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A New Paired Spectral Gradient Method to Improve Unconstrained and Non-Linear Optimization.

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Abstract

The conjugated spectral gradient (SCG) method is an effective method for non-constrained large-scale nonlinear optimization. In this work, a new spectral conjugate gradient method is proposed with a strong Wolfe-Powell line search (SWP). The new proposal is based on using the formula obtained by comparing the proposed algorithm with previously published conjugate gradient algorithms. Under the usual assumptions, the descent properties and overall global convergence of the proposed method are proved. The proposed method is numerically proven to be effective.

1. Introduction:

Conjugate gradient (CG) and SCG methods are the most effective categories for solving large-scale nonlinear unconstrained optimization problems, this is because they have the advantage of fast convergence, low storage and simple iterations [1] [2] [3]. Now consider the nonlinear unconstrained optimization problems [4].

$$\min_{x\in R^n} f(x) \tag{1}$$

where f $\in R^n$ is a smooth function, a gradient vector is usually represented by: $\nabla f(x) = g(x)$. The initial point $x_0 \in R^n$ is usually calculated through iterative process. The new point calculated as follows:

$$x_{k+1} = x_k + a_k d_k \qquad k = 0,1,2,3... \tag{2}$$

The direction of the search is given in:

$$d_{k+1} = \begin{cases} -g_{k+1}, & if \quad k=0 \\ -\theta_{K+1}g_{K+1}+\beta_k d_{K-1}, & if \quad k\geq 1, \end{cases} \tag{3}$$

from which we find:

$$d_{k+1} = -\theta_k g_{k+1} + \beta_k d_K, \qquad if \quad k \geq 2 \tag{4}$$

Where $\beta_k \in \Re$ is the parameter, and $\alpha_K > 0$ is the generated by inexact line search (ILS). In this work, we use (SWP) defined by:

$$\begin{cases} f(x_k+\alpha_k d_k) \leq f(x_k)+\xi\alpha_k g_k^T d_k \\ f(x_k+\alpha_k d_k)^T d_k \geq \rho g_k^T d_k \end{cases} \tag{5}$$

It depends on finding the average for non-linear parameters.

2. New Algorithm and the Descent Property:

The SCG method is obtained by combining the CG search direction and a scalar spectral parameter. Relying on the kindness requirement proposed by the researchers Liu. Jinkui, Du. Xianglin, and Wang Kairong.[5] and [6].

$$\theta_k = 1 - \frac{g_{k+1}^T d_k}{g_k^T d_k} \qquad ,\beta_k = \frac{\|g_{k+1}\|^2}{g_k^T d_k} \tag{6}$$

The parameters proposed by the researchers Xuesha Wu [7] [8]and [9].

$$\theta_k = \frac{d_k^T \gamma_k}{\|g_k\|^2} \quad ,\beta_k = \frac{g_{k+1}^T \gamma_k}{\|g_k\|^2} \tag{7}$$

The parameters proposed by Basim A. Hassan and Hameed M. Sadeq [10].

$$\theta_k = \frac{\|g_{k+1}\|^2 (y_k^T d_k)}{\|g_k\|^2 (y_k^T g_{k+1})}, \quad \beta_k = \frac{\|g_{k+1}\|^2}{\|g_k\|^2} \tag{8}$$

$$\theta_k^{New} = \left(1 - \frac{g_{k+1}^T d_k}{g_{k+1}^T} + \frac{d_k^T \gamma_k}{\|g_k\|^2} + \frac{\|g_{k+1}\|^2 (y_k^T d_k)}{\|g_k\|^2 (y_k^T g_{k+1})}\right) \times \frac{1}{3} \tag{9}$$

$$\theta_k^{New} = \left(\frac{g_k^T d_k - g_{k+1}^T d_k}{3 g_k^T d_k} + \frac{d_k^T \gamma_k}{3\|g_k\|^2} + \frac{\|g_{k+1}\|^2 (y_k^T d_k)}{3\|g_k\|^2 (y_k^T g_{k+1})}\right) \tag{10}$$

