 = 0.1\,\text{mm}$, and $y_1 = 0.1\,\text{mm}$. The phase runs over 2π, with lower values darker.

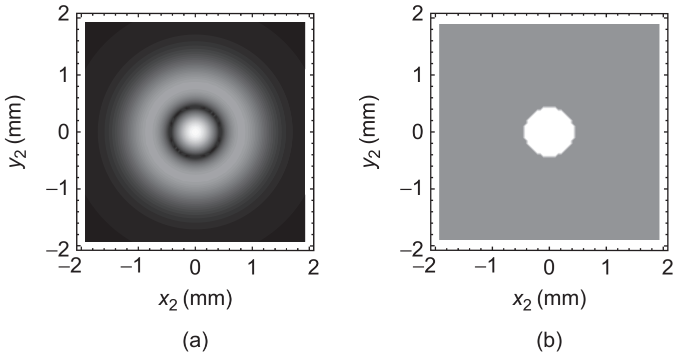

FIGURE 9 The (a) intensity and (b) phase of the cross-spectral density $W(\mathbf{r}_1, \mathbf{r}_2, \omega)$, with $\mathbf{r}_1 = -\mathbf{r}_2$. Here $w_0 = 1.0\,\text{mm}$, $\delta = 0.6\,\text{mm}$, $x_1 = 0.1\,\text{mm}$, and $y_1 = 0.1\,\text{mm}$. The phase runs over 2π, with lower values darker.

The presence of correlation singularities in a variety of optical systems has been noted. Fischer and Visser (2004) observed singularities of the correlation function in the focusing of partially coherent light, and Motsek, Kivshar, Shih, and Swartzlander (2005) noted such singularities in a nonlinear photorefractive crystal. Far-zone characteristics of Schell-model vortex beams were studied by Liu and Lü (2007), and correlation vortices in superpositions of vortex beams were studied by Cheng and Lü (2008). The behavior of correlation vortices in the focusing of partially coherent vortex beams was considered by Liu, Yang, Rong, Wang, and Yan (2010).

Because of the complexity of correlation singularities, a significant amount of effort has been directed toward characterizing them and their propagation and conservation properties. Wang and Takeda (2006) introduced the idea of a "coherence current" and a conservation law of coherence. Swartzlander and Hernandez-Aranda (2007) made an analogy between Rankine vortices of fluid dynamics and correlation singularities. Gbur and Swartzlander (2008) described the complete structure of a correlation vortex in a single transverse plane, a "surface" in the four transverse coordinates. Ren, Zhu, and Duan (2008) characterized the behavior of correlation vortices using their topological characteristics. Maleev and Swartzlander (2008) investigated the evolution of a correlation singularity on propagation, as did van Dijk and Visser (2009). The topology of correlation singularities in the far zone of a quasi-homogeneous source was studied by van Dijk, Schouten, and Visser (2009).

It can be shown that there is a strong relationship between the correlation vortices of a partially coherent field and the optical vortices of the corresponding fully coherent field. This relationship seems to have been first noted by Gbur et al. (2004b), an article which also elaborates on the concept of spectral changes related to vortices. Gbur and Visser (2006) demonstrated that this relationship holds for variable coherence fields in any linear optical system. Additional work by Gu and Gbur (2009) investigated the behavior of higher-order vortices and topological reactions of such vortices.

Gbur et al. (2004b) also observed that zeros of intensity are not generic features of partially coherent fields, but it is still possible to specially prepare fields that are partially coherent and possess such zeros. Gbur, Visser, and Wolf (2004a) determined conditions under which complete destructive interference can be achieved with a partially coherent field in a three-pinhole interferometer; these results were confirmed experimentally both acoustically by Basano and Ottonello (2005) and optically by Ambrosini, Gori, and Paoletti (2005). The phase structure of such nongeneric fields, including both phase and coherence singularities, was studied theoretically by Gan and Gbur (2007). Extending the relationship between different classes of singularities further, Visser and Schoonover

(2008) demonstrated that it is possible to smoothly transform between phase, correlation, and polarization singularities in Young's interferometer, a process referred to as a "cascade" of singularities.

