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Models and Algorithms on Contraflow Evacuation Planning Network Problems

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Abstract **—** After different disasters the evacuation planning support to remove the residence from accidental areas to safe destinations. Contraflow solutions minimize the congestion and make the traffic smooth during evacuation by reversing the required road directions from risk areas to safe places. Mathematical models and solution approaches appear for the maximum, the quickest and the earliest arrival contraflow problems in the literature. The research on contraflow reconfiguration has been motivated from tremendous applications improving both throughput and speed. This note briefly revisits the static and dynamic contraflow problems previously investigated. We also develop models and algorithms that obtain optimal solutions to the lexicographically maximum and the earliest arrival contraflow problems.

Keywords — network optimization, evacuation planning, transportation planning, contraflow, complexity

1. INTRODUCTION

Given an evacuation network, the contraflow problem is a challenging issue of finding a network reconfiguration with ideal lane directions satisfying the given constraints that optimizes the given objective. The developed contraflow evacuation plans, that seek to remove traffic jams and make the traffic systematic and smooth, are emerging to react to different large scale natural and man-made disasters. The application of contraflow is not only limited to evacuation planning but also in traffic planning that reduces congestion and traffic jams during the day-to-day office hours, some accident management cases or some street exhibitions. Various mathematical models, heuristics, optimization and simulation techniques taking into account of macroscopic and microscopic behavioral characteristics deal with contraflow for this transportation network, however an acceptable contraflow solution even approximately is a lacking due to very high computational costs.

A polynomial time solution for the maximum dynamic network flow problem can be found in a fundamental work of (Ford and Fulkerson, 1958). Linking to its properties, a simplest version of the quickest flow problem is solved in (Burkard *et al.*, 1993). Using the concept of flows in negative time that is with a permissible negative flow, to realize former decisions, the transshipment problem is solved with polynomial time in (Hoppe, 1995; Hoppe and Tardos, 2000). Authors in (Wilkinson, 1971; Minieka, 1973) maximize the two-terminal flow simultaneously in each step of time by maintaining the optimal solutions in earlier steps. However, its time complexity is pseudo-polynomial time. A multi-source earliest arrival solution for the given supplies and demand has been achieved in (Baumann and Skutella, 2006).

In addition to a wider class of research on auto-based evacuation, the low-mobility population dominated large cities and developing countries highly demand a research in transit-based and multi-modal evacuation models. Authors in (Pardalos and Arulselvan, 2009) employ a branch and price procedure to solve a path-based bi-modal formulation with buses and cars. With static demands at origins, static travel times on arcs and given bus routes having loading and unloading time zero, they consider an objective of minimizing the costs for frequency of buses and paths of cars. We refer to (Altay and Green, 2006; Chen and Miller-Hooks, 2008; Cova and Johnson, 2003; Dhamala, 2015; Hamacher and Tjandra, 2001; Moriarty *et al.*, 2007; Pel *et al.*, 2012; Schadschneider *et al.*, 2009; Yusoff *et al.*, 2008) for survey papers on auto-based evacuation models. Pyakurel *et al.* (2014) study transit-based evacuation models and present a case study of Kathmandu metropolitan city for transit dependent evacuees as an application. Hua *et al.* (2014) study a multi-modal integrated contraflow model for uncertain arrivals of evacuees in an evacuation region with low mobility population. The transit-based models are initiated with vehicle routine problem whereas the integrated strategy contains non-contraflow to shorten the strategy setup time, full-lane contraflow to minimize the evacuation network capacity and bus contraflow to realize the transit cycle operation.

Contraflow is a widely accepted model for a good solution rather than an optimal one for practical cases. Many authors have presented significant time saving algorithms using contraflow techniques but an acceptable systematic result does not exist yet, see (Dhamala, 2014) for a summary. Kim *et al.* (2008) present a greedy heuristic that finds high quality solutions and a bottleneck relief heuristic for large scale evacuations where the evacuation time has been improved by at least 40 percent with at most 30 percent of the total arc reversals. They showed that the problem of minimizing the evacuation time

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is NP-hard. Authors in (Kim and Shekhar, 2005) present two heuristics for contraflow. The first heuristic solves a minimum cost problem in the time-expanded network in the given time period to record the total number of flow units that pass through each edge during the evacuation time and flips the direction of each edge in favor of the direction of larger flow. The second heuristic is based on simulated annealing iteration that yields a minimum evacuation time locally.

Authors in (Rebennack *et al.*, 2010) give strongly polynomial time algorithms for the contraflow problems of maximizing a static flow, a two-terminal dynamic flow and a two-terminal quickest flow. The latter dynamic contraflows with multi-terminal are NP-hard. Dhamala and Pyakurel (2013) give mathematical model for two-terminal earliest arrival contraflow and solve it polynomially on two-terminal series-parallel graphs, see also (Pyakurel and Dhamala, 2014a). Pyakurel et al. (2014) solve the two-terminal generalized maximum dynamic and earliest arrival contraflow problems on lossy networks. The lexicographically maximum static and dynamic contraflows have been dealt in (Pyakurel and Dhamala, 2014).

Although the contraflow approach increases the flow value and reduces the evacuation time significantly in comparison to the non-contraflow, the arc reversal cost for reconfiguration has to be paid. The authors in (Rebennack *et al.*, 2010) consider the contraflow configuration price in terms of fixed arc switching cost to deliver a feasible amount of flow. The cost of arc orientation for contraflow configuration has been considered as flow cost and time cost separately, and combined both costs together in (Gross, 2014). The orientation cost for contraflow can be at most two third of the flow value. However, the time cost for the orientation is node dependent.

Our objective is to look insights into the contraflow approach and look for more analytical solutions and applications. Although having significantly importance of the contraflow approach, its analytical approach has been considered in the literature relatively less. In this paper we systematically formulated a class of evacuation planning problems with arc reversal possibility which would contribute to better understanding the properties of contraflow reconfiguration for practical use.

