

Development of learning trajectory of perimeter and area of squares and rectangles through various tasks

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Abstract: Learning trajectory on the topic of plane figures (i.e., perimeter and area of squares and rectangles) is necessary for elementary students to facilitate meaningful learning and predict their learning progress. This study aims to develop a learning trajectory through various tasks for the perimeter and area of squares and rectangles that can help students develop understanding and construct mathematical concepts. Study participants were fourth grade elementary students. The method used was design research, which has three stages: preparing for the experiment, teaching, and retrospective analysis. This study used data collection instruments in worksheets, observation sheets, interview guidelines, and field notes. Research results obtained included a learning trajectory that contains adequate learning activities to facilitate learning for elementary students on the perimeter and area of squares and rectangles. Adequate learning trajectory is obtained in hypothetical learning trajectory (HLT) 2, which was previously revised from HLT 1. Learning trajectory developed through a variety of tasks can significantly construct and facilitate elementary students' knowledge to learn the concepts of perimeter and area of squares and rectangles.

Keywords: Learning trajectory, perimeter and area, squares and rectangles, variation tasks

Received 01 March 2022; **Accepted** 24 May 2022; **Published** 02 June 2022

Citation: Kurniawati, L. & Amir, M.F. (2022). Development of learning trajectory of perimeter and area of squares and rectangles through various tasks. *Premiere Educandum : Jurnal Pendidikan Dasar dan Pembelajaran*, 12(1), 54 – 68. Doi.org/10.25273/pe.v12i1.12121



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INTRODUCTION

Geometry is often used to solve problems in everyday life, thereby placing it in the most critical position in mathematics education (Cherif, Gialamas & Stamati, 2017; Özdemir, 2017; Panaoura, 2014; Rofii, Sunardi & Irvan, 2018). Geometry has a major component of the education curriculum in Indonesia that is taught from the elementary school to the tertiary level. Moreover, Geometry has an abstract object of study compared with other learning fields (Prihandoko, 2005). Implicitly, learning geometry materials is able to encourage students to think critically and engage in deductive reasoning, visualization, problem-solving, intuition, logical proof, and argument (Jupri, 2017; Seah, 2015).

Plane figures are among the geometry topics in elementary school mathematics. Plane figures are those that have length and width in one plane. Perimeter and area of squares and rectangles on plane figures are included in subjects that students must master because they are related to real-life (Winarti, Amin, Lukito & Gallen, 2012). Students who have a good understanding of the concept of perimeter will have the initiative to use length in calculating the distance around plane figures to get used to finding the perimeter of plane figures by adding up each side. However, if students do not understand the perimeter well, then they will have difficulty determining the length of sides if an image is not clearly explained (Abadi & Amir, 2022). Meanwhile, if students have spatial ability on plane figures, they will realize that length and width are units of area (Clements et al, 2018; Wickstrom, Fulton & Carlson, 2017). Given that perimeter and area of plane figures are related, students who understand the concept of calculating the perimeter can certainly calculate the area of plane figures.

A previous study has revealed that students tend to merely memorize formulas without knowing the concept (Chintia, Amelia & Fitriani, 2021; Syahbana, 2013). If students do not construct and learn formulas and mathematical contexts, then they will easily forget about the material (Gracia, Rahayu & Hakim, 2020). Elementary school students have difficulty doing minimal tasks with variations (Winarti, Amin & Lukito, 2012). In this case, students have misconceptions on the concepts of perimeter and area of plane figures. They think that plane figures with the same area have the same perimeter (Clements et al, 2018; Yunianto, Prahmana & Crisan, 2021). In learning, teachers explain more without explaining why and how concepts exist. Students are taught more in terms of answering questions than exploring concepts (Fitriani, 2019). In addition, a research in Australia has explained that teachers must focus on student involvement (Clarke, Roche, Clarke & Chan, 2015). Therefore, the researcher focuses on eliminating these misunderstandings through various forms of task. Accordingly, students are expected to understand and apply concepts they have learned to form new understandings that are useful in everyday life (Rohman, Karlimah & Mulyadiprana, 2017).

