# Finance, Markets and Valuation

Asymmetric information modelling in the realized spread: A new simple estimation of the informed realized spread

Modelización de la información asimétrica en el diferencial realizado: Una nueva estimación sencilla del diferencial realizado informado

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#### **Abstract**

The market liquidity plays an authoritative role in the execution of financial transaction. Since the liquidity has immediate impact on the trading, the liquidity risk has been gaining a huge attention in the asset pricing, corporate financing, and risk portfolio management. The bid-ask spread is often reported a significant indicator of the market liquidity and its associated cost in the financial market. This work proposes a new estimation of the bid-ask spread, namely the Informed Realized Spread (IRS). The IRS method is a modified version of the Realized Spread (RS), which exclusively illustrates the asymmetric information effects on the spread size. Despite differences behind the construction of spread proxies, the IRS model is found to be positive and strongly correlated with the RS model. The IRS method is straightforward, computationally less-intensive, and suitable for variety of research in the asset pricing studies.

Keywords: Market Microstructure; Asset Pricing; Informed Realized Spread; Liquidity

Resumen

La liquidez del mercado desempeña un papel fundamental en la ejecución de las transacciones financieras. Dado que la liquidez tiene un impacto inmediato en la negociación, el riesgo de liquidez ha ido ganando una enorme atención en la fijación de precios de los activos, la financiación de las empresas y la gestión de la cartera de riesgos. El diferencial entre la oferta y la demanda suele ser un indicador significativo de la liquidez del mercado y su coste asociado en el mercado financiero. Este trabajo propone una nueva estimación del diferencial entre el precio de compra y el de venta, el Informed Realized Spread (IRS). El método IRS es una versión modificada del Realized Spread (RS), que ilustra exclusivamente los efectos de la información asimétrica sobre el tamaño del spread. A pesar de las diferencias en la construcción de los indicadores del diferencial, el modelo IRS es positivo y está fuertemente correlacionado con el

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modelo RS. El método IRS es sencillo, menos intensivo desde el punto de vista computacional y adecuado para una variedad de investigaciones en los estudios de precios de los activos.

*Palabras clave:* Microestructura del mercado; Fijación de precios de los activos; Diferencial informado realizado; Liquidez.

## 1. Introduction

This study proposes a modified version of the Realized Spread (RS). The bid-ask spread elucidates the easiness and cost for trading an asset. Since the RS model emphasizes on the future value of bid and ask prices, it is often referred to a meaningful measure of the liquidity and trading cost. The waiting period or time delay in selecting the future prices is based on the markets that provide the data of buy and sell transactions. The earlier literature examines the RS model under various waiting periods. The waiting periods for the RS model are reported to 5 minutes (Berkman, Brailsford, & Frino, 2005), 30 minutes (Bacidore & Sofianos, 2003), 24 hours (Bessembinder & Kaufman, 1997), and daily prices (Saleemi, 2020a). The analytical expression of the RS estimator is given as below:

Realized Spread<sub>t</sub> = 
$$\frac{2|\eta_{t+1} - C_t|}{\binom{high_t + low_t}{2}}$$
 (1)

Where,  $high_t$  is the highest price of day t;  $low_t$  refers to the lowest price of day t;  $C_t$  indicates the closing price of day t; and  $\eta_{t+1}$  is a mean value of the following trading-day high and low prices.

$$\eta_{t+1} = (high_{t+1} + low_{t+1}) * \left(\frac{1}{2}\right)$$
(2)

The market makers are likely compensated against the risk of price variation, return uncertainty, and market environment. The RS model elucidates the immediacy cost that the liquidity provider asks against providing the liquidity for an asset.

This work structures the asymmetric information in the RS method, and proposes a simple new estimation of the Informed Realized Spread (IRS). The IRS model assumes, that the asymmetric information about the fundamental value of the asset drives the trading (Glosten & Milgrom, 1985). In this context, the optimistic buyer is more likely to accept the financial inventory at a higher price. Conversely, the pessimistic seller would immediately redeem its position even at a lower price. Therefore, the IRS model assumes that the transaction execution depends on the information about the expected value of an asset.