We investigate the models, methods and algorithms developed in contraflow evacuation planning network problems. Section 2 summarizes the notations, models and solution status of the evacuation planning problems we are using in this paper. We study lexicographically maximum and earliest arrival contraflow problems and present efficient algorithms for them in Sections 3 and 4. A series of investigated contraflow problems have been systematically formulated in both sections. The final section concludes the paper.

2. FUNDAMENTAL CONCEPTS

2.1 Basic Denotations

A building (or a region) to be evacuated is represented by a network $\mathcal{N} = (V, A)$, $|V| = n$ and $|A| = m$ in which the rooms in a building (or places in a region) represent the nodes, and connections between these parts (i.e., doors between rooms, or streets in region) correspond to edges. The source nodes contain evacuees and the sink nodes are the safety. The set of multi-terminals is denoted by the sets of multi-source S and multi-sinks D. Nodes s and d represent the single-source and single-sink. We assume that $A_d^{out} = A_s^{in} = \emptyset$, where $A_v^{out} = \{(v, w) \in A\}$ and $A_v^{in} = \{(w, v) \in A\}$ for the node $v \in V$. The network consists of nonnegative functions of arc capacities $b_A: A \to N$, node capacities $b_V: V \to N$ and arc travel times $\tau: A \to N$. The vectors $\mu(s)$ and $\vartheta(d)$ represent the given supply and demand at each source and sink, respectively. We represent by $b_A(e)$, $\tau(e)$, and $b_V(v)$, the maximum units of flow that may enter the initial node of arc e per time period, the time needed to travel one unit of flow on the arc e from $\text{tail}(e)$ to its $\text{head}(e)$, and the amount of flow allowed to hold at node v respectively. Each arc e may also have a nonnegative cost coefficient $c_A(e)$, for sending one unit of flow through the arc e. The group of evacuees is modeled as a flow which passes through the network over time. Two way network configurations will be allowed in case of lane reversal scenarios.

The nonnegative functions x_{dyna} and x_{stat} define the *dynamic* and *static network flows* on $A \times \boldsymbol{T}$ and A, respectively. Let $\mathcal{N}_{x_{stat}}^R = (V, \vec{A} \cup \overleftarrow{A})$ be the *residual network* of $\mathcal{N}$. Here, $\vec{A} = \{\vec{e} = e | \, x_{stat}(e) < b_A(e)\}$ denotes the set of forward arcs $\vec{e}$ having capacity $b_A(e) - x_{stat}(e)$ and transit time $\tau(e)$, and $\overleftarrow{A} = \{\overleftarrow{e} = (\text{head}(e), \text{tail}(e)) | \, x_{stat}(e) > 0\}$ denotes the set of backward arcs $\overleftarrow{e}$ having capacity $x_{stat}(e)$ and a transit time $-\tau(e)$.

For a *dynamic network* $\mathcal{N} = (V, A, b_A, b_V, \tau, S, D, T)$, the network $\mathcal{N}(T) = (V_T, A_M \cup A_H)$ defines *time-expanded network*. Here, the node set $V_T = \{v(t) | v \in V, \; t = 0, 1, \ldots, T\}$ and the sets of *holdover arcs* with arc capacity $b_V(v)$ and *movement arcs* with arc capacity $b_A(v, w)$, respectively, are defined by

$$A_H = \{(v(t), v(t+1)) | v \in V, t = 0, 1, \ldots, T-1\}$$

$$A_M = \{(v(t), w(t + \tau(v, w))) | (v, w) \in A, t = 0, 1, \ldots, T - \tau(v, w)\}$$

A construction of two-terminal network $\mathcal{N}^*$, called the *extended network*, is generalized to the multi-terminal network $\mathcal{N}$ by adding a super-terminal node $(*)$ and introducing arcs $(*, s_i)$ to each $s_i \in S$ with infinite capacity and zero transit time, and arcs $(d_i, *)$ to each $d_i \in D$ with infinite capacity and transit time $-(T+1)$ for given time period T.

Let the *reversal* of an arc $e = (v, w)$ be $e^{-1} = (w, v)$. Given a dynamic network $\mathcal{N}$ with symmetric travel times, the *auxiliary dynamic network* $\overline{\mathcal{N}} = (V, E, b_E, b_V, \tau, S, D, T)$ consists of the modified arc capacities and travel times, respectively, as

$$b_E(\bar{e}) = b_A(e) + b_A(e^{-1}), \qquad \text{and} \quad \tau(\bar{e}) = \begin{cases} \tau(e) & \text{if } e \in A \\ \tau(e^{-1}) & \text{otherwise} \end{cases}$$

where, an edge $\bar{e} \in E$ in $\overline{\mathcal{N}}$ if $e \vee e^{-1} \in A$ in $\mathcal{N}$. The remaining graph structure and data are unaltered. For a static network $\mathcal{N} = (V, A, b_A, b_V, S, D)$, the *auxiliary static network* $\overline{\mathcal{N}} = (V, E, b_E, b_V, S, D)$ has a similar representation.

Suppose that x_{stat} has a *standard decomposition* into a set of chains $\mathcal{P} = \{p_1, \dots, p_r\}$ with $r \le m$ that allows simple source-sink paths and simple cycles. Let $x_{stat}(p_k)$ be the flow with value $\text{val}(x_{stat}^{pk})$ along p_k. It holds $x_{stat} = \sum_{k=1}^{r} p_k$, where all chains in $\mathcal{P}$ start and end at the terminal nodes and use the arcs in the same direction as x_{stat} does. The lengths of all chains p_k satisfy $\tau(p_k) \le T$ for given T. A Flow decomposition with zero flows on all cycles, known as a path *decomposition*, is also denoted by $\mathcal{P}$. One may assume that there is no flow along any cycle as the positive flow along all cycles could be canceled.

