# CORRIGENDUM TO: APPROXIMATING MINIMUM-AREA RECTANGULAR AND CONVEX CONTAINERS FOR PACKING CONVEX POLYGONS

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Abstract. This note corrects an error in the paper "Approximating minimum-area rectangular and convex containers for packing convex polygons", which appeared in JoCG, Vol. 8(1), pages 1–10.

In an earlier paper [1] we studied the problem of packing a set $P$ of convex polygons into a minimum-area rectangular container. More precisely, we studied the problem of finding a placement of the polygons in $P$, without rotating them, so that the polygons have pairwise disjoint interiors and the axis-aligned bounding box of the polygons has minimum area. We presented an $O(1)$-approximation algorithm for this problem, but unfortunately the proof contains an error. The error is easily fixed, as explained below, but the approximation ratio (as stated in Theorem 2.1 in the paper) increases from 17.45 to 23.78. The error also has an impact on the version of the problem where we want to pack the polygons into a minimum-area convex container; here the approximation ratio (as stated in Theorem 3.1) increases from 27 to 41.56. Below we explain the error, the fix, and the how it impacts the approximation ratios in our results. We assume familiarity with the notation and terminology in our original paper.

**The error and the fix.** In the proof of Lemma 2.1, page 5, line -2, we claimed:

> The middle piece, $Q_j^2$, is bounded by $s(p_j)$ and $s(p_{j+1})$, so $\text{area}(Q_j^2) \leqslant \text{area}(\Delta_j)$.

This is not always correct, however: when the spine $s(p_{j+1})$ ends on the right side of the box $B(S)$ instead of the top side, then $\Delta_j$ is "clipped" by the right side of $B(S)$; see for example regions $\Delta_3$ and $\Delta_4$ in Fig. 1 below. Hence, $Q_j^2$ may not fit completely into $\Delta_j$.

The error can be fixed by modifying the definition of the box $B(S)$, as follows. Let $S$ be the set of spines of the so-called relevant polygons, with their lower endpoints placed at the origin. Originally $B(S)$ was defined as the box of height $h_i$ and minimum width containing the spines in $S$; see page 4 of the original paper, just above Inequality (2). Instead, the width of $B(S)$ should have been scaled by a factor $1/\alpha$, as shown in Fig. 1 below. Since all spines in $S$ have vertical span more than $\alpha h_i$, this new definition guarantees that the extensions of the spines all meet the top side (and not the right side) of the box $B(S)$. Hence, for the

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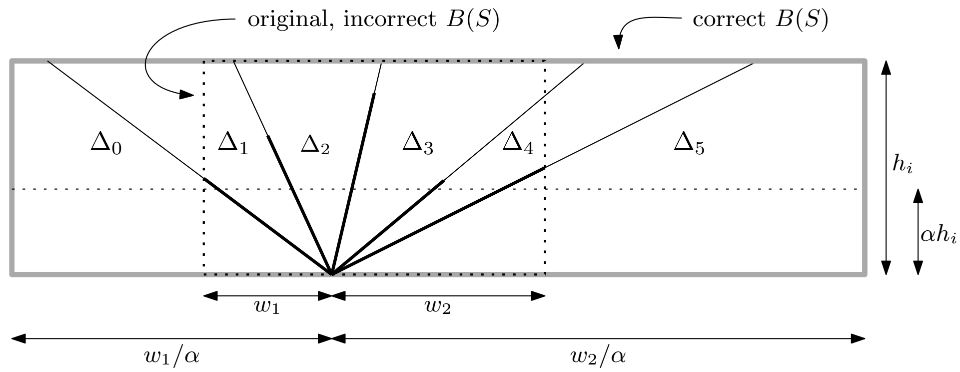


Figure 1: Correct definition of the box $B(S)$. All spines intersect the dotted line at height $\alpha h_i$, so their extensions all meet the top side of $B(S)$.

regions $\Delta_j$ in the new $B(S)$ it is correct that $\text{area}(Q_j^2) \leqslant \text{area}(\Delta_j)$, for $j = 1, ..., t-1$, and, as easily can be verified, it is still correct for $j = 0$ and $j = t$. This modification corrects the logic of the proof of the original paper. The computation of the approximation ratio changes slightly, as detailed below.

**Impact on the approximation ratio in Theorem 2.1.** Since the area for the new box $B(S)$ is larger by a factor of $1/\alpha$ the approximation constants will change. In particular, Inequality (2) in the original paper needs to be changed into

$$\text{area}(B(S)) \leqslant 2/\alpha \cdot h_i \cdot \max_{p \in P_i^*} \text{width}(p).$$

Therefore, Inequality (1) turns into

$$\text{area}(B_i) \leqslant 2/\alpha \cdot \left( \sum_{p \in P_i^*} \text{area}(p) + h_i \cdot \max_{p \in P_i^*} \text{width}(p) \right),$$

and the inequality in Lemma 2.1 is replaced by

$$\text{area}(B_i) \leqslant 2/\alpha \cdot \left( \sum_{p \in P_i} \text{area}(p) + h_i \cdot \max_{p \in P_i} \text{width}(p) \right).$$

Consequently, in Derivation (7) we obtain

$$\text{area}(B) \leqslant \left( \left(1 + \frac{1}{c}\right) \left( \frac{2}{\alpha} + \frac{2}{\alpha(1-\alpha)} \right) + \frac{c+1}{1-\alpha} \right) \cdot \text{OPT}$$

and the term before OPT simplifies to

$$f(c, \alpha) := \left(1 + \frac{1}{c}\right) \cdot \left( \frac{4 + (c-2)\alpha}{\alpha(1-\alpha)} \right),$$

which replaces the definition in (8). The function $f$ attains its minimum at approximately $\alpha = 0.48$ and $c = 2.49$; the minimum is slightly below 23.78.

**Impact on the approximation ratio in Theorem 3.1.** For the approximation of the smallest convex container in Section 3 of the original paper, the computations change as well. In derivation (11) we have to insert the missing factor $1/\alpha$ and obtain

$$\text{area}(B) \leqslant \frac{c+1}{c}\left(\frac{2}{\alpha} + \frac{4}{\alpha(1-\alpha)} + \frac{2c}{1-\alpha}\right) \cdot \text{OPT}.$$

The term before OPT is minimized for approximately $\alpha = 0.45$ and $c = 2.38$ and the minimum is slightly below 41.56.

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## References

[1] H. Alt, M. de Berg and C. Knauer. Approximating minimum-area rectangular and convex containers for packing convex polygons. *Journal of Computational Geometry*, 8(1):1–10, 2017.