Several studies have noted that most students still have difficulty understanding geometric concepts (Bustang, Zulkardi & Darmawijoyo, 2013; Fauzi, 2019; MdYunus & Suraya, 2019). Plane figures are among the concepts where students make numerous mistakes in solving problems; these mistakes are made because students lack practice and do not understand the concept of the problem (Atiqoh, 2019). Consequently, there is a need for a solution using a cognitive approach in the form of task variations. This solution is expected to stimulate students to think critically and have broad insights. Moreover, students will provide good feedback to find out the extent to which they enjoy, understand, and find the benefits of the difficulties they encounter from each type of task. Thus, the use of tasks provides some potential for teachers to determine student learning trajectories. This method will result in active student engagement and provide a challenging context to explore students' mathematical ideas (Widjaja, 2013). Accordingly, the learning trajectory of students is ensured, so that learning is measurable, and

understanding of materials on perimeter and area of square and rectangle material is achieved.

This study aims to develop a learning trajectory that is oriented toward various tasks for elementary school students. The development of this learning trajectory is focused on the perimeter and area of squares and rectangles. The hope is that the development of the learning trajectory can enable students to construct meaningful knowledge and understanding (Daro, Mosher & Corcoran, 2011; Panorkou & Kobrin, 2017), particularly on the perimeter and area of squares and rectangles (Espejo & Deters, 2011; Machaba, 2016; Richit, Tomkelski & Richit, 2021; Rickard, 2005). In addition, the results of this study are expected to be useful in developing the scientific expertise of elementary school mathematics education in teaching mathematics in a meaningful manner (Clarke & Roche, 2018; Ekowati, Azzaha, Saputra, & Suwandayani, 2021; Luis & Moncayo, 2010). Furthermore, researchers and readers can enrich the studies on the development of learning trajectories that can facilitate meaningful learning of mathematics for elementary school students.

METHODS

Research Design

This study develops the preceding learning trajectory using the design research method with three stages: preparing for the experiment, teaching, and retrospective analysis (Gravemeijer & Cobb, 2006; Putri & Zulkardi, 2018; Prahmana, 2017), as shown in **Figure 1**. The stages of developing a learning trajectory have an important aspect, namely, hypothetical learning trajectory (HLT), which was built through local instruction theory (LIT) (Van den Akker, 2006). HLT and LIT lead to learning activities as a learning path taken by students in their learning activities (Prahmana, 2017). HLT comprises activities in learning and students' thinking during the learning process (Gravemeijer & Cobb, 2006).