The IRS model suggests that the informed trader gains from the trading. Assuming the presence for the informed trader, the financial asset has two expected values in the following trading session:

$$\begin{cases} E[v^{ask}] \text{ with probability } \gamma \\ E[v^{bid}] \text{ with probability } \gamma \end{cases} \tag{3}$$

The buyer-initiated trade is expected to be executed if the transaction meets the value at:

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$$E[buy_{t+1}] = E[v^{ask}] = E[ask_{t+1}]$$
 (4)

The seller-initiated trade is assumed to be executed if the transaction meets the value at:

$$E[sell_{t+1}] = E[v^{bid}] = E[bid_{t+1}]$$
(5)

In the following trading session, the informed optimistic buyer enters in the market with probability  $\gamma$ , and informed seller with the bad news is present in the market with probability  $\gamma$ . The market maker provides immediacy service in the absence of buyer or seller. The liquidity provider faces a risk of loss from the informed trader. In this context, the liquidity provider is expected to quote higher ask price for the buyer-initiated trades and lower bid price for the seller-initiated trades.

Assuming the presence of the informed optimistic buyer, the specialist quotes higher ask price as a compensation on the following trading session:

$$E[ask_{t+1}] = high_{t+1}\gamma + \left[ (high_{t+1} + low_{t+1}) * \frac{1}{2} \right] \gamma$$
 (6)

Assuming the presence of the informed pessimistic seller, the specialist quotes lower bid price as a compensation on the following trading session:

$$E[bid_{t+1}] = low_{t+1}\gamma + \left[ (high_{t+1} + low_{t+1}) * \frac{1}{2} \right] \gamma$$
 (7)

In the following trading session, the expected mean value,  $E[\eta_{t+1}]$ , is conditional as:

$$E[\eta_{t+1}] = \frac{E[ask_{t+1}] + E[bid_{t+1}]}{2}$$
 (8)

Inserting the expected mean value,  $E[\eta_{t+1}]$ , in the Equation 1, the IRS model is structured as below:

Informed Realized Spread<sub>t</sub> = 
$$\frac{2|(E[\eta_{t+1}] - c_t)|}{\binom{High_t + Low_t}{2}}$$
 (9)

The IRS model frameworks the possible asymmetric information effects on the following trading session. If there is a higher probability of the informed trader in the market, the following trading-day mean value,  $E[\eta_{t+1}]$ , is expected to be higher than the closing price of day t. This implies, that the liquidity provider tends to increase the ask price for the optimistic buyer-initiated trade. Conversely, the specialist would quote lower bid price against the informed seller. Therefore, the spread size tends to be increased against the risk of loss from the informed counterparty. The IRS model captures the easiness and cost associated with the trading.

The rest of the work is organized as follows. A review of the literature is presented in Section 2. The analytical dataset is presented and discussed in Section 3. The obtained results are discussed in Section 4. The main results of the proposed study are elucidated in Section 5.

#### 2. Review of Literature

In the financial market, a trader tends to estimate and minimize the cost of transaction execution. The bid-ask spread is often used to estimate the market liquidity and its associated cost (Corwin & Schultz, 2012). The liquidity cost elucidates the easiness of executing the transaction in the financial market (Saleemi, 2020b). The financial assets are quoted in pairs. The quoted prices are referred to the ask price and bid price. The liquidity provider tends to accept the financial inventory at a lower bid price. Conversely, the liquidity provider would redeem its position at a higher ask price. This behaviour of the liquidity provider ensures to make profit on the investment. The spread, thereby, refers to the range between ask price and bid price.

The liquidity provider reduces its risk exposure against the future price fluctuations (Amihud & Mendelson, 1980), and thus, quotes a lower bid price at the time of accepting the financial inventory. Meanwhile, the liquidity provider would be compensated against the risk of informed counterparty (Gorton & Metrick, 2010), and thereby, quotes a higher ask price at the time of redemption. The quoted lower bid price or higher ask price elucidates the willingness of the liquidity provider to execute the financial transaction without imposing a cost on the counterparty (Guijarro et al., 2021). In such cases, the quoted lower bid price or higher ask price inclines the spread size. A large size of the spread indicates illiquidity and higher trading cost (Amihud & Mendelson, 1986).