We substitute in the general form $\beta_k = \frac{\|g_{k+1}\|^2}{g_k^T d_k}$

$$d_{k+1} = -\theta_k g_{k+1} + \beta_k d_K, \qquad if \quad k \geq 2$$

After compensation:

$$d_{k+1} = -\theta_k^{New} g_{k+1} + \beta_k d_K, \qquad if \quad k \geq 2 \tag{11}$$

$$d_{k+1} = -\left(\frac{g_k^T d_k - g_{k+1}^T d_k}{3 g_k^T d_k} + \frac{d_k^T \gamma_k}{3\|g_k\|^2} + \frac{\|g_{k+1}\|^2 (\gamma_k^T d_k)}{3\|g_k\|^2 (\gamma_k^T g_{k+1})}\right) g_{k+1} + \left(\frac{\|g_{k+1}\|^2}{g_k^T d_k}\right) d_K \tag{12}$$

3. Algorithm of SCG:

Step 1: Choose an initial point, $x_1 \in R^n, \varepsilon = 10^{-6}, \mu = 0.5$, set $d_1 = -g_1, \quad k = 1$
Step2: Test the convergence if ,$\|g_k\| \leq \varepsilon$, stop or go on.
Step 3: Use the equation:

$$p_1 g_K^T d_k \leq g(x_k + \alpha_k d_k)^T d_k \leq -p_2 g_K^T d_k$$

To calculate the stride length,$\alpha_k > 0$
Step 4: Generate the new point with the equation:
$x_{K+1} = x_k + \alpha_k d_k$
Calculate the gradient, $g_{k+1} = g(x_{k+1}), Test \|g_{k+1}\| \leq \varepsilon$
Stop or go ahead Calculate the spectral parameter.

$$\theta_k = \theta_k^{New} = \left(\frac{g_k^T d_k - g_{k+1}^T d_k}{3 g_k^T d_k} + \frac{d_k^T \gamma_k}{3\|g_k\|^2} + \frac{\|g_{k+1}\|^2 (y_k^T d_k)}{3\|g_k\|^2 (y_k^T g_{k+1})}\right)$$

Calculate the parameter, $\beta_k = \frac{\|g_{k+1}\|^2}{g_k^T d_k}$
Calculate the vector d_{k+1} from the equation:

$$d_{k+1} = -\theta_k g_{k+1} + \beta_k d_K, \quad if \quad k \geq 2$$

step 5: If the return condition for met Powell:
$|g_{k+1}^T \gamma_k| \geq 0.2\|g_{k+1}\|^2$
Put , $d_{k+1} = -g_{k+1}$ and go to step 3 Otherwise, continue.
step 6: Put $k = k+1$ and go to step 3.

4. The Proposed SCG:

After deriving the spectral conjugate gradient coefficient, θ_k^{New}
We will check the sufficient slopes characteristic using Wolff's strong line of research[11] [10].

5. Assumption (A):

1- If f(x) is restricted from below on the levelse:

$$\Psi = \{x : f(x) \leq f(x_0)\} \tag{13}$$

2− If $f(x)$ is continuously differentiable in a certain neighborhood N of Ψ, and its gradient is Lipchitz continuous, i.e., there is a constant.

$$l > 0, \|g(x) - g(y)\| \leq l\|x - y\|, \forall x, y \in N \tag{14}$$

Now using Assumption (A), there exists a positive constant $(\tilde{\omega}, \omega, \tilde{\alpha}, \&\alpha)$, such that:
$0 < \tilde{\omega} \leq \|g_{k+1}\| \leq \omega, 0 < \tilde{\alpha} \leq \|g_k\| \leq \alpha, \forall x \in \psi, \dot{B} \leq \|d_k\| \leq B, \forall x \in \psi$ [12].

Theorem 1:

Suppose that assumption (A) holds. Assuming that the sequences g_k and d_k
α_k be generated by the algorithm SCG, and the step size is obtained by SWP. Then, the proposed method has sufficient descent direction [13].

$$g_{k+1}^T d_{k+1} \leq -\Gamma_1 \|g_{k+1}\|^2, \forall \Gamma_1 > 0 \tag{15}$$

Proof: We will use the property of mathematical induction when $k = 0$ then $g_0^T d_0 = \|g_0\|^2$

Thus, the relationship is true, and now let us suppose that the relationship is true for all values $k \geq 0$
Multiply both sides of the equation 9 at g_{k+1}^T we get.