In contrast to the standard chain-decomposition, a *nonstandard chain-decomposable* flow $\Gamma = \{\gamma_1, \dots, \gamma_{r'}\}$ allows oppositely directed arc flows. For an arc $e = (v, w)$ with travel time $\tau(v, w)$, the reversed arc e^{-1} has a nonpositive travel time $\tau(w, v) = -\tau(v, w)$. A unit of flow on the backward arc e^{-1} that starts from w at time $t + \tau(e)$ and arrives at v at time t cancels a unit of flow sent from v at time t reaching w at time $t + \tau(e)$. This is equivalent to sending one negative unit of flow from v at time t which reaches z at time $t + \tau(e)$. Let γ be a chain that flows along (v, w) in the direction opposite to x_{dyna} and let γ' be another chain flow through (w, v) that cancels the γ flow along (v, w). In order to meet the edge capacity constraints, if γ arrives at v at time t then γ' must arrive at v by time $t' \le t$; and if γ stops using (v, w) at time t then γ' must continue sending flow from w until sometime $t' \ge t$.

2.2. Flow Models

A $s - d$ flow x_{stat} of value $\text{val}(x_{stat})$ in (1) satisfies the flow conservation and capacity constraints (2) and (3), respectively.

$$\text{val}(x_{stat}) = \sum_{e \in A_d^{in}} x_{stat}(e) = \sum_{e \in A_s^{out}} x_{stat}(e) \tag{1}$$

$$\sum_{e \in A_v^{in}} x_{stat}(e) - \sum_{e \in A_v^{out}} x_{stat}(e) = 0, \ \forall v \in V \backslash \{s, d\} \tag{2}$$

$$0 \le x_{stat}(e) \le b_A(e), \ \forall e \in A \tag{3}$$

The flow can be transformed into a *zero circulation* by adding an arc (d, s) with value $\text{val}(x_{stat})$ through it. If x_{stat} is *decomposable* into a set of chains (paths) $\mathcal{P}$, the above *maximum static flow* (MSF) *problem* can be formulated as

$$\max \{\text{val}(x_{stat}) | \sum_{p_k \in \mathcal{P}} \text{val}(x_{stat}^{p_k}) = \text{val}(x_{stat}), \ \sum_{p_k \in \mathcal{P}: e \in p_k} \text{val}(x_{stat}^{p_k}) \le b_A(e) \ \forall e \in A\} \tag{4}$$

In the setting with costs (in place of time), a *minimum cost flow* (MCF) *problem* minimizes $\sum_{e \in A} c_A(e) x_{stat}(e)$, the total cost, to send static flow x_{stat} of fixed value $\text{val}(x_{stat})$.

Let $S_1 \subseteq \dots \subseteq S_q \subseteq S$ and $D_1 \subseteq \dots \subseteq D_r \subseteq D$ be the sets of sources and sinks of a static network, respectively. For a maximal flow, if the greatest units that can enter the sinks in D^* be $\text{maxval}(D^*)$, then a maximal flow that delivers $\text{maxval}(D_k)$ units into each D_k is a *lexicographically maximal flow* on the sinks. A maximal flow that sends greatest units $\text{maxval}(S_j)$ out of each S_j is a *lexicographically maximal flow* on the sources.

A dynamic $s - d$ flow x_{dyna} for given time T satisfies the *flow conservation* and *capacity constraints* (5-7). The inequality flow conservation constraints allow to wait flow at intermediate nodes, however, the equality flow conservation constraints force that flow entering an intermediate node must leave it again immediately.

$$\sum_{\sigma=\tau(e)}^{T} \sum_{e \in A_v^{in}} x_{dyna}(e, \sigma - \tau(e)) - \sum_{\sigma=0}^{T} \sum_{e \in A_v^{out}} x_{dyna}(e, \sigma) = 0, \ \forall v \notin \{s, d\} \tag{5}$$

$$\sum_{\sigma=\tau(e)}^{t} \sum_{e \in A_v^{in}} x_{dyna}(e, \sigma - \tau(e)) - \sum_{\sigma=0}^{t} \sum_{e \in A_v^{out}} x_{dyna}(e, \sigma) \ge 0, \ \forall v \notin \{s, d\}, t \in \boldsymbol{T} \tag{6}$$

$$0 \le x_{dyna}(e, t) \le b_A(e, t), \ \forall e \in A, t \in \boldsymbol{T} \tag{7}$$

The *earliest arrival flow* (EAF) *problem* maximizes the $\text{val}(x_{dyna}, t)$ in (8) for all $t \in \boldsymbol{T}$ satisfying the constraints (5-7). We denote the maximum flow value by $\text{val}_{\max}(x_{dyna}, t)$.

$$\text{val}(x_{dyna}, t) = \sum_{\sigma=0}^{t} \sum_{e \in A_s^{out}} x_{dyna}(e, \sigma) = \sum_{\sigma=\tau(e)}^{t} \sum_{e \in A_d^{in}} x_{dyna}(e, \sigma - \tau(e)) \tag{8}$$

For a given time T the *maximum dynamic flow* (MDF) *problem* maximizes $\text{val}(x_{dyna}, T)$. Remark that a MDF solution maximizes the flow in time T and does not care at earlier time periods. For a given B the *quickest flow* (QF) *problem* looks for a minimal time $\min T = T(B)$ such that the flow value is at least B satisfying the constraints (5-7). For given time T and an ordered set of multi-terminals, the *lexicographically maximum dynamic flow* (LMDF) *problem* finds a feasible flow that lexicographically maximizes the amount leaving each terminal in the given priority.

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2.3. Solutions and Complexities

A primal dual algorithm finds a MCF solution in the underlying static network $\mathcal{N} = (V, A, b_A, \tau, s, d)$ with respect to the time T for a given dynamic network $\mathcal{N} = (V, A, b_A, \tau, s, d, T)$, where arcs transit times are interpreted as cost coefficients. A MDF has been obtained in strongly polynomial time by the *temporally repeated flows* (TRF) of decomposed standard chains over permissible time horizon. A proof on optimality follows from the max-flow min-cut theorem on $\mathcal{N}(T)$ (Ford and Fulkerson, 1958). With a static flow x_{stat} and a path decomposition $\mathcal{P}$, they calculate the MDF value associated to a TRF for given time horizon T as in (9). The sum on the right depends on only the static flow and not on the particular $\mathcal{P}$.

$$\text{val}(x_{dyna}, T) = \sum_{p_k \in \mathcal{P}} (T - \tau(p_k) + 1)\, \text{val}(x_{stat}^{p_k}) = (T+1)\, \text{val}(x_{stat}) - \sum_{e \in A} \tau(e)\, x_{stat}(e) \qquad (9)$$

This MDF can also be obtained by finding a *minimum cost circulation* (MCCF_T) in the static network with an additional edge $(d, s)$of infinite capacity and $-(T+1)$ cost. It is known that a MCCF_T solution has minimum cost if and only if the corresponding residual network does not contain a cycle with negative cost.