The spread is modelled under three major components: asymmetric information cost, inventory holding cost, or order processing cost (Huang & Stoll, 1997). The asymmetric information model constructs the spread in the context of informed trading (Easley & O'Hara, 1992). The informed optimistic buyer would accept the financial inventory at a higher ask price quoted by the market maker. Meanwhile, the informed pessimistic seller would immediately redeem its position at a lower bid price. The informed trading, thereby, determines the market liquidity. There is always a risk of loss for the uninformed trader (Gorton & Metrick, 2010). The liquidity provider reduces its risk exposure against the informed trading, and imposes a cost on the counterparty.

The immediacy cost model constructs the spread in the context of future price fluctuation and return uncertainty (Ho & Stoll, 1981). The market maker acts as a liquidity provider when the actual counterparty is not available to execute the transaction. The market maker provides immediacy service by accepting the risk of future price uncertainty. The liquidity provider reduces risk by imposing a cost on the seller, that is, a lower bid price. The quoted lower bid price reduces the risk future return uncertainty, and increases the ability of the liquidity provider to redeem its position at a higher ask price. Therefore, the immediacy cost determines the market liquidity. The liquidity providers are also compensated against the cost of order processing (Roll, 1984).

The trading cost determines the market liquidity, and its effects are time varying on the financial assets (Degennaro & Robotti, 2007). The market liquidity is reported to be elucidated in a multiple dimension, such as, easiness of executing the transaction, limited trading cost, depth, breadth, and resiliency (Lybek & Sarr, 2002). The immediacy or limited cost of transaction execution is often illustrated in the higher market liquidity (Saleemi, 2020b). The market liquidity is noted to be highly volatile (Guijarro et al., 2019), which can turn into a systemic liquidity risk (Saleemi, 2014). The liquidity risk is time varying (Hasbrouck & Seppi, 2001), which determines by the

information transparency about the fundamental value of the asset. The liquidity risk is priced in returns on the asset (Amihud et al., 2015). The sensitivity of the asset returns to the liquidity shocks increases yields on the investments (Le & Gregoriou, 2020). The yields on the stock market are more sensitive to the liquidity shocks due to the COVID-19 uncertainty (Saleemi, 2021).

The numerous bid-ask spread models are proposed in the asset pricing literature. Under the assumption that a liquidity provider faces order processing cost, the Roll model captures the serial covariance of change in the prices (Roll, 1984). Assuming the probability of the informed traders, the Glosten-Milgrom model argues that the asymmetric information determines the spread size (Glosten & Milgrom, 1985). The spread is further modelled to the proportion of days with zero returns, range between percent buying cost and percent selling cost, and cost parameters by maximizing the likelihood function of daily stock returns (Lesmond, Ogden, & Trzcinka, 1999). Using Gibbs sampler Bayesian estimation of the Roll model, a half spread is proposed in the asset pricing literature (Hasbrouck, 2004).

Based on the price clustering, an Effective Tick spread model is the probability weighted average of each effective spread size divided by average price (Holden, 2009). The CS model estimates the liquidity, trading cost, and volatility for the two consecutive single days (Corwin & Schultz, 2012). The FHT spread model is the simplification of the LOT Mixed estimator (Fong et al., 2017). Using the daily high, low and closing prices, the AR spread estimator modifies the Roll model (Abdi & Ranaldo, 2017). Most recently, the CBML method is proposed on the assumptions, that the liquidity providers tend to be compensated against the risk of informed trading, future price uncertainty, and administration expenses (Saleemi, 2020b).