$$g_{k+1}^T d_{k+1} = -\theta_k^{New} \|g_{k+1}\|^2 + \beta_k g_{k+1}^T d_K \tag{16}$$

$$g_{k+1}^T d_{k+1} = -\left(\frac{g_k^T d_k - g_{k+1}^T d_k}{3 g_k^T d_k} + \frac{d_k^T \gamma_k}{3\|g_k\|^2} + \frac{\|g_{k+1}\|^2 (y_k^T d_k)}{(3\|g_k\|^2 (y_k^T g_{k+1})} \right) + \|g_{k+1}\|^2 + \left(\frac{\|g_{k+1}\|^2}{g_k^T d_k} \right) g_{k+1}^T d_K, \tag{17}$$

We know that:

$$g_{k+1}^T \gamma_k = g_{k+1}^T (g_{k+1} - g_k) = \|g_{k+1}\|^2 - g_{k+1}^T g_k \tag{18}$$

And by substituting one direction from the return condition for Powell [14], which is as follows:

$$|g_{k+1}^T g_k| \geq 0.2 \|g_(k+1)\|^2 \tag{19}$$

$$g_{k+1}^T \gamma_k \leq \|g_{k+1}\|^2 + 0.2\|g_{k+1}\|^2 = 1.2\|g_{k+1}\|^2 \tag{20}$$

Therefore, the proposed algorithm satisfies the sufficient descent condition with SWP conditions.

$$g_{k+1}^T d_{k+1} \leq -\left(\frac{g_k^T d_k - g_{k+1}^T d_k)}{3 g_k^T d_k} + \frac{d_k^T \gamma_k}{(3\|g_k\|^2)} + \frac{\|g_{k+1}\|^2 (y_k^T d_k)}{(3\|g_k\|^2 (y_k^T g_{k+1})} - \frac{g_{k+1}^T d_K}{g_k^T d_k} \right) \|g_{k+1}\|^2 \tag{21}$$

$$g_{k+1}^T d_{k+1} \leq -\left(\frac{g_k^T d_k - p g_k^T d_k}{-3c\|g_k\|^2} + \frac{(-p) g_k^T d_k}{3\|g_k\|^2} + \frac{\|g_{k+1}\|^2 (1+p)(g_k^T d_k)}{3\|g_k\|^2 (0.8)\|g_{k+1}\|^2} - \frac{p g_k^T d_k}{c\|g_k\|^2} \right) \|g_{k+1}\|^2 \tag{22}$$

$$g_{k+1}^T d_{k+1} \leq -\left(\frac{-c(1-p)\|g_k\|^2}{-3c\|g_k\|^2} + \frac{-c(-p)\|g_k\|^2}{3\|g_k\|^2} + \frac{\|g_{k+1}\|^2 (1+p) c \|g_k\|^2}{2.4\|g_k\|^2 \|g_{k+1}^2} + \frac{cp\|g_k\|^2}{c\|g_k\|^2} \right) \|g_{k+1}\|^2 \tag{23}$$

And using the number one hypothesis on the objective function f it's:

$$0 < \tilde{\omega} \leq \|g_{k+1}\| \leq \omega, 0 < \tilde{\alpha} \leq \|g_k\| \leq \alpha, \forall x \in \psi, \dot{B} \leq \|d_k\| \leq B, \tag{24}$$

And from the equation 30 We get

$$g_{k+1}^T d_{k+1} \leq -\left(\frac{(1-p)\alpha^2}{3\alpha^2} + \frac{c(p)\alpha^2}{3\alpha^2} + \frac{(1+p)(c)\alpha^2 \tilde{\omega}^2}{(2.4)\alpha^2 \tilde{\omega}^2} + \frac{cp\alpha^2}{c\alpha^2} \right) \|g_{k+1}\|^2 \leq -\Gamma_1 \|g_{k+1}\|^2$$
$$where \quad \Gamma_1 = \left(\frac{(1-p)\alpha^2)}{3\alpha^2} + \frac{c(p)\alpha^2}{3\alpha^2} + \frac{(1+p)(c)\alpha^2 \tilde{\omega}^2}{(2.4)\alpha^2 \tilde{\omega}^2} + \frac{cp\alpha^2}{c\alpha^2} \right) > 0, \tag{25}$$