Burkard et al. (1993) links the QF to the MDF and to linear fractional programming and present a strongly polynomial time and several polynomial time algorithms for a solution of the QF problem. A natural procedure via the solution of the parametric MCCF_T problem starting with $T = 0$ and continuing until the requirement takes pseudo-polynomial time because of exponential number of slope changes of the optimal value function and the maximum static flow value. By exploiting the properties that $\text{val}_{\max}(x_{dyna}, T)$ is monotone increasing and T attains only integer values, a binary search has been proposed which needs very careful analysis of the search intervals. Their strongly polynomial time algorithm relies on the solution techniques of linear fractional programming presented in (Megiddo, 1979).

Minieka (1973) defines the arrival and departure patterns in maximal flows constructed by (Ford and Fulkerson, 1958), and prove their independence. Given a maximal static flow x_{stat_1} with $\alpha(s)$ units leaving each $s \in S$ and a maximal static flow x_{stat_2} with $\beta(d)$ units arriving each $d \in D$, there exists a maximal flow x_{stat} having these patterns. A proof continues the flow change procedure on the circulations converted from the given maximal flows $x_{stat_1}$and x_{stat_2} until the obtained maximal flow x_{stat} does not obey the required properties. Given a finite graph with integer arc capacity and the priority ordering of the sets of sources and sinks, he establishes an existence a LMSF on the sources and on the sinks. The existence proofs of all LMSF require only the maximum flow minimum cut theorem on Ford-Fulkerson maximum flow algorithm.

For given $t = 0, 1, \ldots, T$, let $D_t = \bigcup_{k=0}^{t} d(k)$ and $S_t = \bigcup_{k=0}^{t} s(k)$ be the priority subsets of sinks and sources in $\mathcal{N}(T)$. Then a LMSF in $\mathcal{N}(T)$ on the sinks (sources) is a MDF in $\mathcal{N}$ with earliest arrival (departure) property.

These results rely on the standard flow decomposition technique (Ford and Fulkerson, 1958). A dynamic flow in $\mathcal{N}$ within T is equivalent to a static flow in $\mathcal{N}(T)$ and vice versa (Ford and Fulkerson, 1958). However, the time dependency of $\mathcal{N}(T)$ does not allow natural extensions of the polynomial time solution procedures. A more advanced nonstandard chain decomposition technique allows a polynomial time solution to certain classes of dynamic flows, (Hoppe and Tardos, 2000), for example, more general LMDF problem. Recall that their decomposition permits arc flows in either directions at different time steps on the availability of such both directions. However, their algorithm is not practical for MDF or QF as it requires a submodular function minimization oracle for a subroutine.

Consider a $\mathcal{N} = (V, A, b_A, \tau, S, D, T, \mu(s), \vartheta(d))$ with specified supply $\mu(s)$ and demand $\vartheta(d)$ at each source and sink, respectively. Given an ordering $u_1, \ldots, u_\delta$ of sources and sinks, the LMDF problem concerns to find a feasible dynamic flow with given T that lexicographically maximizes the amounts leaving the terminals in the given order. This problem is solvable in time complexity $O(\delta \times \text{MCF}(m, n))$, where δ represents the number of terminals and $\text{MCF}(m, n)$ represents the time complexity of the MCF, (Hoppe and Tardos, 2000).

An $s - d$ EAF solution that is optimal for each time $t = 1, \ldots, T$ for given T has been obtained in pseudo-polynomial time, (Wilkinson, 1971; Minieka, 1973). Minieka's proof follows the induction steps over the time periods. His shortest augmenting path algorithm utilizes its results for t_0 time units to extend its results for $t_1 > t_0$ time units by performing only $t_1 - t_0$ iterations. Wilkenson's proof makes an use of the maximum-flow minimum-cut in $\mathcal{N}(T)$. He also recommends the secondary storage of the dynamic flows as the algorithm never executes any operations on dynamic flows. With this, storage of only augmenting paths and their node numbers would be sufficient to recognize the dynamic flows finally. Hoppe (1995) solves the universally maximum flow (also known as the earliest arrival flow) making use of chain decomposable flows. His algorithm is essentially the same as of (Wilkinson, 1971; Minieka, 1973) with same time complexity.

Kamiyama and Katoh (2014) present a polynomial time algorithm for the universally quickest transshipment problem in a single-sink dynamic network that simultaneously maximizes the amount of supplies which have reached the sink at every time step satisfying the uniform path-lengths and fully connected conditions. They first introduce the LMSF problem with hierarchies that is a generalization of classical LMSF problems and present a polynomial time algorithm for this problem. Then, they transform a compressed time expanded network so that a LSMF with hierarchies in this network yields a universally quickest transshipment. Their algorithm improves the complexity in this particular network as the time expanded network of a dynamic network with uniform path-lengths can be compressed so that its size is bounded by a polynomial in the input size.

3. STATIC CONTRAFLOW PROBLEMS

The maximum static contraflow (MSCF) problem has been solved earlier. A proof establishes the fact that a MSCF is equivalent to a MSF problem on an undirected auxiliary graph. In this section, the *lexicographically maximum static contraflow* (LMSCF) *problem* has been introduced and solved efficiently.