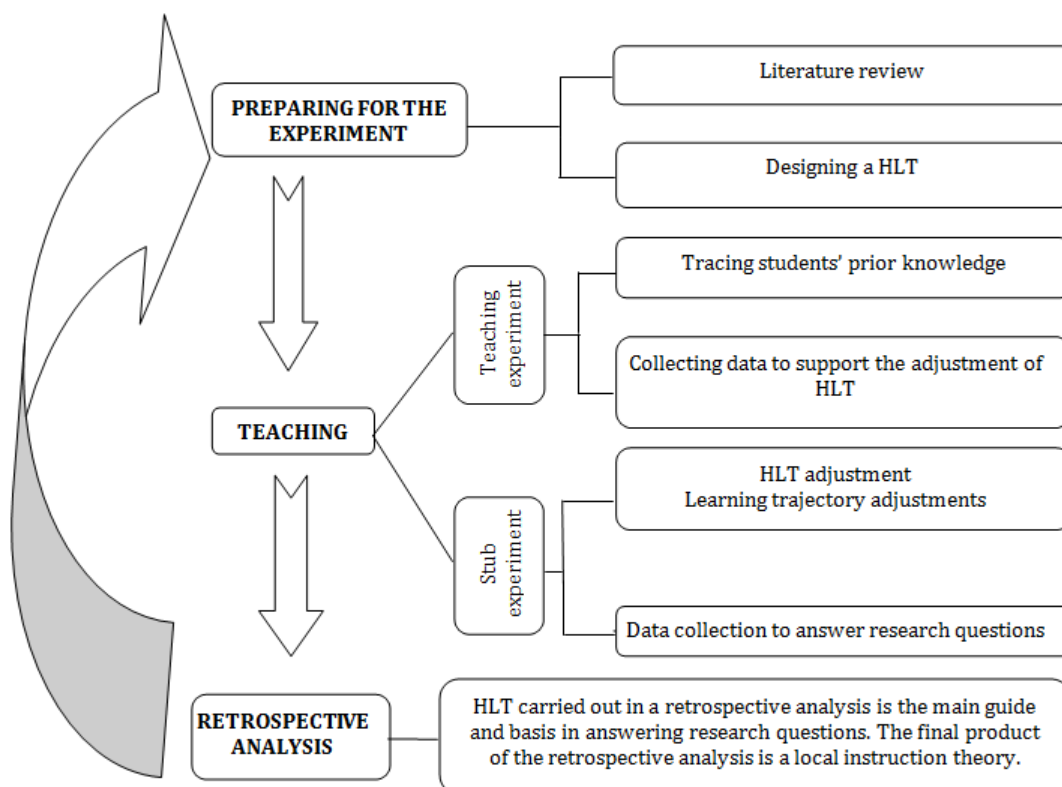


FIGURE 1. Research design stages

Figure 1 describes the three stages of developing a learning trajectory by designing HLT through three stages of preparing for the experiment, teaching, and retrospective analysis, which are carried out through cycles occurring repeatedly. HLT cycle will stop when learning objectives are achieved and research questions can be answered. However, if learning objectives have not been achieved, then the three stages can be repeated (Gravemeijer & Cobb, 2006). The three stages are carried out with the following steps. (1) Preparing for the experiment. Before conducting research, researchers must prepare several things so that the purpose of conducting research is obtained, such as looking for references by reviewing the literature through books and articles, among others. Thereafter, HLT is designed to be tested in the experiment. (2) Teaching design. Teaching design has two stages. The first stage is a teaching experiment, in which HLT that has been designed is tested on students to explore their prior knowledge of the material to be discussed. After conducting the trial, the results obtained are collected to support the adjustment of the learning trajectory plan. In the second stage of the pilot experiment, after the trial was carried out in the previous stage, the researcher adjusted the learning trajectory plan that had been adapted to the students' prior knowledge that was applied to them to obtain data to answer research questions. (3) Retrospective analysis is the final stage in the research design. HLT used becomes the main guide and basis in answering research questions in this stage. In this case, it will produce a product in the form of LIT.

Participants

This study involved 15 fourth-grade elementary school students in Padang, West Sumatra. Composition of the participants was the same throughout the experiment. **Table 1** shows the distribution of participants based on the demographic characteristics of the students involved.

TABLE 1. *Student demographic characteristics*

Demographic Characteristics	Aspects	Number
Sex	Female	9
	Male	6
Age	10 years	11
	11 years	4
Mathematics scores	45–60	3
	60–75	5
	75–87	4
	87–100	3

Material

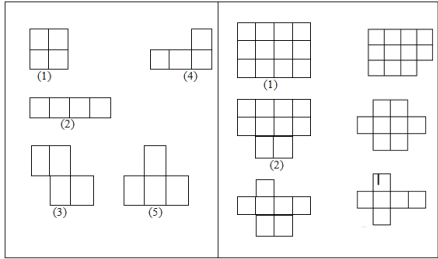
The research instrument in this study consisted of the main and supporting instruments. The main instrument is a variety of tasks called The Task Mathematics Learning (TTML), which was adapted from Clarke & Roche (2017). The current study presents TTML in the form of a worksheet consisting of representation, contextual, and open tasks. TTML indicator grid and samples are shown in **Table 2**.

Representation task focuses on building students' knowledge and understanding of a mathematical concept, although it is not explicitly stated in the objectives of learning mathematics in Indonesia. The importance of representation is implied in the purposes of problem solving and mathematical communication and solving mathematical problems. The objective of this form of task is for students to be able to know the concept of finding the perimeter and area of squares and rectangles.

Contextual task is a form of task that involves physical activities in real life. This task produces all types of proofs of students' mathematical thinking. In this case, students are

encouraged to have two different solutions to challenge them to finish faster, check original conclusions, and relate various aspects of mathematics to one another. The objectives of this form of task are as follows: (1) improve students' mathematical skills and strategies in addressing meaningful problems related to "real world" contexts; (2) improve students' thinking skills; (3) enable students to easily understand interesting and relevant topics to what they are doing, thereby assisting students with difficulty learning mathematics; (4) stimulate students' cognitive aspect to the next level; and (5) combine knowledge and skills, such as tasks requiring measurement, calculation, and logic. In this case, contextual task involves a variety of student knowledge.

