Revista GEOGRÁFICA VENEZOLANA
Volumen 62(2)2021 julio-diciembre
ISSNe 2244-8853 ISSNp 1012-1617
RECIBIDO: agosto, 2020. **ACEPTADO**: diciembre, 2020
pp. 316-329

Validation of the GPS leveling method

through the gradient analysis of the geoidal wave. Case study of Ecuador

Validación del método de nivelación GPS, mediante el análisis de gradiente de ondulación geoidal. Caso de estudio Ecuador

Dennys Enríquez[1]

César Leiva[1]

Santiago Cárdenas[1]

José Carrión[2]

Theofilos Toulkeridis[1]

1 Universidad de las Fuerzas Armadas ESPE, Sangolquí, Ecuador
2 Instituto Geográfico Militar, Quito, Ecuador
daenriquez.hidalgo@gmail.com, caleiva@espe.edu.ec, santiago_cardenasm@hotmail.com, jose.carrion@geograficomilitar.gob.ec, ttoulkeridis@espe.edu.ec
Enríquez: https://orcid.org/0000-0001-5115-9351
Leiva: https://orcid.org/0000-0002-3332-6029
Cárdenas: https://orcid.org/0000-0002-3503-4652
Carrión: https://orcid.org/0000-0003-0894-0104
Toulkeridis: https://orcid.org/0000-0003-1903-7914

Resumen

Validamos la nivelación GPS como alternativa al método tradicional de nivelación geométrica, comparando pendientes geométricas derivadas de la técnica de posicionamiento GNSS, alturas resultantes de campañas de nivelación geométrica y ondulaciones geoidales extraídas del Modelo Geopotencial Global EGM08. Este análisis se realizó en el continente ecuatoriano, donde se identificaron áreas en las que el gradiente de la ondulación geoidal es menos pronunciado. La espacialización del gradiente o variación permitió analizar el desempeño del método de nivelación GPS, bajo la hipótesis de que una menor variabilidad en la ondulación geoide implica menos discrepancias en el desnivel del GPS. Las observaciones GNSS se determinaron en las placas niveladoras pertenecientes a la Red Básica de Control Vertical. Los resultados del estudio se dan con base al error relativo resultante de la comparación del método tradicional de nivelación diferencial con los valores correspondientes obtenidos del posicionamiento GNSS, considerando diferentes distancias para la extensión del desnivel.

PALABRAS CLAVE: geoide; GNSS; EGM08; cálculo de errores.

Abstract

We validated the GPS leveling as an alternative to the traditional geometric leveling method. Validation compares the geometric slopes derived from the GNSS positioning technique, heights resulting from geometric leveling campaigns and geoid undulations extracted from the Global Geopotential Model EGM08. This analysis was performed in the Ecuadorian mainland, where we identified areas in which the gradient of the geoidal undulation is less pronounced. The spatialization of the gradient or variation-based methods allowed to analyze the performance of the GPS leveling method, under the hypothesis that less variability in geoid undulation implies less discrepancies in the GPS unevenness. GNSS observations were determined on the leveling plates belonging to the Basic Vertical Control Network. The results of the study are given based on the relative error resulting from the comparison of the traditional differential leveling method with the corresponding values obtained from the GNSS positioning, considering different distances for the spread of unevenness.

KEYWORDS: geoid; GNSS; EGM08; error calculation.

1. Introduction

In Ecuador, according to the Regulations to the National Cartography Law, the official reference surface of the heights is the mean sea level, determined in the La Libertad tide gauge (Wyss, 1976; Aubrey *et al.*, 1988). From this Geodetic Reference Datum (GRD), the Basic Vertical Control Network (BVCN) has been materialized through the propagation of unevenness obtained by geometric leveling (Lippold, 1980; Grafarend & Ardalan, 1999; Ferrari & Verboven, 2012; Tsmots *et al.*, 2017). On the BVCN circuits, the Ecuadorian Military Geographic Institute (IGM-Ec), has conducted gravimetric campaigns proposing the calculation of geopotential numbers and consequently the determination of physical heights (i.e., normal, orthometric). However, inconsistent information gaps and observations, mainly in the leveling works, have prevented the effect of gravity corrections on the level heights, and therefore the calculation of physical heights (Tierra & Acurio, 2016; Sánchez & Sideris, 2017; Portilla *et al.*, 2021). In this context and for practical purposes, in the current study, level heights (H_n) are considered equivalent to orthometric heights (H) (FIGURE 1; Leiva *et al.*, 2017).

The fundamental input for the determination of vertical coordinates referring to the sea level are the unevenness obtained by geometric leveling, which is a very precise technique, but its realization demands the availability of considerable resources, in addition to favorable atmospheric conditions. A leveling campaign takes several days of field work, which also depends on the distance that needs to be dragged from a BVCN component plate. Furthermore, it is to consider that these surveys are sometimes accompanied by GPS sessions in order to determine the horizontal position of the points (Betti *et al.*, 1999; Eckl *et al.*, 2001; Yigit *et al.*, 2016).

The Global Navigation Satellite System (GNSS), through the use of satellite tracking equipment, allows ellipsoidal heights (h) to be obtained, which, depending on the positioning technique, can reach sub-centimeter accuracies (Erenoglu *et al.*, 2012; Wang, 2013; Scholarin & Awange, 2015; Luna *et al.*, 2017). The determination of these heights, together with geoid undulation values (N, from local or global models), allow the estimation of orthometric heights based on equation (1).

$$H \approx h - N \quad (1)$$

Where:

H: Orthometric height
h: Ellipsoidal height
N: Geoidal undulation

Several satellite missions such as GRACE, GOCE, among others, have allowed the development of Global Geopotential Models (GGMs), which manage to represent the long wavelengths (low frequency) of the Earth's gravity field (Balmino *et al.*, 1999; Pail *et al.*, 2010; Touboul *et al.*, 2012). The spatial resolution of these models is related to their degree of development in spherical harmonics, for example, the combined GGM Earth Gravitational Model 2008 (EGM08), (Pavlis *et al.*, 2008; 2012; 2013; NGA, 2013), is made up of a series of spherical harmonic functions up to degree 2190 and order 2159, corresponding to a spatial resolution of 9.2 km (Pavlis *et al.*, 2008).