Problem 1. *Given $\mathcal{N} = (V, A, b_A, S, D)$, the MSCF problem is to determine a maximum flow from S to D, if the direction of arcs can be reversed.*

Authors in (Rebennack *et al.*, 2010) solve the $s - d$ MSF problem in the auxiliary network $\overline{\mathcal{N}} = (V, E, b_E, s, d)$, decompose the obtained flow into paths and cycles and delete the latter assuring that arcs on either direction will never be used in the optimal flow. An arc $(w, v) \in A$ is reversed if and only if the flow on (v, w) is greater than $b_A(v, w)$, or if there is a nonnegative flow along $(v, w) \notin A$ and the resulting flow is maximum with arc reversals. The flow obtained by their algorithm is feasible and optimal to the $s - d$ MSCF problem, too. It requires $O(h_1(n, m) + h_2(n, m))$ time, where $h_1(n, m) = O(n^2.\sqrt{m})$ and $h_2(n, m) = O(n.m)$ denote the time required to solve the MSF problem and the flow decomposition, respectively. For the general MSF problem, a prior reduction of $\mathcal{N}$ into the $s_0 - d_0$ MSF problem with super-source s_0 connecting to each $s \in S$ having arc capacities equal their respective surplus and connecting each $d \in D$ to super-sink d_0 having arc capacities equal their respective deficits is required.

Theorem 1. (*Rebennack et al., 2010*): *The MSCF Problem 1 is solvable with strongly polynomial time complexity.*

Problem 2. *Given $\mathcal{N} = (V, A, b_A, S, D)$ with ordered sets of sinks and sources, the LMSCF problem at sinks (sources) is to determine a feasible flow that lexicographically maximizes the amounts entering (leaving) the terminals in the given orders, if the direction of arcs can be reversed.*

Algorithm 1 has been presented for an optimal solution to the LMSCF problem.

Algorithm 1: Lexicographically Maximum Static Contraflow (LMSCF)

1. Given, network $\mathcal{N} = (V, A, b_A, S, D)$ with integer inputs.
2. Solve the corresponding LMSF problem on $\overline{\mathcal{N}} = (V, E, b_E, S, D)$ by (Minieka, 1973).
3. Arc $(w, v) \in A$ is reversed, if and only if the flow along arc (v, w) is greater than $b_A(v, w)$ or if there is a nonnegative flow along arc $(v, w) \notin A$ and the resulting flow is in LMSF with the arc reversals for the graph $\mathcal{N}$.
4. Obtain lexicographically maximum static contraflow solution.

Example 1: *Consider a LMSCF problem with 2-sources and 3-sinks in a capacitated static network given in Figure 1(i). Figure 1(iii) represents a LMSCF solution x_{stat_1} with priority ordering of sinks d_3, d_1, d_2. Because of priority on d_1, MSCF solution in Figure 1(ii) has to be re-optimized in this solution. A LMSCF solution x_{stat_2} with priority ordering on the sources s_2 and s_1 is obtained in Figure 1(iv), where MSCF solution in Figure 1(ii) has to be re-optimized as the first priority is on s_2. The final LMSCF solution x_{stat} which satisfies both priority orderings results from the independence of the arrival and departure patterns in a maximal flow as shown in Figure 1(v).*

Theorem 2. Problem 2 can be solved by solving LMSF problem and the flow decomposition in its auxiliary network.
Proof:
We show that a solution given by Algorithm 1 is optimal to Problem 2. We transform N for Problem 2 into the auxiliary network $\overline{N}$. Recall that the arc capacity is increased by adding the capacities of both directions between the nodes, either direction of arc is allowed with modified network, but the priority orderings remain unaltered. An optimal solution to the LMSF problem on $\overline{N}$ could be obtained by solving iteratively the MSF problem in this network with given order of terminals (Minieka, 1973). However, at each iteration a max-flow solution for a given set of terminals is equivalent to the MSCF in N by Theorem 1. Therefore, an optimal solution for the LMSCF problem can be solved by solving the LMSF problem in $\overline{N}$. In addition, a flow decomposition cost should pay for the removal of undesired cycles. This ensures that the direction of arcs take only one but not in both as in the MSCF. ■

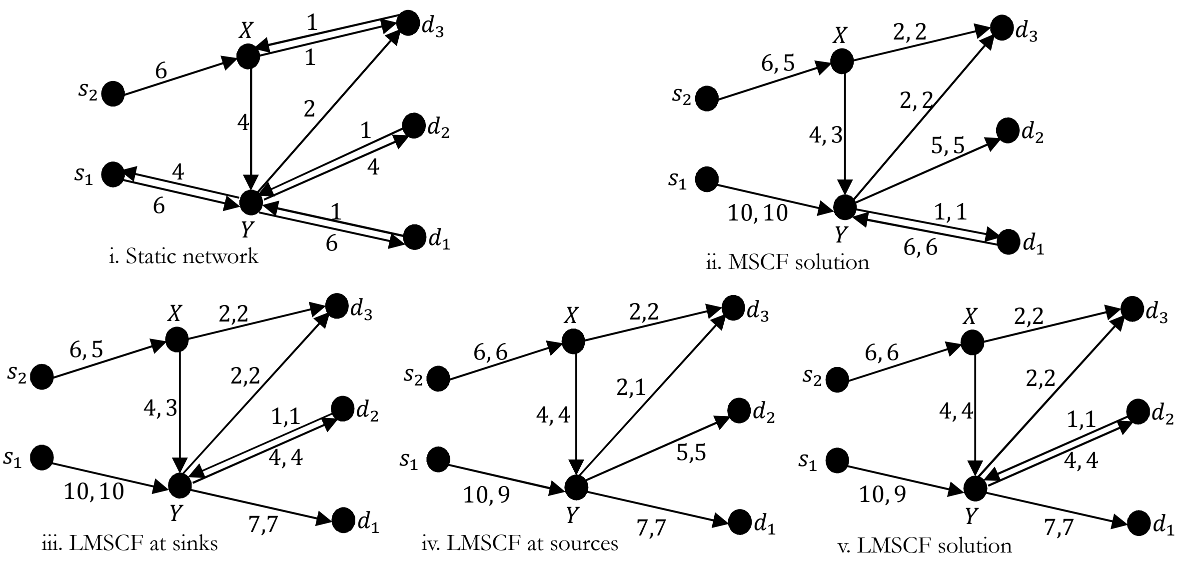

Figure 1: Lexicographically maximum static contraflow solution

Corollary 1. *Problem 2 can be solved with polynomial time complexity for given priority ordering of multi-terminals.*

4. DYNAMIC CONTRAFLOW PROBLEMS

We introduce the lexicographically maximum dynamic contraflow (LMDCF) and the earliest arrival contraflow (EACF) problems, and propose respective algorithms. The maximum dynamic contraflow (MDCF), the quickest contraflow (QCF), the EACF on two-terminal series-parallel graphs and the EACF on lossy network are studied in (Dhamala and Pyakurel, 2013; Pyakurel and Dhamala, 2014a; Pyakurel *et al.*, 2014; Rebennack *et al.*, 2010), respectively.