TABLE 2. *Indicators and sample items*

Indicators	Sample Items
Representation Task	
<ol style="list-style-type: none"> 1. Explaining what they see from the pictures. 2. Estimating the pictures they often encounter. 3. Determining the perimeter and area of the selected shape. 4. Making three squares and rectangles with the same area but different perimeters. 5. Explaining the relationship of the perimeter and area of the plane figures made. 	 <ol style="list-style-type: none"> 1. What do you notice from the given picture? What could be the explanation to your observation? 2. What number do you often encounter in your daily life? Why? What is the perimeter and area? 3. Make three shapes that have the same area but different perimeters on the given paper. 4. What is the relationship between the perimeter and area of the types of shapes you made?
Contextual Tasks	
<ol style="list-style-type: none"> 1. Calculating the perimeter and area of the candy wrapper from the shape obtained. 2. Looking for squares and rectangles and manually calculating the perimeter and area of these shapes thereafter. 	<ol style="list-style-type: none"> 1. Slowly tear the candy wrapper given. Thereafter, calculate the area and perimeter (by considering the provisions) on the paper given. 2. Find objects around you that have square and rectangular shapes. Thereafter, calculate the area and perimeter of the objects you obtain.
Open Tasks	
<ol style="list-style-type: none"> 1. Explaining their knowledge of what L is. 2. Describing the meaning of the perimeter and area of what they know with evidence. 3. Interpreting why it can be referred to as a square and rectangle. 	<ol style="list-style-type: none"> 1. What is L? 2. What could be the perimeter and area? 3. What could be the reason? 4. Why is it a square and rectangle?

Open task is an open task that requires students to think broadly. Therefore, in solving problems in this task type, students must read extensively to increase knowledge. The purpose of this form of task is to familiarize students with critical thinking and enable them to express what they know using their own language. In addition, students realize that different shapes can have the same area but different perimeters, as well as develop skills in calculating area and perimeter.

Supporting instruments consist of observation sheets, interview guidelines, field notes, and learning video recordings. (1) This observation sheet is used to collect data on student activities prior to conducting research. Researchers conducted observations to

know students' initial abilities. (2) Interview guidelines. In conducting interviews, researchers first formulate interview guidelines used as a reference for conducting interviews. (3) Field notes are made to record what is heard, seen, experienced, and thought in the context of data collection. (4) Video recording. This supporting instrument aims to determine whether students are active or inactive in learning according to the designed conjecture and reduce activities that are not in accordance with learning.

Procedure

This research procedure follows the stages of implementing design research: preparing for the experiment, teaching experiment, and retrospective analysis. At the stage of preparing for the experiment, the researchers developed the activity containing the conjecture of students' thinking through HLT. In addition, the researchers conducted classroom observations and interviews with teachers and students to determine students' initial state and abilities. Preparing for the experiment aims to design LIT. Before designing LIT, a literature review (e.g., books and journal articles on square and rectangle) should be conducted first before designing HLT. Meanwhile, HLT aims to provide an overview of the learning process, starting from knowledge possessed by students until the achievement of learning objectives. Stages that must be passed in HLT are designing activities thought to be able to help students gain knowledge and achieve learning goals. Furthermore, learning tools, such as learning implementation plans (LIP) and worksheet, were designed as a reference for implementing HLT. HLT validated and declared valid can be tested in a class called cycle 1.

The teaching experiment stage has two cycles: pilot and teaching experiments as cycles 1 and 2, respectively. The purpose of this stage is to conduct HLT trials and improve the previously designed LIT conjecture in preparing for the experiment, thereby obtaining better HLT. Trial was carried out based on HLT made and carried out based on a reference to the validated LIP and worksheet. On the basis of the trial, an analysis was conducted on the extent to which activities in HLT were implemented in the learning process and their effects on students' understanding.

Retrospective analysis is the final stage of the procedure, in which all data obtained during the study were analyzed. On the basis of this retrospective analysis, parts of HLT are preserved and reduced. The part retained is the one that can positively influence student understanding. By contrast, the reduced part is that considered not to affect the achievement of the learning objectives. Therefore, the cycle in the design research occurs repeatedly. If the research objectives have been achieved, then the cycle will stop.

Data Analysis

Data analysis in this study uses triangulation. Triangulation was carried out by synthesizing research data obtained from data collection through worksheets, observation sheets, interview guidelines, field notes, and learning video recordings. Data analysis was carried out at the final stage of the design research (i.e., retrospective analysis). Results of this analysis indicated that there are parts of HLT that are preserved and reduced. The part retained is the one that can have a positive influence on understanding students. Meanwhile, the reduced part is the one considered not to have an effect on the achievement of learning objectives.