It is also possible to introduce correlation singularities of temporal coherence functions, such as the mutual coherence function $\Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau)$. Swartzlander and Schmit (2004) introduced and experimentally studied singularities of the function $\Gamma(\mathbf{r}, \mathbf{r}, \tau)$, with variable position $\mathbf{r}$ and fixed τ. The resulting singularities have the form of vortices.

Recently, there have been some studies of the polarization singularities of partially coherent beams; this is discussed in Chernyshov, Felde, Bogatyryova, Polyanskii, and Soskin (2009) and Chernyshov, Fel'de, Bogatyreva, Polyanskii, and Soskin (2009).

8. THE COHERENT MODE REPRESENTATION

The coherent mode representation introduced by Wolf (1982)[4] is the expansion of the cross-spectral density function of a partially coherent source or partially coherent field into a diagonal representation of orthogonal modes, of the form

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \sum_n \lambda_n(\omega)\phi_n^*(\mathbf{r}_1, \omega)\phi_n(\mathbf{r}_2, \omega), \tag{8.1}$$

where the eigenvalues $\lambda_n(\omega)$ are non-negative quantities. The modes $\phi_n(\mathbf{r}, \omega)$ are orthogonal with respect to a given domain; for a primary radiation source, the domain is typically taken to be the volume of the source, while for a secondary planar source, the domain is the planar area of the source. The index n may represent multiple indices of summation; the mode decomposition of a field in a region of three-dimensional space, for instance, usually requires two summation indices, while the mode decomposition of a source in three-dimensional space typically requires three.

The coherent mode representation has become an excellent tool for computationally evaluating the propagation of a wave field. Before its introduction, finding the evolution of the cross-spectral density of a wave field on propagation through an optical system required the evaluation of an integral of the form

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \iint W_0(\mathbf{r}_1', \mathbf{r}_2', \omega) G^*(\mathbf{r}_1, \mathbf{r}_1', \omega) G(\mathbf{r}_2, \mathbf{r}_2', \omega)\, \mathrm{d}^2 r_1' \mathrm{d}^2 r_2', \tag{8.2}$$

[4] It is worth noting that a very similar representation was introduced earlier for the mutual intensity function by Gori (1980).

where the integrations are over the input plane of the optical system, $W_0(\mathbf{r}_1, \mathbf{r}_2, \omega)$ is the cross-spectral density of the field on this input plane, and $G(\mathbf{r}, \mathbf{r}', \omega)$ is the Green's function of the optical system.

A numeric solution to this equation would require the evaluation of a four-fold complex integral, a very difficult prospect. Using the coherent mode representation for the field on the input plane, Equation (8.2) may be written as

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \sum_n \lambda_n(\omega) \left[\int \phi_n(\mathbf{r}_1', \omega) G(\mathbf{r}_1, \mathbf{r}_1', \omega)\, \mathrm{d}^2 r_1' \right]^* \times \left[\int \phi_n(\mathbf{r}_2', \omega) G(\mathbf{r}_2, \mathbf{r}_2', \omega)\, \mathrm{d}^2 r_2' \right]. \tag{8.3}$$

The four-fold integral has been reduced to a sum over the product of identical two-fold integrals. Typically a partially coherent field can be represented to a good approximation by a small number of modes; this is illustrated in Figure 10 using the eigenvalues of a Bessel-correlated field. By using the coherent mode representation, the propagation of the cross-spectral density can be evaluated by the determination of a relatively small number of two-fold integrals.

Research related to the coherent mode representation can be broadly broken into four parts: extensions of the basic theory, the determination of mode representations for model fields or sources, methods for determining the mode representation for arbitrary fields, and application of the mode representation to wave propagation problems.

One of the first extensions of the basic theory was by Starikov (1982), who used the representation to define an effective number of degrees of freedom of the source. Wolf (1986a) derived a number of theorems for modes of spatially bandlimited wave fields.

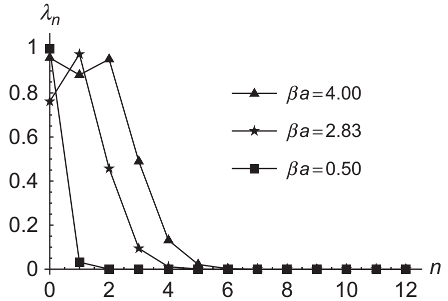

FIGURE 10 Eigenvalues λ_n versus n for a J_0-correlated field such as described by Equation (5.17).