4.1 Maximum Dynamic and Quickest Contraflows

Problem 3. *Given* $\mathcal{N} = (V, A, b_A, \tau, S, D, T)$, *the MDCF problem is to find a maximum flow that can be sent from* S *to* D *in time* T, *if the direction of arcs can be reversed at time zero.*

Dissimilar to the network flows without contraflow, a one-to-one correspondence between MDCF solutions in a contraflow network $\mathcal{N}$ with arc reversals only at starting time is unlikely to the MSCF solutions in the corresponding network $\mathcal{N}(T)$ for multi-terminal networks, see Lemma 1. However, the equality is valid in case of two-terminal networks.

Lemma 1. *An optimal MDCF solution* $\mathcal{N}$ *is not greater than an optimal MSCF solution in the corresponding* $\mathcal{N}(T)$.

In order to find a $s - d$ MSF solution in the auxiliary network $\overline{\mathcal{N}}$, a MCF solution is obtained and the flow is decomposed into paths and removable cycles. An algorithm for $s - d$ MDCF solution determines temporally repeated dynamic flow in $\overline{\mathcal{N}}$ which is an optimal overall dynamic flows in it. An arc $(w, v) \in A$ is reversed if and only if the flow on (v, w) is greater than $b_A(v, w)$, or if there is a nonnegative flow along $(v, w) \notin A$. An optimal solution in $\overline{\mathcal{N}}$ is at most to the optimal solution in $\mathcal{N}$ and Lemma 1 holds. Then exploiting the optimality property of time-expanded network according to Ford-Fulkerson (1958) and Theorem 1, the optimal solutions in $\mathcal{N}$ and $\overline{\mathcal{N}}$ agree.

Theorem 3. (Rebennack *et al.*, 2010): *The* $s - d$ *MDCF problem can be solved in time* $O(h_2(n, m) + h_3(n, m))$, *where* $h_2(n, m) = O(n.m)$ *and* $h_3(n, m) = O(n^2.m^3.\log n)$ *are the time required for the flow decomposition and the MSF problem, respectively.*

The multi-terminal MDCF problem remains NP-hard in the strong sense even with two sources and one sink or vice versa, sees Example 2. The proofs follow by reductions from the problems 3-SAT and PARTITION, (Kim *et al.*, 2008; Rebennack *et al.*, 2010).

Example 2. (Rebennack *et al.*, 2010): *There is no feasible flow within time* $T = 6$ *using only one of the arcs* (X, Y) *or* (Y, X) *in Figure 2. At time 1, one would switch* (Y, X) *in order to increase the capacity and at time 3, one would switch it back again to achieve a feasible flow within a shorter time. Hence, the possibility of using both arcs leads to the problem NP-Complete.*

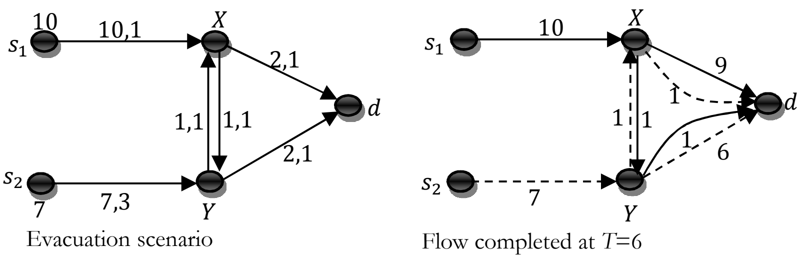

Figure 2: Two sources and single sink MDCF scenario and solution

Problem 4. *Given a dynamic network* $\mathcal{N} = (V, A, b_A, \tau, S, D)$, *the QCF problem is to find the minimum time required to send a given flow value* B *from* S *to* D *having arc reversal capability at time zero.*

The parametric search algorithms yield a solution to the $s - d$ QCF problem (Megiddo, 1979; Burkard *et al.*, 1993). A weakly polynomial time algorithm can be realized by obtaining an upper bound on the quickest time and performing a repeated binary search to the MDCF. The multi-terminal QCF problems are harder than 3-SAT and PARTITION. The MSCF solution method with reduction of the QCF problem back to a supersource-supersink $s_0 - d_0$ QCF problem is not applicable in this case.

Theorem 4. (Rebennack *et al.*, 2010): *The* $s - d$ *QCF problem is solvable in strongly polynomial time, whereas the multi-terminal QCF problem is NP-complete in strong sense.*

Let the given supply-demand vector be given and the arc reversals are allowed back and forth at integer time points. Then the multi-terminal QCF problem is polynomially solvable as it is equivalent to the QF problem. The solution procedure is similar to the MDCF solution algorithm.

4.2 Earliest Arrival Contraflow

We consider the EACF problems with only a single sink as there exists no earliest arrival flow on multiple sinks (Gale, 1959).

Problem 5. *Given* $\mathcal{N} = (V, A, b_A, \tau, S, d, T)$, *the EACF problem is to find a feasible dynamic flow from* S *to* d *that is maximal for all time periods* $0 \leq t \leq T$, *if the directions of arcs can be reversed.*

The more restricted multi-source EACF problem with arc reversals allowed only at zero time must be NP-hard as the MDCF problem is NP-hard. However with arc reversals only once at time zero, an optimal solution to the $s - d$ MDCF (and $s - d$ EACF) problem on a two-terminal series-parallel graph has been obtained by a modification of algorithm in (Rebennack *et al.*, 2010) using the MCCF algorithm of (Ruzika *et al.*, 2011). The main advantage in series-parallel graphs is that every cycle in the residual network has nonnegative cycle length. This solves the MCCF problem introduced in (Ford and Fulkerson, 1958) for the MDF problem in the auxiliary network $\overline{\mathcal{N}}$. The temporally repeated flow thus obtained is an optimal solution to the $s - d$ EACF problem on a two-terminal series-parallel graph.