RESULTS

Preparing for the experiment

In the stage of preparing for the experiment, the researcher designed the learning process in three sessions. Before commencing the session, the researcher first reviewed the

materials on the perimeter and area of squares and rectangles, followed by an explanation of the stages in each type of task.

Teaching experiment

In the experimental preparation stage, the designed HLT was tested on students. The first step is to form groups with their classmates. Thereafter, they discuss to determine the squares and rectangles with the same area and different perimeters, or vice versa, in the figure in the table. Students can determine it correctly. The perimeter and area of plane figures that we often encounter in everyday life are computed. Thereafter, plane figures with the same area but different perimeters, or vice versa, are made. The final step in the first session is to explain the relationship between the perimeter and area of squares and rectangles based on the types of shapes created. In the last step, not all students were able to explain the results of their tasks properly and correctly. The time given by the researcher was 45 minutes starting from eight to eight forty-five minutes. The time given was considered sufficient by the researcher to review the materials, explain the implementation rules, and anticipate other obstacles encountered while working on the task.

In the second session, the activity carried out was to interpret the content of the materials being studied by relating them to everyday-life context. Students were divided into three groups, and the researchers gave them candy thereafter. The researcher instructed them to open the candy slowly. Students trace the candy wrappers that have been opened (Candy wrappers can be perfectly or imperfectly shaped depending on how students tear them). In this step, the students were highly enthusiastic, resulting in the candy wrappers being torn and the shape becoming irregular. Followed by calculating the perimeter and area of squares and rectangles of candy wrappers by paying attention to the conditions that have been given by the researcher on the paper that has been given. In the last step, students were directed to look for square or rectangular objects in the surrounding environment, and calculate the perimeter and area of the plane figures from the objects been obtained. The second session was also given 45 minutes to anticipate unexpected problems. Therefore, must complete the task within 45 minutes.

In the third session, the researcher wanted to know the extent of students' theoretical understanding of the concepts of perimeter and area of plane figures. In this session, the researcher tested such an understanding using open-ended question. In answering, students may use any creative ideas described in their own language. Sample questions are as follows: What is the meaning of a perimeter and an area? Why are objects considered having shapes of square and rectangle? Why is the formula of rectangle $p \times l$? In the last session, students are given 50 minutes, and they are expected to provide the best answers to open-ended questions.

On the bases of the results of the retrospective analysis of the HLT cycle 1, there are several revisions to improve the quality of HLT that will be used in HLT cycle 2, such as multiplying examples of calculating perimeter and area of plane figures based on the concept of plane figures. In addition, providing simple initial examples also provides more complex examples, emphasizing the explanation of the perimeter and area of plane figures that are part of 2D geometry slowly because there are various types of student abilities in one class. On the same day, the validator re-validated the revision to prove the suitability of the learning activities provided and achievement of the learning objectives. After revision, the learning tools can be reused in the trial stage in the same class.

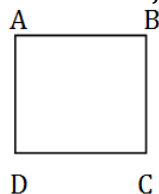
Learning begins at 12 o'clock through questions and answers related to obstacles encountered while working on the previous task. Thereafter, a conceptual explanation is provided on the perimeter and area of plane figures, followed by examples of finding the perimeter and area and rectangles of plane figures ranging from simple to complex (see **Figure 2**).



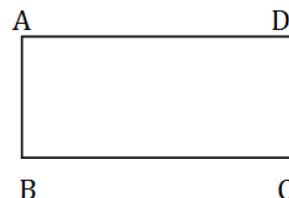
FIGURE 2. Student activity that explains the materials

In the first session, the researcher provides examples and also explains in detail the perimeter and area of plane figures using actual objects or pictures.

- Researcher: Does the task sheet include everything in the plane figures?
 Students : Yes.
 Researcher : What plane figures are there?
 Students : Square and rectangle.
 Researcher: Why is it called a square and a rectangle?
 Student 1 : It's a square because it's a box. A rectangle is elongated.
 Student 2 : All sides of a square are the same, while those of the rectangle are different.
 Student 3 : A square consists of four sides that are all the same. A rectangle consists of different lengths and widths.
 Researcher : All of your answers are correct. This is my explanation.
 A square is a plane figure with four sides of the same length: top, bottom, right side, and left side (or as ABCD in the picture). An example of a concrete object is the lid of the gift box that I brought with me.



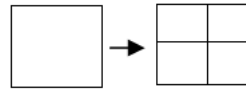
By contrast, a rectangle is a plane figure with four but different sides; the sides consist of a length and a width, each of which is different. The picture shows that sides AD and BC are called length, and sides AB and DC are called width. An example of a concrete object is this book I brought.