The original coherent mode calculations were done for scalar wave fields, but there have been a number of extensions of the theory to fully electromagnetic wave fields. The earliest of these was by Pask and Stacey (1988), who used electromagnetic plane waves as the mode functions. The first general theory for characterizing an electromagnetic coherent mode representation was presented much later by Gori et al. (2003). At about the same time, Tervo, Setälä, and Friberg (2004) constructed a representation based on their theory of coherence for electromagnetic waves; another representation was provided by Kim (2005). Kim and Wolf (2006) introduced a scalar mode representation of an electromagnetic wave field, using a biorthogonal expansion of modes.

A number of alternatives to the original scalar coherent mode representation have also been suggested. Sung, Kim, and Park (1996) introduced a "P-representation" of the cross-spectral density, based on analogy with the expansion of a density operator in coherent state vectors. Withington, Hobson, and Berry (2004) introduced a representation based on an overcomplete set of Gabor basis functions. Martinsson, Lajunen, and Friberg (2007) have suggested the use of "communication modes" of a linear optical system, namely the functions arising from the singular value decomposition of the propagation kernel.

It is, in general, quite difficult to derive the coherent mode representation for a particular wave field; there has been much success, however, in deriving the modal representation for a variety of model wave fields, particularly beams. The most important of these, Gaussian Schell-model beams, was determined by Starikov and Wolf (1982); these beams were interpreted as multimode laser radiation by Gase (1991). The mode representation of Bessel-correlated Schell-model sources was derived by Gori, Guattari, and Padovani (1987). Flat-topped partially coherent beams were treated by Borghi and Santarsiero (1998), and a special class of beams carrying optical vortices was introduced by Ponomarenko (2001). The mode representation of anisotropic Gaussian Schell-model beams was evaluated by Sundar, Makunda, and Simon (1995).

A new and intriguing class of partially coherent Gaussian beams incorporating a "twist" in the azimuthal phase structure was introduced by Simon and Mukunda (1993) and are referred to as twisted Gaussian Schell-model beams; in relatively short order their mode decomposition was derived (Simon & Sundar, 1993) and their propagation characteristics determined (Sundar, Simon, & Mukunda, 1993). An intuitive model of twisted beams as the incoherent superposition of ordinary Gaussian beams was related by Ambrosini, Bagini, Gori, and Santarsiero (1994).

The mode representations of three-dimensional sources and fields have also been considered. Gori, Palma, and Padovani (1989) introduced a modal expansion for blackbody radiation in a spherical cavity. Setälä, Lindberg, Blomstedt, Tervo, and Friberg (2005) applied their aforementioned electromagnetic coherent mode representation to study the full vector properties of blackbody radiation. Gori and Korotkova (2009) have studied the general mode representation of spherical homogeneous sources.

A number of other mode representations of broad physical interest have been derived, including the representation of Lambertian sources (Starikov & Friberg, 1984), the representation of propagation-invariant fields (Ostrovsky, Martinez-Niconoff, & Ramirez-San-Juan, 2001), and that of thin annular sources (Gori, Santarsiero, Borghi, & Li, 2008).

As noted, it is in general difficult to derive the coherent mode representation for an arbitrary wave field. A number of techniques have been proposed, both experimental and theoretical, for extracting the mode behavior from a given partially coherent field. The earliest of these seems to be by Kim and Park (1992), who developed an approximate method for determining the eigenvalues and eigenfunctions of the representation; this method was tested numerically by Hong, Kang, and Kim (1993). For the special case when the modes are known to be of Hermite-Gaussian form, Gori, Santarsiero, Borghi, and Guattari (1998) demonstrated that knowledge of the intensity of the field is sufficient to determine the mode weights. Xue, Wei, and Kirk (2000) further suggested that the weights could be determined by the evolution of the intensity on propagation. Another experimental technique for extracting the mode structure from intensities was recently introduced by Flewett, Quiney, Tran, and Nugent (2009).