Theorem 5. (Dhamala and Pyakurel, 2013): *An optimal solution to the* $s - d$ *EACF problem for two-terminal series-parallel graphs with arc reversal capability at time zero can be obtained in time* $O(nm + m \log m)$.

Looking in depth to a structure of the EAF problem which continues the already obtained flows in earlier steps to forthcoming flows in forward steps, the final solution may change the direction of arcs and obeys the backward flow laws in its processing. Therefore, an introduction of Problem 5 for general graphs is justifiable and solution procedure relaxes the arc reversal capability at a number of times when an EAF solution demands this property. With time to time arc reversals, the optimal solutions in Lemma 1 should agree. For the considered general $s - d$ networks, we present Algorithm 2 that solves Problem 5.

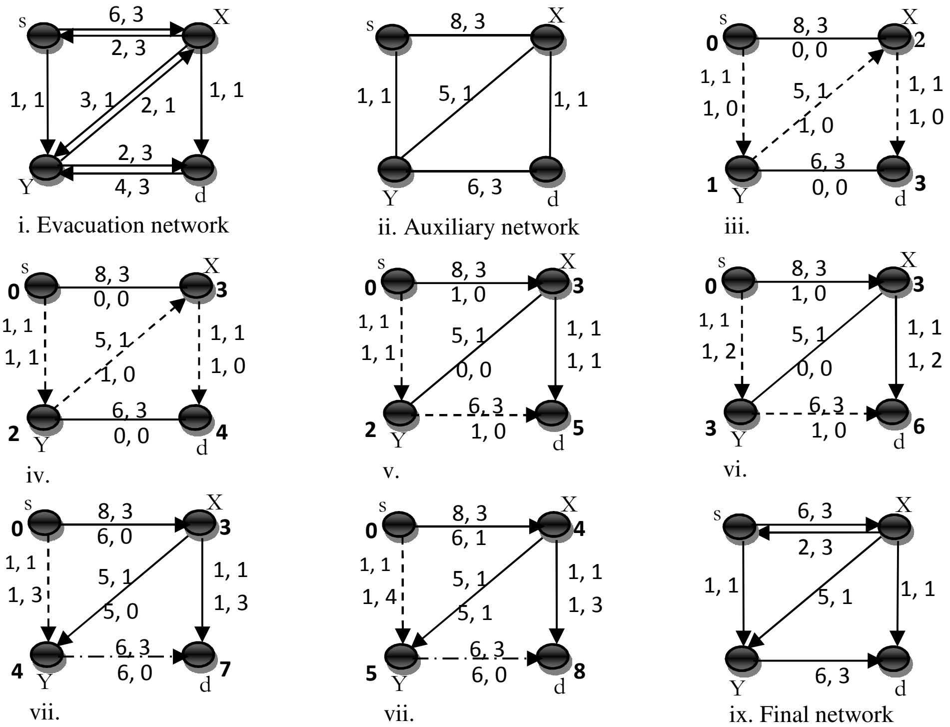

Figure 3: Earliest arrival contraflow solution obtained by Algorithm 2

Algorithm 2: s-d Earliest Arrival Contraflow (EACF)

1. Given, evacuation network $\mathcal{N} = (V, A, b_A, \tau, s, d, T)$ with integer inputs.
2. Transform $\mathcal{N} = (V, A)$ in to the auxiliary graph $\overline{\mathcal{N}} = (V, E, b_E, \tau, s, d, T)$.
3. Solve the EAF Problem in the auxiliary graph $\overline{\mathcal{N}}$ by (Wilkinson, 1971; Minieka, 1973).
4. An arc $(w, v) \in A$ is reversed if and only if the flow along arc (v, w) is greater than $b_A(v, w)$, or if there is a nonnegative flow along arc $(v, w) \notin A$ and the resulting flow is EAF with the arc reversals for the graph $\mathcal{N}$.
5. Obtain an earliest arrival contraflow solution.

Theorem 6. *The EACF Algorithm 2 generates an optimal solution to the EACF Problem 5.*
Proof:

Consider the successive shortest path flows $s - d$ in $\overline{\mathcal{N}}$ by Ford-Fulkerson algorithm (Ford and Fulkerson, 1958) for computing maximal flows for each time period $t = 0, 1, \ldots, T$. In turn they obtain a static flow x_{stat} in the corresponding time expanded network for each time period t that generates a dynamic flow x_{dyna} in $\overline{\mathcal{N}}$ (Wilkinson, 1971; Minieka, 1973). However, by the proof of Theorem 3, the MDCF solution in $\mathcal{N}$ for every time period t equals to the MDF solution in the corresponding auxiliary network. Putting together, the statement follows.■

Corollary 2. *The EACF Problem 5 is solvable in pseudo-polynomial time.*
Proof:

The time complexity of Algorithm 2 is dominated by Step 3. As the EAF solution given by (Wilkinson, 1971) requires pseudo-polynomial time in time expanded network the statement follows. ■

Example 3. *For an evacuation network* $\mathcal{N}$ *in Figure 3* (i), *we apply Algorithm of* (Wilkinson, 1971) *to the auxiliary network* $\overline{\mathcal{N}}$ *in Figure 3*(ii). *We obtain total 13 units of flow at time 7 for* $s - d$ *EACF, see Figures 3*$(iii - vii)$. *If we do not use contraflow in* $\mathcal{N}$, *we obtain only 9 units of flow sent to the sink at this time. As shown in the final Figure 3*(ix), *some of the road lanes are also saved which can be used for other purposes.*

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4.3 Lexicographically Maximum Dynamic Contraflow

A polynomial time algorithm has been presented for the LMDCF problem we have introduced. An alternate expensive solution procedure to this problem would be through the reduction of it into $s_i(0) - s_j(T)$ LMDCF problem corresponding to an orderings of sources s_i and sinks s_j and applying algorithms in the time-expanded network, see also Theorem 2.