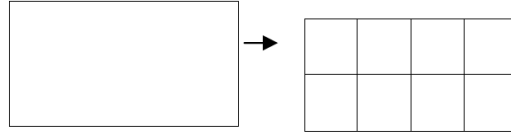
Using this information, you can derive the formula for finding the perimeter and area of squares and rectangles. Who knows the meaning of a perimeter and a area?

- Student 4 : Area is the size of a plane figures
 Researcher : Who else knows?
 Student 5 : Perimeter is the amount that surrounds a plane figures.

Researcher : Thank you for your answer. The teacher will explain that area is what covers a plane figures surface with one unit area.



It takes 4 units of area to cover the square surface. Thus, the area of the square is 4 units of area or 4 cm^2 .



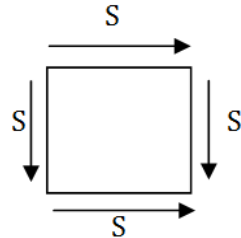
It takes 8 units of area to cover the rectangular surface. Thus, the area of the rectangle is 8 units of area or 8 cm^2 (Sandipan, 2014).

If the area is a surface cover, then the perimeter is obtained by calculating the length of the outer side of the plane figure. For this example, you can use a thread or rope. Who knows the formula of perimeter and area?

Student 6 : Given that a square has four equal sides, the formula for the perimeter of a square is $(4 \times \text{the outermost side})$ and the area $(\text{side} \times \text{side})$.

Student 7 : For a rectangle, the perimeter is $2 \times (p + l)$ while the area is $p \times l$.

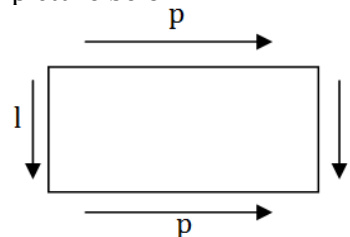
Researcher : Yes, very smart. You can see an example below.



Given that squares have the same sides, calculating the perimeter means adding all the sides or calculating it using a string.

To find the area, the representative of each side is multiplied so that it can cover one plane figures. So we get area of square = side \times side ($s \times s$)

Rectangles have different sides (i.e., length and width). Hence, the formula for the perimeter of a rectangle is $2 \times (p + l)$ or $(p + l + p + l)$. Study the picture below.



The picture shows that the formula for the area of a rectangle is length \times width ($p \times l$).

Calculating the area can also be mapped, similar to the previous example.

After equating the perception of the perimeter and area of plane figures, students can determine the perimeter and area of plane figures of each image. Thereafter, students work on the tasks that have been provided on the representation task sheet given by the researcher. Figure 3 shows an example of student work on the representation task.

In the second session, students traced the candy wrappers in completing contextual tasks similar to the first cycle activity. In the second cycle, to easily count the shape of the candy wrappers, the students had to be careful in tearing them. To calculate the perimeter and area of plane figures, students are free to choose objects that are in the school environment. In the second session, students were able to perform well, as shown in the example of student work in finding the perimeter and area of a pencil case (see **Figure 4**).

In the third session, obtaining satisfactory results entails students understanding every explanation given and have many references from written or unwritten sources in completing the open task. **Figure 5** shows one of the results of the student work on an open task. In this type of task, all students can answer, but not all answers are correct. A development is observed in cycle I.

Tugas representasi bertujuan

KD:

3.9 Menjelaskan dan menentukan keliling dan luas persegi panjang dan persegi.

4.9 Menyelesaikan masalah berkaitan dengan keliling dan luas persegi, dan persegi panjang.

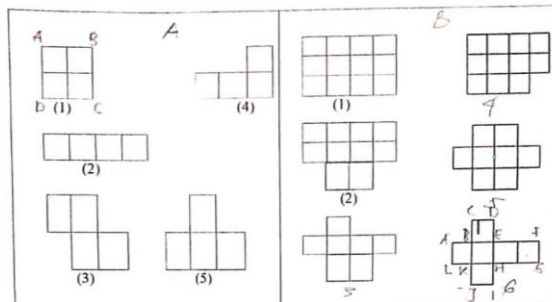


FIGURE 3. Student work on the representation task

Bentuklah kelompok dengan teman sebangkumu, lalu diskusikan.