Based on the estimation of orthometric heights according to equation (1), the EGM08 geoid undulations generate an average error of 0.68 m (Tierra, 2008). The errors of omission and commission related to the MGGs, is able to reach metric dimensions in the representation of the undulations, which limit their use in applications in the engineering field (Grigoriadis *et al.*, 2014; Sjöberg & Bagherbandi, 2012). Knowledge and representation of the topographic surface is, in general, a fundamental requirement in the execution of engineering projects (De Floriani *et al.*, 1996; Darnell *et al.*, 2008). Geodetic marks associated with information corresponding to their position (horizontal component) and height (vertical component) are used as key references for carrying out different types of work (Altamimi *et al.*, 2007; Kotsakis *et al.*, 2012).

Because geometric leveling is a procedure that demands enormous resources and particular conditions for its realization, it causes that in many cases the application of this technique is not feasible. In essence, to know the horizontal and vertical coordinates, two types of topographic and geodetic processes are needed (Plag *et al.*, 2009). In this context, the aim is to validate a method for obtaining unevenness, based on the GNSS positioning technique, and which significantly reduces the necessary resources and minimum conditions for the execution of geometric leveling works (Wu *et al.*, 2019; Tiwari *et al.*, 2020).

Various studies have been conducted worldwide which analyzed the relationship between traditional leveling and obtaining heights using satellite positioning techniques (Li & Goldstein, 1990; Zebker *et al.*, 1994; Dawod *et al.*, 2010; Poitevin *et al.*, 2019). Such is the case of the comparison of the vertical deformations of the profile of the geological fault in Shanyin, Shanxi province in China, based on GNSS observations and precise leveling for multiple periods of time (Qin *et al.*, 2018). The results of the study indicated that the speed in vertical deformation determined with sequential GNSS surveys was 20 mm / year. However, they considered that the deviations in vertical displacement between GNSS and leveling, within a period of three years, show good consistency, with approximately 3-4 mm per year. Therefore, they concluded that the application of GNSS techniques in the control of vertical displacements of the cortex is very feasible.

In South America a methodology has been presented for calculating and analyzing the results of these models in Brazil (Matos *et al.*, 2012). The global model that was used as reference was the EGM08. The research was based on the comparison of 844 GPS / Leveling observations (belonging to the Brazilian geometric leveling network) with geoid heights from different geopotential models. In this way it was determined that using the EGM08, the mean error for the geoid heights was 0.1 m, while applying the MAPGEO2010 (model developed exclusively for Brazil), the mean error was 0.04 meters (Matos *et al.*, 2012; Chuerubim, 2013).

In Ecuador, a recent study had the objective of analyzing the feasibility of using geoid undulations, GNSS observations, traditional leveling and geostatistical techniques to achieve high precision in determining vertical coordinates in engineering projects (Chicaiza *et al.*, 2017). The results obtained indicate that the estimation of the uncertainty was obtained in the interval [-0.5; 0.5] m for errors, and a maximum estimate of the standard deviation of 2 mm in relation to the interpolation method applied. The distribution of the geoid undulation map error obtained in this study has provided a better result than the gravitational models such as EGM08 and EGM96.

However, the application of these global or local geoid models in the calculation of orthometric or level heights are relatively accurate, but because they are determined from interpolation methods, there would be places where they do not fit correctly or have systematic errors. Given all these aforementioned considerations, the objective of the current study has been to validate the GPS leveling method in continental Ecuador, by analyzing geoid undulation variation, and to determine heights referred to the mean sea level being the official heights in the country, using only satellite positioning techniques.

2. GPS leveling

GPS leveling is a term that covers the efficient determination of physical heights or, failing that, geometric heights referred to the mean sea level, this by means of GNSS positioning (therefore, it can also be considered as GNSS leveling) and the inclusion of geoid undulation, taking as a starting point, points belonging to leveling networks (Ferreira *et al.*, 2013; Li *et al.*, 2014; Grigoriadis *et al.*, 2014). This method is affordable for applications where centimeter precision is sufficient or for regions where height reference infrastructure is not available. There are also limitations to the GPS leveling method, which are related to the sensitivity of the type of satellite observations since atmospheric circumstances must be considered, including the state of the ionosphere / troposphere, multipaths and objects blocking signs. Therefore, special attention should be considered to site selection and GNSS measurements (Bovenga *et al.*, 2013; Kenyeres, 2016; Zanutta *et al.*, 2017).