A distinct variety of spread estimators has expanded the literature in the asset pricing and its associated disciplines. However, a few shortcomings are identified in some spread models (Goyenko et al., 2009; Saleemi, 2020b). Under unideal conditions, the CS model produces negative spreads, which is a violation of reality. Since the spread is the range between ask price and bid price, it is assumed a positive value in the asset pricing literature. The Roll spread model, and AR spread estimator are accurate under their ideal conditions. The Roll model fails to compute spreads when the covariance of price change is positive. In such case, the function of square root in the RS model cannot estimate spreads for negative observed values. The function of price variance in the AR model fails to estimate spreads for negative values. The LOT Mixed estimator and Effective Tick spread model require a sophisticated computational procedure.

# 3. Data and Methods

This study introduces a simple new estimation of the IRS measure, that is, a modified version of the RS model. A comprehensive comparison of the newly proposed spread estimator is performed with various spread proxies: (a) Quoted Spread; (b) Effective Spread; and (c) Realized Spread. These spread measures are easier to compute and does not require a sophisticated computational procedure. The analysis is performed on the Australian Securities Exchange (ASX), and the financial data is collected during the period January 01, 2001- April 30, 2021. The financial data is based on daily observations of the high, low, and closing prices.

Among the spread proxies, the Quoted Spread (QS) elucidates the market liquidity and its associated cost in the financial market.

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$$QS_t = \frac{(high_t - low_t)}{\eta_t} \tag{10}$$

Where,  $high_t$  is the highest price of day t;  $low_t$  refers to the lowest price of day t; and  $\eta_t$  is the mean value of high and low prices on day t. Since the closing price is not considered in the QS model, this spread estimator fails to illustrate the real cost that a buyer pays at the time of trade. The Effective Spread (ES) is reported to elucidate the real cost associated with trading. The ES proxy is computed as below:

$$ES_t = \frac{2|C_t - \eta_t|}{\eta_t} \tag{11}$$

Table 1. Descriptive Statistics.

Variables	N	Min	Median	Mean	Max	SD	Skewness
RS	5050	2.30E-16	0.01109	0.01531	0.43841	0.01699	5.64298
IRS	5050	2.30E-16	0.01109	0.01531	0.43841	0.01699	5.64298
QS	5050	0.001197	0.01485	0.01755	0.18281	0.01115	3.57961
ES	5050	2.29E-16	0.00746	0.00974	0.16897	0.00975	4.01687