Problem 6: *Given $\mathcal{N} = (V, A, b_A, \tau, S, D, \mu(s), \vartheta(d), T)$ and ordered multi-terminals, the LMDCF problem is to find a feasible flow that lexicographically maximizes the amount at each priority terminals, if the arc directions can be reversed.*

We illustrate an example which motivates a contraflow configuration for the LMDF problem.
Example 4. *Given a contraflow network in Figure $4(i)$ with priority order $s_1 s_2 d_3 d_2 d_1$ of sources and sinks with given supply-demand vectors (17, 14) and (14, 8, 9) on them, respectively, we construct the auxiliary graph in Figure $4(ii)$. Figure $4(iii)$ gives an optimal solution to the LMDCF problem for time $T = 6$. The solution saves unused lane from sink d_1 to Y. While computing the solution, the lane reversals have to be changed time to time and flow cancellation properties hold. Moreover, the priority orders force in choosing some longer paths even shorter paths are available for less priority terminals. This amount of flow could not be transshipped within this given time without contraflow. For instance, with altered capacity on two-ways lanes between s_1 and Y, Y and d_1, and Y and d_2, flow without contraflow decreases significantly.*

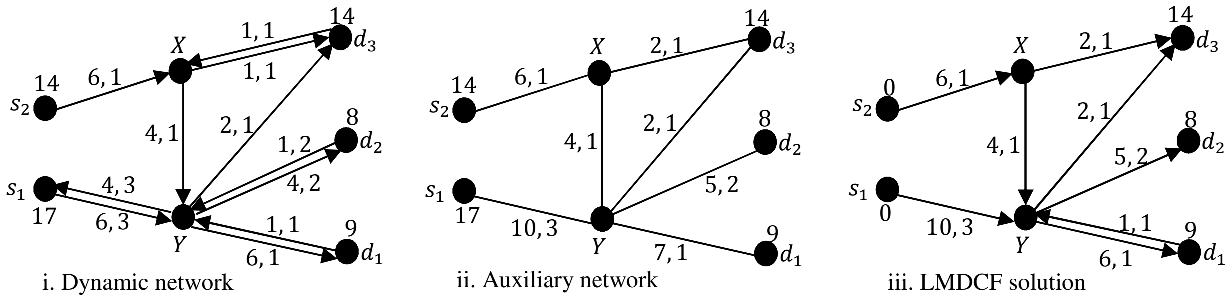

Figure 4: Lexicographically maximum dynamic contraflow solution

An arc reversal capability is assumed to be at each integer time points for the Algorithm 3 which solves the LMDCF problem using chain-decomposable flow of (Hoppe and Tardos, 2000).

Algorithm 3: Lexicographically Maximum Dynamic Contraflow (LMDCF)
1. Given, network $\mathcal{N} = (V, A, b_A, \tau, S, D, \mu(s), \vartheta(d), T)$ with integer inputs.
2. Solve the corresponding LMDF problem on $\overline{\mathcal{N}} = (V, E, b_E, \tau, S, D, \mu(s), \vartheta(d), T)$ by (Hoppe and Tardos, 2000).
3. An arc $(w, v) \in A$ is reversed if and only if the flow along arc (v, w) is greater than $b_A(v, w)$, or if there is a nonnegative flow along arc $(v, w) \notin A$ and the resulting flow is LMDF with the arc reversals for the graph $\mathcal{N}$.
4. Obtain lexicographically maximum dynamic contraflow solution.

Lemma 2. *The LMDCF Algorithm 3 solves the LMDCF Problem 6 correctly.*
Proof:
Recall (Hoppe and Tardos, 2000) that a LMDF solution makes repeated use of the minimum-cost circulation in the residual network. However, the MSCF solution a network is equivalent to the MSF solution in the corresponding auxiliary network, by Theorem 1. Moreover, similar result is valid for the $s - d$ MDCF problem, by Theorem 3. Therefore, the algorithm yields LMDF solution in the auxiliary network which is valid for the LMDCF solution in the original network. ■

Theorem 7. *The LMDCF Algorithm 3 solves the LMDCF Problem 6 in polynomial time complexity.*
Proof:
The construction of auxiliary network and Step 3 are solved in linear time. Then, the complexity of Algorithm 3 depends on Step 2. The LMDF problem on $\overline{\mathcal{N}} = (V, E, b_E, \tau, S, D, \mu(s), \vartheta(d), T)$ can be solved in $O(\delta \times \text{MCF}(m, n))$ time, where δ is the number of iterations and $\text{MCF}(m, n)$ represents the time complexity of the MCF problem in the residual network, (Hoppe and Tardos, 2000). Hence, the total complexity of Algorithm 3 is $O(\delta \times \text{MCF}(m, n))$. ■

5. CONCLUSIONS

In this paper, we introduced the multi-terminal problems LMSCF and LMDCF as a generalization of the problem MSCF, and as a combination of the problems MDCF and LMDF, respectively. We also presented polynomial time algorithms for these problems. Similarly, we generalized the EACF problem and presented an algorithm for this in two-terminal graph.

The flow values obtained by these algorithms increase significantly, although our extended algorithms on contraflow have similar complexity of algorithms without contraflow. Results from existing literature show that the flow value may be doubled for a given time and time required to transship the given value can be two times faster in contraflow than without contraflow. Literature illustrate that a number of emergency and rush hour implementations take benefit from contraflow configuration.

Analytical techniques of contraflow have been considered recently in the literature of evacuation planning by realizing that it improves the solution approaches quite a lot even unturning a lot of arcs on the contraflow reconfiguration. These unturned arcs could be used for other emergency purposes, like the logistics supports. We have illustrated better optimal flows saving some arcs unaltered with at most the same time as conventional algorithms require. A number of applications, insights on solution approaches and their impacts on emergency issues demand a systematic analysis of them.

To the best of our knowledge, these problems we introduced are for the first time in evacuation planning. Further, we are interested on the investigation of the earliest arrival contraflow problems for multi-terminal network with given set of demand-supply at respective nodes. Moreover, we are also interested in implementing integrated contraflow techniques for Kathmandu metropolitan city.

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