The mathematical model on which the GPS leveling method is based is developed below. Let two points be P and Q, where Q has a known level height (FIGURE 1), from equation (1), the height above the mean sea level can be calculated for each point, forming a system of equations.

$$H_n^P = h_P - \eta_P \quad (2)$$

$$H_n^Q = h_Q - \eta_Q \quad (3)$$

Where:

H_n^Q: Level height of point Q (base point)
η_Q: Geoid undulation of point Q (base point)
h_Q: Ellipsoidal height of point Q (base point)
H_n^P: Level height of point P (moving point)
η_P: Geoid undulation of point P (moving point)
h_P: Ellipsoidal height of point P (moving point)

From these two equations, the difference is obtained:

$$H_P^n - H_Q^n = h_P - h_Q - \eta_P + \eta_Q \quad (4)$$

Clearing the level height from point P results in the base equation of the GPS leveling method.

$$H_P^n = H_Q^n + (h_P - h_Q) - (\eta_P - \eta_Q) \quad (5)$$

As mentioned, the level height of point Q is known, ellipsoidal heights are determined in the field by GPS positioning, and geoid undulations are extracted from an available model. FIGURE 1 illustrates the surfaces used and the respective magnitudes for the development of this method. However, in the present study, it was hypothesized that the error in height determined with the GPS leveling method is directly proportional to the distance to the starting plate, and that this error depends on the geoid undulation gradient in the study area (Rapp, 1997; Bouman *et al.*, 2005).

3. Methodology

The methodology followed to test the proposed hypothesis, has been the generation of a geoid undulation gradient map, using the EGM08 model as input. The gradient, or variation, was classified into five zones, qualifying them as low, slight, gentle, moderate and strong. For each zone, a sampling plan was prepared for the component plates of the Basic Vertical Control Network. From this plan, the selected points were positioned by using precision GPS. Following this, the GPS leveling method was applied from one of the positioned plates and the GPS level height was calculated. Finally, errors of this height and that determined with traditional leveling techniques were compared, thus obtaining the typical errors of GPS leveling versus Geometric leveling. The flow chart of the methodology is shown in FIGURE 2.

3.1 Geoid undulation gradient zoning

First, the gradient map or variation of geoid undulation was elaborated as a function of distance or advance, using GIS tools. For this, the EGM08

FIGURE 1. Schematic illustration of the different parts of the GPS levelling

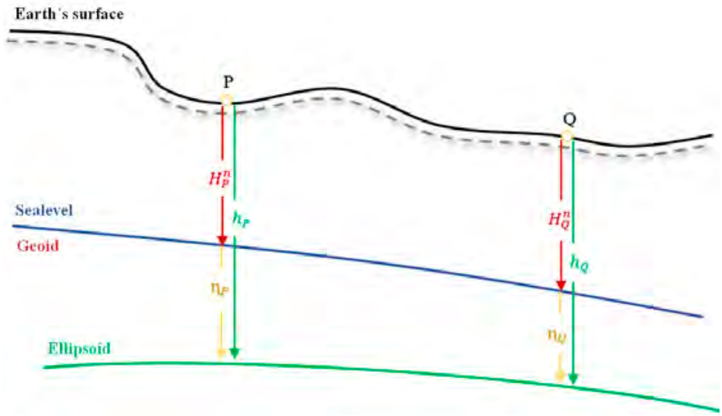
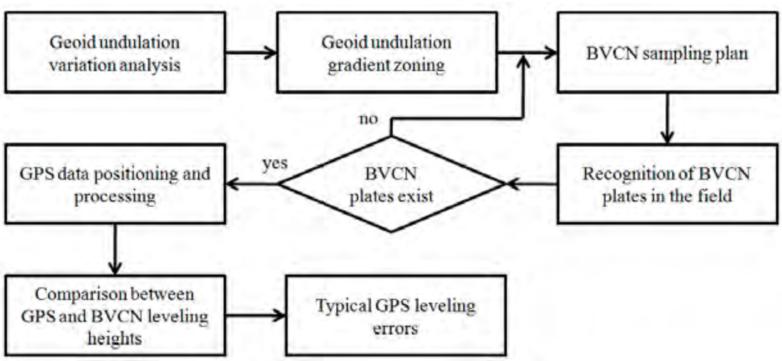


FIGURE 2. Flow chart of the methodology



model was taken as the main input, and the territory of Continental Ecuador was delimited (NGA, 2013). For this zoning or classification of the geoid undulation gradient values, the Natural Breaks method was used, which is based on the Jenks optimization algorithm (North, 2009; Chen *et al.*, 2013; Liu *et al.*, 2016). This method is used to classify the data according to their similarity in different classes according to the breakpoints (Jiang, 2013). As previously mentioned, for this investigation, five classes were defined for the geoid undulation variation (FIGURE 3).

3.2 BVCN sampling plan

The selection of the leveling lines to be sampled was performed by intersecting the geoid undulation gradient zoning map and the BVCN lines. From this intersection, five selected leveling lines resulted, corresponding to each of the zones (FIGURE 4). In each line, a convenient number of plates was selected, considering an analysis distance of 15 km, which made it possible to test the hypothesis regarding the drag distance. The information on these selected lines is detailed in TABLE 1.