Diatas adalah bangun datar yang memiliki luas sama dan keliling yang berbeda, atau luas berbeda keliling sama.

1. Menurutmu gambar nomer berapa yang sering kamu jumpai dalam kehidupan sehari-hari? Mengapa? Dan berapa keliling dan luasnya?

gambar no 1A, dan 6B karena di 1A saya sudah menemukan di buku-buku latihan soal dan TV

1A
 $K = AB + BC + CD + DA$
 $= 2 + 2 + 2 + 2$
 $= 8 \text{ cm}$
 $L = 5 \times 5 = 2 \times 2 = 4 \text{ cm}^2$

6B
 $K = AB + BC + CD + DE + EF + FG + GH + HI + IJ + JK + KL$
 $= 1 + 1 + 1 + 1 + 2 + 1 + 2 + 1 + 1 + 1 + 1 + 1$
 $= 13 \text{ cm}$
 $L = 4 \times 3 = \frac{12}{2} = 6 \text{ cm}^2$

2. Buatlah 3 bentuk yang memiliki luas yang sama tetapi keliling berbeda di kertas yang sudah diberikan!
3. Apa hubungan dari keliling dan luas dari jenis bentuk yang kamu buat?

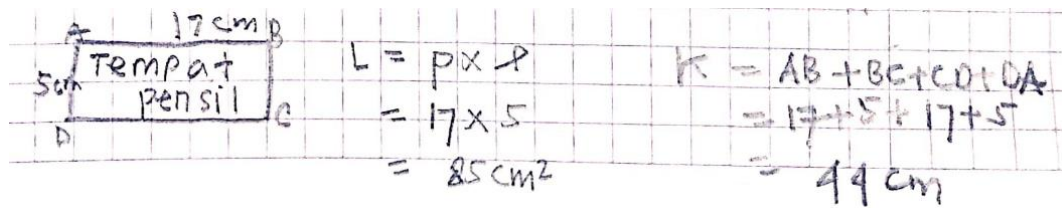


FIGURE 4. Student work on the contextual task

Tugas Terbuka

1. Apa itu L?

Luas

2. Menurut kalian apa itu keliling dan luas?

Keliling adalah pinggiran yang memutar bangun
 Luas adalah total keseluruhan dalam bangun
 $p \times l$ atau $s \times s$

3. Mengapa bisa dikatakan sebagai persegi dan persegi panjang?

karena bentuknya. Jika persegi sisinya sama. Jika persegi panjang bentuk panjang dan lebarnya berbeda.

FIGURE 5. Student work on the open task

Retrospective analysis

Results obtained by students have enabled them to find the perimeter and area of plane figures in squares and rectangles through various forms of tasks, including representation, contextual, and open tasks. On the basis of the teaching experiment, activities in HLT 1 were revised to obtain HLT 2 as follows. (1) It further deepens the explanation of mathematical concepts and provides concrete examples in everyday life. (2) Giving easy examples and giving examples ranging from simple to more complex. (3) Allowing students to explore their environment so that they do not feel burdened in doing mathematics tasks, realize that learning is fun, and develops students' understanding of

the material. (4) An evaluation of their answers is provided to determine which ones are wrong and how they are right.

DISCUSSION

On the bases of the research results, this study uses a learning trajectory using a cognitive approach. No severe obstacles were observed, and students were able to complete the tasks given under the criteria of providing perception, assessment, reasoning, imagination, and capture of meaning related to perimeter and area of squares and rectangles. In the learning trajectory activity developed, students can find solutions carried out by researchers by interpreting their initial ideas on the perimeter and area of squares and rectangles. Students are also able to interpret the contents of the perimeter and area of squares and rectangles. Moreover, they learned by connecting with the context of everyday life, and developing an understanding that students have obtained during perimeter and area of squares and rectangles learning. Students can convey what they know about this type of task by describing it in their own language. In this case, the learning trajectory using a variety of tasks has a real situation or context. To form a hierarchical and meaningful understanding of students, such as ice cubes from the stages of mode of, mode for, and abstract formal (Gravemeijer, 1994; Khairunnisak, Johar, Zubainur & Sasalia, 2021; Risdiyanti & Prahmana, 2021; Syafriandi, Fauzan, Lufri & Armiami, 2020), as shown in **Figure 6**.