Alternative modal decompositions have been shown to be easier to calculate both experimentally and theoretically. Ostrovsky, Zemliak, and Hernández-Garćia (2005) introduced an alternative representation that can be determined experimentally from radiometric measurements. An "elementary source model" was described by Vahimaa and Turunen (2006) for efficient propagation of partially coherent fields. Davis and Schoonover (2009) have introduced a computationally efficient modal decomposition based on the $LDL^{\dagger}$ factorization, while Martinex-Herrero, Mejias, and Gori (2009) have given a nonorthogonal "pseudo-modal" decomposition of the cross-spectral density.

The coherent mode representation has been shown to be directly useful in a number of optical applications, foremost among them, as noted, the propagation of partially coherent wave fields. The first use of coherent modes for propagation appears to be due to Gori (1983), who investigated the free-space propagation of Schell-model sources. Such a strategy has also been applied to the analysis of spectral changes on propagation (Gamliel, 1990), changes in the state of coherence on propagating through an optical system (Shirai & Asakura, 1993), and the evolution of the generalized radiance (Ostrovsky & Rodriguez-Solid, 2000). The mode representation has also been applied to a class of inverse source problems (Habashy, Friberg, & Wolf, 1997).

A modal representation can also be applied to the propagation of fields through random media, as was demonstrated theoretically by Shirai, Dogariu, and Wolf (2003) in a study of the spreading of beams in turbulence. Schwartz and Dogariu (2006) have suggested that a mode-coupling approach could be used to study more general properties of a turbulence-degraded beam.

One interesting observation that has been made is that the coherent mode representation allows for the quasi-geometrical propagation of partially coherent wave fields. Zysk, Carney, and Schotland (2005) have demonstrated that a scalar partially coherent field could be propagated by ray-tracing the individual coherent modes; this method was later extended to electromagnetic partially coherent fields by Schoonover, Zysk, Carney, and Wolf (2008).

Several other applications of the coherent mode representation are worth noting. Gbur and Wolf (1997) used the representation to construct theoretically a primary source that is globally incoherent but produces a fully coherent field. Withington and Murphy (1998) applied the representation to study submillimeter-wave quasi-optical systems. The modal representation has been applied to the development of a near-field measurement technique by Fourestie, Altman, Bolomey, Wiart, and Brouaye (2002).

Although they do not directly apply the coherent mode representation, Salem and Agrawal (2009a) recently described a modal technique for analyzing the coupling of stochastic beams into optical fibers. The technique was used to study the effects of this coupling on the coherence and polarization of the coupled field by Salem and Agrawal (2009b).

9. NUMERICAL SIMULATION OF PARTIALLY COHERENT FIELDS

As already noted, determining the free-space propagation of a partially coherent field typically involves the evaluation of one or more four-fold integrals, a difficult prospect even with modern computing power. The application of the coherent mode representation (described in Section 8) allows one to reduce the problem to a finite sum of two-fold integrals, but even these integrals may be difficult to evaluate – and the coherent mode representation of the field may not be available. Because of this, a variety of techniques have been proposed for the numerical evaluation of the propagation of a partially coherent field.

These techniques may be broadly divided into two classes: efficient simulation of the average properties of the optical field and generation of realizations of a wave field with prescribed statistical behavior. The second class is of interest for evaluating the properties of systems in which detectors can respond fast enough to measure instantaneous field properties.

Of the first class of techniques, a number of methods have been developed for propagating correlation functions using the framework of geometrical optics. We have already noted the propagation of partially coherent fields using geometrical optics and the coherent mode representation, as done by Zysk et al. (2005); this method was later extended by Schoonover et al. (2008) to encompass partially coherent electromagnetic fields. Another ray-based propagation method was introduced by Petruccelli and Alonso (2008), based on their earlier work on generalized radiometry (see Alonso, 2001; Petruccelli & Alonso, 2007, and references therein) that allows the propagation of the cross-spectral density through complex optical systems. Very recently, a third method was developed by Riechert, Dürr, Rohlfing, and Lemmer (2009) to propagate the temporal coherence function by means of rays; this method, however, does not as yet include diffraction effects. Pedersen and Stamnes (2000) introduced another method of propagation based on radiometric concepts.