FIGURE 3. Geoidal Undulation Gradient Zoning Map of Ecuador

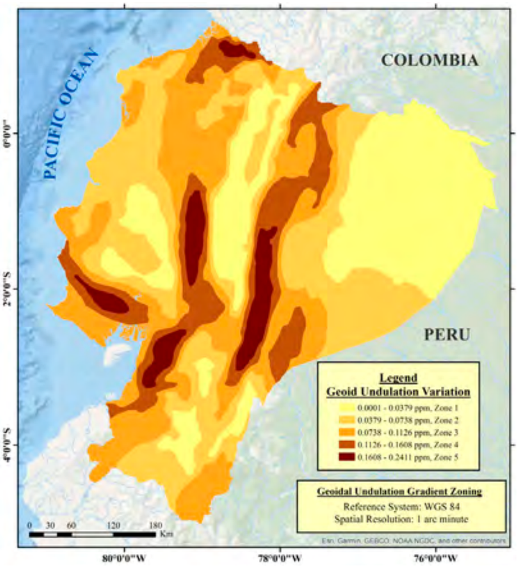


FIGURE 4. Leveling Lines Sampling BVCN of Ecuador

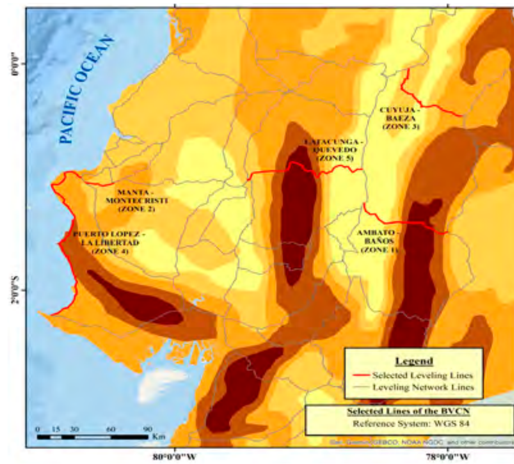


TABLE 1. Leveling Lines Sampling BVCN

Zone of variation	Ring	Line	Location	Plates BVCN
1: Low	XXXI	L1	Ambato - Baños	6
2: Slight	II	L5	Manta - Montecristi	8
3: Gentle	XXIX	L1	Cuyuja - Baeza	5
4: Moderate	I	L1	Puerto López - La Libertad	5
5: Strong	IX	L4	Latacunga - Quevedo	6

3.3 GPS data positioning and processing

GPS positioning was performed in two phases. The first consisted of satellite tracking the base leveling plate using the relative static method, with an approximate time of 8 hours, using a dual-frequency GNSS equipment that guarantees the link to the GNSS Network for Continuous Monitoring of Ecuador (Yunck *et al.*, 1985; Marais *et al.*, 2005; Dow *et al.*, 2009). In the second phase, the other leveling plates were determined through the fast static method, with a tracking time of 1 hour, also using dual-frequency GNSS equipment (Felux *et al.*, 2019; Robustelli *et al.*, 2019).

To process the data, the Trimble Business Center (TBC) 2.99 software was used, verifying compliance with the precision statistics. The coordinates were linked to the GNSS Network for Continuous Monitoring of Ecuador (REGME) in the ITRF Reference Framework 2008, epoch 2016.4. In order to process the data collected in phase 1 of positioning, they were linked to two Continuous Monitoring Stations, using independent sessions and adjusting the network. For phase 2, the data were processed as radial, using the previously determined bases, obtaining as a result the list of coordinates of all these points with their respective precisions.

3.4 GPS Leveling Application

For the application of the GPS leveling method, the mathematical model described in equation 5 was considered, based on a height above the mean sea level of the IGM plate considered as the base, the ellipsoidal heights derived from the GNSS positioning and the geoid undulations of the EGM08 model. TABLE 2 lists the results obtained from the analysis method for each defined geoid undulation variation zone.

TABLE 2. GPS Leveling Data. Zone 1: line Ambato - Baños; Zone 2: line Manta - Montecristi; Zone 3: line Cuyuja - Baeza; Zone 4: line Puerto López - La Libertad; Zone 5: line Latacunga - Quevedo

Plate	Leveled height IGM (m)	Geoid undulation (m)	Ellipsoidal height (m)	Distance (km)	GPS height levelling (m)	Error (cm)
Zone 1						
XXXI-L1-18A	1943.3653	26.01	1970.3225			
XXXI-L1-16A	2006.2484	26.08	2033.2981	2.63	2006.2709	2.25
XXXI-L1-12A	2063.8385	26.1090	2090.8645	6.12	2063.8083	3.02
XXXI-L1-9B	2360.9956	26.0820	2388.0137	8.70	2360.9626	3.29
A-B-15	2464.5428	26.0210	2491.5530	11.71	2464.6395	9.68
A-B-11B	2724.4496	25.933	2751.5258	15.17	2724.6456	19.60
Zone 2						
M-Q-10-A-JA	120.3987	14.657	135.3942			
II-L58-16	113.7232	14.7190	128.7873	0.83	113.7298	0.66
II-L58-14	131.6744	14.857	146.8851	3.06	131.6896	1.52
II-L58-12	184.7069	15.009	200.0997	6.31	184.7522	4.53
II-L58-11	223.0409	15.09	238.5228	8.16	223.0943	5.34
II-L58-10	277.0653	15.132	292.616	9.73	277.1455	8.02
II-L58-8	148.9549	15.254	164.665	12.62	149.0725	11.76
II-L58-7	110.673	15.328	126.573	13.97	110.9065	23.35
Zone 3						
XXIX-L1-48C	2364.8365	26.718	2392.9281			
XXIX-L1-51A	1845.2998	26.3760	2244.8889	4.08	2217.1393	4.45
XXIX-L1-54C	1933.3276	25.9650	1960.6576	8.61	1933.2618	6.58
XXIX-L1-56A	2217.0948	25.479	1846.7662	13.31	1819.9136	18.39
XXIX-L2-2A	2306.3153	25.1820	1871.6275	16.18	1845.0719	22.79
Zone 4						
J-PL-5B	34.7179	15.1300	50.1204			
J-PL-4B	28.8921	15.1970	44.3655	2.95	2217.1393	0.39
L-M-70	14.6718	14.9960	29.9988	5.85	1933.2618	5.85
L-M-66	100.2567	14.5700	115.0364	8.99	1819.9136	15.27
L-M-64	6.5956	14.3660	21.1881	13.19	1845.0719	16.48
Zone 5						
IX-L4-90A	716.2725	22.396	739.1662			
IX-L4-88A	862.4847	22.9390	885.9639	2.90	862.5272	4.25
IX-L4-86A	1012.2921	23.368	1036.2295	5.24	1012.3638	7.17
IX-L4-80A	1618.0884	23.654	1642.4046	9.45	1618.2529	16.45
IX-L4-76A	2065.967	24.315	2091.0894	13.06	2066.2767	30.97
IX-L4-74A	2261.9904	24.531	2287.3465	14.24	2262.3178	32.74

