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Non-Smooth and Complementarity-Based Distributed Parameter Systems

Simulation and Hierarchical Optimization

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Preface

This volume of the *International Series of Numerical Mathematics* presents research results obtained in the first funding phase of the Special Priority Program (SPP) 1962 on *Nonsmooth and Complementarity Based Distributed Parameter Systems: Simulation and Hierarchical Optimization* from 2016 to 2019. The program was funded by the *Deutsche Forschungsgemeinschaft (DFG)*. Within the funding period, 24 projects located at many universities and research institutions across Germany, involving also a tandem project which was co-funded by the Swiss National Fund (SNF) with one project partner in Lugano, unfolded various research activities, leading to more than 100 preprints as well as several workshops and exchange activities in particular for young researchers. The SPP 1962 also co-organized the *International Conference on Continuous Optimization (ICCOPT)* which was held in Berlin from August 5 to 8, 2019, preceded by a summer school for early career researchers from August 3 to 4, 2019. The coordination project for the entire program (supported scientifically by Amal Alphonse and Michael Hintermüller) was located at the Weierstrass Institute for Applied Analysis and Stochastics in Berlin.

The main mathematical theme of the SPP1962 is non-smoothness as many of the most challenging problems in the applied sciences involve non-differentiable structures as well as partial differential operators, thus leading to non-smooth distributed parameter systems. The non-smoothness considered in this SPP typically arises:

(i) Directly in the problem formulation
(ii) Through inequality constraints, nonlinear complementarity, or switching systems
(iii) As a result of competition and hierarchy

In fact, very challenging applications for (i) come from frictional contact problems, or non-smooth constitutive laws associated with physical processes such as Bean's critical state model for the magnetization of superconductors, which leads to a quasi-variational inequality (QVI) problem; for (ii) are related to non-penetration conditions in contact problems, variational inequality problems, or inequality constraints in optimization problems, which, upon proper re-formulation, lead to complementarity problems and further, by means of non-linear complementarity problem (NCP) functions, to non-smooth systems similar to (i); and for (iii) come from multi-objective control systems or leader-follower principles, as they can be found in optimal system design in robotics and biomechanics. Modeling "competition" often leads to generalized Nash equilibrium problems (GNEPs) or partial differential games. Moreover, modeling "hierarchy" results in mathematical programs with equilibrium constraints (MPECs), a class of optimization problems with degenerate, non-smooth constraints. All of these problems are highly nonlinear, lead to QVIs, and represent rather novel mathematical structures in applications based on partial differential operators. In these and related applications, the transition from smoothing or simulation-based approaches to genuinely non-smooth techniques or to multi-objective respectively hierarchical optimization is crucial.

Fundamental difficulties in non-smooth partial differential systems, associated optimization, and hierarchical problems are of analytical as well as algorithmic and numerical nature. For instance, for QVIs, the existence and stability of solutions is a major challenge, whereas MPECs suffer from a lack of existence of Lagrange multipliers due to constraint degeneracy, which hinders the derivation of proper stationarity conditions. Numerical challenges, which are present in all non-smooth problems of this SPP, involve the stability of discretization/model reduction schemes or severe mesh dependence of algorithms. In order to overcome these difficulties, the goals of this SPP are to advance tools from non-smooth and set-valued analysis and to build a basis for stable numerical approximation/discretization schemes that enable the design of algorithms with mesh independent convergence. The SPP 1962 also aims to address the influence of parameters, which enter the above applied problems and which either range within a specified set or result from hierarchy. The former leads to robust optimization in form of deterministic MPECs, which challenge the characterization of stationary points and the development of efficient solvers. Hierarchical problems (or MPECs) contain variables which enter into lower-level problems as parameters. Summarizing, the research program of the SPP leads to a modern treatment of non-smooth problems and will therefore shape future applications in the field.

Corresponding to the above goals, each subsequent section of this volume presents the findings of projects within the SPP.

Berlin, Germany Michael Hintermüller

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