Monte Carlo methods have also been combined with geometric techniques for the efficient propagation of coherence functions. Fischer, Prahl, and Duncan (2008) introduced a Monte Carlo propagation technique based on the Huygens–Fresnel principle; later work (Prahl, Fischer, & Duncan, 2009) investigated the construction of the Green's function for an entire optical system using Monte Carlo methods.

The second class of techniques involves determining realizations, in space and time, of partially coherent fields of given average properties. An early technique of this type was given by Davis, Kim, and Piepmeier (2004), who described a method of generating realizations of stationary electromagnetic random processes in time; the method was later extended to generating full vectorial spatio-temporal realizations by Davis (2007). Another technique for generating spatio-temporal realizations was introduced by Gbur (2006), who constructed the realizations from a collection of pulses of appropriate spatial and temporal shape. Around the same time another method was introduced by Rydberg and Bengtsson (2006) in which realizations are generated from a finite superposition of independent monochromatic fields.

Somewhere between the first and second class of techniques lies the works of Voelz, Bush, and Idell (1997) and Xiao and Voelz (2006), in which the average properties of a partially coherent field are determined by constructing temporal and spatial realizations of the field, respectively, and averaging over these properties.

10. DIRECT APPLICATIONS OF COHERENCE THEORY

The theory of optical coherence plays an important indirect role in many optical applications in which the statistical properties of light must be understood in order to evaluate their effect on system performance. There are also a number of applications, however, based directly on the manipulation of the state of spatial and temporal coherence.

Several of these applications have been touched upon in previous sections. For instance, it has been suggested by Gbur and Visser (2003a), Pu et al. (2004), and van Dijk et al. (2008) that the ability to shape the intensity in the focal region by spatial coherence could be used to develop novel optical trapping and optical manipulation schemes. In particular, Arlt and Padgett (2000) introduced the term "bottle beam" to characterize a focused coherent field with an intensity minimum at the geometric focus; this term has also been adopted for describing partially coherent beams of this type by Pu, Liu, and Nemoto (2005).

The most well-known application involving coherence theory is *optical coherence tomography* (OCT), a low-coherence interferometric technique which can be used to image subsurface features of a biological specimen. OCT has become a very important medical diagnostic tool, and references to it are far too numerous to exhaustively discuss here; we mention the review article by Fercher and Hitzenberger (2002) and the text by Brezinski (2006). It is also worth noting more recent research that improves on the standard OCT modality by treating it as an inverse scattering problem (see Ralston, Marks, Carney, and Boppart, 2006; Marks, Ralston, Carney, and Boppart, 2007 for details).

Laser beams with high spatial and temporal coherence produce speckle patterns on reflection or scattering that can be detrimental for many applications; appropriately chosen partially coherent fields can provide superior performance in many of these cases. Because the high brightness and directionality of laser light is usually required, the strategy for producing partially coherent light is typically to distort a fully coherent laser beam. An overall review of the techniques of speckle reduction was given by Iwai and Asakura (1996); we also list a few illustrative applications. Kato, Nakayama, and Suzuki (1975) used an extended incoherent source to reduce speckle in the recording of holograms; other studies of the effects of coherence in holography include the work of Lurie (1966), Lurie (1968), Wolf, Shirai, Agarwal, and Mandel (1999), and Gopinathan, Pedrini, and Osten (2008). Wang, Tschudi, Halldórsson, and Pétursson (1998) reduced speckle in a laser projection system by introducing a time-varying diffractive optical element. Speckle reduction in OCT was achieved using a partially coherent source by Kim et al. (2005). A number of authors have investigated the usefulness of partially coherent fields in inertial confinement fusion schemes, for instance, Lehmberg and Goldhar (1987), Rothenberg (1997), and Tsubakimoto, Nakatsuka, Miyanaga, and Jitsuno (1998).

Closely related to the goal of speckle reduction is the observation that partially coherent beams are often more useful than their fully coherent counterparts for applications involving propagation through random media such as the turbulent atmosphere. It has been demonstrated that such "prerandomized" beams have less scintillations (intensity fluctuations) in turbulence, making them of interest for free-space laser communications and sensing.