3.5 Comparison of GPS and BVCN Leveling

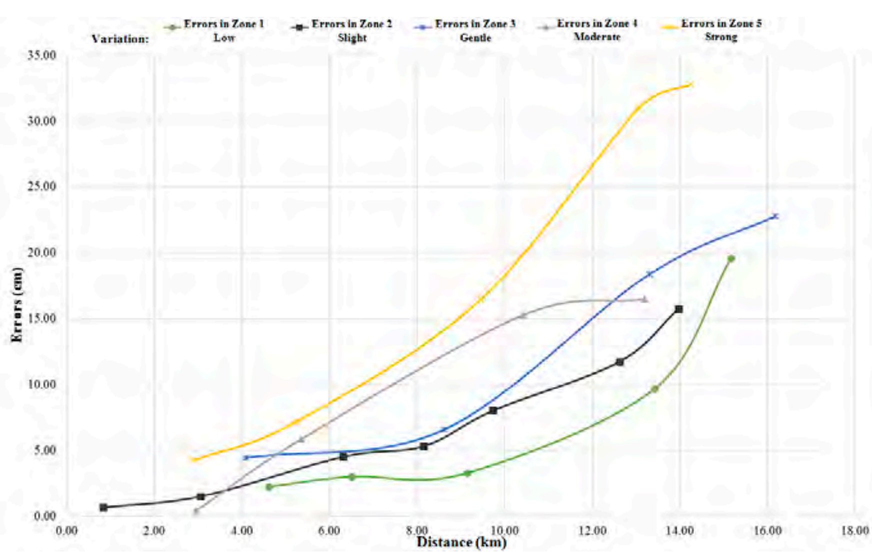
FIGURE 5 demonstrates the GPS leveling error graphs versus distance, corresponding to each of the geoid undulation variation zones. There, in a general context it is illustrated, hat at a greater distance from the motherboard the error also increases. In addition, the errors are greater depending on the area of variation of geoid undulation where it is located.

4. Discussion

The application of the GPS leveling method is an efficient alternative to the traditional geometric leveling technique. This method, due to its differential conception, which can be verified in the term '$(\eta_P - \eta_Q)$' in equation 5, eliminates some systematic errors from the geoid models used. It is additionally evidenced in the direct application of equation (1), for the determination of heights referred to the conventional vertical datum, that has been considered.

The zoning of the geoid undulation gradient in mainland Ecuador is essential to characterize the behavior of this physical property and analyze its influence on the proposed method, under the hypothesis that the greater the variation, the greater the error of the method. The range of variation that was obtained has been 0.0001 - 0.2412 mm/km (ppm), defining five gradient zones, which are detailed below. Zone 1, considered of low variation of geoid undulation, presents an interval between 0.0001 - 0.0379 ppm. It is located in the plains of the Coast, in the inter-Andean valleys and in a large part of the Amazonian Lowland. Zone 2, considered slight variation, presents a range of values between 0.0380 - 0.0739 ppm. It is found mostly on the coast and in the Amazon Lowland. Zone 3, considered of gentle variation, with a range of values between 0.0740 - 0.1126 ppm, is located at the elevations that start the different mountain ranges. Zone 4, considered of moderate variation, presents a range of values between 0.1127 - 0.1608 ppm, being located mostly in the foothills of the mountain ranges. Zone 5, considered a strong

FIGURE 5. Graphical illustration where the GPS leveling errors are summarized



geoid undulation variation, presents a gradient interval between 0.1609 - 0.2412 ppm, which is also located in the foothills of the western and eastern Andes Mountains, with an elongated shape in the north-south direction. In addition, it is present in the Chongón - Colonche mountain range, presenting an elongated east - west shape.

The comparison between heights leveled by the IGM and determined with GPS leveling, allowed calculating the respective method errors and hypothesis testing. The results are able to be analyzed graphically in FIGURE 6, which considers the method error, the drag distance and the variation zone in which it is located. From this graphic analysis, the hypothesis posed could be verified. Indeed, the variation zone and the drag distance have a direct influence on the method error. This is directly proportional to the leveling drag distance and the magnitude of this error depends on what zone of variation it is in, with zone 1 of variation having the least error and zone 5 yielding the greatest error. Finally, In terms of engineering applications, it has been possible to determine the distances for which the typical errors of the 5, 10 and 15 cm method would be obtained, which serve as a guide to decide their use (TABLE 3).

5. Conclusions

The geoid undulation gradient zoning map was generated with five homogeneous zones of variation. The areas with low variation are generally located in plains of the coast, the east and the Inter-Andean Valley, while the areas with the greatest variation are located in the foothills of the Ecuadorian mountain ranges. Each of these zones delimits the area to which it has been able to be leveled by using GPS, depending on the error of the proposed method and considering the distance to the base (IGM leveling plate).