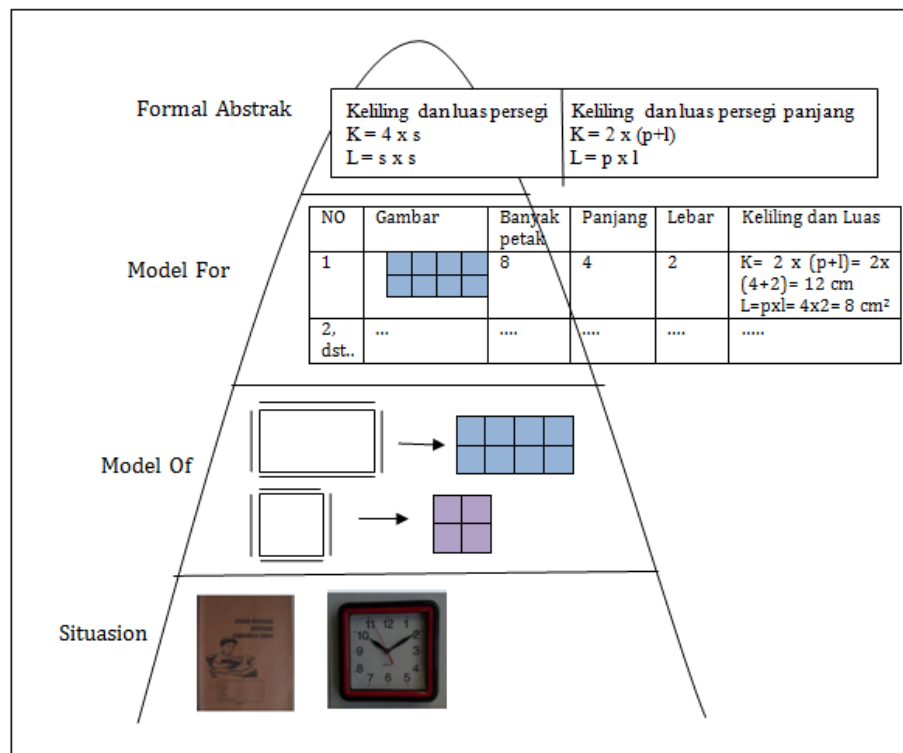


FIGURE 6. Learning trajectory in the form of an iceberg

Conceptual errors include students' misunderstanding questions and formulas, theorems, or definitions that do not adjust to conditions (Ovez, 2012; Widodo, 2013). Indicators of mastery in understanding have two aspects: remembering previous materials and connecting new materials with previous materials (Wulandari & Gusteti, 2020).

Therefore, the design research using a cognitive approach in the form of task variations is based on Bloom's taxonomy aspects, including knowledge (C1), understanding (C2), application (C3), analysis (C4), evaluation (C5), and creating (C6)

(Adesoji, 2018; Beyreli & Sönmez, 2017; Pratiwi, Suparwa, Satyawati & Made, 2021; Sivaraman & Krishna, 2015). The form of task is designed to gradually develop students' knowledge of perimeter and area of squares and rectangles from low to more complex levels so that students discover mathematical concepts in a guided manner (Dumontier, Venugopal, Kumar & Sreenivasa, 2020; Fajari & Chumdari, 2020; Lourdasamy & Magendiran, 2021).

On the bases of the results, students' understanding of the perimeter and area of squares and rectangles can be developed from the informal to formal stage. Students can understand the concept of perimeter and area of squares and rectangles, thereby enabling them to work on various types of tasks given by the researcher. In addition, a retrospective analysis is conducted to develop the planned learning trajectory. Through various tasks given, students are able and accustomed to solving mathematical problems on the perimeter and area of squares and rectangles, ranging from low-level to complex problems. In addition, this variation in tasks trains students to focus, develop skills, become confident, and develop their creative ideas.

CONCLUSION

In developing students' understanding of concepts, researchers provide a variety of tasks in learning that are needed to improve mathematical performance in learning mathematics. The study used the design research method, which consists of three stages: preparing for the experiment, teaching, and retrospective analysis. Subjects in this study were 15 fourth grade students in an elementary school in Padang, West Sumatra.

The learning trajectory developed is a revised learning trajectory from HLT 1 by explaining mathematical concepts and providing concrete examples that exist in everyday life, providing examples ranging from simple to more complex, constructing students' understanding of the material, and providing an evaluation of the answers given. They find out what is wrong and how to justify it. This research is limited in terms of discussion. Apart from squares and rectangles, the perimeter and area of other plane figures can also be developed. Thus, other researchers are expected to develop learning trajectories on different materials.

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