6. The Global Convergence Property:

In this section, we will prove another important condition, called global convergence property. In the following lemma, we review the well-known Zoutendijk condition [15], which plays an important role in the proof of the global convergence analysis of SCG method [16], [17]
Lemma [18]: Let assumption (A) holds. Suppose any iteration method 2 and 3, and α_k is obtained by the SWP. If:

$$\Sigma_{k=1}^{\infty} \frac{1}{\|d_{k+1}\|^2} = \infty \tag{26}$$

Then

$$\lim_{k \to \infty} \quad inf \|g_{k+1}\| = 0 \tag{27}$$

Theorem 2: Consider that assumption (A) is satisfied. The sequences x_k and d_k generated by the algorithm SCG, α_k is obtained by SWP and $d_k$is the descent direction. Then $\lim_{k \to \infty} \quad inf \|g_{k+1}\| = 0$
Proof:
Since the algorithm fulfills the condition of sufficient regression, and $g_{k+1} \neq 0$,[19] [20]
We must prove that $\|d_{k+1}\|$tied from above and take $\| \cdot \|$ to both sides of the equation 9 We get:

$$\|d_{k+1}\| = \| -\theta_k^{New} g_{k+1} + \beta_k d_K \| \tag{28}$$

$$\|d_{k+1}\| \leq | -\theta_k^{New} | \|g_{k+1}\| + |\beta_k| \|d_K\| \tag{29}$$

$$|\theta_k^{New}| = \left|\left(\frac{g_k^T d_k - g_{k+1}^T d_k}{3g_k^T d_k} + \frac{d_k^T \gamma_k}{3\|g_k\|^2} + \frac{\|g_{k+1}\|^2 (y_k^T d_k)}{(3\|g_k\|^2 (y_k^T g_{k+1})}\right)\right| \tag{30}$$

$$|-\theta_k^{New}| \leq \left(\frac{(1-p)\alpha^2}{3\alpha^2} + \frac{c(p)\alpha^2}{3\alpha^2} + \frac{(1+p)(c)\alpha^2\tilde{\omega}^2}{(2.4)\alpha^2\tilde{\omega}^2}\right) = B_1 \tag{31}$$

$$|\beta_k| = \left|\left(\frac{g_{k+1}^T d_K}{g_k^T d_k}\right)\right| \tag{32}$$

$$|\beta_k| \leq \left|\frac{cp\alpha^2}{c\alpha^2}\right| = B_2 \tag{33}$$

compensate for (26),(28) at (24)
We get $\|d_{k+1}\| \leq B_1\|g_{k+1}\| + B_2\|d_K\|$

$$\|d_{k+1}\| \leq B_1\omega + B_2 B = \varphi \tag{34}$$

$$\sum_{k=1}^{\infty} \frac{1}{\|d_{k+1}\|^2} \geq \frac{1}{\varphi^2} \sum_{k=1}^{\infty} 1 = \infty$$

$$\lim_{k\to\infty} \quad inf\|g_{k+1}\| = 0 \quad \Sigma_{k=1}^{\infty} \frac{1}{\|d_{k+1}\|^2} = \infty \tag{35}$$

The new proposed algorithm has achieved global convergence.

7. Results and Discussion:

This algorithm was tested in practice using Fortran 7.7. The program has been tested and this algorithm is practical, using functions in unconstrained optimization. Nonlinear dimensions are used. Table 1 and Figures 1, 2 and 3 includes the numerical results of the algorithm, where the values of. were, $p = 0.9$. The comparison between the proposed algorithm and parameter conjugate gradient algorithms was the First original θ [5], the Second original θ [7], the third original θ [10], and The New θ of the Table 1 includes the numerical results of the second algorithm, where the values of. The comparison between the proposed algorithm and parameter conjugate gradient algorithms. The stop scale used for all message algorithms was: $\|g_{k+1}\| \leq 10^{-6}$. Our average values have been taken (NOI), (NOF) And we symbolized it with the symbol (CPU).

All codes are written in double-precision FORTRAN 77 language and compiled into Visual (Fortran 6.6) (default compiler settings). Under Table 1, we have compiled the names of test functions used and the numerical results between the [5], [7] and [10] algorithm and the SCG algorithm.