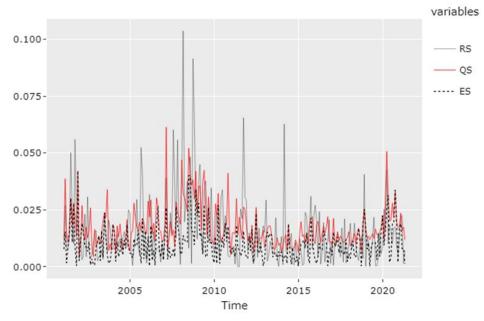


Figure 1. On a monthly basis, the variations in the RS, QS and ES variables

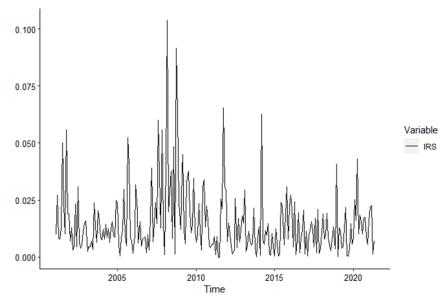


Figure 2. On a monthly basis, the variations in the IRS variable

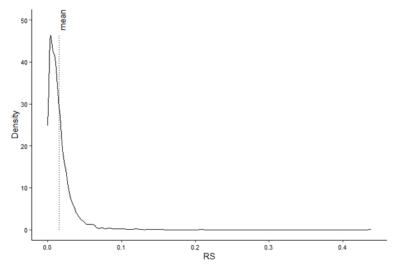


Figure 3. Density plot elucidating skewness for RS variable

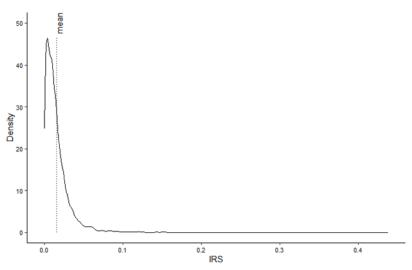


Figure 4. Density plot elucidating skewness for IRS variable

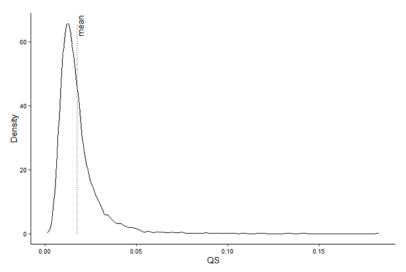


Figure 5. Density plot elucidating skewness for QS variable

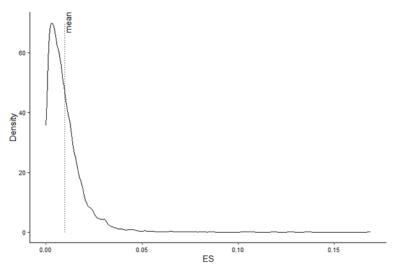


Figure 6. Density plot elucidating skewness for ES variable

# 4. Results

The descriptive statistics are reported in Table 1, and computed on a daily basis. The numerical differences are observed among the spread estimators, excluding the RS and IRS proxies. Since the construction of the spread proxies is based on distinct theoretical assumptions, the adopted spread estimators would possibly impact the computation of liquidity. It is reported that the bid-ask spreads are positively skewed. The positive skewness illustrates the right-skewed distributions with most values to the right of mean. Figures Figure 1 and Figure 2 elucidate variations in the spread proxies on a monthly basis. Despite differences in the theoretical assumptions, the spread estimators are noted to estimate the liquidity and its associated cost.

It is vividly noted that the bid-ask spreads are not constant, and can impose a systemic liquidity risk, as observed during the financial crisis of 2007-2009. However, the liquidity shocks are still occurring over time in the Australian stock market.

Following the concept of kernel density estimation, Figures Figure 3-Figure 6 illustrate the numerical distribution for the spread proxies, and provide important quantity of information. The kernel density approach is a non-parametric technique, which is adopted to visualize the probability density function for spread measures on a daily basis. Density plots vividly report differences in the numerical distributions of the spread estimators. However, the skewed shape of numerical distributions is noted for each variable.

Tubic 2.	Correlation coefficients	arrioring the variables.
DC	IDC	Δ.

Variables	RS	IRS	QS	ES
RS	1	1	0.34	0.23
IRS	1	1	0.34	0.23
QS	0.34	0.34	1	0.76
ES	0.23	0.23	0.76	1

The correlation coefficients are reported in Table 2, and estimated on a daily basis. The adopted spread models have statistically positive relationship, but however, the intensity of the correlation is different among variables. The IRS measure has statistically low correlation with the ES model. The relationship of the IRS method is noted moderate with the QS model. Most importantly, the IRS model has statistically strong relationship with the RS method. Figure Figure 7 further illustrates a strong correlation between IRS and RS models.

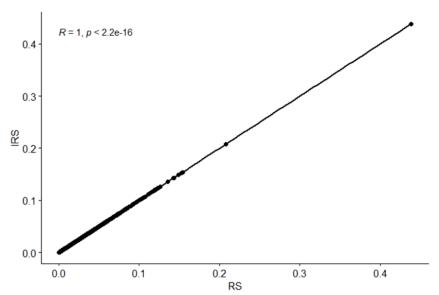


Figure 7. The correlation between RS and IRS models

## 5. Conclusion

This paper provides a new estimation of the bid-ask spread, which is a modified version of the RS model. The newly proposed approach, namely the IRS, assumes that the asymmetric information is one the crucial determinants of the spread size at the time of trade. Using a wider set of information, the IRS model is constructed from daily high, low and closing prices. Despite differences in the theoretical assumptions, the IRS method was noted to be positive and strongly correlated with the RS model. Thereby, the newly proposed method can be applied in the asset pricing studies and its associated disciplines. The IRS model is also easier to compute and does not require a sophisticated computational procedure. The future research can provide a comprehensive comparison of this newly proposed estimation with a larger set of spread proxies, including the volume-based liquidity models.

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