Early articles, such as those by Beran (1966), Taylor (1967), Yura (1972), Belenkii, Kon, and Mironov (1977), and Fante (1981b), looked at the general evolution of the mutual coherence function in turbulence. Other articles, such as those by Kon and Tatarskii (1972) and Leader (1978), studied the propagation characteristics of partially coherent beams. During the same period, the focus of research seems to have gradually shifted to the study of the scintillation characteristics of partially coherent beams. Leader (1979), Fante (1981a), and Banakh, Buldakov, and Mironov (1983) looked at the fluctuations of beams with partial spatial coherence. Fante (1979) noted that a decrease in temporal coherence could also significantly reduce scintillations.

Through the 1990s, the study of such effects seems to have been mostly ignored, with the exception of articles by Wu (1990) and Wu and Boardman (1991) on the propagation and coherence properties of model beams in turbulence.

At the turn of the millenium, interest in partial coherence-based turbulence effects exploded again. Gbur and Wolf (2002) looked theoretically at the spreading of partially coherent beams in random media, and noted that they are "resistant" to turbulence; this was demonstrated experimentally by Dogariu and Amarande (2003). Similar theoretical results were found by Ponomarenko, Greffet, and Wolf (2002) using a Hilbert-space method, and by Shirai et al. (2003) using the coherent mode representation. Long-distance propagation of partially coherent beams was studied soon after by Salem, Shirai, Dogariu, and Wolf (2003). Baykal (2004) investigated the average transmittance of partially coherent beams in turbulence. Many other studies of beam spreading in turbulence have been performed since then for specific classes of beams.

Of most interest to those developing optical communications and sensing applications is the scintillation reduction associated with partially coherent beams. The application of Gaussian Schell-model beams to free-space communication was investigated by Ricklin and Davidson (2002), Ricklin and Davidson (2003), and Korotkova, Andrews, and Phillips (2004).

Much research since then has involved the study of a variety of strategies for reducing scintillation with partial coherence. A "pseudo-partially coherent beam" was introduced by Voelz and Fitzberry (2004) for free-space communication. Kiasaleh (2006) investigated the scintillation of a multiwavelength Gaussian beam; a spectral encoding strategy for scintillation reduction was introduced by Berman, Bishop, Chernobrod, Nguyen, and Gorshkov (2007). Baykal and Eyyuboglu (2007) studied the scintillation of incoherent flat-topped Gaussian beams. It was shown in Korotkova (2006) and Korotkova (2008) that appropriately chosen electromagnetic beams can have appreciably lower scintillation. Furthermore, Gu, Korotkova, and Gbur (2009) have demonstrated that nonuniform, fully polarized beams can have lower scintillation as well, as they become partially polarized on propagation; the propagation characteristics of such beams were studied by Wang and Pu (2008).

One of the most promising partially coherent sources for optical communications is an incoherent array of beams. The possibility of scintillation reduction with beams of different wavelengths was investigated in Peleg and Moloney (2006), Peleg and Moloney (2007), and Polynkin, Peleg, Klein, Rhoadarmer, and Moloney (2007). A study of a partially coherent array of Gaussian beams was performed in Baykal, Eyyuboglu, and Cai (2009), following up on work on coherent arrays done in Eyyuboglu, Baykal, and Cai (2008).

A number of mathematical tools have been developed for studying partially coherent beams in turbulence. The optimization of scintillation and propagation characteristics of beams was investigated in Schulz (2004) and Schulz (2005). An angular spectrum representation for propagation through turbulence was introduced by Gbur and Korotkova (2007); an electromagnetic version of this representation was done by Korotkova and Gbur (2007).

The relation between partial coherence and random media has also been used to probe the structure of the medium itself. Ponomarenko and Wolf (2002) developed a technique for deducing the turbulence correlations using measurements of the correlations of the scattered light. A strategy for determining the scattering parameter of an optically diffusive medium was tested by McKinney, Webster, Webb, and Weiner (2000). A technique called *variable coherence tomography* was introduced in Baleine and Dogariu (2004b) and Baleine and Dogariu (2005), which uses illumination with various states of coherence to deduce structure; this technique was adapted to include polarization effects by Tyo and Turner (2008). More recently, Gu and Gbur (2010) showed that the evolution of correlation singularities in turbulence can be used as a crude measure of turbulence strength.

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