FIGURE 6. Graphical representation of typical GPS leveling errors

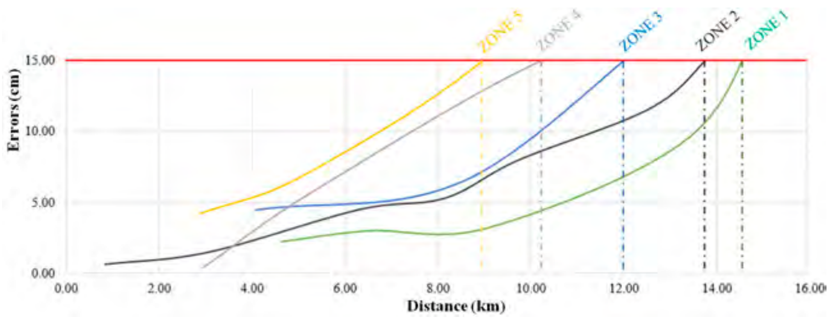


TABLE 3. List of typical GPS leveling errors

Zone of variation	Typical errors		
	5 cm	10 cm	15 cm
1	10.4	13.7	14.6
2	7.8	11.2	13.8
3	6.9	10.1	12.0
4	5.1	7.3	10.1
5	3.8	6.8	8.9

The GPS leveling method allows obtaining heights referred to the mean sea level (official in Ecuador), from the use of IGM level heights, ellipsoidal heights, derived from satellite positioning, and geoid undulations (EGM08). The error presented by this method depends on the distance to the drag base plate, with a directly proportional relationship between error and distance to the base.

The smallest errors of the method belong to the leveled plates in zone 1 of geoid undulation variation, while the greatest errors correspond to zone 5, which justifies the definition of the homogeneous zones of geoid undulation variation realized in the present study.

6. The authors contributions

The authors contributions have been as follows, D.E. and C.L. designed research; D.E., S.C., J.C. and E.F. performed research; D.E., J.C. and T.T. analyzed data; C.L. and T.T. wrote the paper.

7. Data availability

All data generated or analysed during this study are included in this published article [and its supplementary information files].

8. Statement

On behalf of all authors, the corresponding author states that there is no conflict of interest.

9. References quoted

ALTAMIMI, Z.; COLLILIEUX, X.; LEGRAND, J.; GARAYT, B. & C. BOUCHER. 2007. "ITRF2005: A new release of the International Terrestrial Reference Frame based on time series of station positions and Earth Orientation Parameters". *Journal of Geophysical Research: Solid Earth*, 112(B9).

AUBREY, D. G.; EMERY, K. O. & E. UCHUPI. 1988. "Changing coastal levels of South America and the Caribbean region from tide-gauge records". *Tectonophysics*, 154(3-4): 269-284.

BALMINO, G.; PEROSANZ, F.; RUMMEL, R.; SNEEUW, N. & H. SÜNKEL. 1999. "CHAMP, GRACE and GOCE: mission concepts and simulations". *Applicata*, 40: 309-319.

BETTI, B.; BIAGI, L.; CRESPI, M. & F. RIGUZZI. 1999. "GPS sensitivity analysis applied to non-permanent deformation control networks". *Journal of Geodesy*, 73(3): 158-167.

BOUMAN, J.; KOOP, R.; HAAGMANS, R.; MÜLLERR, J.; SNEEUW, N.; TSCHERNING, C. C. & VISSER, P. N. A. M. 2005. "Calibration and validation of GOCE gravity gradients". In: *A window on the future of geodesy*. (pp. 265-270). Springer, Berlin, Heidelberg.

BOVENGA, F.; NITTI, D. O.; FORNARO, G.; RADICIONI, F.; STOPPINI, A. & R. BRIGANTE. 2013. "Using C/X-band SAR interferometry and GNSS measurements for the Assisi landslide analysis". *International Journal of Remote Sensing*, 34(11): 4.083-4.104.

CHEN, J.; YANG, S.; LI, H.; ZHANG, B. & J. LV. 2013. "Research on geographical environment unit division based on the method of natural breaks (Jenks)". *Int. Arch. Photogramm. Remote Sens. Spat. Inf. Sci*, 47-50.

CHICAIZA, E. G.; LEIVA, C. A.; ARRANZ, J. J. & X. E. BUENAŃO. 2017. "Spatial uncertainty of a geoid undulation model in Guayaquil, Ecuador". *Open Geosciences*, 9(1): 255-265.

CHUERUBIM, M. L. 2013. "Use of the software MAPGEO2010 as a teaching resource in the study of the reference surfaces and geodetic reference in geodesy/Utilização do software MAPGEO 2010 como recurso didático no estudo das superficies e referenciais geodésicos adotados em geodesia". *Revista Geográfica Acadêmica*, 7(2): 31-48.

DARNELL, A. R.; TATE, N. J. & C. BRUNSDON. 2008. "Improving user assessment of error implications in digital elevation models". *Computers, Environment and Urban Systems,* 32(4): 268-277.

DAWOD, G. M.; MOHAMED, H. F. & S. S. ISMAIL. 2010. "Evaluation and adaptation of the EGM2008 geopotential model along the Northern Nile Valley, Egypt: Case study". *Journal of Surveying Engineering,* 136(1): 36-40.

DE FLORIANI, L.; MARZANO, P. & E. PUPPO. 1996. "Multiresolution models for topographic surface description". *The Visual Computer,* 12(7): 317-345.

DOW, J. M.; NEILAN, R. E. & C. RIZOS. 2009. "The international GNSS service in a changing landscape of global navigation satellite systems". *Journal of Geodesy,* 83(3-4): 191-198.

ECKL, M. C.; SNAY, R. A.; SOLER, T.; CLINE, M. W. & G. L. MADER. 2001. "Accuracy of GPS-derived relative positions as a function of interstation distance and observing-session duration". *Journal of Geodesy,* 75(12): 633-640.

ERENOGLU, R. C.; YUCEL, M. A.; PIRTI, A. & D. U. SANLI. 2012. "On the performance of GNSS levelling over steep slopes". *Boletim de Ciências Geodésicas,* 18(4): 645-660.