8. Conclusion:

In this work, we prove the property of sufficient proportions and the property of universal convergence of the hadith. Coupled spectral gradient method proposed by strong Wolfe-Powell line search the number. The results show that the SCG algorithm is superior to the conjugated gradient method First original θ [5], Second originalθ[7], Third original θ [10] in terms of the number of Frequency and number Of function In this section, we will discuss how to calculate the percentage improvement percentage for the proposed algorithms compared. With the classic algorithms used in comparison:

$$\left[100 - \frac{Z}{P} \times 100 = D\%\right]$$

Where (Z) represents the (NOI) with respect to (θ^{New}) and (P) represents the (NOI) with respect to one of the classic functions, and (D) represents the final percentage obtained and in the same way the percentages for improving the rest of the algorithms are obtained with the classical algorithms with respect to (NOI) (NOI). Table 2 showing the percentage improvement of the algorithm(θ^{New}) with the classical algorithms used for comparison.Table3 showing the NOF improvement percentage for the algorithm of θ^{New}.Table4 showing the CPU improvement percentage for the algorithm of θ^{New}.In this work, we prove the sufficient descent property and the global convergence property of the newly proposed spectral conjugate gradient method through a strong Wolfe–Powell line search. The numerical results show that the SCG algorithm outperforms [5], [7] and [10] conjugate gradient method in terms of the number of iterations and the number of function evaluations.

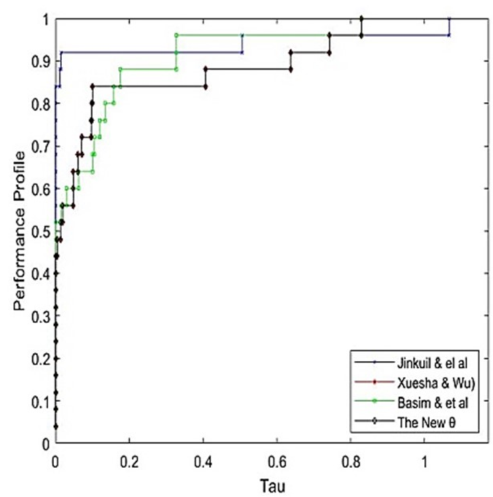

Figure 1. Comparison in terms of (NOI) between First original θ [5] , Second original θ [7], Third original θ [10] and The New θ.

Table 1. A comparison has been made on the basis of the total number of repetitions, symbolized by (NOI), and the total number of computed functions, symbolized by (NOF) and the total number of computed time, symbolized by (CPU).