FELUX, M.; CIRCIU, M. S.; CAIZZONE, S.; ENNEKING, C.; FOHLMEISTER, F. & M. RIPPL. 2019. Towards Airborne Multipath Models for Dual Constellation and Dual Frequency GNSS. *Proceedings of the 2019 International Technical Meeting of The Institute of Navigation* (pp. 62-68). https://doi.org/10.33012/2019.16683.

FERRARI, S. & F. VERBOVEN. 2012. "Vertical control of a distribution network - an empirical analysis of magazines". *The RAND Journal of Economics,* 43(1): 26-50.

FERREIRA, V. G.; ZHANG, Y. & S. R. C. DE FREITAS. 2013. "Validation of GOCE gravity field models using GPS-leveling data and EGM08: a case study in Brazil". *Journal of Geodetic Science,* 3(3): 209-218.

GRAFAREND, E. W. & ARDALAN, A. A. 1999. "World geodetic datum 2000". *Journal of Geodesy,* 73(11): 611-623.

GRIGORIADIS, V. N.; KOTSAKIS, C.; TZIAVOS, I. N. & G. S. VERGOS. 2014. "Estimation of the reference geopotential value for the local vertical datum of continental Greece using EGM08 and GPS/leveling data". In: *Gravity, Geoid and Height Systems*. pp. 249-255. Springer, Cham.

JIANG, B. 2013. "Head/tail breaks: a new classification scheme for data with a heavy-tailed distribution". *The Professional Geographer,* 65(3): 482-494. https://doi.org/10.1080/00330124.2012.700499

KENYERES, A. 2016. "GPS/Leveling". In: E. GRAFAREND (eds.), *Encyclopedia of Geodesy*. Springer, Cham. https://doi.org/10.1007/978-3-319-02370-0_44-1.

KOTSAKIS, C.; KATSAMBALOS, K. & D. AMPATZIDIS. 2012. "Estimation of the zero-height geopotential level W o LVD in a local vertical datum from inversion of co-located GPS, leveling and geoid heights: a case study in the Hellenic islands". *Journal of Geodesy,* 86(6): 423-439.

LEIVA, C.; ARAUJO, N. y X. BUENAÑO. 2017. "Modelo de predicción espacial de ondulación geoidal, para el área urbana de Quito, utilizando técnicas geoestadísticas". *Revista Geoespacial*, 14(1): 83-102.

LI, F. K. & R. M. GOLDSTEIN. 1990. "Studies of multibaseline spaceborne interferometric synthetic aperture radars". *Transactions on Geoscience and Remote Sensing,* 28(1): 88-97.

LI, J.; JIANG, W.; ZOU, X.; XU, X. & SHEN, W. 2014. "Evaluation of recent GRACE and GOCE satellite gravity models and combined models using GPS/leveling and gravity data in China". In: *Gravity, Geoid and Height Systems*. pp. 67-74. Springer, Cham.

LIPPOLD Jr., H. R. 1980. "Readjustment of the national geodetic vertical datum". *Eos, Transactions American Geophysical Union*, 61(24): 489-491.

LIU, X.; GAO, Z.; NING, J.; YU, X. & Y. ZHANG. 2016. "An improved method for mapping tidal flats based on remote sensing waterlines: a case study in the Bohai Rim, China". *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, 9(11): 5.123-5.129.

LUNA, M. P.; STALLER, A.; TOULKERIDIS, T. & H. PARRA. 2017. "Methodological approach for the estimation of a new velocity model for continental Ecuador". *Open Geosciences*, 9(1): 719-734.

MARAIS, J.; BERBINEAU, M. & M. HEDDEBAUT. 2005. "Land mobile GNSS availability and multipath evaluation tool". *IEEE Transactions on Vehicular Technology*, 54(5): 1.697-1.704.

MATOS, A. C. O. C. D.; BLITZKOW, D.; GUIMARÃES, G. D. N.; LOBIANCO, M. C. B. e S. M. A. COSTA. 2012. "Validação do MAPGEO2010 e comparação com modelos do geopotencial recentes". *Boletim de Ciências Geodésicas*, 18(1): 101-122.

NATIONAL GEOSPATIAL-INTELLIGENCE AGENCY (NGA). 2013. Available in: Gravitational Model 2008 (http://earth-info.nga.mil).

NORTH, M. A. 2009. A method for implementing a statistically significant number of data classes in the Jenks algorithm. *Sixth International Conference on Fuzzy Systems and Knowledge Discovery* (Vol. 1, pp. 35-38). Institute of Electrical and Electronics Engineers (IEEE).

PAIL, R.; GOIGINGER, H.; SCHUH, W-D.; HÖCK, E.; BROCKMANN, J. M.; FECHER, T.; GRUBER, T.; MAYER-GÜRR, T.; KUSCHE, J.; JÄGGI, A. & D. RIESER. 2010. "Combined satellite gravity field model GOCO01S derived from GOCE and GRACE". *Geophysical Research Letters*, 37(20).

PAVLIS, N. K.; HOLMES, S. A.; KENYON, S. C. & J. K. FACTOR. 2008. An earth gravitational model to degree 2160: EGM2008. Presented at the *2008 General Assembly of the European Geoscience Union*, Vienna, Austria, April 13–18, 84(1): 2-4.

PAVLIS, N. K.; HOLMES, S. A.; KENYON, S. C. & J. K. FACTOR. 2012. "The development and evaluation of the Earth Gravitational Model 2008 (EGM2008)". *Journal of Geophysical Research: Solid Earth*, 117(B4).