No.	Test Functions	N	First original θ [5]			Second original θ [7]			Third original θ [10]			The New θ		
			NOI	NOF	CPU	NOI	NOF	CPU	NOI	NOF	CPU	NOI	NOF	CPU
1	Wood	100	3078	3564	0.27	3078	3564	0.27	3733	4337	0.36	2876	3355	0.25
		500	100	194	0.05	100	194	0.05	100	194	0.05	100	194	0.06
		900	4799	5464	0.36	4799	5464	0.35	4734	5317	0.36	4673	5224	0.36
2	Cubic	100	8004	8745	0.56	8004	8745	0.55	8004	8903	0.56	8004	8620	0.54
		500	1865	2643	0.18	1865	2643	0.18	1943	2734	0.18	1742	2519	0.16
		900	50	143	0.01	50	143	0.01	50	143	0.01	50	143	0.01
3	Helica	100	1397	1923	0.49	1397	1923	0.48	1502	2045	0.51	1330	1810	0.46
		500	2989	3796	0.91	2989	3796	0.91	2501	3201	0.98	2132	2844	0.97
		900	288	535	0.13	288	535	0.12	291	535	0.13	291	535	0.13
4	Rosen	100	3465	3722	0.32	3465	3722	0.31	2115	2385	0.16	2071	2333	0.16
		500	95	198	0.06	95	198	0.07	95	198	0.06	95	198	0.06
		900	4137	4519	0.51	4137	4519	0.51	4137	4522	0.48	2229	2610	0.23
5	Powell	100	179	394	0.02	179	394	0.02	179	394	0.02	167	380	0.02
		500	33	119	0.03	33	119	0.05	33	119	0.04	33	119	0.04
		900	91	215	0	91	215	0.01	91	215	0	91	215	0.02
6	OSP	100	5462	6087	1.59	5462	6087	1.56	3513	4108	0.99	4160	4695	1.69
		500	8004	8634	0.52	8004	8634	0.52	8004	8539	0.5	8004	8602	0.5
		900	6419	7385	0.41	6419	7385	0.43	6590	7529	0.44	6001	6896	0.39
7	DENSCHNB	100	8004	8062	1.56	8004	8062	1.55	8004	8062	1.58	8004	8062	1.58
		500	6952	7688	0.54	6952	7688	0.53	6952	7803	0.65	6952	7667	0.56
		900	1808	2596	0.27	1808	2596	0.27	1884	2658	0.29	1733	2514	0.25
8	Miele	100	6835	7755	1.1	6835	7755	1.1	7413	8314	1.23	6410	7450	1.17
		500	3111	3802	0.48	3111	3802	0.46	3715	4459	0.53	3082	3800	0.49
		900	44	142	0.03	44	142	0.03	44	142	0.03	44	142	0.03
9	Wolfe	100	3278	3764	0.55	3478	3964	0.56	3933	4537	0.58	2676	3155	0.52
		500	2665	3216	0.26	2496	2811	0.18	3514	3896	0.23	2613	2960	0.22
		900	3807	3775	0.22	3715	4132	0.25	3373	3721	0.21	2713	3060	0.22
10	DENSCHNF	100	6003	6425	0.36	6003	6437	0.37	6003	6418	0.36	6003	6542	0.36
		500	265	453	0.02	264	452	0.03	265	453	0.01	281	462	0.02
		900	81	201	0.01	81	201	0.00	81	201	0.00	81	201	0.00
11	DIXMAANI	100	1116	1719	0.11	1144	1722	0.11	1348	1850	0.12	1178	1760	0.11
		500	2574	3230	0.89	2838	3017	0.79	1850	2207	0.54	1727	2278	0.58
		900	969	1484	0.30	917	1293	0.28	1200	1475	0.35	1051	1433	0.31
12	Ex-Freudenstein	100	208	390	0.10	208	390	0.09	208	390	0.10	208	390	0.09
		500	101	190	0.01	101	190	0.02	101	190	0.01	101	190	0.01
		900	472	652	0.04	472	652	0.03	472	652	0.05	472	652	0.04
13	NONDIA Shanno-78	100	79	155	0.06	79	155	0.05	79	155	0.05	79	155	0.05
		500	2203	2495	0.28	2203	2495	0.30	2203	2495	0.29	2203	2495	0.28
		900	70	163	0.01	120	279	0.01	120	279	0.02	120	279	0.02
14	DIXMAANA	100	70	163	0.01	70	163	0.01	70	163	0.00	70	163	0.01
		500	2617	3171	0.64	2936	3394	0.63	2303	2411	0.58	1627	1178	0.58
		900	6003	6591	0.35	6003	6520	0.34	6003	6459	0.35	6003	6069	0.34
15	Shall wo	100	5331	6212	0.33	5701	6386	0.28	5956	5745	0.27	1203	2495	0.28
		500	6003	6061	1.06	6003	6061	1.07	6003	6061	1.08	6003	6061	1.06
		900	4952	5554	0.38	4951	5525	0.36	4951	5507	0.37	4451	5189	0.37
16	Recipe	100	1515	2252	0.12	1531	2098	0.14	1607	2178	0.11	1491	2178	0.12
		500	19	93	0.02	19	93	0.00	19	93	0.00	19	93	0.00
		900	4597	5346	0.46	4606	5375	0.44	4561	5196	0.45	4549	5188	0.45
17	Diagonal 4	100	163	155	0.01	5745	163	0.05	6410	1760	0.01	8004	8745	0.56
		500	3394	2495	0.63	6061	2411	0.28	3082	2278	0.58	1865	2643	0.18
		900	6520	279	0.34	5507	6459	0.02	44	1433	0.34	50	143	0.01
18	Extended	100	6419	7385	0.41	6419	7385	0.43	6590	7529	0.44	6001	6896	0.39
		500	101	6061	0.00	6061	3394	0.58	2495	6061	0.02	6061	5701	0.33
		900	472	5554	0.45	5525	6520	0.34	279	5554	0.46	5507	6003	1.06
19	Summ	100	8004	8062	1.56	8004	8062	1.55	8004	8062	1.58	8004	8062	1.58
		500	8634	8745	0.27	7688	7688	0.28	8634	8062	0.27	8004	6087	0.30
		900	7385	2643	1.08	2596	2596	1.07	7385	7688	1.08	6419	8634	0.01
20	Rosen	100	6952	7688	0.54	6952	7688	0.53	6952	7803	0.65	6952	7667	0.56
		500	3796	2989	0.91	3796	1397	0.97	1923	2045	0.91	1397	1923	101
		900	535	288	0.12	535	2989	0.13	3796	3201	0.13	2989	3796	472

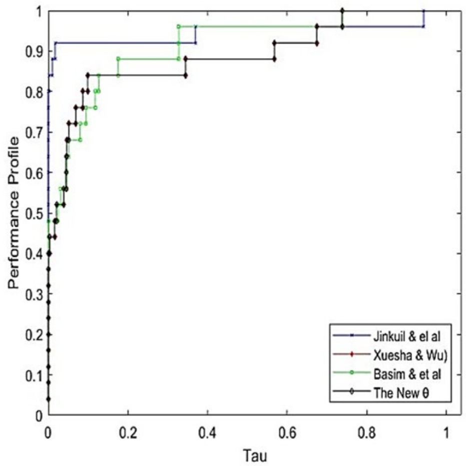

Figure 2. Comparison in terms of (NOF) between First original θ [5], Second original θ [7], Third original θ [10] and The New θ.

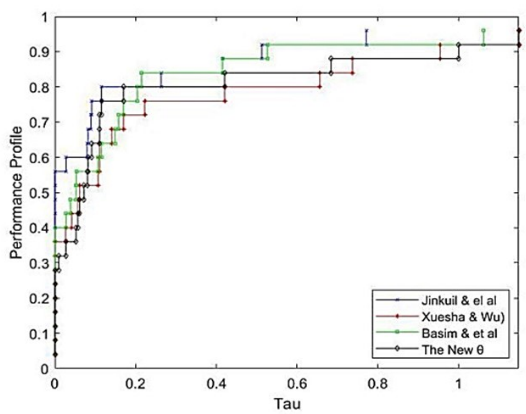

Figure 3. Comparison in terms of (CPU) between First original θ [5], Second original θ [7], Third original θ [10] and The New θ.

Table 2. NOI improvement percentage for the algorithm θ^{New}.

Optimization ratio between algorithm New and [5]	Optimization ratio between New algorithm and [7]	Optimization ratio between algorithm New and [10]
17.83%	18.03%	16.86%

Table 3. NOF improvement percentage for the algorithm θ^{New}.

Optimization ratio between algorithm New and [5]	Optimization ratio between algorithm New and [7]	Optimization ratio between algorithm New and [10]
16.21 %	16.32%	15.49%

Table 4. CPU improvement percentage for the algorithm θ^{New}.

Optimization ratio between algorithm New and [5]	Optimization ratio between algorithm New and [7]	Optimization ratio between algorithm New and [10]
2.73 %	2.29 %	0.65%

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Declarations:

Conflict of interest: The authors declare that they have no conflict of interest.

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طريقة التدرج الطيفي المقترن الجديدة لتحسين الامثلية غير المقيدة وغير الخطية.

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الخلاصة

طريقة التدرج الطيفي المقترن (*SCG*) هي طريقة فعالة لتحسين غير المقيدة غير الخطية واسعة النطاق. في هذا العمل ، تم اقتراح طريقة تدرج طيفي مترافق جديدة مع بحث قوي عن خط *Wolfe − Powell*(*SWP*) . المقترح الجديد يعتمد على استخدام الصيغة التي تم الحصول عليها من خلال مقارنة الخوارزمية المقترحة مع خوارزميات التدرج المتوافق المنشورة سابقا، تم إثبات خصائص النسب والتقارب العالمي الشامل للطريقة المقترحة، وقد ثبتت فعالية الطريقة المقترحة عدديًا.

الكلمات الدالة : التحسين غير المقيد ، طريقة التدرج المترافق ، خاصية النسب الكافية.

التمويل: لا يوجد.
بيان توفر البيانات: جميع البيانات الداعمة لنتائج الدراسة المقدمة يمكن طلبها من المؤلف المسؤول.
اقرارات:
تضارب المصالح: يقر المؤلفون أنه ليس لديهم تضارب في المصالح.
الموافقة الأخلاقية: لم يتم نشر المخطوطة أو تقديمها لمجلة أخرى، كما أنها ليست قيد المراجعة.