PAVLIS, N. K.; HOLMES, S. A.; KENYON, S. C. & J. K. FACTOR. 2013. "Correction to 'The development and evaluation of the Earth Gravitational Model 2008 (EGM2008)'". *Journal of Geophysical Research: Solid Earth*, 118(5): 2.633-2.633.

PLAG, H. P.; ROTHACHER, M.; PEARLMAN, M.; NEILAN, R. & C. MA. 2009. The global geodetic observing system. *Advances in Geosciences*, (Volume 13: Solid Earth (SE; pp. 105-127).

POITEVIN, C.; WÖPPELMANN, G.; RAUCOULES, D.; LE COZANNET, G.; MARCOS, M. & L. TESTUT. 2019. "Vertical land motion and relative sea level changes along the coastline of Brest (France) from combined space-borne geodetic methods". *Remote Sensing of Environment*, 222: 275-285.

PORTILLA, Ó.; LEIVA, C.; LUNA, M.P. & T. TOULKERIDIS. 2021. "Elaboration of a local projection system in mainland Ecuador for the generation of detailed cartography". *Revista Geográfica Venezolana*, 62(1): 176-196.

QIN, S.; WANG, W. & S. SONG. 2018. "Comparative study on vertical deformation based on GPS and leveling data". *Geodesy and Geodynamics*, 9(2): 115-120.

RAPP, R. H. 1997. "Use of potential coefficient models for geoid undulation determinations using a spherical harmonic representation of the height anomaly/geoid undulation difference". *Journal of Geodesy*, 71(5): 282-289.

ROBUSTELLI, U.; BAIOCCHI, V. & G. PUGLIANO. 2019. "Assessment of dual frequency GNSS observations from a Xiaomi Mi 8 Android smartphone and positioning performance analysis". *Electronics*, 8(1): 91.

SÁNCHEZ, L. & M. G. SIDERIS. 2017. "Vertical datum unification for the international height reference system (IHRS)". *Geophysical Journal International*, 209(2): 570-586.

SHOLARIN, E. A. & J. L. AWANGE. 2015. "Global navigation satellite system (GNSS)". In: *Environmental Project Management*. pp. 177-212. Springer, Cham.

SJÖBERG, L. E. & M. BAGHERBANDI. 2012. "Quasigeoid-to-geoid determination by EGM08". *Earth Science Informatics*, 5(2): 87-91.

TIERRA A. 2008. "Evaluación del EGM08 y EGM96 en el Ecuador a partir de datos de GPS y nivelación geométrica". *Revista Geoespacial*, (6): 72-80.

TIERRA A. & V. ACURIO. 2016. "Modelo neuronal para la predicción de la altura geoidal local en el Ecuador". *Revista Geoespacial*, (13): 59-71

TIWARI, A.; NARAYAN, A. B.; DWIVEDI, R.; DIKSHIT, O. & B. NAGARAJAN. 2020. "Monitoring of landslide activity at the Sirobagarh landslide, Uttarakhand, India, using LiDAR, SAR interferometry and geodetic surveys". *Geocarto International*, 35(5): 535-558.

TOUBOUL, P.; FOULON, B.; CHRISTOPHE, B. & J. P. MARQUE. 2012. "CHAMP, GRACE, GOCE instruments and beyond". In: *Geodesy for Planet Earth*. pp. 215-221. Springer. Berlin/Heidelberg, Germany.

TSMOTS, I.; SKOROKHODA, O.; IGNATYEV, I. & V. RABYK. 2017. Basic vertical-parallel real time neural network components. *12th International Scientific and Technical Conference on Computer Sciences and Information Technologies (CSIT)*, (Vol. 1, pp. 344-347). Institute of Electrical and Electronics Engineers (IEEE).

WANG, G. 2013. "Teaching high-accuracy global positioning system to undergraduates using online processing services". *Journal of Geoscience Education*, 61(2): 202-212.

WU, H.; ZHAO, X.; PANG, C.; ZHANG, L. & B. FENG. 2019. "Multivariate Constrained GNSS Real-time Full Attitude Determination Based on Attitude Domain Search". *The Journal of Navigation*, 72(2): 483-502.

WYSS, M. 1976. "Local changes of sea level before large earthquakes in South America". *Bulletin of the seismological society of America*, 66(3): 903-914.

YIGIT, C. O.; COSKUN, M. Z.; YAVASOGLU, H.; ARSLAN, A. & Y. KALKAN. 2016. "The potential of GPS Precise Point Positioning method for point displacement monitoring: a case study". *Measurement*, 91: 398-404.

YUNCK, T. P.; MELBOURNE, W. G. & C. L. THOENTON. 1985. "GPS-based satellite tracking system for precise positioning". *IEEE transactions on geoscience and remote sensing*, (4): 450-457.

ZANUTTA, A.; NEGUSINI, M.; VITTUARI, L.; CIANFARRA, P.; SALVINI, F.; MANCINI, F.; STERZAI, P.; DUBBINI, M.; GALEANDRO, A. & A. CAPRA. 2017. "Monitoring geodynamic activity in the Victoria Land, East Antarctica: Evidence from GNSS measurements". *Journal of Geodynamics*, 110: 31-42. https://doi.org/10.1016/j.jog.2017.07.008.

ZEBKER, H. A.; ROSEN, P. A.; GOLDSTEIN, R. M.; GABRIEL, A. & C. L. WERNER. 1994. "On the derivation of coseismic displacement fields using differential radar interferometry: The Landers earthquake". *Journal of Geophysical Research: Solid Earth*, 99(B10): 19.617-19.634.

Lugar y fecha de finalización del artículo:
Sangolquí, Pichincha, Ecuador; julio, 2020